Business Conditions, Market Volatility and Option Prices

Christian Dorion*

Department of Finance, HEC Montreal, Montreal, Canada (christian.dorion@hec.ca)

July 14, 2011

First Draft: February 13, 2009

EFA 2011 Stockholm Meetings Paper

Abstract Options are bets on volatility. Volatility is highly counter-cyclical, but the volatility models used to value options typically disregard macroeconomic risk when predicting future volatility. The asset pricing properties of these models suffer from this omission; their option pricing errors tend to increase as business conditions deteriorate. We propose the use of a model in which the conditional expected level of volatility evolves with business conditions and, as a result, that accounts for macroeconomic risk. This risk is quantified using a mixed data sampling (MIDAS) structure to account for changes in the recently introduced Aruoba-Diebold-Scotti (ADS) Business Conditions Index. The new model outperforms existing ones in explaining asset returns and in the pricing of options, especially when business conditions are at their worst.

Keywords Business conditions; GARCH; Macroeconomic risk; Mixed data sampling; Option valuation; Volatility.

JEL Classifications: C22, E32, G13

*I would like to thank to George Tauchen, Eric Ghysels, Jan Ericsson, Alexandre Jeanneret, Chayawat Ornthanalai, Redouane Elkamhi, Vadim di Pietro, and seminar participants at McGill, HEC Montreal, HEC Paris, HEC Lausanne, FCEE-Católica, U. of Exeter, Rouen Business School, Copenhagen Business School, Université Laval, Bank of Canada, and at the Long Term Volatility & Economic Fundamentals conference at the Volatility Institute (NYU Stern) for their comments. Special thanks to Peter Christoffersen and Kris Jacobs for their invaluable suggestions, and to the Institut de Finance Mathématique de Montréal (IFM2) for their financial support.

Introduction

The pricing of options crucially depends on expectations regarding the distribution of future volatility. While volatility is widely documented to be counter-cyclical,[1] the volatility models used to value options do not explicitly account for any such cyclicality. When predicting future volatility, these models do not account for any macroeconomic variable and let volatility revert to a constant mean level that is independent of current business conditions. The option pricing errors of these models are counter-cyclical across all maturities and moneyness levels. We propose a model that removes this cyclicality in pricing errors by allowing the conditional expected level of volatility, here referred to as the fundamental volatility level, to change with business conditions.

This study extends that of Engle, Ghysels, and Sohn (2009, henceforth EGS). EGS develop a GARCH-MIDAS model in which volatility varies around a fundamental volatility level determined by economic fundamentals.[2] These authors find that this fundamental volatility process is significantly related, even at short horizons, to such factors as inflation or industrial production growth. They also show that accounting for these economic fundamentals significantly improves volatility forecasts. We build on their work and analyze the asset pricing properties of volatility models accounting for changes in business conditions.

Central to our analysis is the new business conditions index recently introduced by Aruoba, Diebold, and Scotti (2009, henceforth ADS). Published by the Federal Reserve Bank of Philadelphia since January 9, 2009, this daily index summarizes a variety of macroeconomic variables. The cur-

[1]Among many others, see Schwert (1989), Engle and Rangel (2008), David and Veronesi (2009) and Engle, Ghysels, and Sohn (2009).

[2]On MIDAS models, see, for instance, Ghysels, Santa-Clara, and Valkanov (2005), Forsberg and Ghysels (2007), Ghysels, Sinko, and Valkanov (2007), and Engle, Ghysels, and Sohn (2009). Note that Engle, Ghysels, and Sohn (2009) refer to the fundamental volatility process as the secular volatility process. Regime-switching models, *à la* Hamilton and Susmel (1994), provide another approach to allow for different mean-reversion levels. However, using economic fundamentals has the advantage of identifying the determinants of gradual changes in conditional volatility expectations.

rent version of the index is updated each time one of the following is: initial jobless claims, payroll employment, industrial production, personal income less transfer payments, manufacturing and trade sales, and quarterly real GDP. Within a model *à la* EGS, we use changes in the ADS index in lieu of inflation or industrial production growth; the fundamental volatility in our model hence accounts for the most recent information regarding the macroeconomic situation.

In their work, EGS first consider a version of the GARCH-MIDAS model with a fundamental volatility based exclusively on the historical volatility (HV) of the financial series. EGS use this model, which we coin the HV-MIDAS model, as a benchmark for specifications in which macroeconomic variables substitute for HV measures. Rather than replacing HV by the ADS index, we let fundamental volatility depend both on historical volatility and on changes in business conditions. As a result, our model, the Macro-MIDAS, enables us to quantify the impact of business conditions beyond their impact on past volatility, which is especially relevant given the forward-looking nature of options.

Model parameters are estimated by maximum likelihood. The Macro-MIDAS model nests Duan's (1995) model and significantly outperforms it in fitting asset returns and stock market volatility. These results are consistent with the growing consensus that two-factor volatility processes better capture the time-series properties of volatility by separately accounting for transient and high-persistence volatility shocks.[3] While two-factor models mostly rely on two latent, autoregressive volatility factors, our approach instead specifies that macroeconomic determinants impact on conditional volatility expectations. The Macro-MIDAS model also does significantly better than the nested two-factor HV-MIDAS model in explaining stock market volatility. These results demonstrate that changes in business conditions are an important determinant of the fundamental volatility process. This confirms the results of Engle and Rangel (2008) and Engle,

[3] On two-factor models, see, amongst others, Engle and Lee (1999); Andersen, Bollerslev, Diebold, and Ebens (2001); Alizadeh, Brandt, and Diebold (2002); and Engle and Rangel (2008).

Ghysels, and Sohn (2009) who extensively study physical volatility processes and find them to be counter-cyclical.

Based on a no-arbitrage argument, we derive a risk neutralization of the Macro-MIDAS model that accounts for the correlation between financial returns and changes in business conditions. This risk-neutralized form of the model enables a cross-sectional analysis of option-pricing errors on twenty years of weekly option data, one of the most extensive data sets analyzed in the option pricing literature. Duan's (1995) model exhibits counter-cyclical pricing errors. The Macro-MIDAS model reduces pricing errors throughout the sample, and especially when business conditions are at their worst. The model also outperforms the HV-MIDAS, a nested two-component model in which business conditions are constrained not to contribute. Notably, allowing for a second volatility component is not sufficient in itself to remove the the pricing-error cyclicality; accounting for business conditions is. These results illustrate the importance of accounting for macroeconomic risk when valuing options.

While this study is based on a reduced-form model, the empirical results further the understanding of the links between business conditions and option prices. It complements work by David and Veronesi (2011) who develop an equilibrium model with learning investors. In their model, the implied volatility of at-the-money options is endogenously counter-cyclical. In the data, we find that this holds true for the whole cross-section of options. In our model, this cyclicality emerges as a result of that of the physical volatility, which is amplified through the risk neutralization. While the benchmark models capture this cyclicality in sample, they neglect it in their volatility forecasts, yielding the counter-cyclicality in their pricing errors.

Empirically, David and Veronesi (2011) find that implied volatility leads variables such as industrial capacity utilization and short term interest rates. Bekaert, Hoerova, and Duca (2011) find that option prices significantly covaries with lead and lag measures of monetary policy. Bollerslev, Gibson, and Zhou (2011) relate the volatility premium to a set of macro-finance variables, some of which are included in the ADS index. They find that the premium rises sharply during recessions and that sudden increases in the premium tend to exhibit strong persistence. In our model, the highly persistent fundamental volatility is, by construction, a significant determinant of the difference between implied and physical volatility. In Section 2.4, we find that the macroeconomic risk proxy influencing our fundamental volatility explains a non-trivial part of the variation of the VIX over twenty years, providing intuition as to why our model substantially reduces pricing errors when business conditions deteriorate.

In summary, this paper shows that a simple dynamic volatility model is able to draw on the informational content of the ADS Business Conditions Index to better capture and understand properties of stock market volatility and of option prices. This ability could prove highly relevant in better understanding the risk inherent to option portfolios throughout the business cycle. In cross sections of option returns, Aramonte (2009) finds macroeconomic uncertainty to be a priced factor, and his results are robust to controlling for a variety of relevant factors such as market and liquidity factors, higher moments of intra-daily returns, and the SMB and HML factors. Our paper suggests that these results are, at least in part, a consequence of the expected level of stock market volatility varying with business conditions.

The paper is organized as follows. Section 1 presents the Macro-MIDAS model. Section 2 briefly discusses the ADS Business Conditions Index, compares it to other macroeconomic series of interest, and discusses the estimation of the Macro-MIDAS model using maximum likelihood. Section 3 uses the maximum likelihood estimates to price twenty years of option data and analyzes the impact of business conditions on the model's implied volatilities. Finally, Section 4 concludes.

1 The Macro-MIDAS Model

1.1 The Model's Foundations

Dynamic volatility models can be divided into two categories. In a stochastic volatility model, the volatility process is driven by unobservable shocks that are imperfectly correlated with shocks to the return process. In a GARCH model, the shocks to the volatility process are a deterministic transformation of the return innovations. We observe returns and can estimate volatility, but never observe the latter. In this way, stochastic volatility models are somewhat more realistic.

However, by assuming a single source of randomness, GARCH models offer a framework where, given the observable return process $R_t = \log(S_t/S_{t-1})$, where S_t is the stock price, the filtration of the return shocks is trivial. In stochastic volatility models, inferring two unobservable shocks using a single observable is a more demanding task. For simplicity, we here choose to cast our study in a GARCH framework. The return process is given by

$$R_{t+1} = \mu_{t+1} + \sqrt{h_{t+1}}\varepsilon_{t+1}\,, \qquad \varepsilon_{t+1} \overset{\mathbb{P}}{\sim} \mathcal{N}(0,1)\,, \tag{1.1}$$

where μ_{t+1}, the conditional expected return, and h_{t+1}, the conditional variance of returns, are $\mathcal{F}_t-$measurable.

Most dynamic volatility models eventually mean revert to a constant volatility level, a somewhat undesirable property. The newly introduced GARCH-MIDAS model of Engle, Ghysels, and Sohn (2009, henceforth EGS) was introduced to capture a simple intuition: the stock market volatility process should mean revert to different levels depending on macroeconomic conditions. Consider the following multiplicative variance specification, suggested by Engle and Rangel (2008) and EGS:

$$h_{t+1} = g_{t+1}\tau_{t+1} \, , \tag{1.2}$$

$$g_{t+1} = (1 - \alpha - \beta) + \alpha g_t \varepsilon_t^2 + \beta g_t \, , \tag{1.3}$$

where τ_t, which refers to the fundamental volatility process, can be interpreted as a time-varying conditional expectation for the level of stock market volatility. The g_t process, which has an unconditional mean of one, accounts for transient shocks to the volatility process by allowing short-run volatility to diverge from fundamental volatility.

Historical volatility is defined as the sum of squared daily returns over a given horizon,

$$HV_t = \sum_{n=0}^{N-1} R_{t-n}^2 \, , \tag{1.4}$$

and provides a consistent estimate of stock market volatility.[4] EGS thus suggest the following specification for the fundamental volatility process:[5]

$$\log(\tau_{t+1}) = m + \theta_{hv} \sum_{k=0}^{K-1} \phi_k (w_{hv}) HV_{t-k} \, . \tag{1.5}$$

Rather than focusing solely on the last historical volatility measure, this specification loads smoothly on recent observations in the MIDAS spirit. We assume that ϕ_k follows a Beta weighting scheme,

$$\phi_k(w) = \frac{(1 - k/K)^{w-1}}{\sum_{j=0}^{K-1} (1 - j/K)^{w-1}} \, ,$$

[4] EGS refer to the estimate of Equation (1.4) as a realized volatility (RV) estimate. Strictly speaking, the estimator is indeed a RV estimate, but some readers may associate RV with the intraday, high-frequency version of the estimator in Equation (1.4). We use the historical volatility (HV) terminology to highlight the low-frequency nature of the RV estimator used here. For more on realized volatility, see, amongst many others, Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Christoffersen, and Diebold (2006), Liu and Maheu (2008), and Andersen and Benzoni (2008).

[5] Equation (1.5) is based on distributed lags of historical volatility measures that are positive by construction. Hence, there is no need to model the *logarithm* of the fundamental variance. However, EGS show that there is little impact from doing so, and the log specification has the advantage of allowing for negative values to enter the smoothing function, which proves handy when it comes to using macroeconomic series.

which discards past observations at a rate controlled by w; the larger the w, the faster past historical volatility levels are discarded.[6] In their analysis, EGS find quarterly historical volatilities ($N = 63$ trading days) computed on each day of the past four years ($K = 1008$) to be the best performing time spans. As our data set largely overlaps theirs, we use these time spans in our analysis.

1.2 Macro-MIDAS: Historical Volatility and Business Conditions

We define the Macro-MIDAS model as follows:

$$\mu_{t+1} = r + \lambda \sqrt{h_{t+1}} - \frac{1}{2} h_{t+1} \,, \tag{1.6}$$

$$g_{t+1} = (1 - \alpha(1+\gamma^2) - \beta) + \alpha g_t (\varepsilon_t - \gamma)^2 + \beta g_t \,, \tag{1.7}$$

$$\log(\tau_{t+1}) = m + \theta_{hv} \sum_{k=0}^{K-1} \phi_k (w_{hv}) HV_{t-k} + \theta_m \sum_{k=0}^{K-1} \phi_k (w_m) \Delta x_{t-k} \,, \tag{1.8}$$

where x_t is a daily indicator of the quality of business conditions, and where $\Delta x_t = x_t - x_{t-N}$ denotes a measure of the improvement (or deterioration) of business conditions over the last N business days. We use the ADS Business Conditions Index, discussed in Section 2 below, as a measure of x_t. While the MIDAS framework is, first and foremost, useful for dealing with data sampled at mixed frequencies, it still proves relevant here even though HV_t and Δx_t are both available on a daily basis. Indeed, the functional form of Equation (1.8) allows for a rich lag structure that enables the model to combine past observations of historical volatilities and business conditions in a non-trivial way.

EGS estimate the GARCH-MIDAS model of Equations (1.2)–(1.5) replacing historical volatilities by measures of inflation or of industrial production growth. In the 1953–2004 period, they find that the level of these variables explains 35% and 17% of expected volatility, respectively. We suggest that changes in business conditions constitute a source of risk that contributes to expected

[6] Beta weights are usually parameterized by two parameters; we omitted the one allowing for hump-shaped weightings for the sake of parsimony as preliminary experiments showed it provided little benefit. Engle, Ghysels, and Sohn (2009) do the same in their analysis of historical volatility.

volatility levels *beyond* what is measured by recent historical volatility levels. In this way, we are essentially combining the informational content of historical volatilities and business conditions. In order to evaluate the relevance of accounting jointly for both variables, we also consider two restricted versions of the Macro-MIDAS model: the HV-MIDAS model, in which we constrain θ_m to be zero, and the PureMacro-MIDAS model, in which we constrain θ_{hv} to be zero.

Note that we introduce the γ parameter in the short-run variance specification of Equation (1.7) to allow for the well documented "leverage effect" (Black 1976), which is particularly important when considering the option-valuation properties of a model.[7] Now, by fixing τ_t to the constant value $e^m = \omega/(1 - \alpha(1 + \gamma^2) - \beta)$, one retrieves the nested non-affine GARCH model of Duan (1995), hereafter, NGARCH, in which h_t simply varies around a constant expected variance level parameterized by ω,[8]

$$h_{t+1} = \omega + \alpha h_t(\varepsilon_t - \gamma)^2 + \beta h_t \,. \tag{1.9}$$

2 Estimating the Model using the ADS Business Conditions Index

The fundamental variance process of the Macro-MIDAS model, defined in Equation (1.8), requires a measure of business conditions. Before discussing the estimation of the model, this section presents the ADS Business Conditions Index and argues that it is well suited to fulfill the role implied by our characterization of the fundamental variance process.

2.1 The ADS Business Conditions Index

On January 9, 2009, the Federal Reserve Bank of Philadelphia introduced the *ADS Business Conditions Index*, an index that is built on the work of Aruoba, Diebold, and Scotti (2009, henceforth ADS).

[7] See, for instance, Nandi (1998), Heston and Nandi (2000), Chernov and Ghysels (2000), Christoffersen and Jacobs (2004), and Christoffersen, Heston, and Jacobs (2006).

[8] Our model could have been designed to nest the affine model of Heston and Nandi (2000).

In their paper, ADS develop a model that infers latent business conditions from daily term spread observations, weekly initial jobless claims, monthly (non-agricultural) payroll employment, and quarterly real GDP.[9] The ADS procedure filter out a daily autoregressive process

$$x_t = \varphi x_{t-1} + v_t \,, \qquad v_t \overset{\mathbb{P}}{\sim} \mathcal{N}(0,1)\,, \tag{2.1}$$

that is referred to as the business conditions index. The ADS approach accommodates missing data, temporal aggregation, complex lag structures and time trends to ultimately obtain a linear state-space representation.

The average value of the ADS index, $\mathbb{E}[x_t]$, is zero, and progressively larger positive values indicate progressively better conditions. The converse is true for negative values. The v_t innovations are assumed to have unit variance. The first column of Table 1 reports summary statistics on the index, and the lower right panel of Figure 1 plots its value through time, with shaded regions highlighting the NBER recessions. All deep troughs of the index coincide with NBER recessions. In that sense, the index clearly seems to adequately captures the business conditions' relative quality level through time. Note that while the NBER typically announces that the economy reached a peak or a trough several months after it actually occurred, the ADS index value can be updated each time one of its input series is updated. For instance, the ADS Index captured the U.S. economy's December 2007 downturn in real time, while the NBER officially announced it 12 months later, on December 1, 2008.

ADS have to rely on the very simple dynamics of Equation (2.1) for the x_t business conditions index; in particular, the homoskedasticity assumption is necessary for identification. The second column of Table 1, however, makes it clear that v_t innovations not i.i.d. The third and fourth

[9] The term spread is defined here as the difference between ten-year and three-month Treasury yields. The index published by the Federal Reserve Bank of Philadelphia is based on the ADS paper, but includes some modifications. However, we use the data as kindly provided by Aruoba, Diebold, and Scotti; the series was computed on April 7^{th}, 2008.

columns of the same table are obtained by fitting a simple GARCH(1,1) to the v_t innovations of Equation (2.1),

$$v_t = \sqrt{h_t^x} u_t \,, \qquad u_t \overset{\mathbb{P}}{\sim} \mathcal{N}(0,1) \,, \tag{2.2}$$

$$h_t^x = \omega_x + \alpha_x v_{t-1}^2 + \beta_x h_{t-1}^x \,. \tag{2.3}$$

That is, given the filtered index values, we relax the homoskedasticity assumption and allow innovations to the business conditions index to have a time-varying variance h_t^x. While the u_t are still far from normally distributed, as indicated by the value of the Jarque-Bera statistic, their likelihood and moments are nonetheless more reasonable. The model of Equations (2.1)–(2.3) is, thus, preferable for modeling purposes.

2.2 The ADS Index and Macroeconomic Time Series

To justify the use of the ADS Business Conditions Index in our model, this subsection demonstrates that this index conveys some of the informational content usually attributed to indicators such as inflation and industrial production growth. Figure 1 reports some key macroeconomic series throughout the time period considered in this paper.

The lower left panel of Figure 1 plots Ang and Piazzesi's (2003) inflation factor. This factor is computed from the principal component of three inflation measures based on the consumer price index (CPI), the production price index for finished goods (PPI), and spot market commodity prices as given by the CRB Spot Index (PCOM). For these three indices, we follow Ang and Piazzesi in computing a growth measure, $\log\left(\frac{P_t}{P_{t-12}}\right)$, where P_t is the index level. The resulting series, which are used to compute the principal component, are displayed above the inflation factor in Figure 1. Analogously, the central panels plot the series that are used to compute Ang and Piazzesi's real activity factor. These series are the growth rate—$\log\left(\frac{I_t}{I_{t-12}}\right)$, where I_t is the level—of employment

(EMPLOY) and of industrial production (IP), and the unemployment rate (UE). We refer the reader to Ang and Piazzesi (2003) for more details.[10]

Table 2 reports the correlations between the business conditions index, the rate on the three-month treasury bill, the term spread, the growth rate of employment, the unemployment rate, the (detrended) real GDP, [11] and Ang and Piazzesi's factors. Daily series are sampled monthly for monthly correlations; daily and monthly series are sampled quarterly for quarterly correlations.

The index is strongly and positively correlated with the growth rate of employment (72.4%), with the real activity factor (79.4%), and, to a lesser extent, with real GDP (14.6%). As the term spread is the sole daily driver of the index, it is not surprising that it has a relatively strong correlation with the index at the daily level (−27%); interestingly, this correlation remains mainly unchanged by sampling the series monthly (−27.6%). As expected, the index has a strong negative correlation with the unemployment rate (−47.5%). Finally, the index's correlation with the inflation factor is negative (−7.3%) but, surprisingly, insignificant.

In sum, the business conditions index seems to covary intuitively with many macroeconomic series of interest, while offering the great advantage of accounting for the most recent macroeconomic news.

2.3 Model Estimation: Asset Returns and Stock Market Volatility

Equipped with the ADS Index as a measure of business conditions, we can now estimate the Macro-MIDAS model given by Equations (1.6)–(1.8). Table 3 reports the maximum likelihood estimates obtained using S&P 500 returns between January 1968 and December 2007 for the

[10] The series were obtained from Federal Reserve and the Commodity Research Bureau websites. This paper does not account for the Index of Help Wanted Advertising in Newspapers in its replication of Ang and Piazzesi's real activity factor. At first glance, this omission does not yield any notable qualitative difference.

[11] We are interested in the index's correlation with these four series (TS, EMPLOY, UE and GDP) since they are closely related to the index's inputs. Yet, note that the EMPLOY, UE, and GDP series are, here, processed as in Ang and Piazzesi (2003) while Aruoba, Diebold, and Scotti (2009) use levels directly in a much more sophisticated approach.

Macro-MIDAS model, for the nested HV- and PureMacro-MIDAS constrained versions, as well as for the NGARCH benchmark from Duan (1995).[12]

All MIDAS models significantly outperform the nested NGARCH model. Decomposing the variance into two components allows each component to take on one of the two fundamentally different roles that must unduly be assumed by the single component in the NGARCH model. The fundamental variance process captures the long-memory-like properties of stock market volatility. The Macro-MIDAS model's fundamental variance process, for instance, has a persistence of over 0.99. The θ_{hv} and θ_m loadings on historical volatilities and business conditions are positive and negative, respectively. The former captures the persistence of variance following financial turmoils; fundamental variance strongly and positively loads on recent historical variance levels. Fundamental variance loads negatively on recent changes in business conditions, thus capturing the counter-cyclical nature of volatility; when business conditions deteriorate, the conditional expected variance level rises.

Note that the θ parameters are smaller (in absolute terms) in the Macro-MIDAS model than in the HV- and PureMacro-MIDAS models. This highlights that the historical volatility levels are not independent from changes in business conditions; when the latter deteriorate, the historical volatility levels tend to rise. Along the same line, both w parameters rise when recent volatilities and changes in business conditions are paired. That is, the Macro-MIDAS model weights the recent values of both signals more than the nested models that discard the older values more slowly. While overlapping, the informational content of both signals is clearly not the same; tests based on the likelihoods reported in Table 3 strongly reject the HV- and PureMacro-MIDAS nested

[12] This study uses S&P 500 data (SPX) because of its availability over a long horizon and because options on the SPX have been actively traded for a long time. Returns on major indexes and their volatility are usually highly correlated, and volatility tends to be higher in recessions regardless of the index being considered. We are thus confident that the results obtained in this paper would also obtain using data for other stock indexes.

models in favor of the Macro-MIDAS model—the likelihood ratio statistics and their p-values are not reported, but the latter are below 1e-6.

Figure 2 illustrates how the Macro-MIDAS model blends both fundamental volatility processes implied by the nested HV- and PureMacro-MIDAS models. As a reference, we plot a horizontal line at 16.31%, the NGARCH model's unconditional variance level implied by $\sqrt{252\,\mathbb{E}\left[h_t\right]} = \left(252\omega/(1-\alpha(1+\gamma^2)+\beta)\right)^{\frac{1}{2}}$. The Macro-MIDAS model's fundamental volatility process ranges from 11.3% to 25%, at times driven by the contribution of historical volatilities, at times by the contribution of changes in business conditions. The contribution of the ADS Index is particularly strong near and during recessions. The contribution of historical volatilities is most important around the October 1987 crash. Besides, historical volatilities have a surprisingly modest impact in the late 90s, given the relatively high level of volatility observed during the Russian/LTCM crisis.

Including a fundamental variance component gives the short-run variance component the flexibility to allow for greater volatility of variance and to better capture the leverage effect. Indeed, the value of β, the autoregressive variance coefficient, is lower for MIDAS models than for the NGARCH, and lower for the Macro-MIDAS than for the two nested ones. Similarly, values of α and γ are higher for the three MIDAS models than for the GARCH(1,1) benchmark, and even more so in the Macro-MIDAS case. Altogether, our Macro-MIDAS model allows for an 18% higher volatility of variance than that of the NGARCH model (1.862 vs. 1.576) and yields a correlation of -74.3% between the returns and variance processes, about 4.6% greater in magnitude than that of the NGARCH model. By way of comparison, between January 1990 and December 2007, the correlation between excess returns on the S&P 500 and changes in the VIX is -74.1%; when considering changes in variance, i.e. ΔVIX^2, the correlation is -73.0%.

Table 3 also reports, for both models that account for business conditions, the correlation between total market innovations and innovations to the business conditions index. This correlation,

$\mathrm{Corr}_t(\varepsilon_{t+1}, u_{t+1})$, is about 5% under both the PureMacro-MIDAS and the Macro-MIDAS models. That the observed correlation is positive is consistent with the preliminary analysis of Section 2.2, which shows that the business conditions index is negatively correlated with Ang and Piazzesi's (2003) inflation factor, but positively with their real activity factor.[13] Five percent may seem low, but the business condition index evolves relatively smoothly through time and does not distinguish between expected and unexpected movements of the underlying business conditions. Obtaining a low, positive correlation here suggests that increases in the business conditions index reflect heightened expectations about the state of the economy rather than the arrival of unexpected positive news.

2.4 Fundamental Volatility, Macroeconomic Risk and the VIX

The Macro-MIDAS model splits volatility into a time-varying conditional expectation and a short-run, excess volatility component. Moreover, the model allows us to easily isolate the contribution of macroeconomic risk to conditional volatility expectations. We now exploit this feature of the model to better understand the impact of macroeconomic risk on the level of the CBOE Volatility Index (VIX). On a given date, the VIX is the square root of the one-month forward price of variance implicit in the cross section of S&P 500 options. By measuring the impact of changes in business conditions on this index, we can thus roughly assess the economic role played by macroeconomic risk in the evolution of the perceived uncertainty on one of the most important market in the world.

[13]Bodie (1976) finds that stock returns covary negatively with both anticipated and unanticipated inflation. Fama (1981) suggests that this negative relationship is driven by real variables covarying positively with stock returns, but negatively with inflation. Yet, the impact of real macro variables on equity returns has found mitigated support for many years. Flannery and Protopapadakis (2002) note that Chen, Roll, and Ross (1986) express their *"embarrassment"* with the situation and that Chan, Karceski, and Lakonishok (1998) *"are at a loss to explain"* the poor performance of macroeconomic factors in explaining stock returns. However, in their own work, Flannery and Protopapadakis, estimating a GARCH model of equity returns, find that these returns are affected by announcements of nominal and real macroeconomic factors.

Table 4 reports the results of linear regressions in which the regressand is the VIX. The data spans from June 1988 to December 2007.[14] As explanatory variables, we consider the total volatility of the Macro-MIDAS model and its different components: the short-run component and the fundamental volatility. The fundamental component can be further divided in a component solely driven by historical volatilities (setting $\theta_m = 0$) or in a component accounting only for macroeconomic risk (setting $\theta_{hv} = 0$).

An NBER dummy, set to one on recession days, is used as a control. The VIX, which is on average 19.03% over the 20 years considered, is found to be 6.78% higher on an average recession day. This estimate is statistically significant at any reasonable level and is also of economic significance; a volatility swap is approximately 35.6% more expensive during a recession.[15] The total (physical) volatility, $\sqrt{252 * h_t}$, explains 80% of the variations of the VIX.[16] It also seems to explain most of the VIX's counter-cyclicality as the statistical significance of the NBER dummy vanishes when controlling for total volatility. The short-run component of volatility alone, $\sqrt{g_t}$, can explain close to 58% of the variations of the VIX, but falls short of explaining its counter-cyclical behaviour.

This highlights the crucial role played by time-varying expectations in the Macro-MIDAS model. Fundamental volatility explains 34% of the variations in the VIX and, most importantly, it explains the counter-cyclical nature of the fear index. Indeed, the statistical significance of the NBER dummy vanishes when controlling for time-varying expectations. Regressing the VIX on

[14] This period is that for which we consider option data in the next section. From June 1988 to December 1989, we use VXO values (often referred to as the "old" VIX) that are based on OEX options rather than SPX ones and that are *not* model free. While the VXO is most likely a biased proxy of the value that the VIX would have taken over this early period, we are confident that this bias has little impact on the overall results obtained using 20 years of data.

[15] As the VIX is the square root of the one-month forward price of variance, the relationship between the VIX and the value of a volatility swap is affected by Jensen's inequality, whence the "approximately". Qualitatively, however, the approximation does not affect the results.

[16] Variations in the volatility risk premium could easily explain the remaining 20%. See Eraker (2008), Bollerslev, Tauchen, and Zhou (2009), Bollerslev, Gibson, and Zhou (2011) or Christoffersen, Heston, and Jacobs (2011) for interesting discussions of the premium.

the restricted HV and macroeconomic components of the fundamental volatility process confirms that our macroeconomic risk proxy plays a significant role in explaining the counter-cyclicality of the VIX. When restricting to the HV component of fundamental volatility, the p-value associated with the NBER dummy is virtually zero; when considering only its macroeconomic component, the p-value increases to 5%; when considering the unrestricted fundamental volatility, the p-value reaches 11%.

In our settings, macroeconomic risk alone explains 13% of the variations in the VIX. An increase of 1% in the macroeconomic component of fundamental volatility leads, on average, to an increase in the VIX of 2.36%, meaning that a volatility swap is approximately 12.4% more expensive for each 1% increase in our macroeconomic risk proxy. These are probably lower bounds on the impact of macroeconomic risk. Indeed, Bollerslev, Gibson, and Zhou (2011) remark that "the volatility risk premia has plausible business cycle variation [and that their] estimated risk premium rises sharply during the two NBER-dated macroeconomic recessions." It is also likely that the intensity of jumps rises in bad economic times. These are two examples of channels here unaccounted for through which macroeconomic risk is likely to affect the level of the VIX.

In sum, these regressions highlight the economic significance of the impact of macroeconomic risk on expected future volatility. In the next section, we take this analysis further by considering the impact of macroeconomic risk on the full cross section of options rather that only on the single moneyness and maturity VIX level.

3 Option-Valuation

Accounting for business conditions in modeling the physical volatility process does improve the model's ability to describe the past and expected distribution of the volatility of returns. Now, we address whether accounting business conditions improves the asset pricing properties of the

Macro-MIDAS model on the cross section of option prices. We consider twenty years of call option prices from 1988 to 2007, one of the most extensive data sets in the option pricing literature. Our data set covers two recessions, those in the early 1990 and in 2001.

3.1 Risk Neutralization

In order to analyze the option-pricing properties of the Macro-MIDAS model, a risk-neutral form of the model is needed. Typically, GARCH volatility models include a single source of randomness, the ε_{t+1} innovation of Equation (1.1). However, accounting for time-varying business conditions introduces a second source of randomness in the Macro-MIDAS model, that is the macroeconomic u_{t+1} innovation of Equation (2.2). Moreover, as observed in Section 2.3, the correlation between the two innovation processes, $\text{Corr}_t(\varepsilon_{t+1}, u_{t+1})$, is non-zero: market returns are correlated with macroeconomic news. This correlation is however imperfect, i.e., there is a "pure-market" innovation process $z_{t+1} \overset{\mathbb{P}}{\sim} \mathcal{N}(0,1)$, independent from u_{t+1}, such that

$$\varepsilon_{t+1} = \rho u_{t+1} + \sqrt{1-\rho^2} z_{t+1} , \tag{3.1}$$

where $\rho = \text{Corr}_t(\varepsilon_{t+1}, u_{t+1})$ by construction.[17]

Our risk neutralization, building on Christoffersen, Elkamhi, Feunou, and Jacobs (2010), relies on the assumption that the equity risk premium on macroeconomic risk is subsumed by the premium on volatility risk and by the contribution of macroeconomic conditions to the volatility process. This assumption was carried over to the model by maintaining λh_{t+1} as sole determinant of the equity risk premium in the expected return specification of Equation (1.6). We show in the appendix that, under this assumption, the correlation structure of Equation (3.1) leads to the

[17] This correlation structure implies that market movements do not feed back into the real economy; this implicit assumption is most likely violated in practice (see, for instance, Bernanke, Gertler, and Gilchrist (1999) on the financial accelerator hypothesis), but is made here for the sake of simplicity. Corradi, Distaso, and Mele (2010) rely on a similar assumption in a continuous-time setting.

following risk neutralization of the macroeconomic and pure-market innovation processes:

$$u_t^* = u_t + \rho\lambda \tag{3.2}$$

$$z_t^* = z_t + \sqrt{1-\rho^2}\lambda\ . \tag{3.3}$$

The mean shift on each process is proportional to the conditional correlation of that process with total market innovations. Interestingly, if ρ=0, that is, if market shocks and macroeconomic shocks were uncorrelated, the latter would be unaffected by the risk neutralization.

Given Equations (3.2) and (3.3), the risk-adjusted returns of the Macro-MIDAS model are given by

$$R_{t+1} = r - \frac{1}{2}\sqrt{\tau_{t+1}g_{t+1}} + \sqrt{\tau_{t+1}g_{t+1}}\left(\rho u_{t+1}^* + \sqrt{1-\rho^2}z_{t+1}^*\right) \tag{3.4}$$

$$g_{t+1} = (1-\alpha(1+\gamma^2)-\beta) + \alpha g_t\left(\rho u_t^* + \sqrt{1-\rho^2}z_t^* - \gamma - \lambda\right)^2 + \beta g_t \tag{3.5}$$

$$\log(\tau_{t+1}) = m + \theta\sum_{k=0}^{K-1}\phi_k(w_{hv})\,HV_{t-k} + \theta_m\sum_{k=0}^{K-1}\phi_k(w_m)\,\Delta x_{t-k} \tag{3.6}$$

$$x_t = \varphi x_{t-1} + \sqrt{h_t^x}\left(u_t^* - \rho\lambda\right) \tag{3.7}$$

$$h_t^x = \omega_x + \alpha_x h_{t-1}^x\left(u_{t-1}^* - \rho\lambda\right)^2 + \beta_x h_{t-1}^x\ , \tag{3.8}$$

where u_t^* and z_t^* are independent and serially uncorrelated standard normal innovations under the risk-adjusted measure $\mathbb{Q}$.

3.2 Option Valuation Results

Using the parameter estimates from Table 3 and the above risk neutralization, it is easy to evaluate call option prices via Monte Carlo. We consider weekly cross sections of call options on the S&P 500 index from June 1988 to December 2007. This data set is assembled from two different data bases: from June 1988 to December 1995, we use Berkeley data; from January 1996 to December 2007, we use OptionMetrics data. All options were quoted on the CBOE. For Berkeley data, we follow

Bakshi, Cao, and Chen (1997). For OptionMetrics data, the midpoint between bid and ask prices is used as the option price, and the dividend yield provided by OptionMetrics is used to infer an ex-dividend index level to be used in the option pricing. We also filter zero-volume quotes, and we apply the filtering rules suggested in Bakshi, Cao, and Chen (1997).

Then, for each model, on each Wednesday t_w, we perform Monte Carlo simulations using 2,000 paths of $\{z^*_{t_w+\tau,k}\}$ and, when needed, of $\{u^*_{t_w+\tau,k}\}$ in order to price options quoted on week t_w. The shocks are generated using Sobol sequences and we perform Duan and Simonato's (1998) empirical martingale adjustment, which ensures that each simulated path has a drift of r.[18] Simulations are performed using only the information set up to time t_w with the notable exception that we use parameter estimates from Section 2.3. As these parameters were estimated using *physical* data spanning from 1968 to 2007, and as they are used to price options between 1988 and 2007, this is not strictly an out-of-sample exercise. Yet, as the models were estimated without using option data, this exercise is still a stringent exercise in terms of analyzing a model's capacity to properly describe the expected distribution of future returns.

Aggregate option-valuation metrics are reported in Table 5. The Macro-MIDAS model performs, overall, better than the three nested benchmarks. On short- and medium-term options, MIDAS models better capture the volatility smirk than does the benchmark NGARCH model. For long-term options, however, the Macro-MIDAS model is outperformed by its benchmark; we will return to this result shortly.

Figure 3 further illustrates how these results vary through time and break up along the maturity and moneyness dimensions. The figure displays, throughout the sample, 13-week moving averages of the forecast improvement in IVRMSE terms of the Macro-MIDAS model over the NGARCH

[18] Christoffersen, Dorion, Jacobs, and Wang (2010) illustrate the accuracy of these simulation settings by comparing the quasi-Monte Carlo results with the exact results computed using the quasi-analytical solutions for the affine model of Heston and Nandi (2000).

model. We plot 13-week moving averages because the weekly measures are very noisy, especially for short-term options and for ITM options. The averages reported with each subplot are based on the weekly measures. First, the benchmark NGARCH model exhibits strongly counter-cyclical IVRMSE's across all maturities and moneyness levels, as shown in Figure 5. The Macro-MIDAS model removes this cyclicality, especially (i) for medium-term options and (ii) as we go from ITM to OTM calls.

As previously noted, in IVRMSE terms, it is only on long-term options that the Macro-MIDAS model shows a worse fit to implied volatilities than its benchmark does. However, the Macro-MIDAS model actually fits the implied volatilities of long-term options dramatically better than its benchmark near and during recessions.

The Macro-MIDAS model outperforms the NGARCH in 1988 and slightly less so in 1989. Looking at the performance of the nested HV-MIDAS and PureMacro-MIDAS models in these two years, we find that the performance of the Macro-MIDAS model is driven by the persistent contribution of past historical volatilities; in the NGARCH single component model, the volatility impact of the Black Monday wears off too quickly. In 1990 and 1991, the Macro-MIDAS model again offers a better fit to option prices, but this time draws on the informational content of the business conditions index. The same observation holds around the second recession in our sample; Section 3.3 offers a refined analysis of the impact of business conditions on the pricing results.

Interestingly, regardless of the maturity or moneyness level, the Macro-MIDAS model performs at par with the NGARCH model during the late 90s, the models performing equally bad as the VIX reaches all-time highs during the Russian/LTCM crisis. As Figure 2 displays, the time-varying volatility expectations captured by the fundamental volatility process is rising very slowly. It is likely that the fundamental volatility process, as specified, is too smooth to account for drastic changes in the market's expectations about future volatility. Besides, while stock market volatility is relatively high during this period, business conditions are better than average. As defined here, the fundamental volatility process simply sums, in the log-volatility domain, the impact of both historical volatilities and recent changes in business conditions. It is possible that, during the late 90s, this simple sum puts too much emphasis on the better-than-average business conditions and too little on recent volatility levels.

3.3 The Impact of Business Conditions

3.3.1 The worse the business conditions, the greater the improvement

Panel A of Table 6 reports, in its first three columns, the average improvement of the Macro-MIDAS model over its benchmark, conditioned on whether a given week falls within an expansion or a recession period. Average improvements over recessions are highly statistically significant. However, of the 1020 weeks in our data set of options, only 72 (approximately 7.1%) fall within a recession. To further study the extent of the Macro-MIDAS model's improvements over the benchmark and detail the role of business conditions in these improvements, Panel A of Table 6 also reports statistics conditional on the contemporary level of the ADS Index. That is, instead of relying on a NBER recession dummy to determine that a week falls within a period of bad business conditions, we compute centered quarterly moving averages of the ADS on each Wednesday t,

$$x_t^{(63)} = \frac{1}{63} \sum_{s=t-31}^{t+31} x_s \,. \tag{3.9}$$

As the index average is theoretically zero, we will say that a week is in the middle of a quarter with "bad" business conditions when $x_t^{(63)} < 0$, "severe" business conditions when $x_t^{(63)} < -1$, "extreme" business conditions when $x_t^{(63)} < -1.5$. In our sample, 380 weeks (37.3% of the 1020 weeks) are exposed to bad business conditions, 178 (17.5%) to severe ones, and 45 (4.4%) to extreme ones.

Panel A of Table 6 reports that the improvement brought by the Macro-MIDAS model increases as business conditions deteriorate. The conclusion is clear: the worse the business conditions, the greater the improvement from the Macro-MIDAS model. While in expansion the average improvement is a modest 3.35%, it rises to 12.88% in bad times, to 22.18% under severe business conditions, and to 29.75% under extreme conditions. This pattern is observed across all maturities and moneyness levels; for medium- and long-term calls, and for ATM and OTM calls the pattern is even more striking. Accounting for business conditions, for these options, reduces the IVRMSE by approximately 40%.

3.3.2 Synthetic at-the-money options

Even if we don't have option data prior to 1988, we can compute a model price for an at-the-money option through time. On each day from January 1968 to December 2007, we use the NGARCH and Macro-MIDAS models to price a 30-day option with its strike equal the the index value on that day. The time series of NGARCH implied volatilities for such an option is reported in the upper-left panel of Figure 4, and statistics for this time series are reported in the first row of Table 6's Panel B. Statistics on the difference between the Macro-MIDAS model's implied volatilities and those of the NGARCH are reported on the second row of Panel B and are broken down in the mid- and lower-left panels of Figure 4. The right-column panels of Figure 4 and the remainder of Table 6's Panel B report the same results but in dollar-price units.[19]

On actual data, the benchmark NGARCH model consistently underprices options in bad times. Table 6's Panel B reports that, on average, the Macro-MIDAS model predicts higher implied volatilities than those of NGARCH. Interestingly, this implied-volatility difference is increasing

[19] The average NGARCH IV reported in Panel B is higher in recessions than in expansion, as expected. Interestingly, the average call price is higher in expansion. While this might seem contradictory, it is actually due to the fact that the underlying, the S&P 500 index is, on average, higher in expansion than in recessions.

as business conditions deteriorate. For example, under extreme business conditions, the 30-day, at-the-money implied volatility of the Macro-MIDAS model is 1.6% higher than that implied by the NGARCH model. In terms of option prices, this translates into a 9.1% higher option price on average, a difference of sizable economic importance.

3.3.3 Improvements over the nested HV-MIDAS

Figure 5 sheds further light on the role played by business conditions in the Macro-MIDAS model's option-valuation performance. In this figure, the Macro-MIDAS model is compared to the HV-MIDAS, rather than only the NGARCH, in order to better grasp the marginal impact of accounting for changes in business conditions. The data is here divided in five buckets corresponding to the quintiles of $x_t^{(63)}$ conditional on business conditions being bad, i.e. $x_t^{(63)} < 0$. In -Q1, business conditions are moderately bad, in -Q5, business conditions are at their very worst. The difference between both models' average improvement is given by the solid black line. The uppermost panel illustrates that the incremental importance of accounting for changes in business conditions is almost monotonic with the deterioration of business conditions; it rises from 0.46% in -Q1 to 15.30% in -Q5. As evidenced in the six other panels of Figure 5, this pattern is rather robust to maturities and moneyness levels.

In sum, while the benchmark NGARCH model exhibits strongly counter-cyclical IVRMSE's across all maturities and moneyness levels, the Macro-MIDAS model removes this cyclicality and this last figure highlight the crucial role played by business conditions in the ability of the Macro-MIDAS model to better describe the cross-sections of option prices.

4 Conclusion

This paper introduces the Macro-MIDAS model, a dynamic volatility model accounting for both financial and macroeconomic sources of fundamental volatility. The new model is shown to outperform the NGARCH benchmark in fitting asset returns and pricing options, especially around the 1990-1991 and 2001 recessions. In particular, the Macro-MIDAS model improves on the benchmark's option-valuation abilities by mitigating the counter-cyclicality of its implied-volatility bias, across all maturity and moneyness levels. The Macro-MIDAS model also allows us to isolate the contribution of macroeconomic risk to the level of volatility, and this contribution is found to account for a sizeable 13% of the variation in the VIX through time.

This work offers several avenues for further research. For instance, conducting our analysis in a stochastic volatility framework would allow us to assess the extent of the relationship between the macroeconomic shocks entering our fundamental volatility process and the unobservable volatility shocks inherent to stochastic volatility models. Besides, incorporating analyst forecasts or survey results in the business conditions' forecasting model could further improve the abilities of the Macro-MIDAS model to explain observed option prices. Buraschi, Trojani, and Vedolin (2009) also suggest that dispersion in analyst forecasts is strongly related to implied volatility levels. Otherwise, once it is established that business conditions impact option prices, option data could eventually be used to infer market expectations of future business conditions.

Another line of investigation would be to refine how the informational content of historical volatilities and business conditions are combined to model the fundamental volatility process. Our work uses historical volatilities based on daily returns; when intraday data are available, intraday realized volatilities could prove more reactive to current market conditions. Moreover, the Macro-MIDAS model simply sums the impact of historical volatilities and business conditions in the log-volatility domain. However, given that responses to macroeconomic news differ depending on the current state of the economy, it is likely that an approach allowing for further nonlinearities would prove fruitful. Finally, a study of how higher moments of the stock returns' distribution evolve with changing business conditions could further our understanding of the volatility premium and of its time-series properties.

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Appendix

Risk Neutralization

We consider a GARCH model of the form

$$R_{t+1} = r + \lambda_{t+1} \sqrt{h_{t+1}} - \tfrac{1}{2}h_{t+1} + \sqrt{h_{t+1}}\Big(\rho u_{t+1} + \sqrt{1-\rho^2} z_{t+1}\Big) \tag{A.1}$$

$$h_{t+1} = f\,(\,\cdot \mid \Theta, \mathcal{F}_t\,)\ , \tag{A.2}$$

where λ_{t+1} and h_{t+1} are $\mathcal{F}_t$-measurable, and where $\{\, u_t \,\}$ and $\{\, z_t \,\}$ are independent and serially uncorrelated innovation processes. To formally demonstrate the risk neutralization of u_t and z_t as introduced in Equations (3.2) and (3.3), we here draw on Christoffersen, Elkamhi, Feunou, and Jacobs (2010, henceforth CEFJ) treatment of two-shocks stochastic volatility models (see CEFJ's Section 7). Note that our model is, however, fundamentally different from a stochastic volatility model in that, here, the "second" shock, u_{t+1}, does not contemporaneously impact the variance but the mean of the return process. We will return to the implications of this fundamental difference shortly.

First, we write the risk neutralization of our return process in terms of the risk neutralization of the bivariate, uncorrelated normal innovations $\{\, u_t, z_t \,\}$ using the following Radon-Nikodym derivative:

$$\xi_\tau \equiv \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right| \mathcal{F}_\tau = \exp\left\{-\sum_{t=1}^{\tau}\left(\eta_{u,t}u_t + \eta_{z,t}z_t + \Psi_t^{u,z}\left(\eta_{u,t},\eta_{z,t}\right)\right)\right\}, \tag{A.3}$$

where $\Psi_t^{u,z}$ is natural logarithm of the moment-generating function of the $\{\, u_t, z_t \,\}$ pairs, that is,

$$\Psi_t^{u,z}\left(\eta_u,\eta_z\right) = \tfrac{1}{2}\left(\eta_u^2 + \eta_z^2\right) . \tag{A.4}$$

For the probability measure $\mathbb{Q}$ defined by Radon-Nikodym derivative of Equation (A.3) to be an equivalent martingale measure (EMM), it must be the case that

$$\begin{aligned} 1 &= \mathbb{E}_{\text{t-1}}^{\mathbb{Q}}\left[\frac{S_t}{S_{t-1}}\Big/\frac{B_t}{B_{t-1}}\right] = \mathbb{E}_{\text{t-1}}^{\mathbb{P}}\left[\frac{\xi_t}{\xi_{t-1}}\frac{S_t}{S_{t-1}}\Big/\frac{B_t}{B_{t-1}}\right] \\ &= \mathbb{E}_{\text{t-1}}^{\mathbb{P}}\left[\exp\left\{-\eta_{u,t}u_t - \eta_{z,t}z_t - \Psi_t^{u,z}\left(\eta_{u,t},\eta_{z,t}\right)\right\}\exp\left\{\lambda_t\sqrt{h_t} - \tfrac{1}{2}h_t + \sqrt{h_t}\left(\rho u_t + \sqrt{1-\rho^2}z_t\right)\right\}\right] \end{aligned} \tag{A.5}$$

or, equivalently,

$$0 = \Psi_t^{u,z}\left(\eta_{u,t} - \rho\sqrt{h_t}\,, \eta_{z,t} - \sqrt{(1-\rho^2)h_t}\right) - \Psi_t^{u,z}\left(\eta_{u,t},\eta_{z,t}\right) + \lambda_t\sqrt{h_t} - \tfrac{1}{2}h_t\,, \tag{A.6}$$

which boils down to

$$\rho\eta_{u,t} + \sqrt{1-\rho^2}\eta_{z,t} = \lambda_t\,. \tag{A.7}$$

Equation (A.7) admits an infinity of solutions. Yet, as highlighted above, our model has the specificity that both shocks affect the mean of the return process. Thus, the bivariate normal shocks can be seen as blending into a single stream of standard normal innovations $\{\, \varepsilon_t \,\}$ and Equation (A.1) is equivalent to

$$R_{t+1} = r + \lambda_{t+1}\sqrt{h_{t+1}} - \tfrac{1}{2}h_{t+1} + \sqrt{h_{t+1}}\varepsilon_{t+1}\,. \tag{A.8}$$

This last equation is that of Duan's (1995), which is a special case of CEFJ for which the (linear) Radon-Nikodym derivative can be written as

$$\xi_\tau \equiv \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right| \mathcal{F}_\tau = \exp\left\{-\sum_{t=1}^{\tau}\left(\eta_t\varepsilon_t + \Psi_t^{\varepsilon}\left(\eta_t\right)\right)\right\}, \tag{A.9}$$

where Ψ_t^{ε} is natural logarithm of the moment-generating function of the $\{\varepsilon_t\}$ innovations, that is, $\Psi_t^{\varepsilon}(\eta) = \frac{1}{2}\eta^2$. Again, for the $\mathbb{Q}$ measure defined by Equation (A.9) to be an EMM, it must be that

$$\begin{aligned} 1 &= \mathbb{E}^{\mathbb{Q}}_{\text{t-1}}\left[\frac{S_t}{S_{t-1}}\Big/\frac{B_t}{B_{t-1}}\right] = \mathbb{E}^{\mathbb{P}}_{\text{t-1}}\left[\frac{\xi_t}{\xi_{t-1}}\frac{S_t}{S_{t-1}}\Big/\frac{B_t}{B_{t-1}}\right] \\ &= \mathbb{E}^{\mathbb{P}}_{\text{t-1}}\left[\exp\left\{-\eta_t\varepsilon_t - \Psi_t^{\varepsilon}(\eta_t)\right\}\exp\left\{\lambda_t\sqrt{h_t} - \tfrac{1}{2}h_t + \sqrt{h_t}\varepsilon_t\right\}\right] \end{aligned} \tag{A.10}$$

$$\Leftrightarrow 0 = \Psi_t^{\varepsilon}\left(\eta_t - \sqrt{h_t}\right) - \Psi_t^{\varepsilon}(\eta_t) + \lambda_t\sqrt{h_t} - \tfrac{1}{2}h_t\,, \tag{A.11}$$

which implies that $\eta_t = \lambda_t, \forall t$. Now, for Equations (A.3) and (A.9) to describe the same Radon-Nikodym derivative, it must be that

$$\begin{aligned} \xi_\tau &= \exp\left\{-\sum_{t=1}^{\tau}\left(\lambda_t\varepsilon_t + \tfrac{1}{2}\lambda_t^2\right)\right\} \\ &= \exp\left\{-\sum_{t=1}^{\tau}\left(\lambda_t\rho u_t + \lambda_t\sqrt{1-\rho^2}z_t + \overbrace{\tfrac{1}{2}\left(\rho^2\eta_{u,t}^2 + 2\rho\sqrt{1-\rho^2}\eta_{u,t}\eta_{z,t} + (1-\rho^2)\eta_{z,t}^2\right)}^{\text{Using Equation (A.7)}}\right)\right\} \\ &= \exp\left\{-\sum_{t=1}^{\tau}\left(\eta_{u,t}u_t + \eta_{z,t}z_t + \Psi_t^{u,z}(\eta_{u,t},\eta_{z,t})\right)\right\} \end{aligned}$$

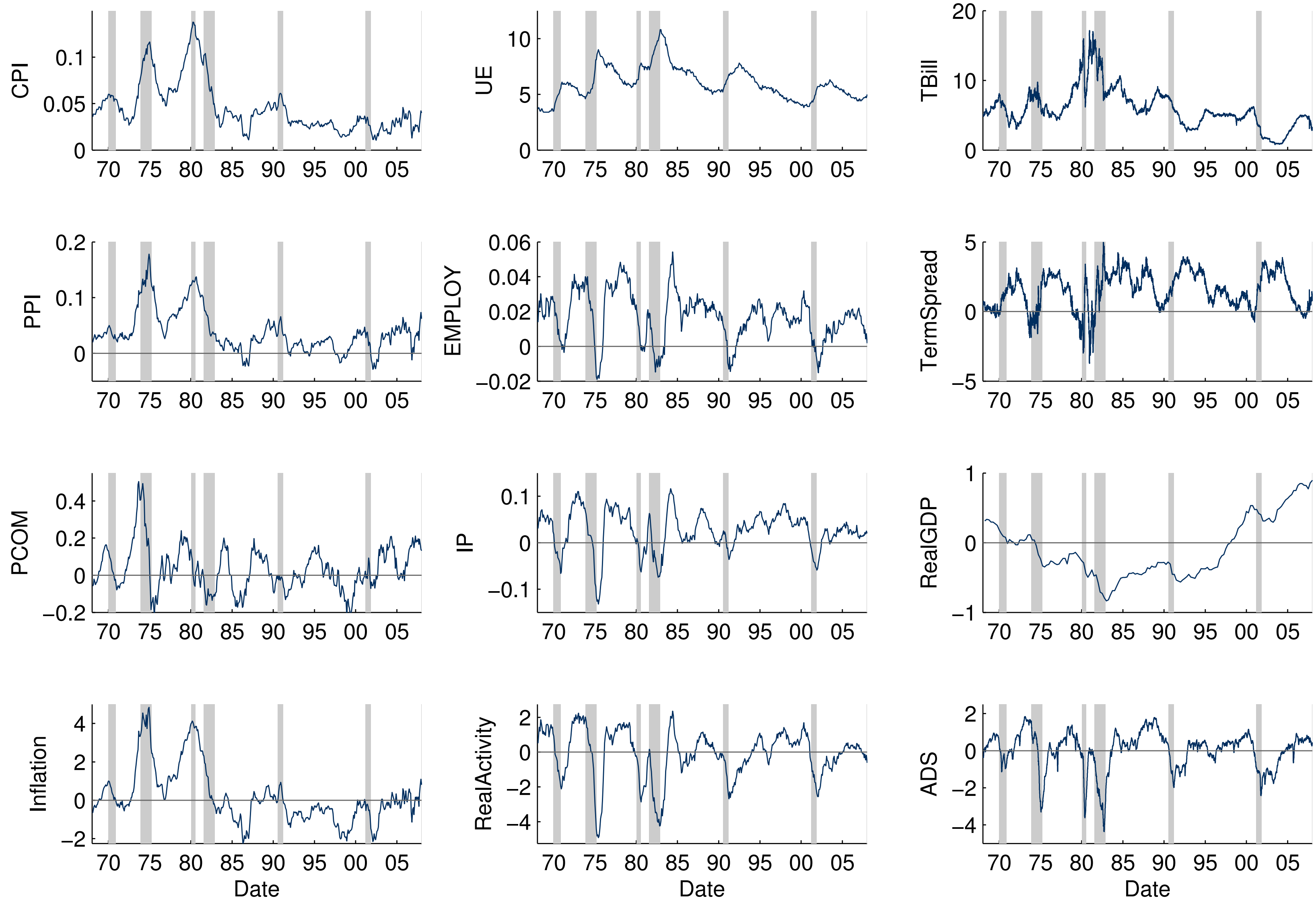
where the last equality holds if, and only if, for all t,

$$\eta_{u,t} = \rho\lambda_t \quad \text{and} \quad \eta_{z,t} = \sqrt{1-\rho^2}\lambda_t\,. \tag{A.12}$$

We thus have that $u_t^* = u_t + \rho\lambda_t$, $z_t^* = z_t + \sqrt{1-\rho^2}\lambda_t$, and

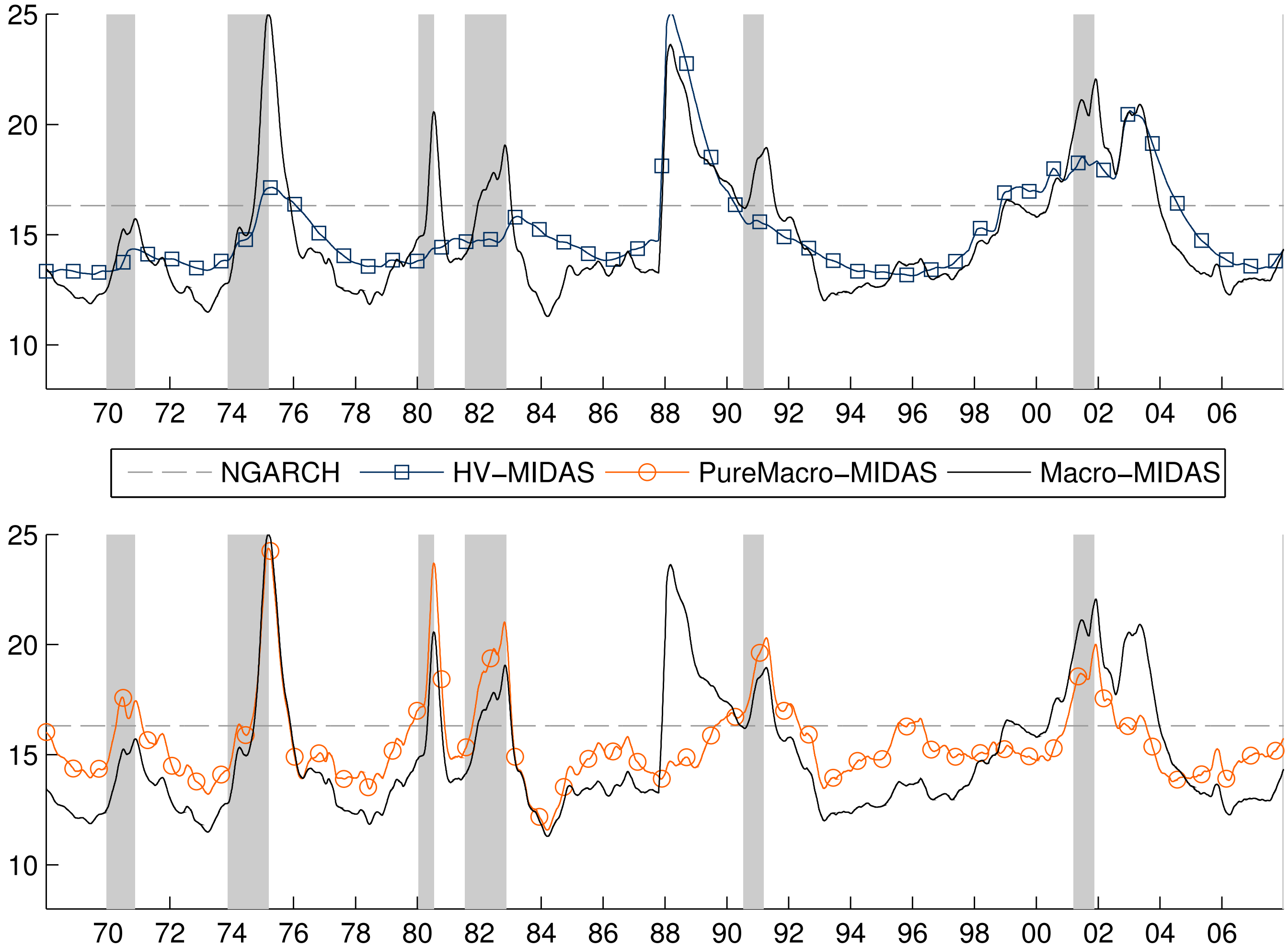
$$\varepsilon_t^* = \varepsilon_t + \lambda_t = \rho u_t^* + \sqrt{1-\rho^2}z_t^*\,. \tag{A.13}$$

Figure 1: *Macroeconomic Series*



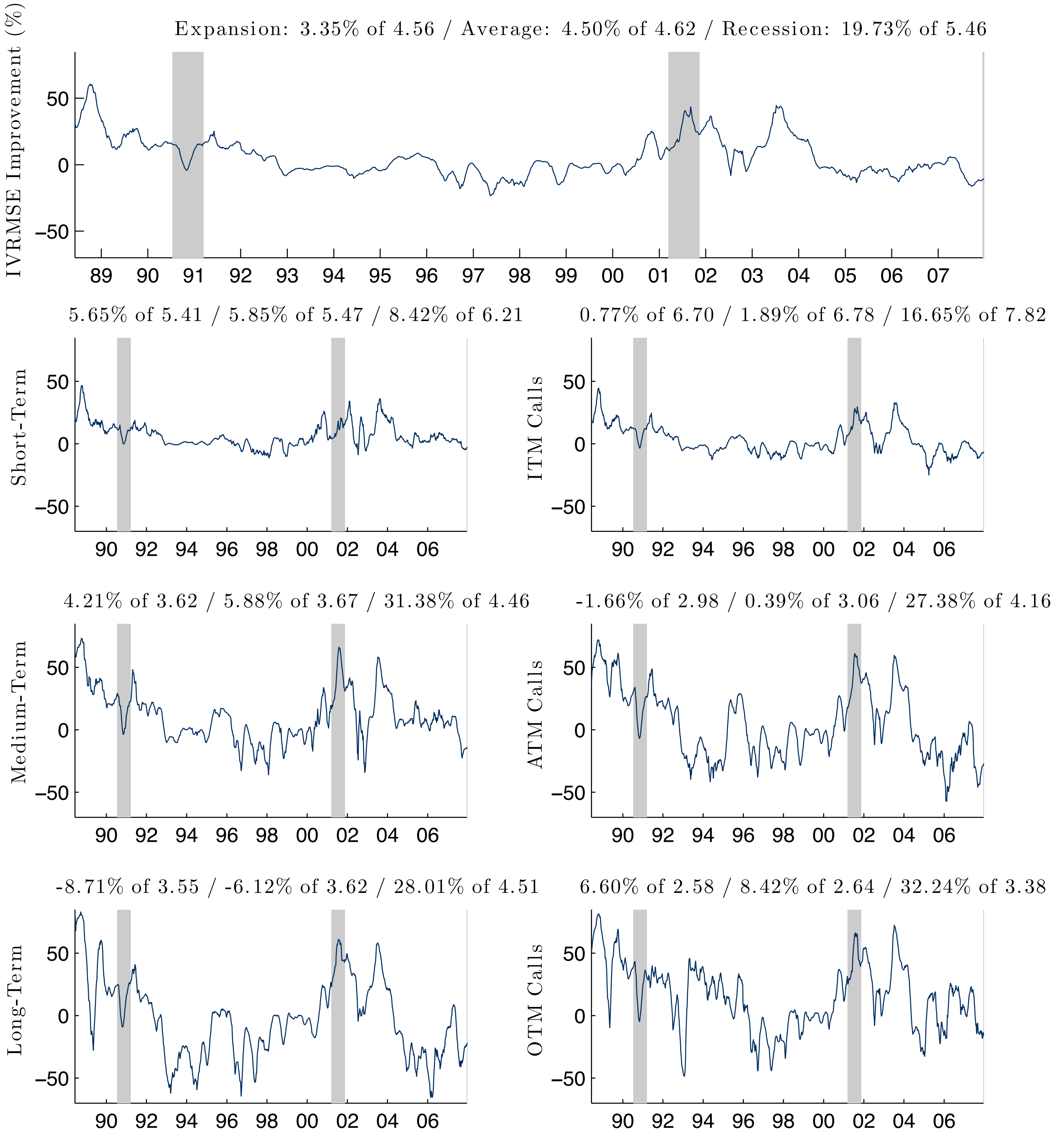
In the bottom row of the left and central columns are Ang & Piazzesi's monthly inflation and real activity indicators; their respective input series are reported in the first three rows. The right column displays the T-Bill rates, the term spread, the detrended real GDP, and the business conditions index.

Figure 2: *Fundamental Variance Processes*



In both panels, we plot, as a solid black line, the annualized fundamental volatility level of the Macro-MIDAS model along with a dashed horizontal line at 16.31%, which corresponds to the NGARCH model's unconditional volatility, $\sqrt{252\omega/(1-\alpha(1+\gamma^2)-\beta)}$. *In the upper panel, we superimpose the fundamental volatility level obtained for the HV-MIDAS model; in the lower panel, we superimpose the volatility level obtained for the PureMacro-MIDAS model.*

Figure 3: *Macro-MIDAS Model's IVRMSE Improvement Over the NGARCH Model*

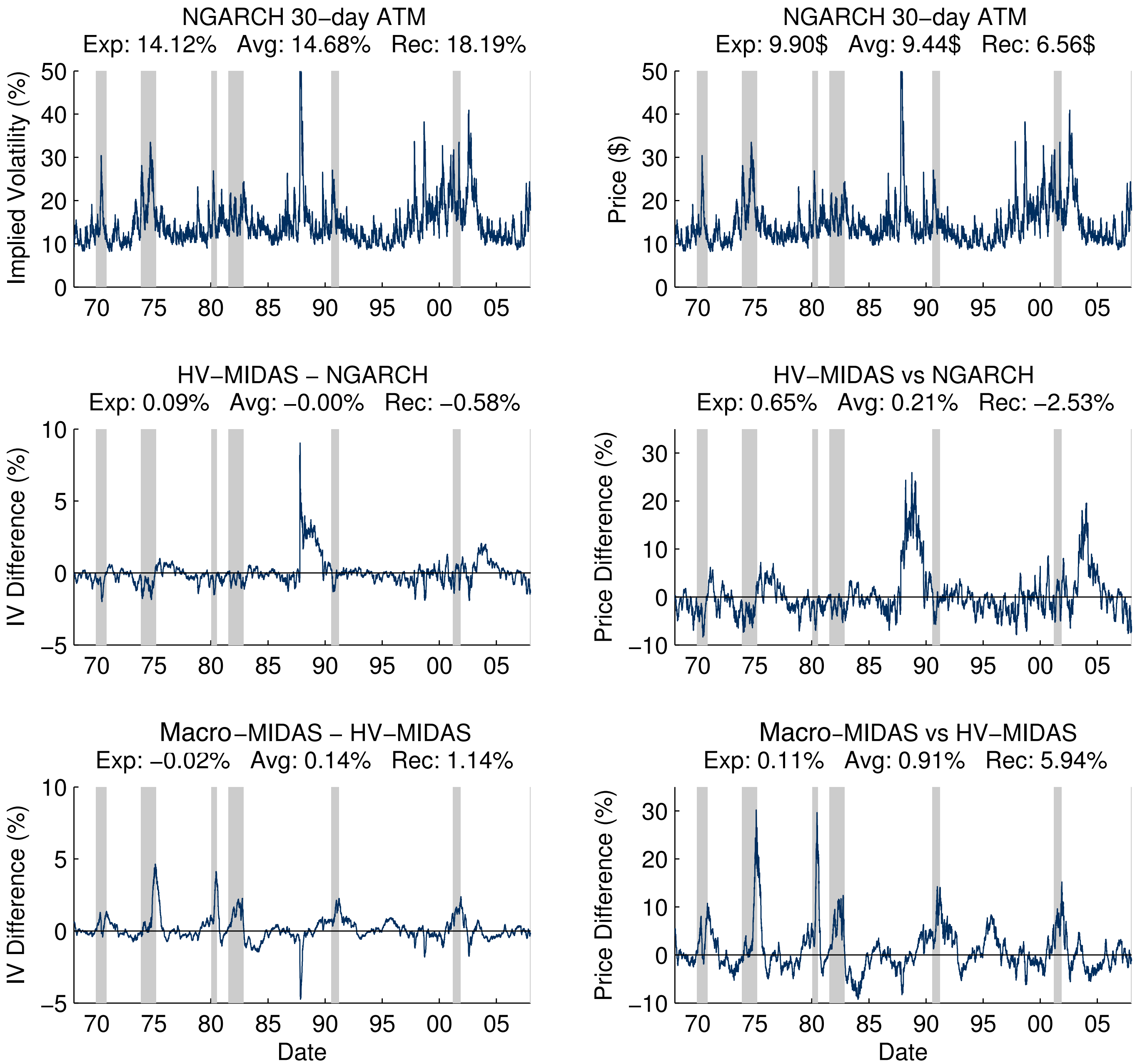


Using parameter estimates in Table 3, we price options and compute weekly IVRMSE measures for the NGARCH model, $IVRMSE_w^{\text{NG}}$*, and for the Macro-MIDAS model,* $IVRMSE_w^{\text{HV},\Delta x}$*. As title to each panel of this figure are summary statistics over* Expansion *periods, the complete sample* (Average)*, and* Recession *periods. The statistics are reported as*

$$\widehat{\mathbb{E}}\left[\left(IVRMSE_w^{\text{NG}} - IVRMSE_w^{\text{HV},\Delta x}\right)\Big/ IVRMSE_w^{\text{NG}}\right] \qquad \text{of} \qquad \widehat{\mathbb{E}}\left[IVRMSE_w^{\text{NG}}\right],$$

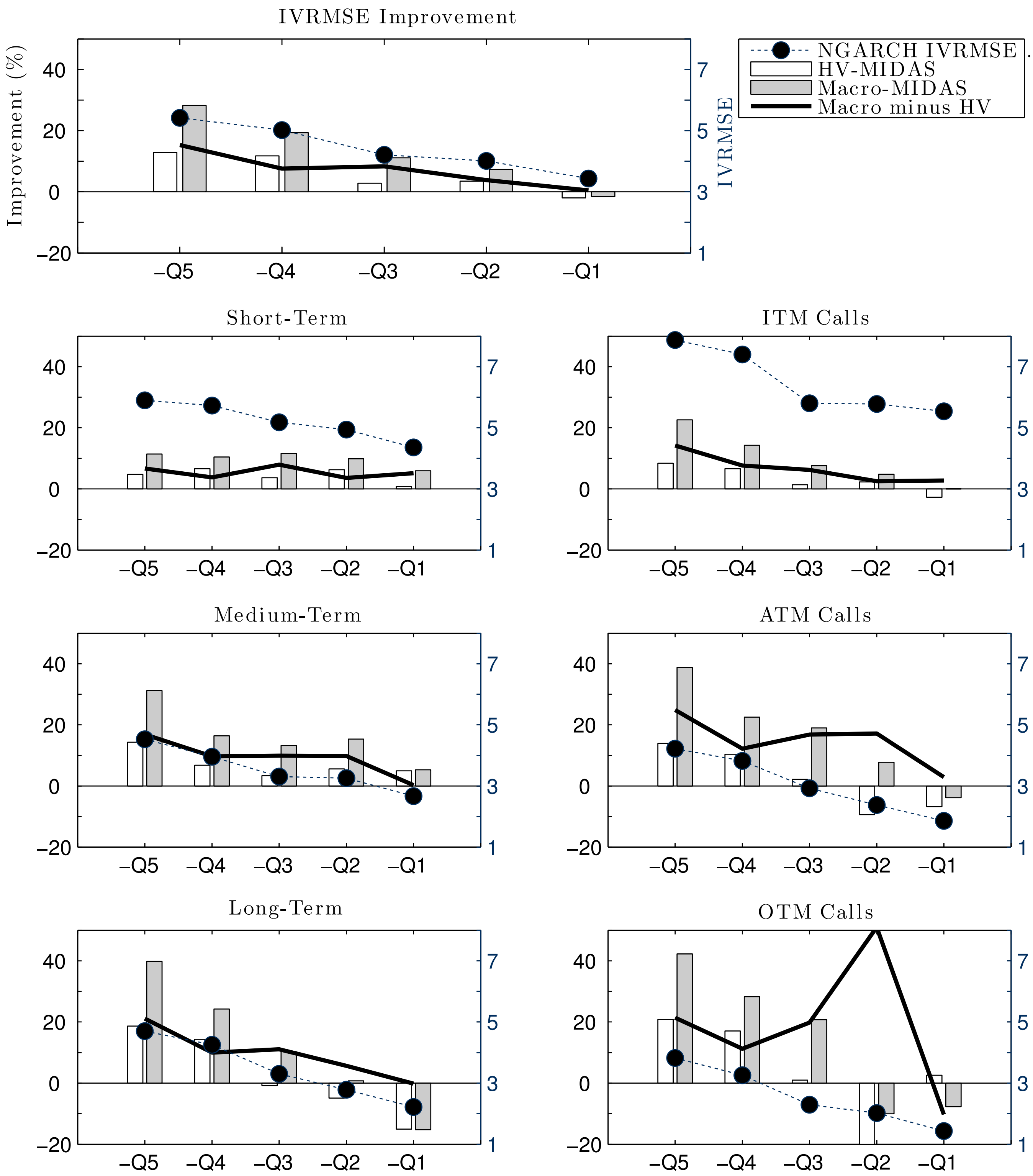
that is the average relative improvement of the Macro-MIDAS over the NGARCH along with the average IVRMSE of the latter model. The improvement time series is displayed. On the left-hand side, results are divided along the options' maturity: 45 days to maturity (DTM) or less, between 46 and 90 DTM, or more than 90 DTM. On the right-hand side, results are divided along options' moneyness: $K/S \leq 0.975$, $0.975 < K/S < 1.025$, *and* $K/S \geq 1.025$.

Figure 4: *Synthetic Options*



The upper-left panel of this figure reports the time series of implied volatilities obtained using the NGARCH model to price synthetic, at-the-money options with 30 days to maturity, daily from January 1968 to December 2007. These options are created assuming that their strike is equal to the index value on each given day. The mid-left panel reports the difference between the HV-MIDAS model's implied volatilities for these synthetic options and the NGARCH implied volatilities. Similarly, the lower-left panel reports difference in implied volatilities when comparing the Macro-MIDAS and HV-MIDAS models. The right-column panels report the data of the left column but in the price domain. Note that price differences are in relative terms, e.g. $\left(C_t^{\mathrm{HV}} - C_t^{\mathrm{NGARCH}}\right)/C_t^{\mathrm{NGARCH}}$.

Figure 5: *The Impact of Accounting for Changes in Business Conditions*



This figure reports the average pricing improvements, $\widehat{\mathbb{E}}\left[\left(IVRMSE_w^{\text{NG}} - IVRMSE_w^{\mathcal{M}}\right)\big/ IVRMSE_w^{\text{NG}}\right]$, *of the HV-MIDAS and Macro-MIDAS models; the difference between both models' average improvement is given by the solid black line. The data is divided in five buckets corresponding to the quintiles of* $x_t^{(63)}$ *conditional on business conditions being bad, i.e.* $x_t^{(63)} < 0$. *In -Q1, business conditions are moderately bad, in -Q5, business conditions are at their very worst. The right-hand side y-axis is used only for the NGARCH's average IVRMSE reported, for each quintile, by dots connected with a dotted line. Panels are divided along the same lines as in Figure 3. Note that, in the* OTM Calls *panel, the underperformance of the HV-MIDAS model is truncated and actually reaches -61.04%.*

Table 1: *Descriptive Statistics on the Business Conditions Index*

	Index: x_t	Residuals: v_t	GARCH Residuals: u_t	GARCH Variances: $h_t \times 1e4$
Min	-4.36	-0.36	-9.16	0.13
Mean	0.02	0.00	0.02	4.91
Max	1.83	0.21	6.52	171.50
Std. Dev.	1.04	0.02	1.06	9.31
Skewness	-1.21	-0.92	-0.26	6.95
Kurtosis	4.84	23.51	7.88	76.91
Log-Likelihood		-9254.3	27298.5	
Jarque-Bera		177850.0	10099.2	

This table reports summary statistics on the processes of Equation (2.1), (2.2) *and* (2.3) *:*

$$x_t = \varphi x_{t-1} + v_t \,, \qquad v_t = \sqrt{h_t^x} u_t \,, \qquad h_t^x = \omega_x + \alpha_x v_{t-1}^2 + \beta_x h_{t-1}^x \,.$$

Maximum likelihood parameter estimates are $\varphi = 0.999999$, $\omega_x = 1.4117e-06$, $\alpha_x = 0.1183$ *and* $\beta_x = 0.8817$, *implying a variance persistence of* 0.9999.

Table 2: *Correlations between Macroeconomic Series*

		ADS	TBill	TS	UE	EMPLOY	Inflation	RA
Monthly	**TBill**	-0.026*						
	Term Spread	-0.276	-0.436					
	UE	-0.475	0.304	0.532				
	EMPLOY	0.724	0.142	-0.204	-0.282			
	Inflation	-0.073*	0.556	-0.456	0.090$^{4.8\%}$	0.141		
	Real Activity	0.794	-0.059*	-0.286	-0.585	0.903	-0.023	
Quarterly	**Real GDP**	0.170$^{3.1\%}$	-0.502	-0.350	-0.733	0.012*	-0.074*	0.264

This table reports the correlations between eight of the series displayed in Figure 1. For the first six rows, the three daily series are sampled monthly; daily correlations between these are similar to the ones reported here. The last row reports the correlations of the first seven series, sampled quarterly, with the detrended real GDP; again, unreported quarterly correlations are very similar to the monthly ones. Correlations marked with an asterisk are not *statistically significant at the 90% level; the exponent, whenever there is one, is the correlation's p-value; all other correlations are significant at least to the 99% level.*

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Table 3: *Maximum Likelihood Estimates*

	NGARCH	MIDAS HV	MIDAS PureMacro	MIDAS Macro
$\boldsymbol{\lambda}$	0.0151	0.0156	0.0149	0.0155
	(2.31E-08)	(2.48E-08)	(2.23E-09)	(1.24E-08)
$\boldsymbol{\omega}$, **m**	1.03E-06	-9.714	-9.255	-9.730
	(1.30E-12)	(9.78E-07)	(3.26E-07)	(2.17E-07)
$\boldsymbol{\alpha}$	0.0567	0.0624	0.0589	0.0629
	(1.29E-09)	(4.09E-08)	(3.66E-09)	(8.30E-09)
$\boldsymbol{\beta}$	0.9067	0.8846	0.8951	0.8725
	(4.77E-08)	(1.72E-08)	(2.16E-08)	(5.23E-08)
$\boldsymbol{\gamma}$	0.6873	0.7283	0.7278	0.7850
	(4.33E-07)	(1.10E-06)	(2.29E-07)	(4.44E-07)
$\boldsymbol{\theta}_{\mathbf{hv}}$		68.234		63.825
		(5.27E-05)		(3.21E-06)
$\mathbf{w}_{\mathbf{hv}}$		2.931		3.203
		(2.68E-06)		(1.38E-06)
$\boldsymbol{\theta}_{\mathbf{m}}$			-1.238	-1.009
			(1.54E-06)	(5.53E-07)
$\mathbf{w}_{\mathbf{m}}$			3.370	3.722
			(7.31E-07)	(1.82E-06)
SR Persistence	0.9902	0.9801	0.9852	0.9741
1.0 − LR ($\times 10^4$)		0.6616	0.7376	0.7036
SR VoV ($\times 10^4$)	1.576	1.786	1.677	1.862
LR VoV ($\times 10^4$)		0.0338	0.0483	0.0533
Corr($\mathbf{R}_{t+1}, \mathbf{h}_{t+2}$)	-69.70%	-71.75%	-71.72%	-74.30%
Corr($\varepsilon_{t+1}, \mathbf{u}_{t+1}$)			5.04%	5.03%
Log-Likelihood	33791.3	33806.7	33803.0	33822.0
BIC	-6.7080	-6.7093	-6.7085	-6.7105

This table reports maximum likelihood estimates for the NGARCH model as well as those of MIDAS models for three different specifications: (i) HV-MIDAS: quarterly historical volatilities computed from daily returns; (ii) PureMacro-MIDAS: quarterly differences of the Business Conditions Index values; and (iii) MacroHV-MIDAS: a combination of both (i) and (ii). Below each parameter estimate, we report its Bollerslev-Wooldridge standard error. The Bayesian information criterion (BIC) values account for the number of parameters in each model and for the length of the time series of S&P 500 returns between January 1968 and December 2007.

Short-run (SR) persistence and annualized volatility of variance (VoV) are $\alpha(1+\gamma^2)+\beta$ and the average of $\sqrt{252\,\alpha^2(2+4\gamma^2)h_{t+1}^2}$, respectively. For the long-run (LR) component, we approximate the persistence and volatility of variance by fitting an AR(1) to the fundamental volatility process, i.e.,

$$\tau_t = \phi_0 + \phi_1 \tau_{t-1} + \sqrt{v} e_t ,$$

where e_t is white noise. The volatility of variance is approximated by $\sqrt{252v}$, while the long-run persistence is approximated by ϕ_1 and is very close to one for all MIDAS models; we here report $10^4 \times (1-\phi_1)$.

Table 4: *The Macro-MIDAS Model's Volatility and the VIX*

	NBER	Total Vol		Short-Run Vol		Fundamental Volatility					
						HV		PureMacro		Macro	
cst	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03
	(31.56)	**(80.04)**	**(80.34)**	**(47.84)**	**(51.09)**	**(36.56)**	**(38.54)**	**(32.63)**	**(32.81)**	**(39.22)**	**(39.46)**
Total Vol		1.01	1.00								
		(24.23)	**(23.66)**								
Short-Run Vol				4.71	4.54						
				(16.98)	**(16.41)**						
Fundamental Vol						1.44	1.37	2.36	1.83	1.32	1.23
						(5.77)	**(5.80)**	**(4.55)**	**(2.99)**	**(8.41)**	**(7.18)**
NBER	6.78		1.31		4.64		5.57		3.24		2.54
	(5.67)		(1.30)		**(5.17)**		**(4.01)**		**(1.96)**		(1.59)
$\mathbf{R^2}$	9.1%	80.3%	80.6%	58.1%	62.2%	27.0%	33.1%	13.0%	14.4%	34.0%	35.2%

We consider the VIX over 4936 business days, 347 (7%) of which fall within an NBER recession. Coefficients reported for the NBER *dummy are obtained from* δ_N *in, e.g., the following regression*

$$\mathrm{VIX}_t = \mathrm{cst} + \delta_N \mathrm{NBER}_t + e_t\ .$$

The VIX is expressed in annualized percentage terms. As in all the regressions that follow, regressors are centered around a zero mean so that the constant cst *captures the mean of the VIX, 19.03%. The reported coefficient on the NBER is the increase in the VIX on an average recession day, here 6.78%.*

In the Total Vol *regressions,*

$$\mathrm{VIX}_t = \mathrm{cst} + \delta_h \sqrt{h_t} + \delta_N \mathrm{NBER}_t + e_t\ ,$$

the total volatility $\sqrt{h_t}$ *is expressed in annualized percentage terms and centered around zero.*

In the Short-Run Vol *regressions,*

$$\mathrm{VIX}_t = \mathrm{cst} + \delta_g \sqrt{g_t} + \delta_N \mathrm{NBER}_t + e_t\ ,$$

the short-run volatility $\sqrt{g_t}$ *is standardized so that the reported coefficient can be interpreted as the increase in the VIX caused by an increase of the short-run volatility one standard deviation away from its mean.*

In the Fundamental Volatility *regressions,*

$$\mathrm{VIX}_t = \mathrm{cst} + \delta_\tau \sqrt{\tau_t} + \delta_N \mathrm{NBER}_t + e_t\ ,$$

the fundamental volatility $\sqrt{\tau_t}$ *is expressed in annualized percentage terms and centered around zero. The* HV *regressions consider the component of fundamental volatility that is solely driven by historical volatilities. The* PureMacro *regressions consider the component of fundamental volatility that is solely driven by changes in business conditions. The* Macro *regressions consider the fundamental volatility driven by both measures.*

Parenthesized below each coefficient are t-stats computed using Newey-West standard errors with a lag of 63, corresponding to one quarter of trading days. The choice of a quarter-long lag is arbitrary and intended to be very conservative; in our case $T = 4936$, *so that* $\lfloor 4(T/100)^{0.25} \rfloor = 10$ *would be the lag suggested by Newey and West (1994).*

Table 5: *Option-Pricing Results*

	Moneyness	N	Avg.	NGARCH	MIDAS HV	MIDAS PureMacro	MIDAS Macro
Log-Likehood				33791.3	33806.7	33803	**33822**
IVRMSE		68923	18.41	5.23	5.14	5.21	**5.10**
DTM ≤ 45	**(0.33, 0.95)**	5478	29.83	12.61	12.52	12.49	**12.39**
	[0.95,1.00)	7464	17.81	4.29	4.19	4.18	**4.08**
	[1.00, 1.05)	8431	15.03	2.67	2.55	2.62	**2.47**
	[1.05, 1.87]	3147	18.28	3.81	3.57	3.72	**3.41**
45 < DTM ≤ 91	**(0.33, 0.95)**	3295	23.02	7.08	6.98	6.82	**6.77**
	[0.95,1.00)	3854	17.27	3.84	3.79	3.72	**3.67**
	[1.00, 1.05)	5282	15.13	3.01	2.96	3.02	**2.93**
	[1.05, 1.87]	4112	16.01	2.96	2.72	2.98	**2.72**
91 < DTM ≤ 182	**(0.33, 0.95)**	3221	21.54	5.36	5.43	**5.23**	5.34
	[0.95,1.00)	2519	17.88	3.68	3.72	**3.61**	3.63
	[1.00, 1.05)	2924	16.49	3.36	3.32	3.40	**3.31**
	[1.05, 1.87]	4018	16.02	3.37	3.11	3.43	**3.07**
DTM > 182	**(0.33, 0.95)**	2950	20.90	**4.95**	5.21	5.21	5.41
	[0.95,1.00)	2572	18.40	**4.06**	4.11	4.30	4.27
	[1.00, 1.05)	3105	17.89	4.08	**4.00**	4.35	4.18
	[1.05, 1.87]	6551	16.60	3.73	**3.31**	3.96	3.43
RMSE		68923	$40.44	39.34	32.35	38.23	**30.55**

For each model, we first recall its log-likehood as estimated in Section 2.3. Then, we report overall IVRMSE values, followed by IVRMSE values obtained over maturity/moneyness buckets of options. For completeness, we also report the overall RMSE values; IVRMSEs and RMSEs are computed as follows:

$$IVRMSE = \sqrt{\frac{1}{N}\sum_{t,k}\left(\mathrm{IV}\left(C_{t,k}\right) - \mathrm{IV}\left(C_{t,k}^{\text{MODEL}}\right)\right)^2} \quad \textit{and} \quad RMSE = \sqrt{\frac{1}{N}\sum_{t,k}\left(\frac{C_{t,k} - C_{t,k}^{\text{MODEL}}}{C_{t,k}}\right)^2}.$$

Apart from the average call price ($40.44), all entries are percentage points. In each row, the entry for the best performing model for that row is in bold font.

Table 6: *The Macro-MIDAS Model and Business Conditions*

Panel A: Macro-MIDAS IVRMSE Improvement Over the NGARCH Model

	Expansion	**Overall**	**Recession**	$x_t^{(63)} < 0$	$x_t^{(63)} < -1$	$x_t^{(63)} < -1.5$
# Weeks	948	1020	72	380	178	45
All Calls	3.35	4.50	19.73	12.88	22.18	29.75
	(0.69)	(0.68)	(2.58)	(1.02)	(1.60)	(2.64)
Short-Term	5.65	5.85	8.42	9.84	10.63	23.45
	(1.28)	(1.24)	(4.93)	(1.81)	(3.61)	(3.45)
Medium-Term	4.21	5.88	31.38	16.22	21.35	39.06
	(1.17)	(1.14)	(3.79)	(1.88)	(3.68)	(3.20)
Long-Term	-8.71	-6.12	28.01	11.95	30.07	42.92
	(1.27)	(1.25)	(3.61)	(1.60)	(1.87)	(3.09)
ITM Calls	0.77	1.89	16.65	9.85	16.65	24.21
	(0.67)	(0.66)	(2.46)	(0.95)	(1.64)	(2.68)
ATM Calls	-1.66	0.39	27.38	16.86	29.45	42.71
	(1.28)	(1.23)	(3.33)	(1.57)	(2.01)	(2.54)
OTM Calls	6.60	8.42	32.24	15.24	34.23	45.16
	(2.26)	(2.12)	(3.51)	(4.80)	(2.43)	(3.58)

Panel B: Synthetic 30-DTM, ATM Options

	Expansion	**Overall**	**Recession**	$x_t^{(63)} < 0$	$x_t^{(63)} < -1$	$x_t^{(63)} < -1.5$
# Days	8684	10068	1384	3833	1640	819
NGARCH IV	14.12	14.68	18.19	15.57	17.86	17.01
	(0.06)	(0.06)	(0.13)	(0.08)	(0.12)	(0.13)
IV Difference	0.06	0.13	0.56	0.57	1.02	1.60
	(0.01)	(0.01)	(0.04)	(0.02)	(0.04)	(0.05)
NGARCH Price ($)	9.90	9.44	6.56	8.88	11.41	6.73
	(0.11)	(0.10)	(0.25)	(0.16)	(0.27)	(0.29)
Relative Price Difference (%)	0.77	1.12	3.31	4.00	6.26	9.05
	(0.07)	(0.07)	(0.19)	(0.11)	(0.20)	(0.30)

In Panel A, using parameter estimates in Table 3, we price options and compute weekly IVRMSE measures for the NGARCH model, $IVRMSE_w^{NG}$, *and for the Macro-MIDAS model,* $IVRMSE_w^{HV,\Delta x}$. *Panel A reports averages of the relative improvement resulting from using the latter model over the former, that is,* $(IVRMSE_w^{NG} - IVRMSE_w^{HV,\Delta x})/IVRMSE_w^{NG}$. *The average over all 1020 weeks in the option data set is reported under the* Overall *column. The first and third columns report averages when restricting to weeks falling in periods of expansion or recession. The last three columns report the average when restricting to Wednesdays for which the centered quarterly moving average of the ADS Index* $\left(x_t^{(63)} = \frac{1}{63}\sum_{s=t-31}^{t+31} x_s\right)$ *is below 0, -1 or -1.5. All entries are in percentage points, and standard errors are parenthesized below each average. Similarly, Panel B reports the average NGARCH implied volatility of a synthetic 30-DTM, at-the-money call (obtained by setting the call's strike at the index level on each day in our sample), from January 1968 to December 2007 (the time series is reported in the upper-left panel of Figure 4). The difference between the Macro-MIDAS model's implied volatility and that of the NGARCH is then reported. For comparison, the NGARCH average price is reported for the different subsamples (the only entries in dollar terms), along with the Macro-MIDAS – NGARCH price difference in relative terms, i.e. based on* $\left(C_t^{HV,\Delta x} - C_t^{NG}\right)\Big/C_t^{NG}$.