

# Bond Variance Risk Premia\*

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## Abstract

In this paper, we first propose a new fear measure for Treasury markets, similar to the VIX for equities. This implied variance measure relies on the strike of simple variance swaps which are robust to the inclusion of jumps. We then construct a Treasury bond variance risk premium as the difference between this implied variance and expected variance estimate using autoregressive models. Variance risk premia in bond markets behave very differently from equity markets in many respects: Firstly, while equity variance risk premia remain mostly positive and display pronounced spikes, bond market variance risk premia are highly volatility and change sign quite often. Secondly, we show that bond variance risk premia predict (i) bond returns, (ii) stock returns in the cross-section, and (iii) corporate credit spreads and the predictive power is stronger than for the equity variance risk premium. The return predictability is remarkably robust to the inclusion of standard predictors suggested in the literature and after addressing several econometric concerns. Furthermore, we show that uncertainty about macroeconomic variables is an important determinant of the variance risk premia.

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During the recent financial crisis only one sector generated significant profits for the leading investment banks: Volatility arbitrage trading in forex, fixed income, and commodities. According to a BIS survey on foreign exchange and derivatives markets activity, the interest rate derivatives market has grown by 24% over the last three years to reach an average daily turnover of USD 2.1 trillion.<sup>1</sup> As a consequence, both market and academic interest in equity-index volatility measures and their associated risk premia has grown rapidly. For instance, the VIX index—also dubbed the “investors’ fear index”—is believed to be a good proxy of aggregate uncertainty or risk aversion.<sup>2</sup> The VIX is also shown to be a good predictor for the cross-section of stocks (Ang, Hodrick, Xing, and Zhang, 2006), corporate credit spreads (Collin-Dufresne, Martin, and Goldstein, 2001) and bond excess returns (Baele, Bekaert, and Inghelbrecht, 2010). Furthermore, the associated variance risk premium extracted from equity markets also predicts stock market index and bond excess returns (Drechsler, 2010 and Drechsler and Yaron, 2011), as well as corporate credit spreads (Wang, Zhou, and Zhou, 2010). Given this extensive literature for equity markets, it is rather surprising that no effort has been undertaken to measure these risk premia in fixed income markets. Filling this gap is the goal of this paper.

To further motivate our paper, imagine an investor who was long a synthetic variance swap on the 30 year or 10 year Treasury futures over the past 20 years. Figure 1 plots the payoff on this long position in the variance swap from January 1990 to September 2010. As is evident from the figure, the strategy exhibits large variability during times of crisis like the two crisis periods indicated by the gray shaded areas. Interestingly, the strategy yields attractive annualized Sharpe ratios of almost 1.32 and 1.87, respectively. These numbers are twice as large compared to holding Treasury futures themselves and three times as large compared to investing in the equity market index. Overall, the numbers motivate a deeper examination of these risk premia and their impact on asset prices.

**[Insert Figure 1 approximately here.]**

We contribute to the literature in the following way. First, we construct and document variance risk premia for the term structure of Treasury bonds. Second, we calculate an implied

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<sup>1</sup>See BIS (2010).

<sup>2</sup>See, e.g., Bollerslev, Gibson, and Zhou (2007) and Korteweg and Polson (2010), among others.

volatility measure comparable to the VIX for the Treasury market, the TIV measure. Third, we investigate whether the bond variance risk premium helps forecasting bond and stock returns, and corporate credit spreads. Finally, we investigate the economic determinants of bond variance risk premia.

To the best of our knowledge, we are the first who construct a term structure of variance risk premia for the bond market. Even though there is ample evidence of priced variance risk in both the index and single stock equity market, we know surprisingly little about the compensation for variance risk in bond markets. The variance risk premium is defined as the difference between the risk-neutral and physical variance. While most of the literature has mainly focussed on either Black and Scholes (1973) implied volatility or a model-free approach, we follow Martin (2011), who constructs implied variance measures from so called simple variance swaps. The advantage of this approach is that it leads to a genuine measure of implied variance, whereas, the so-called model-free approach contains higher order moments of the risk-neutral distribution once the assumption of the underlying asset being an Itô process is relaxed. This is particularly useful, as we study a period where assets are subject to sudden jumps.

While the risk-neutral expectation may be estimated in a completely model-free fashion using a cross-section of options written on the underlying, the calculation of the objective expectation requires some mild auxiliary modeling assumptions. A priori, it is not clear, what the best proxy for this objective expectation should be. Andersen, Bollerslev, and Diebold (2007) show that simple autoregressive type of models estimated directly for the realized volatility often perform better than parametric modeling approaches designed to forecast the integrated volatility. In calculating our benchmark bond variance risk premium we thus use a version of the HAR-RV model proposed by Corsi (2009) augmented by including lagged implied variance as additional regressors.<sup>3</sup>

The implied variance measures we derive in both equity and bond markets are remarkably similar and the unconditional correlation is around 60% measured over the last 20 years.<sup>4</sup>

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<sup>3</sup>Bollerslev, Sizova, and Tauchen (2010) use the simple HAR-RV model to construct the stock market variance risk premium while Busch, Christensen, and Nielsen (2011) use the same approach as we do to improve forecasts of realized volatility.

<sup>4</sup>The correlation between the implied variance measures for 30 year Treasury futures and equities is as high as 69% whereas the correlation between the implied variance measure for 5 year Treasury futures and equities is around 53%.

The VIX index is often dubbed as a fear of gauge index as it spikes in crisis periods. We find a similar pattern for the bond market using the Martin (2011) approach. Implied volatilities in bond markets spike when uncertainty is high. On the other hand, we find that the variance risk premia in bond and equity markets display very different patterns. While the variance risk premium in the equity market is essentially always positive (i.e. it acts as an insurance premium), the variance risk premium in the Treasury market switches the sign quite often. This suggests that investors do not always perceive volatility in the Treasury market as risk. This is also manifested in the predictive regressions we run. We find a large and statistically significant predictive power of the bond variance risk premium for bond and stock excess returns, and corporate credit spreads. Unlike results we find for the equity market, however, the sign is mostly negative, indicating that a higher bond variance risk premium requires lower expected returns on average. This is in contrast to the usually positive slope coefficient found for the equity variance risk premium (see Zhou, 2010 or Mueller, Vedolin, and Zhou, 2011).

Our findings can be summarized as follows: First, a one standard deviation change in the 30 year Treasury bond variance risk premium induces a 0.4 standard deviation negative change in bond excess returns, increases the stock market excess return by 0.4 to 0.6 standard deviations and induces a negative 0.25 to 0.66 standard deviation change in high grade corporate credit spreads across different maturities. In contrast, the equity variance risk premium on the other only has significant forecasting power for low grade credit spreads.

Since the predictive power of the bond variance risk premia is not driven out by either term structure or macro factors, we ask ourselves what the drivers of these variance risk premia could be. One obvious candidate are proxies for uncertainty. Many papers have documented that the equity variance risk premium is affected by proxies of uncertainty extracted from survey data (see Drechsler and Yaron, 2011 among others). We test this hypothesis using forecasts on a variety of macro and term structure variables from Blue Chip and find high  $R^2$  of around 30% for the equity and up to 60% for the bond variance risk premium. Uncertainty of nominal (inflation) risk is a significant for both bond and equity variance risk premia. In addition, variance risk premia also load on proxies for real (consumption) risk. Bond variance risk premia are also driven by uncertainty about Fed interventions.

We remain agnostic about possible theoretical underpinnings of our findings. While most papers that study the predictability from the variance risk premium onto other factors find a positive slope coefficient, we find negative signs for bond excess returns and credit spread changes. The interpretation of these papers is usually that the variance risk premium is a proxy for economic uncertainty which loads on the stochastic discount factor and therefore has to require a positive premium. Our intuition is that bond and equity variance risk premia give us an indication of so called flight to quality effects. Stock and bond correlation is highly time-varying and while the average unconditional correlation is positive, it can switch sign quite often and has done so increasingly often during the past couple of years. Negative stock and bond correlation is often related to flight to quality (see Baele, Bekaert, and Inghelbrecht, 2010). Intuitively, we would expect flight to quality to occur in periods of high uncertainty. Indeed, running simple rolling regressions from stock and bond correlation onto equity and bond variance risk premia, we often find opposing effects, especially in periods of distress.

We are not the first to study variance risk premia. While a plethora of literature has focussed on the stock market variance risk premium, less attention has been given to the variance risk premia of individual stocks or commodities.<sup>5</sup> Rather surprisingly, to the best of our knowledge, almost no paper has looked at variance risk premia in the fixed income market. In a contemporaneous study, Trolle and Schwartz (2011) empirically study the swaption cube using swap data on 10 year Treasury notes.

The rest of the paper is organized as follows. Section **I.** describes our data set and section **II.** describes the econometric methods used to estimate the TIV measure and the variance risk premia. Section **III.** presents the results of our empirical study and section **IV.** concludes. The Appendix contains a more detailed description of the data and alternative methods to estimate the implied and realized variance measures.

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<sup>5</sup>For literature on the stock market variance risk premium, see, e.g., Driessen, Maenhout, and Vilkov (2009), Bollerslev, Gibson, and Zhou (2009), Carr and Wu (2009), Cremers, Halling, and Weinbaum (2010) and Todorov (2010), among others. Bakshi and Kapadia (2003) and Vedolin (2010) for example study the variance risk premia of individual stocks and Trolle and Schwartz (2009) investigate variance risk premia in commodity markets.

## I. Data

### A. *Treasury Futures and Options Data*

To calculate our implied and realized volatility measures for Treasury bonds, we use futures and options data from the Chicago Mercantile Exchange (CME). Our Treasury futures and options data runs from October 1982, May 1985 and May 1990 to September 2010 for the 30 year, 10 year, and 5 year Treasury bond futures and options, respectively. We use a monthly frequency throughout this paper and thus have 336, 305, and 245 observations available, respectively. We use high-frequency intra-day price data of the 30 year U.S. Treasury bond futures and the 10 year and 5 year U.S. Treasury notes futures and we use end-of-day prices of options written on the underlying futures.

Treasury futures are traded electronically as well as by open outcry. While the quality of electronic trading data is higher, the data only becomes available in August 2000. To maximize our time span, we therefore use data from electronic as well as pit trading sessions. We only consider trades that occur during regular trading hours (07:20–14:00) when the products are traded side-by-side in both markets, as liquidity in the after-hours electronic market is significantly smaller.

The contract months for the Treasury futures are the first three (30 year Treasury bond futures) or five (10 year and 5 year Treasury notes futures) consecutive contracts in the March, June, September, and December quarterly cycle. This means that at any given point in time up to five contracts on the same underlying are traded. To get one time series, we roll the futures on the 28<sup>th</sup> of the month preceding the contract month.

For options, the contract months are the first three consecutive months (two serial expirations and one quarterly expiration) plus the next two (30y futures) or four (10y and 5y futures) months in the March, June, September, and December quarterly cycle. Serials exercise into the first nearby quarterly futures contract, quarterlies exercise into futures contracts of the same delivery period. We roll our options data consistent with the procedure applied to the futures.

## B. Other Data

### Treasury Data:

We use the Fama and Bliss discount bond database from CRSP to compute yields, returns, and forward rates for two to five year bonds. Yields and returns are computed in logs. Yield spreads and excess returns are constructed relative to the one year bond. We denote by  $r_{t+1}^{(\tau)} = p_{t+1}^{(\tau-1)} - p_t^{(\tau)}$ , the return on a  $\tau$  year bond with log price  $p_t^{(\tau)}$ . The excess bond return is defined as:

$$rx_{t+1}^{(\tau)} \equiv r_{t+1}^{(\tau)} - y_t^{(1)},$$

where  $y_t^{(1)}$  is the one year yield. From the same data, we also construct a tent-shaped factor from forward rates, the Cochrane and Piazzesi (2005) factor, CP. Wright and Zhou (2009) document the strong predictive power of the mean jump size for bond risk premia. In line with those authors, we measure the rolling realized jump mean,  $\tilde{J}$ , using five minute frequency data on the 30 year Treasury bond futures. We use a 24-month rolling window due to the assumption that jumps are large and rare. Summary statistics are reported in Table 1, Panel A and Table 2, Panel A.

### Stock Index Futures and Options Data:

To calculate the implied and realized variance measures for the stock market we use futures and options on the S&P 500 index from CME. The sample period is from January 1983 to September 2010. As additional implied variance measure and to check our results that are obtained using futures data we also use the VIX and VXO measures that are calculated using options on the S&P 500 cash index instead of S&P 500 futures.<sup>6</sup> The VIX is available starting in January 1990 and the VXO is available since January 1986.

### Stock Market Data:

To proxy for the market portfolio we use the S&P 500 index. The one year market excess return  $rx_t^{(m)}$  is defined as the annual return on the market portfolio less the one year Treasury yield. We also calculate the excess returns on a portfolio of value and growth stocks ( $rx_t^{(v)}$  and  $rx_t^{(g)}$ ), respectively. These portfolios are constructed using stocks traded on NYSE, AMEX, and NASDAQ from CRSP. Summary statistics are reported in Table 1, Panel B.

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<sup>6</sup>The VIX is the implied volatility calculated using a model free approach whereas the VXO is calculated using the Black and Scholes (1973) implied volatility.

### Credit Spreads:

We construct a term structure of corporate credit spreads for different ratings (AAA, BBB, and B). To calculate credit spreads we use the difference between corporate and Treasury par yields obtained from Bloomberg’s option free fair market curves. Summary statistics for selected credit spreads are reported in Table 1, Panel C.

### Macroeconomic Data:

We compute the eight static macroeconomic factors  $\hat{F}_j, j = 1 \dots, 8$  from Ludvigson and Ng (2009, 2010) for an updated data set.<sup>7</sup> We also estimate volatility proxies for inflation and consumption. We calculate these by estimating a GARCH process for monthly CPI inflation and consumption (non-durables and services). The macroeconomic data is from Global Insight and the Federal Reserve Economic Data (FRED). We present summary statistics in Table 2, Panel B.

### Forecast Data:

We use forecast data from BlueChip Economic Indicators (BCEI) to calculate proxies of uncertainty about macroeconomic variables. BCEI collects monthly forecasts of twelve key financial and macroeconomic indicators from about fifty professional economists in leading financial and economic advisory firms.<sup>8</sup> The forecasts are made for different time horizons. This data exhibits strong seasonality and thus we adjust the series using a 12-period ARIMA filter. We use the cross sectional standard deviation of the filtered panel data within each month as the monthly gauge of uncertainty. We calculate the time series of the cross sectional standard deviation using the forecasts for the current and the subsequent calendar year for each forecast variable  $i$ . Thus, for each variable we have two time series reflecting the uncertainty of the forecaster. Our uncertainty proxy  $\hat{U}^i$  is the first principal component extracted from these two time series.

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<sup>7</sup>The original data set was previously used in Stock and Watson (2002). Some of the macroeconomic variables are no longer available after 2007. Consequently, we use 125 instead of 132 macroeconomic time series. In addition, we exclude all stock market and interest rate time series and work with a set of 104 variables. We also use the full data set with 125 variables and the original factors for shorter sample period ending in 2007 as a robustness check. Our results remain unchanged. A detailed description of the macroeconomic data is provided in Appendix B. and Table 8.

<sup>8</sup>The twelve series are real gross domestic product (RGDP), the GDP chained price index (GDPI), the consumer price index (CPI), industrial production (IP), real disposable personal income (DPI), non-residential investment (NRI), the unemployment rate (UNEM), housing starts (HS), corporate profits (CP), total US auto and truck Sales (AS), the three-month secondary market bank discount basis (SR) and the 10 year constant maturity Treasury yield (LR).



To construct the Fed uncertainty factor we follow Ulrich (2011). Each month, we run a cross-sectional Taylor rule regression using the the predictions of the individual forecasters:

$$r_t^j = c_t + \beta_t^g g_t^j + \beta_t^\pi \pi_t^j + \epsilon_t^j,$$

where  $r_t^j$  is the forecast of the three month discount rate of forecaster  $j$ ,  $g_t^j$  the forecast of real GDP growth, and  $\pi_t^j$  the forecast of inflation (CPI) and  $\epsilon_t^j$  is thus the anticipated Fed intervention.  $c_t$ ,  $\beta_t^g$  and  $\beta_t^\pi$  are assumed to be time-varying to account for the forward-looking bias. The cross-sectional variance of  $\epsilon_t^j$  is then a proxy for so called Fed uncertainty.

The summary statistics of the uncertainty factors for CPI inflation, real disposable income, housing starts and the Fed uncertainty are displayed in Table 2, Panel A.

[Insert Tables 1 and 2 approximately here.]

## II. Estimation of Bond Variance Measures and Variance Risk Premia

In this section we describe the methods used to estimate the expected risk-neutral and objective variance,  $\mathbb{E}_t^{\mathbb{Q}} \left( \int_t^T \sigma_u^2 du \right)$  and  $\mathbb{E}_t^{\mathbb{P}} \left( \int_t^T \sigma_u^2 du \right)$ . Since we do not want to take an a-priori stance on the “best” proxy for the risk-neutral and objective variance, we first use various methods to calculate both the variance under the risk-neutral and the physical probability measure. We then document the differences among the methods and we choose the appropriate measures to calculate the bond variance risk premia.

We essentially use two different methods to approximate  $\mathbb{E}_t^{\mathbb{Q}} \left( \int_t^T \sigma_u^2 du \right)$ , the expected risk-neutral variance:

1. *MIV* denotes the model-free implied variance.
2. *SMIV* denotes the risk-neutral variance of simple returns.

For both methods we consider various interpolation methods to find the expected risk-neutral variance for a one month horizon.

To approximate the expected objective variance,  $\mathbb{E}_t^{\mathbb{P}} \left( \int_t^T \sigma_u^2 du \right)$ , we also use different approaches, two of which are presented in this section:

1.  $RV^{(k \min)}$  denotes the realized variance using data sampled at a  $k$  minute interval. We consider 5, 25 and 60 minute intervals.
2.  $RV^{(IV)}$  denotes the heterogeneous autoregressive realized variance estimator estimator augmented by including lagged implied variance as additional regressors.

Further methods that can be used to approximate the expected objective variance are described in Appendix A..<sup>9</sup>

In the last part of this section we define a *Treasury Implied Volatility* or “TIV” measure in the spirit of the well known VIX index that is calculated by CBOE for the S&P 500 index. Our proposed TIV measure is the 30 year Treasury bond futures implied volatility. The 30 year Treasury futures and options have the longest available history and they are significantly more liquid than the 10 year and 5 year Treasury futures. We propose that this measure be used as the analogue to the VIX for the Treasury market. We calculate a daily TIV measure going back to October 1982. Unconditionally, the TIV measure has a correlation of 50% with the VIX for the common sample period since 1983.<sup>10</sup>

#### A. *Implied Variance*

Following standard practice, we use options to back out a proxy for the expected variance under the risk-neutral measure. The simplest way to calculate the implied variance would be to invert the standard Black (1976) formula. Black’s model is often used to value interest rate options.<sup>11</sup> However, one of the relevant assumptions underlying the model is constant volatility which is inconsistent with the application to forecasting changes in volatility.

Britten-Jones and Neuberger (2000) propose a method that does not require this assumption and thus does not suffer from this inconsistency. Moreover, their approach is completely

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<sup>9</sup>Additional measures include  $RV^{(HAR)}$ , the standard HAR-RV estimator,  $BV^{(k \min)}$ , the realized bi-power variation using data sampled at a  $k$  minute interval,  $RV^{(AC1)}$ , the first-order autocorrelation-adjusted realized variance estimator,  $RV^{(TS)}$ , the two scale realized variance estimator and  $RV^{(TSadj)}$ , the two scale realized variance estimator that adjusts for bias introduced by microstructure noise.

<sup>10</sup>The time series for the TIV measure is available upon request from the authors. The data will also be made available on the authors’ website. Note that we construct our own VIX measure which is based on options on S&P500 future options rather than on the underlying cash index. The reason being that futures options data on the S&P500 date back to the 1980s with high trading volumes. For the common sample period since 1990, our VIX measure has a correlation with the published VIX of 99.4% and the root mean squared error is below 1%.

<sup>11</sup>E.g., Busch, Christensen, and Nielsen (2011) use this measure to study the forecasting power of implied volatility for realized volatility of Treasury bond futures.

model free and only requires current option prices. However, interest rates are assumed to be non-stochastic, which again is inconsistent as they are assumed to be stochastic when it comes to calculating the payoff of the option (which is written on a futures that is dependent on an underlying interest rate process). Moreover, both approaches assume European instead of American options and forward instead of futures contracts as the underlying asset.

One application of the model-free implied variance is the VIX which is the implied volatility index of S&P500 options. Neuberger (1994) shows that the VIX corresponds to the quadratic variation of the forward price of the S&P500 index under the risk-neutral measure. One issue with the model-free implied variance is that it heavily relies on the Itô assumption for the underlying process. In the presence of skewness, Carr and Lee (2009) show that the VIX will be upward biased compared to the true risk-neutral quadratic variation. Recently, Martin (2011) introduced the simple variance swap for which the realized leg can be computed from simple returns of the underlying index and the index forward. The SVIX, as the VIX, can be approximated as a portfolio of out-of-the-money options. As he shows, the SVIX can be constructed under weaker assumptions and in particular, even in the presence of jumps.

To implement the two methods for calculating the implied variance, we treat the American options as European.<sup>12</sup> Furthermore, we assume that the short risk-free rate is non-stochastic (or at least not too volatile) such that the forward and futures prices coincide.

To obtain the model-free implied variance  $MIV$ , we follow Demeterfi, Derman, Kamal, and Zhou (1999) and Britten-Jones and Neuberger (2000). They show that if the underlying asset price is continuous, the risk-neutral expectation of total return variance is defined as an integral of option prices over an infinite range of strike prices. Since in practice, the number of traded options for any underlying asset is finite, the available strike price series is a finite sequence. Suppose the available strike prices of the call options belong to  $[\underline{K}^{call}, \overline{K}^{call}]$ , where  $\overline{K}^{call} \geq \underline{K}^{call} \geq 0$ . As shown in Jiang and Tian (2005), a truncated version of the integral over the infinite range of strike prices can be used to evaluate the model-free implied

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<sup>12</sup>Jorion (1995) shows that early exercise premia are small for short maturity at-the-money options on futures, while Overdahl (1988) demonstrates that early exercise of options on Treasury futures happens about 0.1% of the time and happens both with calls and puts but only with options that are significantly in the money. In the empirical implementation we use only out-of-the money options and thus assume that the early exercise option will not distort the option price.

volatility. Denote  $C(T, K)$  the spot call price with strike price  $K$  expiring at time  $T$  and  $F_t$  the forward price. We use the trapezoidal rule to numerically calculate the integral:

$$2 \int_{\underline{K}^{call}}^{\overline{K}^{call}} \frac{C(T, K) - \max(0, F_t - K)}{K^2} dK \approx \frac{\overline{K}^{call} - \underline{K}^{call}}{m} \sum_{i=1}^m [g_{t,T}(K_i^{call}) + g_{t,T}(K_{i-1}^{call})].$$

where

$$g_{t,T}(K_i^{call}) = \frac{C(T, K_i^{call}) - \max(0, F_t - K_i^{call})}{(K_i^{call})^2}, \quad (1)$$

and  $K_i^{call}$  is the  $i^{th}$  largest strike price for the call option. To implement the trapezoidal rule, we now need the option prices  $C(T, K_i^{call})$ , for  $i = 1, \dots, m$ . Since some of these prices are not available, we apply a cubic spline interpolation method as proposed in Forsythe, Malcolm, and Moler (1977) to obtain the missing values.<sup>13</sup>

Let

$$MIV_t^{(\tau)} = \frac{\overline{K}^{call} - \underline{K}^{call}}{m} \sum_{i=1}^m [g_{t,T}(K_i^{call}) + g_{t,T}(K_{i-1}^{call})], \quad (2)$$

where  $\tau = T - t$  denotes the time horizon or time to maturity. As mentioned above, we replace  $F_t$  in equation (1) by the futures price. Since in-the-money and at-the-money options are less liquid, (2) is evaluated for out-of-the money call options whose strike prices are no less than  $0.94 \times F_t$ . Finally, We set  $m = 100$  and restrict  $MIV_t^{(\tau)} = 0$  when  $t = T$ .

In addition to the model free implied variance,  $MIV_t^{(\tau)}$ , we then calculate the implied variance from simple returns,  $SMIV_t^{(\tau)}$ . The calculations follow the same steps as the calculations for the model-free implied variance. The  $SMIV_t^{(\tau)}$  is defined as:

$$SMIV^{(\tau)} = \frac{2e^{rT}}{T} \left( \int_0^{F_t} \frac{1}{S_0^2} P(T, K_i^{put}) dK_i^{put} + \int_{F_t}^{\infty} \frac{1}{S_0^2} C(T, K_i^{call}) dK_i^{call} \right). \quad (3)$$

We estimate the model-free and the simple return implied variances at the end of each month for a 30 day horizon to get our monthly time series, denoted  $MIV_t^{(i)}$  and  $SMIV_t^{(i)}$ , where

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<sup>13</sup>Jiang and Tian (2005) take a different approach: They first calculate the implied volatilities of available options with the Black and Scholes formula, and then use the interpolation method to obtain the Black and Scholes implied volatilities of the unavailable options. Using these implied volatilities, they use the Black and Scholes formula again to obtain the continuum of option prices. They claim that their method can avoid the nonlinearity problem in the option prices. However, we find a direct use of the interpolation method on the option prices to be more robust.

$i = \{30y, 10y, 5y, E\}$  stands either for the 30 year, 10 year or 5 year Treasuries or the equity index.

### B. Realized Variance

To estimate  $\mathbb{E}_t^{\mathbb{P}} \left( \int_t^{t+1} \sigma_u^2 du \right)$ , the daily expected variance we first consider realized variance  $RV_t$ , which is defined as

$$RV_t = \sum_{i=1}^M r_{t,i}^2, \quad (4)$$

where

$$r_{t,i} = \log P \left( t - 1 + \frac{i}{M} \right) - \log P \left( t - 1 + \frac{i-1}{M} \right)$$

is the intra-daily log return in the  $i^{\text{th}}$  sub-interval of day  $t$  and  $P(t - 1 + i/M)$  is the asset price at time  $t - 1 + i/M$ . The estimator is consistent for  $\int_{t-1}^t \sigma_u^2 du$  if the log price process does not have jump components and under some mild regularity conditions. For each day, we take  $r_{t,i}$  between 7:25 and 14:00. We use three different sampling frequencies for  $r_{t,i}$ , namely, we use  $k = \{5, 25, 60\}$  minute intervals to calculate  $RV_t^{(k \text{ min})}$ .

The normalized monthly realized variation  $RV_{t,1m}$  is defined by the average of the 22 daily measures. The normalized weekly realized variation  $RV_{t,1w}$  is correspondingly defined by the average of the 5 daily measures:

$$RV_{t,1w} = \frac{1}{5} \sum_{j=0}^4 RV_{t-j}, \quad \text{and} \quad RV_{t,1m} = \frac{1}{22} \sum_{j=0}^{21} RV_{t-j}.$$

To better capture the long memory behavior of volatility, we use the daily, weekly and monthly realized variance estimates to estimate the heterogenous autoregressive model of realized volatility (HAR-RV) proposed by Corsi (2009). The daily HAR-RV model is expressed as

$$RV_{t+1} = \alpha + \beta_D RV_t + \beta_W RV_{t,1w} + \beta_M RV_{t,1m} + \varepsilon_{t+1}.$$

This simple method avoids some difficulties in long memory time series modeling and the parameters can be consistently estimated by OLS. However, a Newey-West correction is needed to make appropriate statistical inference. Moreover, such a HAR type model can

be easily modified, for example, by adding extra covariates that contain predictive power. It is also possible to apply the model to integrated squared variance measures other than the simple realized variance. Andersen, Bollerslev, and Diebold (2007) extend the model to show that the predictability for  $RV_{t+h}$  over different time intervals almost always comes from the continuous component of the total price variation, rather than the discontinuous jump component.

We aim to obtain the monthly estimates directly, so we replace the daily realized variance  $RV_{t+1}$  by the monthly measure  $RV_{t+22,1m}$ . Moreover, we include lagged estimates of implied volatility to improve the realized volatility forecasts. Hence, we then run the following OLS regression for the projection:

$$RV_{t+22,1m} = \alpha + \beta_D RV_t + \beta_W RV_{t,1w} + \beta_M RV_{t,1m} + \beta^{(i)}(h) \mathbf{IV}(\mathbf{L})_t + \varepsilon_{t+22,1m}, \quad (5)$$

where  $\mathbf{IV}(\mathbf{L})_t$  contains lagged implied variance up to lag  $L$ .<sup>14</sup> We use intra-day data sampled at the 5-minute interval to calculate the daily realized variance, i.e.  $RV_t = RV_t^{(5 \text{ min})}$ . We implement the regression using a rolling window of 180 trading days. This allows us to obtain real-time forecasts  $\widehat{RV}_{t+22,1m}$  for  $RV_{t+22,1m}$  using the rolling parameter estimates.<sup>15</sup>

As the HAR-RV predictor for  $\int_t^{t+\tau'} \sigma_u^2 du$  we use

$$RV_t^{(\text{HAR})} = 22 \times \widehat{RV}_{t+22,1m}. \quad (6)$$

We denote the augmented version of our monthly HAR-RV predictor  $RV^{(\text{IV})}$ . To obtain the monthly estimates for the realized variance, we sum the daily estimates over the last month. Subsequently, the realized variance ( $RV^{(k \text{ min})}$ ) estimator always refers to the aggregated monthly measure.

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<sup>14</sup>We choose the lag length to be four using the Akaike and Bayesian information criteria.

<sup>15</sup>We use daily realized variance estimates from the first 223 trading days as the input for initial estimation: Daily realized variances from day 1 to day 201 are used to construct  $RV_t$ ,  $RV_{t,1w}$ , and  $RV_{t,1m}$ . Daily realized variances from day 23 to day 223 are used to construct  $RV_{t+22,1m}$ . On day 223, the first out of sample forecast  $\widehat{RV}_{t+22,1m}$  from the fitted model is constructed by using  $RV_{223}$ ,  $RV_{223,1w}$ , and  $RV_{223,1m}$  as the input data to the initial fitted model. The same method is applied for day 224, 225, ... with the corresponding window parameters. We estimate the expected realized variance using an expanding data set to avoid a look ahead bias.

Figure 2 shows time series plots for the monthly variance measures for the 30 year, 10 year, and 5 year Treasury bond futures, respectively as well as the S&P 500. For the S&P 500, we use options on S&P 500 futures to be consistent with our calculations of the Treasury implied variances. Note, however, that the calculation of the VIX for example requires options on the cash index itself. Table 3, Panels A and B present summary statistics of implied and realized volatility measures (which are simply obtained by taking the square root of the corresponding variance measures). The numbers shown are annualized and expressed in percent. The average size of the variance measures for the long-term futures is consistently larger than for the short-term futures. This is intuitive, as agents who invest in longer term bonds want to be compensated for holding these bonds. The equity implied volatilities are almost twice as large as the implied volatilities for 30 year futures and almost three and more than four times as large as the implied volatilities for 10 year and 5 year futures, respectively. Realized volatilities are on average lower than the corresponding implied volatilities implying a variance risk premium that is positive on average for bonds and the equity index. The summary statistics for the VIX index are included in Panel A for comparison purposes. As noted earlier, the equity index model-free implied variance (or volatility in this particular case) backed out from futures options is very highly correlated with the VIX and consequently, the summary stats are very similar.

**[Insert Figure 2 approximately here.]**

With regards to the statistical properties of different variance measures, the realized variance  $RV$  is on average larger than the bipower variation  $BV$ , since the former includes both continuous and discontinuous parts of the integrated variance, while the latter is a robust estimation for the continuous part only. Quantities of the realized variance estimates with lower frequency data (25 and 60 minutes) are smaller than those obtained with higher frequency data (5 minutes), but they are more stable (i.e. they have lower sample standard deviations). This may suggest that the lower frequency versions underestimate the true integrated variance. For the 30 year and 10 year bond futures, skewness and kurtosis for robust realized variance estimators when microstructure noise is present are much higher than those for equally spaced-time estimators. But for the 5 year bond futures, empirical distributions of the robust realized variance estimators do not show too much skewness and

fat-tail behavior.<sup>16</sup> Overall, we find the realized variance estimates from the HAR-RV and augmented HAR-RV model’s projections to have relatively low skewness and kurtosis. However, the method to obtain  $RV^{(IV)}$  does not guarantee its positiveness. As for the implied variance measures, the implied volatility measures using the model-free approach are higher than using the simple returns approach. The intuition is the same as in Martin (2011). If the return distribution is skewed, simple return implied variance tends to be lower than for standard variance swaps. The reason being that the implied variance from the latter contain all the higher order moments of the return distribution. Overall, the implied variance measures have lower skewness than their realized variance counterparts.

**[Insert Table 3 approximately here.]**

### C. Variance Risk Premia

We define the variance risk premium from day  $t$  to  $T$  as follows:

$$VRP_{t,\tau} \equiv \mathbb{E}_t^{\mathbb{Q}} \left( \int_t^T \sigma_u^2 du \right) - \mathbb{E}_t^{\mathbb{P}} \left( \int_t^T \sigma_u^2 du \right),$$

where  $\tau = T - t$  denotes the time horizon.<sup>17</sup> Economic theory suggests that the variance risk premium should be positive in order to compensate investors who bear risks from expected price fluctuations. The general positiveness of the variance risk premia can also be confirmed empirically from comparing the means of the different variance (or volatility) measures.

From an econometrician’s point of view however, it is not a priori clear what variance measures should be used. While the expected variance under the risk-neutral measure can be estimated in a completely model-free fashion, the calculation of the objective expectation, requires some mild auxiliary modeling assumptions. Andersen, Bollerslev, and Diebold (2007) argue that simple autoregressive type models estimated directly for the realized variance typically perform equally well as, and often better, than parametric modeling approaches designed to forecast the integrated variance. Therefore, we use the  $RV^{(IV)}$  measure as our benchmark. This proxy of the variance risk premium has recently been used in Bollerslev,

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<sup>16</sup>These results are not reported.

<sup>17</sup>For notational simplicity, we subsequently drop the subscript  $\tau$  as we always consider the one month horizon.



Sizova, and Tauchen (2010). In Figure 3 we plot the annualized variance risk premia (expressed in squared percent) defined as the difference between the model-free implied variance ( $MIV^{(i)}$ ) and the  $RV^{(IV,i)}$  for the 30 year Treasury bond ( $VRP^{(30y)}$ ) as well as the 10 year and 5 year Treasury notes ( $VRP^{(10y)}$  and  $VRP^{(5y)}$ ). As we can see, the three time-series share a lot of co-movement: The unconditional correlations between the 5 year, 10 year and 30 year bond variance risk premia ranges between 55% and 82%. We also note that  $VRP^{(30y)}$  displays the largest volatility, especially during crisis periods indicated by the shaded areas. The bond variance risk premia are positive on average but they change sign quite often. In contrast, the equity variance risk premium  $VRP^{(E)}$  (also plotted in Figure 3) is always positive and on average significantly higher in magnitude. The correlation between the bond and the equity variance risk premia is about 45% to 50%. The summary statistics of the annualized variance risk premia expressed in squared percent are reported in Table 3, Panel C.

[Insert Figure 3 approximately here.]

#### D. Treasury Implied Volatility (TIV)

In this section we introduce a measure for *Treasury Implied Volatility* in the spirit of the VIX. To construct the TIV measure we use the model free implied variance obtained by a linear interpolation using the nearest two expiration dates,  $MIV^{(i)}$ . The TIV measure is the square root of the  $MIV_t^{(30y)}$  for the futures on 30 year Treasuries. Figure 4 (lower panel) plots the annualized TIV measure and our VIX measure (backed out from options on futures) for the common sample period 1983 to 2010.<sup>18</sup> The unconditional correlation of the two monthly time series is 46%. The unconditional correlation between the TIV measure and the original VIX for the period 1990 to 2010 is 62%.<sup>19</sup> In the upper panel, we also depict the STIV and SVIX calculated using the simple variance swap approach. Again, the two series almost move in lock-step, the unconditional correlation being 50%.

We also calculate the implied volatility using simple returns, STIV and SVIX, which are robust to jumps. The correlation for the full sample period is 49% and the correlation since

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<sup>18</sup>However, we calculate the TIV measure going back to October 1982, the start date of our data.

<sup>19</sup>The correlation between the TIV and our VIX measure for the same time period is exactly the same, which is not surprising given the near perfect correlation between the original VIX calculated using options on the cash index and our measure calculated using options on futures.

1990 is 64%. As before, we can also compare the SVIX calculated using options on the cash index and the futures. As in Martin (2011) we use data from Optionmetrics to calculate the SVIX backed out from options on the cash index. This data is available starting in January 1996. The correlation between the two SVIX measures is again nearly perfect and reaches 99.7%. The correlation between the STIV and the cash SVIX (as well as the futures SVIX) for the period from 1996 to 2010 is 68%.

In the next section we document the strong predictive power of the bond variance risk premia for excess bond and stock returns, and corporate credit spreads. We use the implied variance measures based on simple returns (STIV and SVIX) because they are robust to jumps. However, our results are not sensitive to the choice of implied variance measure.

[Insert Figure 4 approximately here.]

### III. Empirical Evidence

In this section, we study the predictive power of the Treasury bond variance risk premia for annual Treasury bond excess returns, stock excess returns and corporate credit spread changes. We do this univariate and multivariate, i.e. we run regressions using only various variance risk premia measures as regressors before including additional explanatory variables. We find that estimated coefficients of bond variance risk premia are both economically and statistically significant even if we include standard predictors suggested in the literature. We then also investigate the economic determinants of bond variance risk premia. We calculate the variance risk premia using the methods described in the previous section. Namely, the variance risk premium is the difference between the model-free implied variance and the augmented HAR-RV projection. We use the implied variance calculated from simple returns

as it has slightly more desirable properties and is robust to the presence of jumps. However, none of the results in this section are sensitive to the choice of implied variance measure.<sup>20</sup>

### A. Predictability

We start by assessing the in-sample predictive ability of bond variance risk premia for asset returns. To do this, we run the following type of regression:

$$rx_{t+h}^{(i,\tau)} = \beta^{(i,\tau)}(h)\mathbf{VRP}_t + \gamma^{(i,\tau)}\mathbf{M}_t + \epsilon_{t+h}^{(i,\tau)} \quad (7)$$

where  $rx_{t+h}^{(i,\tau)}$  denotes excess returns on two to five year nominal Treasury bonds (with a  $h = 1$  year holding period), the market (S&P 500 index), value and growth stock excess returns (six months and one year holding period), or corporate credit spread changes for spreads in rating category  $i = \{AAA, BBB, B\}$ , respectively.  $\mathbf{VRP}_t$  is a vector containing the equity market variance risk premium  $\text{VRP}_t^{(E)}$ , and the Treasury bond variance risk premia for 5, 10 and 30 year maturities ( $\text{VRP}_t^{(30y)}$ ,  $\text{VRP}_t^{(10y)}$  and  $\text{VRP}_t^{(5y)}$ ).  $\mathbf{M}$  denotes a vector of additional predictor variables and  $\epsilon^{(i,\tau)}$  is the error term. Note that we run standardized regressions, meaning, for all regressors and regressands, we de-mean and divide by the standard deviation. This makes coefficients easier comparable across different predictors.

For the Treasury bond excess return regressions,  $\mathbf{M}$  includes the rolling mean jump size  $\tilde{J}$  constructed from high frequency data on 30 year Treasury futures (see Wright and Zhou, 2009), the Cochrane and Piazzesi (2005) factor, CP, and the eight macro factors from Ludvigson and Ng (2009, 2010),  $\hat{F}_j, j = 1 \dots, 8$ . For the stock market excess return regressions we include the dividend yield  $DY$ , the earnings/price ratio  $E/P$ , the book-to-market ratio  $B/M$  and  $NTIS$ , the net equity expansion as additional regressors as in Goyal and Welch (2008). The regression results are presented in Tables 4 to 6. Coefficients are estimated with ordinary-least squares and standardized and  $t$ -statistics are calculated using Newey and West (1987) standard errors. The sample period is from July 1992 to December 2010.

[Insert Table 4 approximately here.]

In Table 4 we report the regression results excluding the additional control variables. Panel A contains the results for Treasury bond excess returns. The coefficient for the 30 year

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<sup>20</sup>In fact, the results even hold if the implied variance is calculated by inverting the Black (1976) formula.

variance risk premium is significant and negative for all different maturities, implying that higher bond variance risk premium leads to lower excess returns. The coefficient for the 10 year bond variance risk premium on the other hand is significantly positive for all bond maturities. This result is robust to using a residual from a regression of the 10 year bond variance risk premium on the 30 year bond variance risk premium instead, implying that not all bond variance risk premia contain the same information that is relevant for forecasting. The average adjusted  $R^2$  is around 6%. Unlike the bond variance risk premia, the equity variance risk premium does not seem to contain any relevant information for forecasting bond excess returns at an annual horizon.<sup>21</sup>

Univariate regression results for the stocks and credit spreads are reported in Panels B and C of Table 4, respectively. Overall, the dominating factor for the predictability is again the 30 year bond variance risk premium: Estimated coefficients are positive and statistically significant for the market, growth, and value excess returns for the six and twelve month horizons with adjusted  $R^2$  ranging from 9% to 13%. Again, the equity variance risk premium is not statistically significant for any stock portfolio. For monthly credit spread changes, the estimated coefficients for the 30 year bond variance risk premium are negative and highly statistically significant at any maturity for investment grade bonds (AAA and BBB) with  $t$ -statistics between 2.33 and 4.53. For B rated bonds however, Treasury bond variance risk premia are no longer significant. For the equity variance risk premium the pattern seems reversed. The significance improves as the credit quality deteriorates and the equity variance risk premium is a highly significant predictor of B credit spread changes. Overall, the adjusted  $R^2$  range between 8% for short maturity AAA bonds and 31% for long maturity BBB bonds.

To check the robustness of our univariate results, we add different established predictors of bond and equity risk premia. The results are reported in Tables 5 and 6.

**[Insert Tables 5 and 6 approximately here.]**

To summarize, the results from the univariate regressions with respect to the 30 year bond variance risk premia are remarkably robust to the inclusion of a host of control variables.

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<sup>21</sup>These findings echo the results in Mueller, Vedolin, and Zhou (2011) who find that the equity variance risk premium heavily loads on short-term bond risk premia but does not predict excess returns at the annual horizon.

The coefficients for the Treasury excess returns are still significantly negative, while the coefficients for the bond excess returns remain significant and positive. Including the CP factor, the jump measure and the macro factors increases the adjusted  $R^2$  to roughly 45% across all maturities for the Treasury bond regressions. The 10 year bond variance risk premium is driven out and the equity variance remains insignificant. As in Ludvigson and Ng (2009), the macro factors explain a significant fraction of the variation in bond excess returns over the sample period, while the CP factor is not significant. The jump measure is the most significant predictor with  $t$ -statistics ranging between 6.9 and 7.2. Wright and Zhou (2009) report that the implied variance extracted from equity options loses its predictive power when the regression is augmented by the jump measure. While results are different from theirs in many respects, it is important to note that our bond variance risk measures are significant even when adding the mean jump size.

We report results from regressing stock excess returns for a six and twelve month holding period on variance risk premia and a series of predictor variables in Table 6. We augment the univariate regressions with the dividend yield, earnings to price ratio, book to market ratio (all for the S&P 500) and net equity expansion as in Goyal and Welch (2008). Except for NTIS, none of the additional factors is statistically significant in the regressions. As is the case with the bond regressions, the 30 year bond variance risk premium remains significant for all horizons and portfolios. In addition, the equity market variance risk premium has predictive power for the market and the growth portfolio but not the value portfolio. Overall, the adjusted  $R^2$  raise to roughly 30% for the six month horizon and up to 40% for the annual excess returns.

We currently do not report extended regressions for the corporate credit spreads as none of the other suggested factors have been significant. The results are available upon request.

In summary, we find that excess returns on bonds, the stock market and corporate credit spreads are predictable using bond variance risk premia. The reported in-sample predictability is strong both statistically and economically.

## B. What Drives Bond Variance Risk Premia?

In this section, we explore the economic drivers of the bond variance risk premia in more detail. Equilibrium models that study variance risk premia focus on the equity market only. Drechsler and Yaron (2011) link the variance risk premium of the market index to uncertainty about fundamentals. In particular, time variation in economic uncertainty and a preference for early resolution of uncertainty are required to generate a positive variance premium that is time varying and predicts excess stock market returns. Drechsler (2010) reports a high correlation between the variance risk premium and the dispersion in the forecasts of next quarter's real GDP growth from the Survey of Professional Forecasters. It is natural to assume that variance risk premia are associated with higher uncertainty. Options provide investors with a hedge against high variance in the underlying returns and high variance usually occurs when unexpected shocks affect macroeconomic variables. The premium that investors are willing to pay or receive against such events is related to their uncertainty.

To test the hypothesis that uncertainty affects variance risk premia, we run regressions from the monthly variance risk premia on uncertainty factors about the real and the nominal side of the economy as well as about monetary policy actions. The uncertainty factors are constructed from BCEI forecast data. We proxy for uncertainty about the real and nominal side of the economy by the cross sectional standard deviation of the forecasts of CPI ( $\widehat{U}^{CPI}$ ), real disposable income ( $\widehat{U}^{RDPI}$ ), and housing starts ( $\widehat{U}^{HS}$ ) for the current and the next calendar year, respectively. The Fed uncertainty factor  $\widehat{U}^{FED}$  is the cross sectional variance of the residual from a Taylor rule regression of the short rate forecast on real GDP growth and inflation (CPI) forecasts.<sup>22</sup> In addition to the uncertainty measures, we also include two variables that measure the time-varying volatility of inflation and consumption ( $\sigma_\pi$  and  $\sigma_g$ , respectively) and two macro factors that can be interpreted as a real ( $\widehat{F}_1$ ) and a nominal (price) factor  $\widehat{F}_2$ . The macro volatilities are calculated by estimating a GARCH process using monthly CPI and per capita consumption (non-durables and services). The macro factors are constructed using the first two principal components of a large set of macro variables as in Ludvigson and Ng (2009, 2010).<sup>23</sup>

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<sup>22</sup>See section I. for details.

<sup>23</sup>Given that the factors are principal components, the economic interpretation is not straightforward. We calculate the marginal correlations (instead of marginal  $R^2$  as in Ludvigson and Ng, 2009) of the individual time series with the respective factors for our data set. As in Ludvigson and Ng (2009), it is reasonable to

Hence, we run the following regression:

$$VRP_t^{(i)} = \beta^U \widehat{\mathbf{U}}_t + \beta^F \widehat{\mathbf{F}}_t + \gamma^S \widehat{\mathbf{S}}_t + \epsilon_t^{(i)}, \quad (8)$$

where  $VRP_t^{(i)}$  is the equity or bond variance risk premium for a particular maturity ( $T = 30, 10$  and  $5$  years) at time  $t$ ,  $\widehat{\mathbf{U}}_t$  is a vector of the uncertainty measures at time  $t$ ,  $\widehat{\mathbf{F}}_t$  contains the real and nominal macro factors, and  $\widehat{\mathbf{S}}_t$  contains the macro volatilities  $\sigma_\pi$  and  $\sigma_g$ .  $\epsilon_t^{(i)}$  is the error term. Again, all coefficients are estimated with ordinary-least squares and standardized, and  $t$ -statistics are in brackets and are calculated using Newey and West (1987) standard errors. The sample spans the period from July 1992 to December 2009.

[Insert Table 7 approximately here.]

The results are presented in Table 7. In summary, the results confirm that uncertainty variables have relevant explanatory power for variance risk premia but there are cross sectional differences. First, we run regressions using only the uncertainty proxies as explanatory variables and then we include both the levels and volatilities of measures for the real and nominal side of the economy.

The four uncertainty factors alone explain almost 60% and 50% of the variation in 30 year and 10 year bond variance risk premia, respectively but only about 30% of equity variance risk premia and roughly 22% of 5y bond variance risk premia.<sup>24</sup>

Uncertainty about monetary policy actions is highly significant for 30 year bond variance risk premia but at most marginally for some of the other measures. The most important and robust uncertainty measure is uncertainty about inflation (CPI) that is significant for bond and variance risk premia and robust to including additional regressors. Uncertainty about the real side of the economy proxied by uncertainty about real disposable income is significant for all variance risk premia measures as long as no other controls are included.<sup>25</sup>

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interpret the first factor as a real factor. The second factor can be interpreted as an inflation factor. See Appendix B. for additional information.

<sup>24</sup>There is possibly too much noise in the time series of the 5y bond variance risk premia as the 5 year futures and options are much less liquid compared to the instruments for 30 year and 10 year bonds and notes. This interpretation is also supported by the less than stellar predictability of the 5 year bond variance risk premium as shown in section A..

<sup>25</sup>We also have a measure of uncertainty about real GDP growth. This measure is seems less powerful than uncertainty about real disposable income and is less robust. However, the results are robust to including uncertainty about real GDP growth in the regressions.

Uncertainty about the housing sector seems relevant for the equity variance risk premium but not for bond variance risk premia in general.

Adding levels and volatilities of macro variables to the regression only marginally improves the adjusted  $R^2$  for bond variance risk premia. Macro volatilities are not significant for any of the bond variance risk premia while the real factor  $\widehat{F}_1$  enters with a positive sign and is significant. For equity variance risk premia, the additional four variables help raise the adjusted  $R^2$  from 31% to 38%, the additional explanatory power mainly driven by the real factor  $\widehat{F}_1$ .

Note that in the current specification of regression (8) we include a limited set of uncertainty factors as we try to parsimoniously capture measures of the real and nominal side of the economy and uncertainty thereabout. In a kitchen sink regression of additional uncertainty measures (results currently not reported) we find that several of them are significant. On the other hand, only the real factor turns out to be relevant for explaining the variation in variance risk premia whereas all other macro factors are not. This further supports the notion that the variance risk premia are driven by uncertainty and not by actual macro fundamentals.

It is also noteworthy that some loadings on uncertainty factors are estimated to be negative. A priori it might seem odd that higher uncertainty implies a *lower* risk premium. However, one thing to keep in mind is that the variance risk premium can change its sign. This happens more often for Treasury variance risk premia than for the equity variance risk premium. It is conceivable that investors' preferences with regards to risky assets are time varying. The uncertainty measurements are all derived from forecasts of macroeconomic variables. Some of these variables are pro-cyclical, such as real GDP and real disposable income. A divergence of predictions about pro-cyclical indicators (or increasing uncertainty about these variables) is often thought to reflect a future downturn in the economy. A negative economic shock will drive investors to seek safe investments in a flight to quality and into Treasuries. As a result, prices of Treasuries will go up and yields will fall. Changing perceptions of the risks associated with Treasury securities will most likely also have a negative impact on required risk premia.

Obviously, forecast dispersions about macroeconomic variables may not always reflect a view that the overall economic situation is worsening. Patton and Timmermann (2010) show that



the time-series correlation between the consensus forecast and the dispersion in forecasts is strongly negative for GDP growth while it is positive for inflation. The dispersion in beliefs about the GDP growth rate seems to be strongly counter-cyclical whereas the dispersion of beliefs about inflation does not exhibit an equally clear pattern. These empirical results may be used to reconcile the existence of both negative and positive coefficients for different uncertainty variables in regression (8).

#### IV. Conclusion

We construct variance risk premia for U.S. Treasury notes and bonds with 5 year, 10 year and 30 year maturities, respectively. While the existing literature has studied the equity market variance risk premium, variance risk premia of individual stocks or even commodities, this is the first paper to examine these premia in the fixed income markets. The Treasury variance risk premia are correlated with each other but the correlation is far from perfect, suggesting that they may contain different information depending on the maturity.

We find that the bond variance risk premia are positive on average and display a large, often counter-cyclical variability. We construct a new measure of Treasury implied volatility measure similar to the well established VIX index for the index equity market.

We show that the Treasury bond variance risk premia possess significant predictive power for both bond and stock excess returns as well as credit spread changes for different credit qualities. This predictability is remarkably robust to including various other predictors such as Treasury jump measures, the Cochrane and Piazzesi (2005) factor or the Ludvigson and Ng (2009, 2010) macro factors.

Finally, we investigate the economic drivers of the bond variance risk premium. Recent literature has suggested that economic uncertainty is a potential determinant of variance risk premia and we show corroborating evidence that uncertainty about the real and the nominal side of the economy is important for explaining the bond variance risk premia. At the same time, macroeconomic variables themselves have little effect.

This paper is a first attempt at measuring and quantifying variance risk premia in fixed income markets, exploring their determinants and assessing their importance for predicting

asset prices and risk premia. However, we have remained agnostic about the structural drivers of these risk premia. Prima facie, it is not clear why risk premia from fixed income markets should have such a strong predictive power for corporate bonds or stocks. Chen, Joslin, and Tran (2010) provide a theoretical foundation that the jump risk premium in the market equity risk premium is mainly driven by agents' disagreement about either the intensity of disasters or their impact. Starting from this foundation, one could extend their model for learning and study the implications of both learning and heterogeneous beliefs on the term structure of interest rates and the corresponding risk premia for both stochastic volatility and jumps. In such a setting, these risk premia emerge quite naturally as they compensate agents for holding assets that are subject to both volatility and jump risk.

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# Appendix

## A. Bond Variance and Jump Measures

In this section we describe additional methods used to estimate the expected risk-neutral and objective variance,  $\mathbb{E}_t^{\mathbb{Q}}\left(\int_t^T \sigma_u^2 du\right)$  and  $\mathbb{E}_t^{\mathbb{P}}\left(\int_t^T \sigma_u^2 du\right)$ . We use various methods to calculate both the variance under the risk-neutral and the physical probability measure. Furthermore, we calculate and analyze measures of jump risk for the Treasury bond futures.

As discussed in Section II., we essentially use two different methods to approximate  $\mathbb{E}_t^{\mathbb{Q}}\left(\int_t^T \sigma_u^2 du\right)$ , the expected risk-neutral variance:

1. *MIV* denotes the model-free implied variance.
2. *SMIV* denotes the risk-neutral variance of simple returns.

In addition, we also calculate the Black (1976) implied variance. Further below, we describe the additional interpolation methods to find the expected risk-neutral variance for a one month horizon.

To approximate the expected objective variance,  $\mathbb{E}_t^{\mathbb{P}}\left(\int_t^T \sigma_u^2 du\right)$ , we use seven different approaches in total:

1.  $RV^{(k \min)}$  denotes the realized variance using data sampled at a  $k$  minute interval. We consider 5, 25 and 60 minute intervals.
2.  $BV^{(k \min)}$  denotes the realized bi-power variation using data sampled at a  $k$  minute interval. Again, we consider 5, 25 and 60 minute intervals.
3.  $RV^{(AC1)}$  is the first-order autocorrelation-adjusted realized variance estimator.
4.  $RV^{(TS)}$  is the two scale realized variance estimator.
5.  $RV^{(TSadj)}$  denotes the  $RV^{(TS)}$  estimator that adjusts for bias introduced by microstructure noise.
6.  $RV^{(HAR)}$  denotes the heterogeneous autoregressive realized variance estimator estimator.
7.  $RV^{(IV)}$  denotes the heterogeneous autoregressive realized variance estimator estimator augmented with lagged implied variance terms.

### A.1. Implied Variance

In addition to the two model free implied variances,  $MIV_t^{(\tau)}$  and  $SMIV_t^{(\tau)}$ , we also use the inverted Black (1976) formula to calculate the implied variance  $BIV_t^{(\tau)}$ . For every day  $t$  we start by calculating the implied variance measures for all available option maturities  $\tau_i, i = 1 \dots, N$ . Having calculated  $MIV_t^{(\tau_i)}$  and  $BIV_t^{(\tau_i)}$ , we consider four different methods to construct the monthly estimates.<sup>26</sup>

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<sup>26</sup>To save space we just show the formulas for *MIV*.

1. First, we consider the available  $MIV_t^{(\tau_i)}$  and use the Forsythe, Malcolm, and Moler (1977) cubic spline method to interpolate over  $\tau = 20, 30, \dots, 180$  days. As our time  $t$  estimates for the 30 day horizon we pick the interpolated value  $MIV_t^{(30)}$ .
2. Second, we average over the seventeen *interpolated* values from above, i.e.  $MIV_t^{(\tau)}$  with  $\tau = 20, 30, \dots, 180$  and normalize to a one month horizon.

$$MIV_t^{(\text{int avg})} = \frac{30}{17} \sum_{\tau=20,30,\dots,180} \frac{MIV_t^{(\tau)}}{\tau}. \quad (-1)$$

3. The third method is a simple average over the implied variances of the *available* options, i.e. without interpolation. Unlike the second method, this average is calculated using a potentially different number of observations  $N$  at each point in time  $t$ . Again, we normalize to a one month horizon.

$$MIV_t^{(\text{sim avg})} = \frac{30}{N} \sum_{i=1,\dots,N} \frac{MIV_t^{(\tau_i)}}{\tau_i}. \quad (-2)$$

4. The final method is similar to the approach used to calculate the daily VIX index.<sup>27</sup> At each point in time  $t$  we consider the two sets of options with expiration dates  $\tau_1$  and  $\tau_2$  closest to the desired 30 day horizon so that  $\tau_1 < 30 < \tau_2$ . We calculate  $MIV_t^{(\text{lin})}$  as the weighted average, i.e. a linear interpolation between the implied variances of the two options:

$$MIV_t^{(\text{lin})} = \frac{\tau_2 - 30}{\tau_2 - \tau_1} MIV_t^{(\tau_1)} + \frac{30 - \tau_1}{\tau_2 - \tau_1} MIV_t^{(\tau_2)}. \quad (-3)$$

Using these four methods, we estimate the model-free ( $SMIV$  and  $MIV$ ) and the Black (1976) implied variances at the end of each month for a 30 day horizon to get our monthly time series. Our benchmark method is  $MIV_t^{(\text{lin})}$ , which is used without subscript in the main text.

## A.2. Realized Variance

To estimate  $\mathbb{E}_t^{\mathbb{P}} \left( \int_t^{t+1} \sigma_u^2 du \right)$ , the daily expected variance we first consider realized variance  $RV_t$  and realized bi-power variation  $BV_t$ .  $BV_t$  is defined as

$$BV_t = \frac{\pi}{2} \left( \frac{M}{M-1} \right) \sum_{i=1}^M |r_{t,i}| |r_{t,i+1}|, \quad (-4)$$

where

$$r_{t,i} = \log P \left( t - 1 + \frac{i}{M} \right) - \log P \left( t - 1 + \frac{i-1}{M} \right)$$

is the intra-daily log return in the  $i^{\text{th}}$  sub-interval of day  $t$  and  $P(t - 1 + i/M)$  is the asset price at time  $t - 1 + i/M$ . Just as  $RV_t$ ,  $BV_t$  is consistent for  $\int_{t-1}^t \sigma_u^2 du$  if the log price process does not have jump components and under some mild regularity conditions. For each day,

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<sup>27</sup>See the VIX White Paper, Whaley (1993).

we take  $r_{t,i}$  between 7:25 and 14:00. We use three different sampling frequencies for  $r_{t,i}$ , namely we use  $k = \{5, 25, 60\}$  minute intervals to calculate  $BV_t^{(k \min)}$ .

The third estimator for  $\mathbb{E}_t^{\mathbb{P}} \left( \int_t^{t+1} \sigma_u^2 du \right)$  is the first-order autocorrelation-adjusted realized variance estimator  $RV_t^{(\text{AC1})}$  described in Zhou (1996). We use a data frequency of 5 minutes and apply the practical method described in Hansen and Lunde (2006) to construct the estimator.

$$RV_t^{(\text{AC1})} = \sum_{i=2}^{M-1} r_{t,i}^2 + \sum_{i=1}^{M-1} r_{t,i} r_{t,i+1} + \sum_{i=2}^M r_{t,i-1} r_{t,i}. \quad (-5)$$

The fourth estimator is the two scale realized variance proposed by Zhang, Mykland, and Ait-Sahalia (2005). As before, we use 5-minute data to construct the variance estimate. Following their method, we first define the set of time grids on day  $t$  as  $\mathcal{G} = \{1, \dots, M\}$ . Let  $\mathcal{G}^{(k)} = \{k, k+K, k+2K, \dots, k+aK\}$ ,  $k = 1, \dots, K$ , and  $K+aK = M$ . Obviously,  $\mathcal{G}^{(k)} \cap \mathcal{G}^{(k')} = \emptyset$  if  $k \neq k'$ ,  $k = 1, \dots, K$ .

Define

$$RV_t^{(k)} = \sum_{i=1}^a \left( r_{t,i}^{(k)} \right)^2,$$

where

$$r_{t,i}^{(k)} = \log P \left( t-1 + \frac{k+iK}{M} \right) - \log P \left( t-1 + \frac{k+(i-1)K}{M} \right).$$

Then, the two scale realized variance estimator is defined as

$$RV_t^{(\text{TS})} = RV_t - \frac{\overline{M}}{M} RV_t^{(\text{avg})}, \quad (-6)$$

where  $\overline{M} = (M-K+1)/K$  and  $RV_t^{(\text{avg})} = \frac{1}{K} \sum_{k=1}^K RV_t^{(k)}$ . In the case of equally spaced 5-minute data, and since the returns  $r_{t,i}$  are measured between 7:25 and 14:00,  $M = 80$ . We set  $K = 5$  and thus  $a = 15$ .

We consider the following bias-corrected  $RV^{(\text{TS})}$  as our fifth estimator:

$$RV_t^{(\text{TSadj})} = \left( 1 - \frac{\overline{M}}{M} \right)^{-1} RV_t^{(\text{TS})}. \quad (-7)$$

Zhang, Mykland, and Ait-Sahalia (2005) claim that  $RV_t^{(\text{TSadj})}$  performs better than  $RV_t^{(\text{TS})}$ ,  $RV_t^{(\text{avg})}$  and  $RV_t$  in estimating  $\int_{t-1}^t \sigma_u^2 du$  in the presence of microstructure noise.

Using the five methods described above, we obtain daily variance estimates for each trading day  $t$ . The normalized monthly realized variation  $RV_{t,1m}$  is defined by the average of the 22 daily measures. The normalized weekly realized variation  $RV_{t,1w}$  is correspondingly defined by the average of the 5 daily measures:

$$RV_{t,1w} = \frac{1}{5} \sum_{j=0}^4 RV_{t-j}, \quad \text{and} \quad RV_{t,1m} = \frac{1}{22} \sum_{j=0}^{21} RV_{t-j}.$$

The final estimator is the HAR-RV predictor for  $\int_t^{t+\tau'} \sigma_u^2 du$ ,  $RV^{(\text{HAR})}$ .



To obtain the monthly estimates for the first five measures, we sum the daily estimates over the last month. Subsequently, the realized variance ( $RV^{(k \min)}$ ), bi-power variation ( $BV^{(k \min)}$ ), first order auto-correlation-adjusted realized variance ( $RV^{(AC1)}$ ), two scaled realized variance ( $RV^{(TS)}$ ) and adjusted two scaled realized variance ( $RV^{(TSadj)}$ ) estimators always refer to the aggregated monthly measures.

### A.3. Jump Risk Measures

Wright and Zhou (2009) document that the average jump size in Treasury bonds is a good predictor for bond excess returns and in particular they show that Black and Scholes (1973) implied volatility loses its predictive power in the presence of such jump measures. In the following, we also calculate empirical measures for intra-daily jump components which can be easily constructed using the realized variance and the bi-power variation,  $RV_t$  and  $BV_t$ . Barndorff-Nielsen and Shephard (2004, 2006) show that the difference between the realized variance,  $RV_t$  and the bi-power variation,  $BV_t$  can be used to consistently estimate the discontinuous jump variation in the price process. Furthermore, if some regularity conditions hold, the joint distribution of  $RV_t$  and  $BV_t$  converges asymptotically to a bivariate Normal distribution, and this statistical property can be used to test whether price jumps occur on a given day  $t$ . Following Huang and Tauchen (2005) and Zhang, Zhou, and Zhu (2009), we use the ratio type statistic:

$$Z_{ratio,t} = \frac{\sqrt{M} \frac{RV_t - BV_t}{RV_t}}{\sqrt{A \max(1, B)}}, \quad (-8)$$

where  $A = (\pi/2)^2 + \pi - 5$  and

$$B = \frac{\int_{t-1}^t \sigma_u^4 du}{(\int_{t-1}^t \sigma_u^2 du)^2},$$

for the jump test. Under the null of no jump occurring on day  $t$ , (-8) converges to a standard Normal  $\mathcal{N}(0, 1)$ . For estimating,  $B$ , the denominator can be approximated by  $BV_t^2$ , and the integrated quarticity  $\int_{t-1}^t \sigma_u^4 du$  can be estimated by the realized tri-power quarticity ( $TP_t$ ) or quadpower quarticity ( $QP_t$ ),

$$\begin{aligned} TP_t &= \mu_{\frac{4}{3}}^{-3} \left( \frac{M^2}{M-2} \right) \sum_{i=1}^{M-2} (|r_{t,i}| |r_{t,i+1}| |r_{t,i+2}|)^{\frac{4}{3}}, \\ QP_t &= \mu_1^{-4} \left( \frac{M^2}{M-3} \right) \sum_{i=1}^{M-3} |r_{t,i}| |r_{t,i+1}| |r_{t,i+2}| |r_{t,i+3}|, \end{aligned}$$

where  $\mu_a = \mathbb{E}(|Z|^a)$ ,  $Z \sim \mathcal{N}(0, 1)$ . We then use:

$$J_t^{ratio} = (RV_t - BV_t) \times \mathbf{1}\{\Phi(Z_{ratio,t}) \geq \alpha\}, \quad (-9)$$

as the empirical measure for the jump variation on day  $t$ , where  $\Phi(x)$  is the cumulative distribution function of the standard Normal, and we fix the type I error at 0.001 ( $\alpha = 0.999$ ). After obtaining daily jump variation estimations, we aggregate them to a monthly basis to extract monthly jump variation estimates in that month. Finally, we calculate a maturity weighted average Treasury jump measure  $J^{(T)}$  from the measures for the 30 year Treasury bonds and 10 year and 5 year Treasury notes.

## B. Macroeconomic Data

We compute the eight static macroeconomic factors  $\widehat{F}_j, j = 1 \dots, 8$  from Ludvigson and Ng (2009, 2010) for an updated data set.<sup>28</sup> Some of the macroeconomic variables are no longer available after 2007. Consequently, we use 125 instead of 132 macroeconomic time series. Furthermore, we exclude all stock market and interest rate time series and work with a set of 104 variables. However, we also calculate the macroeconomic factors using all 125 series (denoted  $\widehat{F}_j^{(125)}$ ). The main data source for the macroeconomic data is Global Insight. The complete list and description of the macro variables is presented in Table 8. The transformations are the same as in Ludvigson and Ng (2009, 2010).  $\log$  denotes logarithm,  $\Delta \log$  and  $\Delta^2 \log$  are the first and second differences of the logarithm, respectively.  $lev$  denotes the level of the series and  $\Delta lev$  is the first difference in levels.

The updated factors are very similar to the original factors. The correlations for the overlapping time period are in the range of 75% to 99% (absolute values) although some factors switch ranks (i.e.  $\widehat{F}_2^{(125)}$  with the updated data is highly correlated with the original  $\widehat{F}_3^{LN}$  and vice versa; the same applies to  $\widehat{F}_7$  and  $\widehat{F}_8$ ). The correlations with the factors from the data set without bond and equity time series are slightly smaller except for  $\widehat{F}_1$ , where the correlation remains at 98%. Given that the factors are principal components, the economic interpretation is not straightforward. The macro variables are arranged in seven groups: output and income (OI); consumption, orders and inventories (COI); labor market (LM); housing (H); money and credit (MC); bond and exchange rates (BE); and prices (P). The stock market category is excluded here. We analyze the factors using the marginal correlations of the individual time series with the respective factors. As in Ludvigson and Ng (2009), the first factor can be interpreted as a real factor. The second factor can be interpreted as an inflation factor. The third factor could be interpreted as a housing/real factor, the fourth seems to pick up variation in exchange rates and again in housing variables. None of the factors have a mentionable correlation with the the Cochrane and Piazzesi (2005) factor, whereas the third factor using all data  $\widehat{F}_3^{(125)}$  (the second factor in the original data set,  $\widehat{F}_2^{LN}$ ) exhibits a correlation of -0.45%. This is to illustrate that our factors only pick up information about macro variables, whereas the original data contains a lot of information from the term structure that is also picked up by the CP factor.

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<sup>28</sup>Originally, the data set was used in Stock and Watson (2002).

**Table 1**  
**Summary Statistics of Excess Returns and Credit Spreads**

This table presents summary statistics for Treasury excess returns (Panel A), stock excess returns (Panel B) and credit spreads (Panel C) for the time period July 1992 to December 2010. The excess returns for 2, 3, 4, and 5 year bonds are calculated for a one year holding period and the stock market excess returns (S&P 500 index, growth and value portfolio) are for an annual holding period. All numbers are annualized and expressed in percent.

**PANEL A: BOND EXCESS RETURNS**

	2y	3y	4y	5y
Mean	0.72	1.42	1.99	2.35
StDev	1.31	2.57	3.60	4.50
Min	-2.37	-5.24	-6.88	-8.37
Max	3.52	7.31	10.30	12.54
Skewness	-0.03	-0.13	-0.18	-0.24
Kurtosis	2.19	2.39	2.43	2.51
AC(1)	0.94	0.94	0.93	0.92

**PANEL B: STOCK EXCESS RETURNS**

	S&P 500		Value		Growth	
	6m	12m	6m	12m	6m	12m
Mean	0.75	1.50	0.53	1.11	1.34	2.54
StDev	4.33	6.39	4.71	6.78	4.94	6.67
Min	-18.83	-19.80	-17.38	-16.86	-25.61	-23.60
Max	12.05	15.27	11.38	14.63	17.05	20.56
Skewness	-1.40	-1.03	-0.86	-0.65	-1.61	-0.93
Kurtosis	6.80	4.00	4.67	3.13	9.97	5.16
AC(1)	0.87	0.94	0.85	0.92	0.86	0.94

**PANEL C: CREDIT SPREADS**

	AAA			BBB			B		
	1y	5y	10y	1y	5y	10y	1y	5y	10y
Mean	0.36	0.49	0.44	1.09	1.26	1.25	3.73	4.22	4.23
StDev	0.26	0.28	0.26	0.76	0.76	0.65	2.09	1.90	1.75
Min	0.00	0.17	0.00	0.36	0.43	0.39	1.06	2.15	2.28
Max	1.84	1.77	1.32	4.48	4.54	3.81	12.17	12.77	12.25
Skewness	2.47	2.07	0.76	2.45	2.18	1.81	1.57	2.18	2.38
Kurtosis	11.12	8.17	3.09	9.68	8.76	7.09	6.45	8.88	9.94
AC(1)	0.88	0.92	0.90	0.98	0.98	0.97	0.97	0.97	0.96

**Table 2**  
**Summary Statistics of Macro and Uncertainty Factors**

Panel A reports the summary statistics of the Cochrane and Piazzesi (2005) factor, CP, the mean jump size,  $\tilde{J}$ , the uncertainty measures  $\hat{U}^{(i)}$ , and the annualized inflation and consumption volatilities  $\sigma_\pi$  and  $\sigma_g$ . Panel B contains the summary statistics for the eight macro factors from Ludvigson and Ng (2009, 2010),  $\hat{F}_j$ ,  $j = 1, \dots, 8$ .

PANEL A: CP AND JUMP FACTORS AND UNCERTAINTY PROXIES								
	CP	$\tilde{J}$	$\hat{U}^{FED}$	$\hat{U}^{CPI}$	$\hat{U}^{RDPI}$	$\hat{U}^{HS}$	$\sigma_\pi$	$\sigma_g$
Mean	0.70	0.01	0.03	-0.50	-0.46	0.46	1.35	1.00
StDev	1.44	0.08	0.02	0.97	1.13	1.04	0.46	0.15
Min	-4.69	-0.21	0.00	-1.70	-1.99	-6.26	0.77	0.72
Max	4.85	0.16	0.10	5.66	3.97	1.97	3.52	1.30
Skewness	0.18	-0.53	1.20	3.02	1.38	-1.77	1.88	0.18
Kurtosis	3.71	2.90	5.55	15.14	4.87	10.65	7.76	2.03
AC(1)	0.88	0.97	0.63	0.84	0.83	0.75	0.94	0.99

PANEL B: MACRO FACTORS								
	$\hat{F}_1$	$\hat{F}_2$	$\hat{F}_3$	$\hat{F}_4$	$\hat{F}_5$	$\hat{F}_6$	$\hat{F}_7$	$\hat{F}_8$
Mean	0.33	0.05	0.06	-0.08	0.16	-0.02	0.17	0.12
StDev	4.74	3.59	2.74	2.10	1.81	2.02	1.80	1.71
Min	-7.02	-13.06	-11.72	-8.11	-4.40	-9.58	-3.89	-5.73
Max	22.63	15.65	6.58	12.06	5.46	13.03	4.91	5.33
Skewness	1.93	0.69	-1.02	0.73	0.06	0.35	0.31	0.10
Kurtosis	8.06	6.66	5.47	8.11	3.45	14.33	2.81	3.58
AC(1)	0.85	-0.12	0.42	0.49	0.23	-0.19	0.21	-0.10

**Table 3**  
**Summary Statistics of Implied and Realized Volatility and Variance Risk Premia**

Panels A and B report summary statistics for different implied and realized volatility measures. SMIV and MIV denote the model-free implied variance for a one month horizon. The measures are a linear interpolation using call options of the nearest two expiration dates on day  $t$ .  $RV^{(5 \text{ min})}$  denotes realized variance sampled at the 5 minute frequency, and  $RV^{(IV)}$  denotes the heterogeneous autoregressive model realized variance estimator augmented with lagged implied variance terms. All quantities are annualized and expressed in percent. Panel C presents summary statistics for bond and equity variance risk premia. The variance risk premia are annualized and expressed in squared percentage terms. They are calculated as the difference of the model free implied variance and the projected value from the heterogeneous autoregressive model realized variance estimator augmented with lagged implied variance terms. All data is monthly and the sample spans the period from July 1992 to December 2010.

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**PANEL A: IMPLIED VOLATILITY**

	30y Treasury		10y Treasury		5y Treasury		S&P500		
	SMIV	MIV	SMIV	MIV	SMIV	MIV	SMIV	MIV	VIX
Mean	10.22	10.03	7.07	6.91	4.68	4.49	18.86	20.25	20.34
StDev	2.65	2.43	1.68	1.70	1.16	1.24	7.61	8.61	8.32
Min	6.26	6.03	3.99	2.20	1.84	0.65	9.74	9.97	10.42
Max	24.06	21.96	13.51	13.26	9.26	9.53	52.84	58.46	59.89
Skewness	2.22	1.93	0.86	0.53	0.52	0.63	1.49	1.50	1.57
Kurtosis	10.34	8.79	4.35	3.97	3.75	4.23	6.19	6.28	6.62
AC(1)	0.87	0.85	0.85	0.72	0.75	0.65	0.88	0.88	0.87

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**PANEL B: REALIZED VOLATILITY**

	30y Treasury		10y Treasury		5y Treasury		S&P500	
	$RV^{(IV)}$	$RV^{(5 \text{ min})}$	$RV^{(IV)}$	$RV^{(5 \text{ min})}$	$RV^{(IV)}$	$RV^{(5 \text{ min})}$	$RV^{(IV)}$	$RV^{(5 \text{ min})}$
Mean	9.50	8.38	6.38	5.49	3.89	3.71	17.03	14.80
StDev	1.44	2.05	0.93	1.47	0.63	1.06	6.02	8.27
Min	6.81	4.77	4.52	2.87	2.84	1.96	9.66	5.35
Max	15.01	18.68	9.54	10.93	7.12	7.55	40.84	77.23
Skewness	1.16	1.21	0.65	0.88	1.16	0.88	1.35	3.04
Kurtosis	5.44	5.87	4.10	3.99	5.74	3.74	5.29	19.13
AC(1)	0.81	0.70	0.83	0.65	0.74	0.60	0.85	0.75

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**PANEL C: VARIANCE RISK PREMIA**

	30y Treasury		10y Treasury		5y Treasury		S&P500	
	SMIV	MIV	SMIV	MIV	SMIV	MIV	SMIV	MIV
Mean	1.66	1.32	0.92	0.77	0.62	0.50	93.22	163.92
StDev	3.98	3.11	1.33	1.33	0.55	0.67	130.11	216.58
Min	-2.04	-2.00	-1.32	-2.25	-0.80	-1.41	-15.11	8.24
Max	30.47	21.82	7.61	7.54	2.92	4.56	1124.57	1830.00
Skewness	4.21	3.68	1.75	1.54	1.05	2.16	4.69	4.27
Kurtosis	25.20	20.66	7.47	7.68	5.10	11.81	33.76	28.36
AC(1)	0.71	0.60	0.74	0.52	0.69	0.43	0.75	0.80

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**Table 4**  
**Bonds, Stocks and Credit Spreads Predictability**

Each month we run the following regression:

$$rx_{t+h}^{(i,\tau)} = \beta^{(i,\tau)}(h)\mathbf{VRP}_t + \epsilon_{t+h}^{(i,\tau)}$$

For bonds,  $rx_{t+h}^{(i,\tau)}$  is the one year ( $h = 12$ ) excess return on  $\tau = 24, 36, 48, 60$  month Treasury bonds. For stocks,  $rx_{t+h}^{(i,\tau)}$  is the 6 month or 1 year excess return on the market, value or growth portfolio, respectively. For credit spreads,  $rx_{t+h}^{(i,\tau)}$  is the monthly change in the credit spread for rating category  $i = \{AAA, BBB, B\}$  and maturity  $\tau = 1, 5, 10$  years.  $\mathbf{VRP}_t$  is a vector containing the equity market variance risk premium  $VRP_t^{(E)}$ , and the bond variance risk premia  $VRP_t^{(30y)}$ ,  $VRP_t^{(10y)}$  and  $VRP_t^{(5y)}$ . Regressions are standardized, meaning all variables are de-measured and divided by their standard deviation. Coefficients are estimated with ordinary-least squares and standardized.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) standard errors. The sample spans the period from July 1992 to December 2010.

**PANEL A: FAMA BLISS TREASURY BONDS**

	2y	3y	4y	5y
$VRP^{(30y)}$	-0.387 (-2.52)	-0.385 (-2.70)	-0.406 (-3.27)	-0.362 (-3.10)
$VRP^{(10y)}$	0.482 (2.32)	0.509 (2.53)	0.515 (2.59)	0.506 (2.48)
$VRP^{(5y)}$	-0.080 (-0.58)	-0.086 (-0.69)	-0.053 (-0.45)	-0.039 (-0.32)
$VRP^{(E)}$	0.141 (1.03)	0.138 (1.05)	0.089 (0.70)	0.066 (0.52)
<b>AdjR<sup>2</sup></b>	<b>0.06</b>	<b>0.07</b>	<b>0.07</b>	<b>0.07</b>

**PANEL B: STOCKS**

	S&P 500		Growth		Value	
	6m	12m	6m	12m	6m	12m
$VRP^{(30y)}$	0.498 (3.88)	0.621 (3.81)	0.453 (3.47)	0.568 (3.31)	0.310 (2.72)	0.407 (2.89)
$VRP^{(10y)}$	-0.218 (-1.16)	-0.714 (-2.61)	-0.182 (-0.97)	-0.693 (-2.62)	0.243 (1.27)	-0.196 (-0.64)
$VRP^{(5y)}$	-0.199 (-0.84)	0.393 (1.69)	-0.186 (-0.91)	0.421 (2.03)	-0.524 (-2.28)	0.106 (0.37)
$VRP^{(E)}$	-0.038 (-0.37)	-0.112 (-1.13)	0.032 (0.33)	-0.030 (-0.29)	-0.090 (-1.23)	-0.129 (-1.56)
<b>AdjR<sup>2</sup></b>	<b>0.10</b>	<b>0.12</b>	<b>0.09</b>	<b>0.12</b>	<b>0.13</b>	<b>0.07</b>

**PANEL C: CREDIT SPREADS**

	AAA			BBB			B		
	1y	5y	10y	1y	5y	10y	1y	5y	10y
$VRP^{(30y)}$	-0.496 (-3.50)	-0.666 (-4.53)	-0.396 (-3.07)	-0.257 (-2.33)	-0.443 (-4.32)	-0.415 (-2.87)	0.045 (0.21)	-0.016 (-0.08)	-0.014 (-0.09)
$VRP^{(10y)}$	0.567 (2.10)	0.368 (1.99)	-0.030 (-0.12)	0.118 (0.67)	-0.028 (-0.13)	-0.172 (-0.92)	-0.155 (-0.75)	-0.211 (-0.95)	-0.231 (-1.09)
$VRP^{(5y)}$	-0.192 (-1.75)	0.018 (0.11)	0.054 (0.31)	-0.014 (-0.10)	0.164 (0.83)	0.299 (1.97)	0.089 (0.52)	0.170 (0.90)	0.170 (1.00)
$VRP^{(E)}$	-0.044 (-0.58)	0.216 (2.53)	0.332 (2.06)	0.400 (4.18)	0.608 (3.03)	0.578 (2.81)	0.428 (5.95)	0.480 (5.46)	0.463 (4.57)
<b>AdjR<sup>2</sup></b>	<b>0.08</b>	<b>0.14</b>	<b>0.12</b>	<b>0.11</b>	<b>0.31</b>	<b>0.31</b>	<b>0.17</b>	<b>0.21</b>	<b>0.19</b>

**Table 5**  
**Treasury Bonds Predictability**

Each month we run the following regression:

$$rx_{t+h}^{(\tau)} = \beta^{(\tau)} \mathbf{VRP}_t + \gamma' \mathbf{M}_t + \epsilon_{t+h}^{(\tau)},$$

where  $rx^{(\tau)}$  is the one year excess return on  $\tau = 24, 36, 48, 60$  months Treasury bonds.  $\mathbf{VRP}_t$  is a vector containing the equity market variance risk premium  $\text{VRP}_t^{(E)}$ , and the bond variance risk premia  $\text{VRP}_t^{(30y)}$ ,  $\text{VRP}_t^{(10y)}$  and  $\text{VRP}_t^{(5y)}$ .  $\mathbf{M}$  includes the Cochrane and Piazzesi (2005) factor, the mean jump size,  $\tilde{J}$ , and the macro factors  $\hat{F}_j, j = 1 \dots, 8$  from Ludvigson and Ng (2009). Regressions are standardized, meaning all variables are de-measured and divided by their standard deviation. Coefficients are estimated with ordinary-least squares and standardized.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) standard errors. The sample spans the period from July 1992 to December 2010.

	2y	3y	4y	5y
$\text{VRP}^{(30y)}$	-0.296 (-2.46)	-0.291 (-2.50)	-0.312 (-2.83)	-0.263 (-2.44)
$\text{VRP}^{(10y)}$	0.083 (0.52)	0.115 (0.73)	0.136 (0.89)	0.168 (1.10)
$\text{VRP}^{(5y)}$	0.081 (0.97)	0.060 (0.72)	0.067 (0.83)	0.052 (0.61)
$\text{VRP}^{(E)}$	-0.038 (-0.48)	0.002 (0.02)	0.006 (0.06)	0.021 (0.22)
CP	-0.035 (-0.28)	-0.007 (-0.05)	0.039 (0.29)	0.057 (0.44)
$\tilde{J}$	-0.658 (-6.92)	-0.633 (-7.08)	-0.603 (-7.23)	-0.587 (-7.23)
$\hat{F}_1$	0.196 (1.99)	0.151 (1.53)	0.105 (1.11)	0.037 (0.39)
$\hat{F}_2$	0.040 (1.33)	0.033 (1.07)	0.036 (1.13)	0.045 (1.34)
$\hat{F}_3$	-0.162 (-1.72)	-0.175 (-2.04)	-0.203 (-2.61)	-0.207 (-2.92)
$\hat{F}_4$	-0.227 (-2.60)	-0.266 (-3.20)	-0.318 (-4.08)	-0.368 (-4.86)
$\hat{F}_5$	0.194 (3.60)	0.166 (3.05)	0.150 (2.77)	0.150 (2.74)
$\hat{F}_6$	0.116 (1.69)	0.136 (2.10)	0.162 (2.63)	0.177 (3.06)
$\hat{F}_7$	-0.011 (-0.17)	0.037 (0.53)	0.067 (0.96)	0.094 (1.37)
$\hat{F}_8$	-0.066 (-1.26)	-0.012 (-0.23)	0.025 (0.46)	0.043 (0.80)
AdjR <sup>2</sup>	0.47	0.45	0.45	0.46

**Table 6**  
**Stock Excess Returns Predictability**

Each month we run the following regression:

$$rx_{t+h}^{(i)} = \beta^{(i)}(h)\mathbf{VRP}_t + \gamma'\mathbf{M}_t + \epsilon_{t+h}^{(i)},$$

where  $rx^{(i)}$  are the annual excess returns on the S&P 500 index, the value and growth portfolio, respectively.  $\mathbf{VRP}_t$  is a vector containing the equity market variance risk premium  $\text{VRP}_t^{(E)}$ , and the bond variance risk premia  $\text{VRP}_t^{(30y)}$ ,  $\text{VRP}_t^{(10y)}$  and  $\text{VRP}_t^{(5y)}$ .  $\mathbf{M}$  includes the dividend yield DY, the earnings/price ratio E/P, the book-to-market ratio B/M and the net equity expansion NTIS from Goyal and Welch (2008). Regressions are standardized, meaning all variables are de-measured and divided by their standard deviation. Coefficients are estimated with ordinary-least squares and standardized.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) standard errors. The sample spans the period from July 1992 to December 2010.

	S&P 500		Growth		Value	
	6m	12m	6m	12m	6m	12m
$\text{VRP}^{(30y)}$	0.451 (2.89)	0.522 (3.15)	0.399 (2.61)	0.443 (2.89)	0.304 (1.90)	0.420 (2.39)
$\text{VRP}^{(10y)}$	-0.052 (-0.29)	-0.536 (-2.22)	-0.019 (-0.10)	-0.551 (-2.41)	0.316 (1.62)	-0.079 (-0.29)
$\text{VRP}^{(5y)}$	-0.409 (-2.16)	0.163 (0.96)	-0.385 (-2.27)	0.219 (1.49)	-0.649 (-3.23)	-0.058 (-0.26)
$\text{VRP}^{(E)}$	0.217 (2.31)	0.181 (2.67)	0.272 (2.74)	0.242 (2.60)	0.070 (0.86)	0.073 (0.93)
DY	0.218 (1.25)	0.264 (1.68)	0.206 (1.20)	0.241 (1.61)	0.069 (0.39)	-0.018 (-0.09)
E/P	-0.003 (-0.02)	-0.021 (-0.14)	0.013 (0.07)	-0.066 (-0.47)	-0.088 (-0.46)	-0.070 (-0.33)
B/M	0.054 (0.45)	0.107 (0.88)	0.060 (0.47)	0.152 (1.29)	0.089 (0.68)	0.191 (1.34)
NTIS	0.477 (2.69)	0.480 (3.11)	0.423 (2.87)	0.419 (3.32)	0.405 (2.22)	0.520 (3.01)
AdjR <sup>2</sup>	0.34	0.41	0.28	0.37	0.27	0.30



**Table 7**  
**Economic Drivers of Bond Variance Risk Premia**

The table reports the results from regressing the respective variance risk premia on uncertainty measures  $\widehat{\mathbf{U}}_t$ , latent macro factors  $\widehat{\mathbf{F}}_t$  and macro volatility measures  $\widehat{\mathbf{S}}_t$ :

$$VRP_t^{(i)} = \beta^U \widehat{\mathbf{U}}_t + \beta^F \widehat{\mathbf{F}}_t + \beta^S \widehat{\mathbf{S}}_t + \epsilon_t^{(i)}.$$

The uncertainty variables are defined as the cross sectional standard deviation of the forecasts of CPI  $\widehat{U}^{(CPI)}$ , real disposable income  $\widehat{U}^{(RDPI)}$  and housing starts  $\widehat{U}^{(HS)}$ . We estimate the uncertainty about FED actions  $\widehat{U}^{(FED)}$  from regressing forecasts of the short rate on CPI and real GDP forecasts. The macro variables  $\widehat{F}_j, j = 1, \dots, 8$  are estimated as the first eight principal components from a data set of 104 macroeconomic variables. In the regression, we include the first and the second factor that can be interpreted as a real and a (nominal) price factor. The macro volatilities  $\sigma_\pi$  and  $\sigma_g$  are estimated using a GARCH process for inflation and per capita consumption (non durables and services). Regressions are standardized, meaning all variables are de-measured and divided by their standard deviation. Coefficients are estimated with ordinary-least squares and standardized.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) standard errors. The sample spans the period from July 1992 to December 2009.

	$VRP^{(30y)}$			$VRP^{(10y)}$			$VRP^{(5y)}$			$VRP^{(E)}$		
$\widehat{U}^{FED}$	-0.194 (-3.31)	-0.170 (-2.65)	-0.180 (-2.84)	-0.090 (-1.56)	-0.118 (-1.71)	-0.136 (-1.96)	-0.041 (-0.56)	-0.110 (-1.26)	-0.138 (-1.59)	-0.026 (-0.34)	0.018 (0.21)	-0.040 (-0.63)
$\widehat{U}^{CPI}$	0.530 (3.18)	0.512 (3.00)	0.426 (2.25)	0.274 (2.91)	0.287 (2.89)	0.165 (1.74)	0.295 (2.07)	0.344 (2.48)	0.225 (1.67)	0.463 (2.60)	0.447 (2.81)	0.288 (2.37)
$\widehat{U}^{RDPI}$	0.197 (2.12)	0.184 (1.78)	0.145 (1.41)	0.414 (4.96)	0.383 (4.37)	0.331 (3.79)	0.264 (2.32)	0.288 (2.36)	0.253 (2.06)	0.194 (2.03)	0.261 (2.27)	0.241 (2.15)
$\widehat{U}^{HS}$	-0.113 (-1.59)	-0.068 (-1.15)	-0.051 (-0.95)	-0.133 (-1.72)	-0.144 (-1.75)	-0.127 (-1.51)	0.116 (1.16)	0.000 (0.00)	-0.015 (-0.13)	0.220 (2.34)	0.222 (1.90)	0.145 (1.68)
$\sigma_\pi$		0.063 (0.60)	0.027 (0.28)		0.050 (0.40)	-0.006 (-0.05)		-0.148 (-1.02)	-0.220 (-1.51)		-0.124 (-0.89)	-0.253 (-1.74)
$\sigma_g$		-0.036 (-0.58)	-0.046 (-0.77)		0.098 (1.32)	0.087 (1.22)		0.118 (1.07)	0.114 (1.06)		-0.174 (-1.46)	-0.164 (-1.58)
$\widehat{F}_1$			0.188 (2.64)			0.259 (2.83)			0.234 (1.99)			0.274 (2.98)
$\widehat{F}_2$			-0.029 (-0.62)			-0.024 (-0.56)			0.056 (0.96)			0.219 (1.74)
AdjR <sup>2</sup>	0.59	0.59	0.60	0.49	0.49	0.51	0.22	0.24	0.25	0.31	0.32	0.38

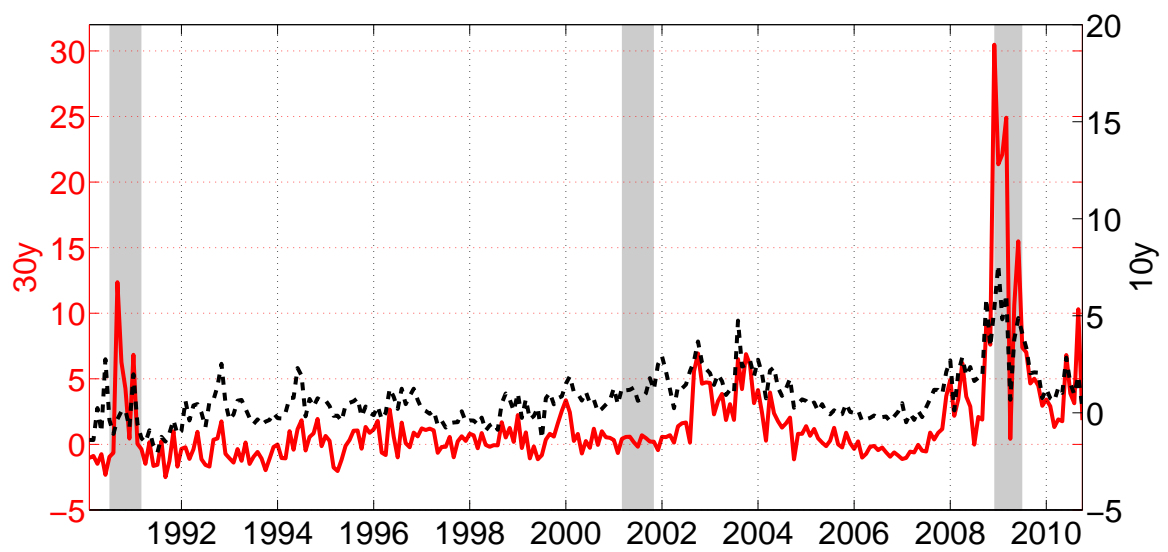
**Table 8**  
**Macro variables: data description and transformation**

The table lists the name of each macro time series, its mnemonic, the description and the source. We start with the data set used in Ludvigson and Ng (2009, 2010) consisting of 132 time series. Some of the macroeconomic variables are no longer available after 2007 and we only have 125 series available throughout the end of 2009. The time series are arranged in the eight groups used by Ludvigson and Ng (2009, 2010): output and income (OI); consumption, orders and inventories (COI); labor market (LM); housing (H); money and credit (MC); bond and exchange rates (BE); prices (P); and stock market (SM). We exclude stock market and interest rate variables (series numbers 82 to 102 in Ludvigson and Ng (2009, 2010)) for a total of 104 macro time series. The transformations are the same as in Ludvigson and Ng (2009, 2010).  $\log$  denotes logarithm,  $\Delta \log$  and  $\Delta^2 \log$  are the first and second differences of the logarithm, respectively.  $lev$  denotes the level of the series and  $\Delta lev$  is the first difference in levels. The data source is Global Insight except for series 97 to 100 (BEA) and series 104 (University of Michigan). All data are available for the full sample period from January 1990 to September 2010.

Series No.	Short Name	Mnemonic	Transformation	Description	Group
1	PI	ypr	$\Delta \log$	Personal Income (AR, Bil. Chain 2000 \$)	Output and Income
2	PI less transfers	a0m051	$\Delta \log$	Personal Income Less Transfer Payments (AR, Bil. Chain 2000 \$)	Output and Income
3	M&T sales	mtq	$\Delta \log$	Manufacturing and Trade Sales (Mil. Chain 1996 \$)	Consumption, Orders and Inventories
4	Retail sales	a0m059	$\Delta \log$	Sales of Retail Stores (Mil. Chain 2000 \$)	Consumption, Orders and Inventories
5	IP: total	ips10	$\Delta \log$	Industrial Production Index - Total Index	Output and Income
6	IP: products	ips11	$\Delta \log$	Industrial Production Index - Products, Total	Output and Income
7	IP: final prod	ips299	$\Delta \log$	Industrial Production Index - Final Products	Output and Income
8	IP: cons gds	ips12	$\Delta \log$	Industrial Production Index - Consumer Goods	Output and Income
9	IP: cons dble	ips13	$\Delta \log$	Industrial Production Index - Durable Consumer Goods	Output and Income
10	IP: cons nondble	ips18	$\Delta \log$	Industrial Production Index - Nondurable Consumer Goods	Output and Income
11	IP: bus eqpt	ips25	$\Delta \log$	Industrial Production Index - Business Equipment	Output and Income
12	IP: matls	ips32	$\Delta \log$	Industrial Production Index - Materials	Output and Income
13	IP: dble matls	ips34	$\Delta \log$	Industrial Production Index - Durable Goods Materials	Output and Income
14	IP: nondble matls	ips38	$\Delta \log$	Industrial Production Index - Nondurable Goods Materials	Output and Income
15	IP: mfg	ips43	$\Delta \log$	Industrial Production Index - Manufacturing (Sic)	Output and Income
16	IP: res util	ips307	$\Delta \log$	Industrial Production Index - Residential Utilities	Output and Income
17	IP: fuels	ips306	$\Delta \log$	Industrial Production Index - Fuels	Output and Income
18	NAPM prodn	pmp	$lev$	Napm Production Index (Percent)	Output and Income
19	Cap util	utlb00004	$\Delta lev$	Capacity Utilization (Mfg.)	Output and Income
20	Help wanted indx	lhelvr.M	$\Delta lev$	Index of Help-Wanted Advertising in Newspapers (1967=100;Sa)	Labor Market
21	Help wanted/emp	lhelx	$\Delta lev$	Employment: Ratio; Help-Wanted Ads: No. Unemployed Clf	Labor Market
22	Emp CPS total	lhem	$\Delta \log$	Civilian Labor Force: Employed, Total (Thous.,Sa)	Labor Market
23	Emp CPS nonag	lhmag	$\Delta \log$	Civilian Labor Force: Employed, Nonagric. Industries (Thous.,Sa)	Labor Market
24	U: all	lhur	$\Delta lev$	Unemployment Rate: All Workers, 16 Years & Over (%;Sa)	Labor Market
25	U: mean duration	lhu680	$\Delta lev$	Unemploy. By Duration: Average (Mean) Duration in Weeks (Sa)	Labor Market
26	U ; 5 wks	lhu5	$\Delta \log$	Unemploy. By Duration: Persons Unempl. Less than 5 Wks (Thous.,Sa)	Labor Market
27	U 5-14 wks	lhu14	$\Delta \log$	Unemploy. By Duration: Persons Unempl. 5 to 14 Wks (Thous.,Sa)	Labor Market
28	U 15+ wks	lhu15	$\Delta \log$	Unemploy. By Duration: Persons Unempl. 15 Wks + (Thous.,Sa)	Labor Market
29	U 15-26 wks	lhu26	$\Delta \log$	Unemploy. By Duration: Persons Unempl. 15 to 26Wks (Thous.,Sa)	Labor Market
30	U 27+ wks	lhu27	$\Delta \log$	Unemploy. By Duration: Persons Unempl. 27 Wks + (Thous.,Sa)	Labor Market
31	Unemp claims	ICSA	$\Delta \log$	Initial Claims	Labor Market
32	Emp: total	ces002	$\Delta \log$	Employees on nonfarm Payrolls: Total Private	Labor Market
33	Emp: gds prod	ces003	$\Delta \log$	Employees on nonfarm Payrolls: Goods-Producing	Labor Market
34	Emp: mining	ces006	$\Delta \log$	Employees on nonfarm Payrolls: Mining	Labor Market
35	Emp: const	ces011	$\Delta \log$	Employees on nonfarm Payrolls: Construction	Labor Market
36	Emp: mfg	ces015	$\Delta \log$	Employees on nonfarm Payrolls: Manufacturing	Labor Market
37	Emp: dble gds	ces017	$\Delta \log$	Employees on nonfarm Payrolls: Durable Goods	Labor Market
38	Emp: nondble gds	ces033	$\Delta \log$	Employees on nonfarm Payrolls: Nondurable Goods	Labor Market
39	Emp: services	ces046	$\Delta \log$	Employees on nonfarm Payrolls: Service-Providing	Labor Market
40	Emp: TTU	ces048	$\Delta \log$	Employees on nonfarm Payrolls: Trade, Transportation, and Utilities	Labor Market
41	Emp: wholesale	ces049	$\Delta \log$	Employees on nonfarm Payrolls: Wholesale Trade	Labor Market
42	Emp: retail	ces053	$\Delta \log$	Employees on nonfarm Payrolls: Retail Trade	Labor Market
43	Emp: FIRE	ces088	$\Delta \log$	Employees on nonfarm Payrolls: Financial Activities	Labor Market
44	Emp: Govt	ces140	$\Delta \log$	Employees on nonfarm Payrolls: Government	Labor Market

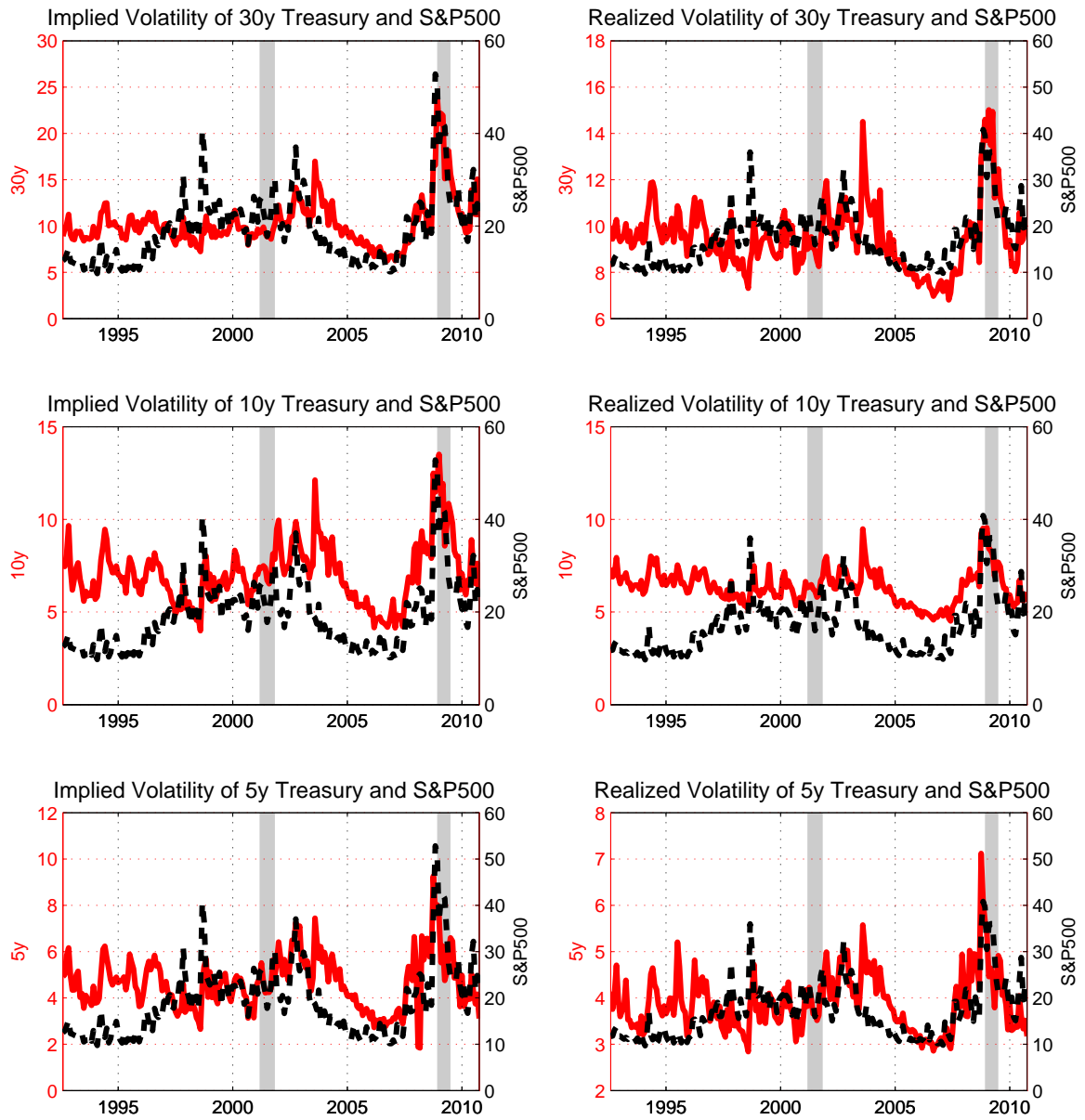
Table 8: Macro variables: data description and transformation (cont.)

45	Avg hrs	ces151	lev	Avg Weekly Hrs of Prod or Nonsup Workers on Private Nonfarm Payrolls: Goods-Producing	Labor Market
46	Overtime: mfg	ces155	$\Delta lev$	Avg Weekly Hrs of Prod or nonsup Workers on Private Nonfarm Payrolls: Mfg Overtime Hours	Labor Market
47	Avg hrs: mfg	a0m001	lev	Average Weekly Hours, Mfg. (Hours)	Labor Market
48	NAPM empl	pmemp	lev	Napm Employment Index (Percent)	Labor Market
49	Starts: nonfarm	hsfr	log	Housing Starts:Nonfarm(1947-58); Total Farm&Nonfarm(1959-) (Thous.,Saar)	Housing
50	Starts: NE	hsne	log	Housing Starts:Northeast (Thous.U.)S.A.	Housing
51	Starts: MW	hsmw	log	Housing Starts:Midwest (Thous.U.)S.A.	Housing
52	Starts: South	hssou	log	Housing Starts:South (Thous.U.)S.A.	Housing
53	Starts: West	hswst	log	Housing Starts:West (Thous.U.)S.A.	Housing
54	BP: total	hsbr	log	Housing Authorized: Total New Priv Housing Units (Thous.,Saar)	Housing
55	BP: NE	hsbne	log	Houses Authorized by Build. Permits:Northeast (Thou.U.)S.A.	Housing
56	BP: MW	hsbmw	log	Houses Authorized by Build. Permits:Midwest (Thou.U.)S.A.	Housing
57	BP: South	hsbsou	log	Houses Authorized by Build. Permits:South (Thou.U.)S.A.	Housing
58	BP: West	hsbwst	log	Houses Authorized by Build. Permits:West (Thou.U.)S.A.	Housing
59	PMI	pmi	lev	Purchasing Managers' Index (Sa)	Consumption, Orders and Inventories
60	NAPM new ordrs	pmno	lev	Napm New Orders Index (Percent)	Consumption, Orders and Inventories
61	NAPM vendor del	pmndel	lev	Napm Vendor Delivered Index (Percent)	Consumption, Orders and Inventories
62	NAPM Invent	pmnv	lev	Napm Inventories Index (Percent)	Consumption, Orders and Inventories
63	Orders: cons gds	x213	$\Delta$ log	Mfrs' New Orders, Consumer Goods and Materials (Bil. Chain 1982 \$)	Consumption, Orders and Inventories
64	Orders: dble gds	a0m007	$\Delta$ log	Mfrs' New Orders, Durable Goods Industries (Bil. Chain 2000 \$)	Consumption, Orders and Inventories
65	Orders: cap gds	a0m027	$\Delta$ log	Mfrs' New Orders, Nondefense Capital Goods (Mil. Chain 1982 \$)	Consumption, Orders and Inventories
66	Unf orders: dble	a1m092	$\Delta$ log	Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 2000 \$)	Consumption, Orders and Inventories
67	M1	fm1	$\Delta^2$ log	Money Stock: M1(Curr,Trav.Cks,Dep,Other Ck'able Dep) (Bil\$,Sa)	Money and Credit
68	M2	fm2	$\Delta^2$ log	Money Stock: M2(M1+O'nite Rps,Euro\$,G/P&B/D Mmfs&Sav&Sm Time Dep)(Bil\$,Sa)	Money and Credit
69	MB	fmfba	$\Delta^2$ log	Monetary Base, Adj tor Reserve Requirement Changes (Mil\$,Sa)	Money and Credit
70	Reserves tot	fmrna	$\Delta^2$ log	Depository Inst Reserves:Total, Adj. tor Reserve Req Chgs (Mil\$,Sa)	Money and Credit
71	Reserves nonbor	fmrnba	$\Delta^2$ log	Depository Inst Reserves:Nonborrowed,Adj. Res Req Chgs (Mil\$,Sa)	Money and Credit
72	C&I loans	fclnbw	$\Delta^2$ log	Commercial & Industrial Loans Outstanding in 1996 Dollars (Bci)	Money and Credit
73	dC&I loans	fclbmc	$\Delta^2$ log	Wkly Rp Lg Com'l Banks:Net Change Com'l & Indus Loans (Bil\$,Saar)	Money and Credit
74	Cons credit	ccinrv	$\Delta^2$ log	Consumer Credit Outstanding - Nonrevolving (G19)	Money and Credit
75	Inst cred/PI	ccipy	$\Delta lev$	Ratio, Consumer Installment Credit to Personal Income (Pct.)	Money and Credit
76	Ex rate: avg	exrus	$\Delta$ log	United States;Effective Exchange Rate (Merm) (Index No.)	Bond and Exchange rates
77	Ex rate: Switz	exrsw	$\Delta$ log	Foreign Exchange Rate: Switzerland (Swiss Franc Per U.S.\$)	Bond and Exchange rates
78	Ex rate: Japan	exrjan	$\Delta$ log	Foreign Exchange Rate: Japan (Yen per U.S.\$)	Bond and Exchange rates
79	Ex rate: UK	exruk	$\Delta$ log	Foreign Exchange Rate: United Kingdom (Cents per Pound)	Bond and Exchange rates
80	Ex rate: Canada	exrcan	$\Delta$ log	Foreign Exchange Rate: Canada (Canadian\$ per U.S.\$)	Bond and Exchange rates
81	PPI: fin gds	pwfsa	$\Delta^2$ log	Producer Price Index: Finished Goods (82=100,Sa)	Prices
82	PPI: cons gds	pwfcsa	$\Delta^2$ log	Producer Price Index: Finished Consumer Goods (82=100,Sa)	Prices
83	PPI: int mat'ls	pwimsa	$\Delta^2$ log	Producer Price Index: Intermed Mat.Supplies & Components (82=100,Sa)	Prices
84	PPI:crude mat'ls	pwcmsa	$\Delta^2$ log	Producer Price Index: Crude Materials (82=100,Sa)	Prices
85	Spot market price	psscom	$\Delta^2$ log	Spot market price index: bls & crb: all commodities (1967=100)	Prices
86	NAPM com price	pmcp	lev	Index Of Sensitive Materials Prices (1990=100) (Bci-99a)	Prices
87	CPI-U: all	punew	$\Delta^2$ log	Napm Commodity Prices Index (Percent)	Prices
88	CPI-U: apparel	pu83	$\Delta^2$ log	Cpi-U: All Items (82-84=100,Sa)	Prices
89	CPI-U: transp	pu84	$\Delta^2$ log	Cpi-U: Apparel & Upkeep (82-84=100,Sa)	Prices
90	CPI-U: medical	pu85	$\Delta^2$ log	Cpi-U: Transportation (82-84=100,Sa)	Prices
91	CPI-U: comm.	puc	$\Delta^2$ log	Cpi-U: Medical Care (82-84=100,Sa)	Prices
92	CPI-U: dbles	pucd	$\Delta^2$ log	Cpi-U: Durables (82-84=100,Sa)	Prices
93	CPI-U: services	pus	$\Delta^2$ log	Cpi-U: Services (82-84=100,Sa)	Prices
94	CPI-U: ex food	puxf	$\Delta^2$ log	Cpi-U: All Items Less Food (82-84=100,Sa)	Prices
95	CPI-U: ex shelter	puxhs	$\Delta^2$ log	Cpi-U: All Items Less Shelter (82-84=100,Sa)	Prices
96	CPI-U: ex med	puxm	$\Delta^2$ log	Cpi-U: All Items Less Medical Care (82-84=100,Sa)	Prices
97	PCE: cons	N/A	$\Delta^2$ log	Pce (BEA 2.3.4U), Impl Pr Defl:Pce (1987=100)	Prices
98	PCE: durables	N/A	$\Delta^2$ log	Pce (BEA 2.3.4U), Impl Pr Defl:Pce; Durables (1987=100)	Prices
99	PCE: nondurables	N/A	$\Delta^2$ log	Pce (BEA 2.3.4U), Impl Pr Defl:Pce; Nondurables (1987=100)	Prices
100	PCE: services	N/A	$\Delta^2$ log	Pce (BEA 2.3.4U), Impl Pr Defl:Pce; Services (1987=100)	Prices
101	AHE: goods	ahpgp	$\Delta^2$ log	Avg Hourly Earnings of Prod or Nonsup Workers on Private Nonfarm Payrolls: Goods-producing	Labor Market
102	AHE: const	ahpcon	$\Delta^2$ log	Avg Hourly Earnings of Prod or Nonsup Workers on Private Nonfarm Payrolls: Construction	Labor Market
103	AHE: mfg	ahpmf	$\Delta^2$ log	Avg Hourly Earnings of Prod or Nonsup Workers on Private Nonfarm Payrolls: Manufacturing	Labor Market
104	UMICE		$\Delta lev$	U. of Mich. Index of Consumer Sentiment (Expected Index)	Consumption, Orders and Inventories



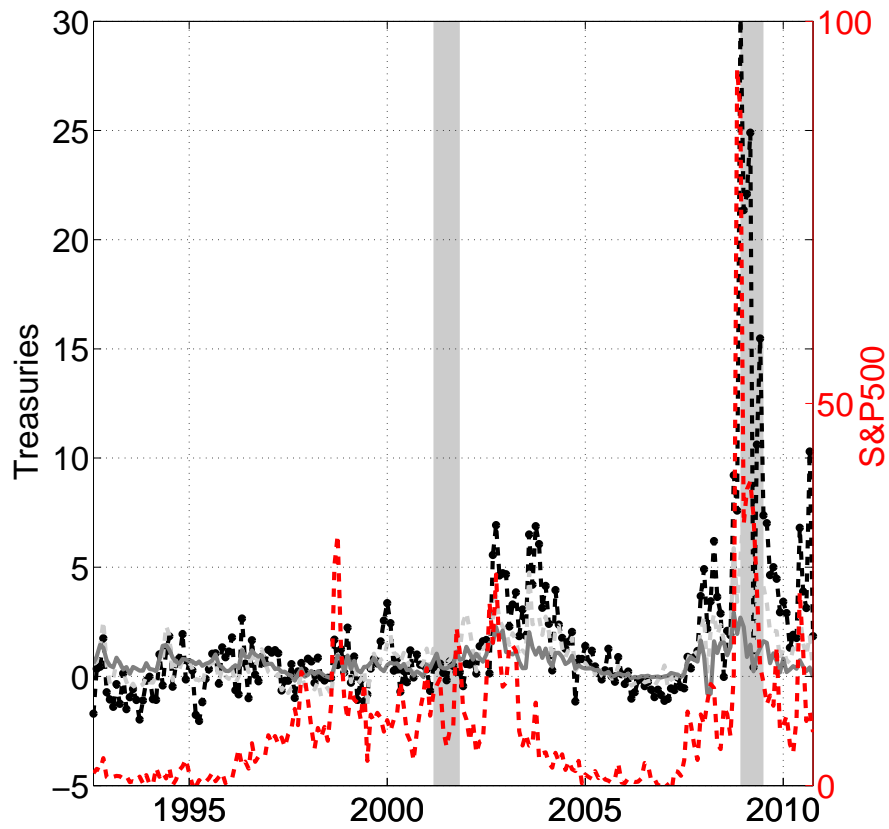
**Figure 1. Synthetic Variance Swap on U.S. Treasuries**

This figure plots the time-series of the payoffs on long positions in variance swaps on the 30 year Treasury bond futures (bold line) and the 10 year Treasury notes futures (dotted line). Shaded areas correspond to recessions as defined by the NBER. Data runs from January 1990 to September 2010.



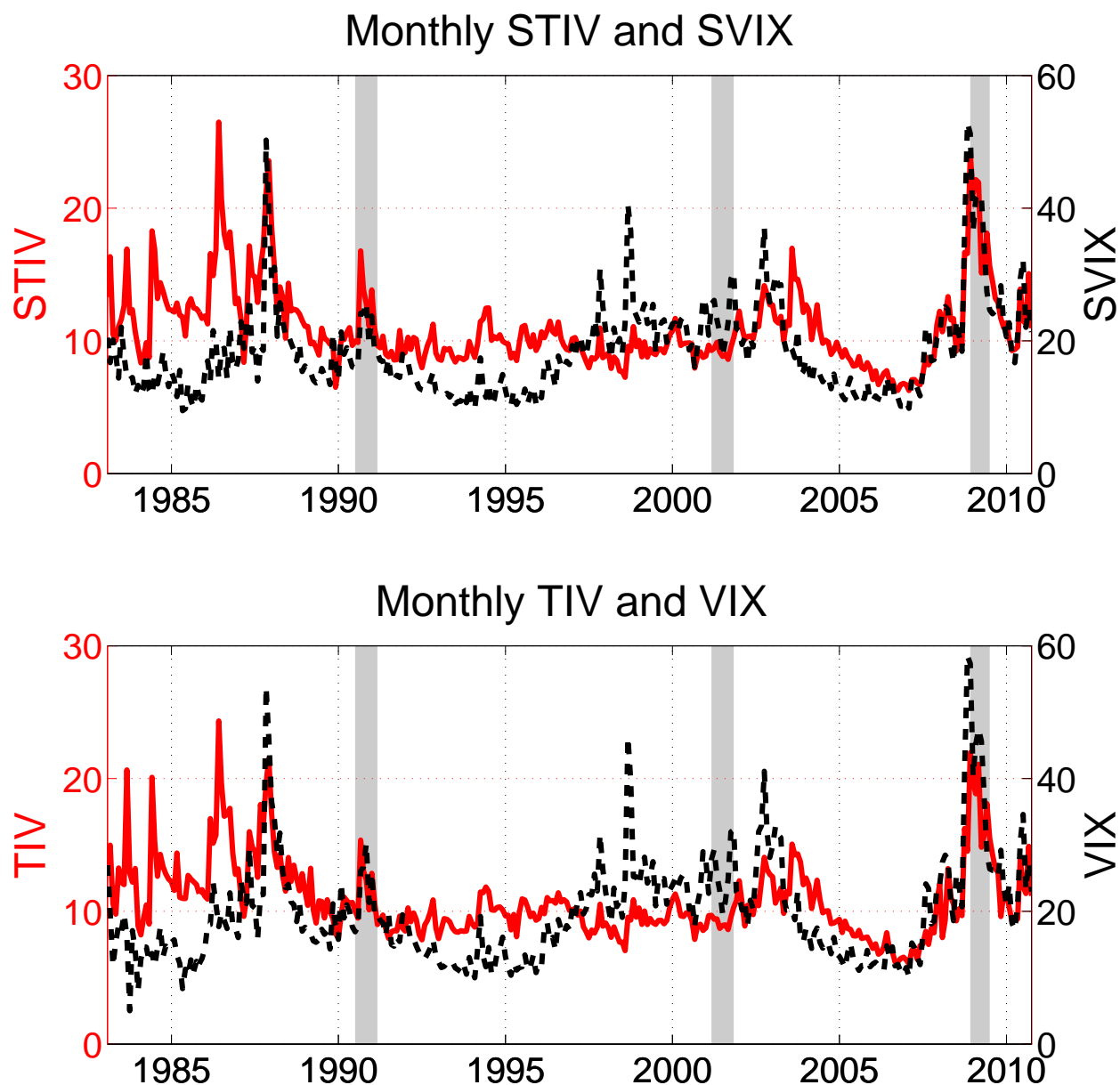
**Figure 2. Realized and Implied Volatility of 30y, 10y and 5y Treasuries**

This figure plots monthly realized (right panels) and implied (left panels) volatility measures for the 30 year, 10 year and 5 year Treasury futures (left axis, bold line) and the S&P 500 futures (right axis, dashed line). Monthly realized volatility measures are obtained from aggregating the daily realized volatility estimations over that month and the monthly implied volatility estimates are the end of month observations. All numbers are annualized in percent. Shaded areas correspond to recessions as defined by the NBER. The data runs from July 1992 to September 2010.



**Figure 3. Variance Risk Premia Treasury Bonds and S&P500**

This figure plots annualized variance risk premia for the 30 year Treasury bonds (left axis) and the S&P500 index (right axis). Shaded areas correspond to recessions as defined by the NBER. Data is monthly and runs from July 1992 to September 2010.



**Figure 4. Simple Treasury IV and SVIX**

This figure plots the Treasury implied volatility measure STIV and the SVIX (upper panel) and TIV and VIX (lower panel) for the period 1983 to 2010. The STIV (SVIX) measure is calculated using daily options on 30 year Treasury bond futures (S&P500 futures) using the simple variance approach in Martin (2011). The TIV (VIX) measure is calculated using daily options on the 30 year Treasury bond futures (S&P500 futures) using the model-free approach in Britten-Jones and Neuberger (2000). The unconditional correlation between STIV and the SVIX (TIV and VIX) is 50% (46%).