Idiosyncratic Risk and the Cross-Section of Stock Returns

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Abstract

Idiosyncratic volatility has received considerable attention in the recent financial literature. Whether average idiosyncratic volatility has recently risen, whether it is a good predictor for aggregate market returns and whether it has a positive relationship with expected returns in the cross-section are still matters of active debate. We revisit these questions from a novel perspective, by taking the cross-sectional variance of stock returns as a measure of average idiosyncratic variance. Two key advantages of this measure are its model-free nature and its observability at any frequency, which allows us to present new results on the properties of daily idiosyncratic volatility series. Through central limit arguments, we formally show that the cross-sectional dispersion of stock returns can be regarded as a consistent and asymptotically efficient estimator for idiosyncratic volatility. We empirically confirm that the cross-sectional measure provides a very good proxy for average idiosyncratic risk as implied by standard asset pricing models and that it predicts well aggregate returns, especially at the daily frequency. The predictability power of idiosyncratic risk is further increased when adding a measure of cross-sectional skewness to the cross-sectional variance factor. We finally provide evidence that idiosyncratic risk is a positively rewarded risk factor.

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1 Introduction

The recent financial literature has paid considerable attention to idiosyncratic volatility. Campbell et al. (2001) and Malkiel and Xu (2002) document that idiosyncratic volatility increased over time, while Brandt et al. (2009) show that this trend completely reversed itself by 2007, falling below pre-1990s levels and suggest that the increase in idiosyncratic volatility through the 1990s was not a time trend but rather an "episodic phenomenon". Bekaert et al. (2008) confirm that there is no trend both for the United States and other developed countries. A second fact about idiosyncratic volatility is also a source of contention. Goyal and Santa-Clara (2003) put forward that idiosyncratic volatility has forecasting power for future excess returns, while Bali et al. (2005) and Wei and Zhang (2005) find that the positive relationship is not robust to the sample chosen. Finally, while some economic theories suggest that idiosyncratic volatility should be positively related to expected returns, Ang et al. (2006) find that stocks with high idiosyncratic volatility have low average returns.

An underlying issue in all these studies is the measurement of idiosyncratic volatility. Campbell et al. (2001) use a value-weighted sum of individual firm idiosyncratic variances, computed as the variances of residuals of differences between individual firm returns and the return of an industry portfolio to which the firm belongs.[1] In addition to this measure, Bekaert et al. (2008) use also the individual firm residuals of a standard Fama and French three-factor model to compute a value-weighted aggregate idiosyncratic volatility.[2]

We revisit the issues regarding the dynamics and forecasting power of idiosyncratic variance by using instead the cross-sectional dispersion of stock returns. Through central limit arguments, we provide the formal conditions under which the cross-sectional variance (CSV) of stock returns asymptotically converges towards the average idiosyncratic variance.[3] One key advantage of this measure is obviously its observability at any frequency, while the previous approaches have used monthly measures based on time series of daily returns. A second important feature is that this measure is model-free, since we do not need to obtain residuals from a particular model to compute it.

We confirm empirically that the cross-sectional variance is an excellent proxy for the idiosyncratic variance obtained from the CAPM or the Fama-French models, as done in the previous literature. Correlations between the CSV measure and the model-based measures estimated monthly, are always above 99%, whether we consider equally-weighted or

[1] This amounts to imposing unit beta restrictions in an industry-market model.

[2] This is also the approach followed in Ang et al. (2006).

[3] Goyal and Santa-Clara (2003) argue informally that their measure can be interpreted as a measure of cross-sectional dispersion of stock returns, but do not establish a formal link between the two. In the practitioners' literature (see DiBartolomeo (2006)), cross-sectional dispersion of returns is called *variety* and is used in risk management and performance analysis.

capitalization-weighted measures of idiosyncratic variance. We also estimate a regime-switching model for CSV time series at both daily and monthly frequencies and find remarkably coherent results in terms of parameter estimates. If we were to build a daily series of model-based idiosyncratic variance, we will roll a window of one-month of daily data, which will result in a very persistent time series. We construct such a daily series but could not find any regimes. This reinforces the usefulness of the CSV to capture idiosyncratic volatility at high frequency.

The regime-switching model indicates clearly that the CSV is counter-cyclical, the dispersion of returns being high and quite variable when economic growth subsides. We analyze further the relation between CSV and economic and financial variables. In particular, we find that there exists a substantial correlation between the equal-weighted CSV and consumption growth volatility. This is consistent with Tédongap (2010) who provides strong evidence that consumption volatility risk explains a high percentage of the cross-sectional dispersion in average stock returns for the usual set of size and book-to-market portfolios that have been used in tests of asset pricing models. In intertemporal asset pricing models of Bansal and Yaron (2004), Bollerslev et al. (2009) and Bollerslev et al. (2009), consumption growth volatility is a measure of economic uncertainty, which is a priced risk factor that affects returns, therefore providing a rationale for the observed correlation between CSV and consumption growth volatility.

On the debate about predictability of aggregate returns by the idiosyncratic variance, we first verify empirically that the CSV measure leads to the same conclusions that other studies (in particular Goyal and Santa-Clara (2003) and Bali et al. (2005)) have reported at the monthly frequency. Then, we report new results at the daily frequency. Specifically, we show that the predictive power of idiosyncratic volatility is much stronger both quantitatively and statistically at the daily frequency than at the monthly frequency. This relationship is robust to the inclusion of return variance and option-implied variance as additional variables in the predictive regressions.

We find that the relation is much stronger and stable across periods between the equally-weighted measure of aggregate idiosyncratic volatility and the returns on the equally-weighted index than for the market-cap weighted equivalents. Economic sources of heterogeneity between firms, as diverse as they can be, are better reflected in an equally weighted measure, all other things being equal. This argument is consistent with previous findings in Bali et al. (2005), who argue that the relationship between equal-weighted average idiosyncratic risk and the market-cap weighted index on the sample ending in 1999:12 is mostly driven by small stocks traded in the NASDAQ. Of course, when the bubble burst, the market capitalization of dot.com small firms was relatively more affected causing the relationship to break down in 2000 and 2001. This effect is not prevalent in an equally-weighted index, for which the relationship remains strong.

However, the frequency at which predictive regressions are run has an important impact on the results, since at lower frequencies we find little evidence of predictability for the equally-weighted measure of CSV. At quarterly and annual frequencies, we find that the capitalization-weighted measure of CSV is a very strong predictor of the aggregate value-weighted returns. When using CSV^{CW} alone as a predictor we obtain remarkable R^2s of 4% and 26% at quarterly and annual frequencies, respectively. Adding the implied variance brings the R^2s to almost 19% and 29%. In all these predictability regressions, the sign of the CSV^{CW} variable is negative. We relate these results to potential explanations in terms of missing factors, Guo and Savickas (2008), or dispersion of investors' opinions, Cao et al. (2005).

Finally, we unveil an asymmetry in the relationship between idiosyncratic variance and returns and show that the predictive power of specific risk is substantially increased when a cross-sectional measure for idiosyncratic skewness is added as explanatory variable. In fact, this is yet another key advantage of our measure that it lends itself to straightforward extensions to higher-order moments.

The statistical significance of the moments of the cross-sectional distribution in these predictive regressions of future returns is not the same as the cross-sectional pricing of stocks or portfolios. However, as emphasized in Goyal and Santa-Clara (2003), the two pieces of evidence are related. Using a Fama-MacBeth procedure with several sets of portfolios, we find support for a positive and significant price of risk for the exposure to the idiosyncratic variance risk. Theoretical rationalizations of a positive relation between idiosyncratic risk and expected returns can be found in the asset pricing literature. Levy (1978), Merton (1987) and Malkiel and Xu (2002) pricing models relate stock returns to their beta with the market and their beta to market-wide measures of idiosyncratic risk. In these models, an important portion of investors' portfolios may differ from the market. Their holdings may be affected by corporate compensation policies, borrowing constraints, heterogeneous beliefs and include *non-traded* assets that add background risk to their traded portfolio decisions (e.g. human capital and private businesses). These theoretical predictions are also in line with Campbell et al. (2001)'s argument that investors holding a limited number of stocks hoping to approximate a well-diversified portfolio would end up being affected by changes in idiosyncratic volatility just as much as by changes in market volatility. More recently, Guo and Savickas (2008) argue that changes in average idiosyncratic volatility provide a proxy for changes in the investment opportunity set and that this proxy is closely related to the book-to-market factor[4].

Ang et al. (2006) and Ang et al. (2009) find results that are opposite to our findings and to these theories since stocks with high idiosyncratic volatility have low average returns but cannot fully rationalize this result. However, Huang et al. (2009) find that the negative sign in the relationship between idiosyncratic variance and expected returns at the stock level becomes positive after controlling for return reversals. Similarly, Fu (2009) documents that high idiosyncratic volatilities of individual stocks are contemporaneous with high returns, which tend to reverse in the following month.

The rest of the paper is organized as follows. In Section 2, we provide a formal argument for choosing the cross-sectional variance of returns as a measure of average idiosyncratic volatility, explore its asymptotic and finite-distance properties, as well as the assumptions behind its use, and compare it to other measures formerly selected in the literature. Section 3 provides an empirical implementation of the concept, again in comparison with other measures, by studying its time-series behavior, outlining the presence of regimes and a counter-cyclical property. In Section 4, we provide new results on the predictability of returns by idiosyncratic volatility, and we also extend the analysis to idiosyncratic skewness. Section 5 focuses on the analysis of the cross-sectional relationship between idiosyncratic risk and expected returns. Section 6 concludes and a technical appendix collects proofs and more formal derivations.

2 The Cross-sectional Variance as a Measure of Idiosyncratic Variance

Let N_t be the total number of stocks in a given universe at day t, and assume with no loss of generality a conditional single factor model for excess stock returns.[5] That is, we assume that for all $i = 1, ..., N_t$, the return on stock i in excess of the risk-free rate can be written as:

$$r_{it} = \beta_{it} F_t + \varepsilon_{it}. \tag{1}$$

where F_t is the factor excess return at time t, β_{it} is the beta of stock i at time t, and ε_{it} is the residual, with $E(\varepsilon_{it}) = 0$ and $cov(F_t, \varepsilon_{it}) = 0$. We assume that the factor model under

[4] Alternative explanations of the relation between idiosyncratic risk and return are the firm's assets' call-option interpretation by Merton (1974) where equity is a function of total volatility as in Black and Scholes (1973) as well as Barberis et al. (2001) prospect theory asset pricing model with loss aversion over (owned) individual stock's variance.

[5] Assuming a single factor structure is done for simplicity of exposure only and the results below can easily be extended to a multi-factor setting.

consideration is a strict factor model, that is $cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ for $i \neq j$.[6]

Given T observations of the stock returns and the factor return, one can use the residuals of the regression to obtain a measure of the idiosyncratic variance of asset i by: $\sigma_i^2 = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}^2$. An average measure of idiosyncratic variance over the T observations (say a month) can be obtained by averaging across assets such individual idiosyncratic variance estimates. This is the approach that has been followed by most related papers with observations of the returns at a daily frequency to compute monthly idiosyncratic variances.

We propose instead to measure at each time t the cross-sectional variance of observed stock returns. Using formal central-limit arguments, we show that, under mild simplifying assumptions, this cross-sectional measure provides a very good approximation for average idiosyncratic variance. In contrast with most previous measures of average idiosyncratic variance, the CSV offers two main advantages: it can be computed directly from observed returns, with no need to estimate other parameters such as betas, and it is readily available at any frequency and for any universe of stocks.

2.1 Measuring the cross-sectional variance

To see this, first let $(w_t)_{t\geq 0}$ be a given weight vector process. The return on the portfolio defined by the weight vector process (w_t) is denoted by $r_t^{(w_t)}$ and given by:

$$r_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it} r_{it}. \tag{2}$$

We restrict our attention to non-trivial weighting schemes, ruling out situations such that the portfolio is composed by a single stock. We also restrict the weights to be positive at every given point in time. Hence, a weighting scheme (w_t) is a vector process which satisfies $0 < w_{it} < 1 \; \forall \; i, t$.

The cross-sectional variance measure is defined as follows.

Definition (***CSV***): The ***cross-sectional variance*** measure under the weighting scheme (w_t), denoted by $\boldsymbol{CSV}_t^{(w_t)}$, is given by

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it} \left(r_{it} - r_t^{(w_t)} \right)^2. \tag{3}$$

[6] This assumption is made in the single index or diagonal model of Sharpe (1963) and in the derivation of the APT in Ross (1976). It implies that all commonalities are explained by the factor model in place. One should notice that the very definition of idiosyncratic risk relies precisely on the assumption of orthogonal residuals: assuming that the model is the "true" factor model implies that the"true" idiosyncratic risk is the one measured with respect to that model, which in turn implies that no commonalities should be left after controlling for the common factor exposure.

A particular case of interest is the ***equally-weighted CSV*** (or EW CSV), denoted by CSV_t^{EW} and corresponding to the weighting scheme $w_{it} = 1/N_t \ \forall \ i, t$:

$$CSV_t^{EW} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left(r_{it} - r_t^{EW}\right)^2, \tag{4}$$

where r_t^{EW} is the return on the equally-weighted portfolio.

Another weighting scheme of interest is the cap weighting scheme. If we denote by c_{it} be the market capitalization of stock i at the beginning of the month corresponding to day t, $C_t = \sum_{i=1}^{N_t} c_{ti}$ the total market capitalization and r_t^{CW} the return on the market capitalization-weighted portfolio, the ***cap-weighted (CW)*** (or CW CSV) is defined as:

$$CSV_t^{CW} = \sum_{i=1}^{N_t} w_{it}^{CW} \left(r_{it} - r_t^{CW}\right)^2, \tag{5}$$

where $w_{it}^{CW} = \sum_{i=1}^{N_t} \frac{c_{it}}{C_t}$.

For any given weighting scheme (in particular EW or CW), the corresponding cross-sectional measure is readily computable at any frequency from observed returns. This stands in contrast with the previous approaches that have used monthly measures based on time series regressions on daily returns. The second important feature of the CSV is its model-free nature, since we do not need to specify a particular factor model to compute it.[7]

2.2 A Formal Relationship between CSV and Idiosyncratic Variance

The following proposition establishes a formal link between CSV and idiosyncratic variance. It is an asymptotic result ($N_t \to \infty$) obtained under the assumptions of homogeneous betas and residual variances across stocks, i.e. $\beta_{it} = \beta_t = 1 \ \forall \ i$, $E(\varepsilon_{it}^2) = \sigma_\varepsilon^2(t) \ \forall i$. These assumptions will be relaxed below.

Proposition 1 ***(CSV as a proxy for idiosyncratic variance - asymptotic results)****:*
Assume $\beta_{it} = \beta_t = 1 \ \forall \ i$ (homogeneous beta assumption) and $E(\varepsilon_{it}^2) = \sigma_\varepsilon^2(t) \ \forall i$ (homogeneous residual variance assumption), then for any strictly positive weighting scheme, we have that:

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it} \left(r_{it} - r_t^{(w_t)}\right)^2 \underset{N_t \to \infty}{\longrightarrow} \sigma_\varepsilon^2(t) \ \textit{almost surely.} \tag{6}$$

[7]While Goyal and Santa-Clara (2003) and Wei and Zhang (2005)consider the equally-weighted CSV in conjunction with other measures, they do not provide a thorough discussion about the conditions under which it can be interpreted as a proxy for idiosyncratic variance nor their empirical validity in the data, as we provide in this paper.

Proof See Appendix A.

This result is important because it draws a formal relationship between the dynamics of the cross-sectional dispersion of realized returns and the dynamics of idiosyncratic variance. Note that this asymptotic result $CSV_t^{(w_t)} \longrightarrow \sigma_\varepsilon^2(t)$ holds for any weighting scheme that satisfies $0 < w_{it} < 1 \; \forall \; i, t$. Of course, at finite distance, different weighting schemes will generate different proxies for idiosyncratic variance. In the empirical analysis that follows, we shall focus on the equally-weighted scheme, while also considering the cap-weighted scheme for comparison purposes. Formal justification for our focus on the equally-weighted scheme is provided in the next section, where we show that the EW CSV is the best estimator for idiosyncratic variance within the class of CSV obtained under a strictly positive weighting scheme.

2.3 Properties of CSV as an Estimator for Idiosyncratic Variance

First, we derive in Proposition 2 the bias and the variance of the CSV as an estimator of idiosyncratic variance. Then we study their asymptotic limits as the number of firms grows large and conclude that the equally-weighted CSV is the best among all-positively-weighted estimators.

Proposition 2 ***(Bias and variance of CSV)***:
Maintaining the homogenous beta assumption ($\beta_{it} = \beta_t = 1 \; \forall \; i, t$) and the homogeneous residual variance assumption ($E(\varepsilon_{it}^2) = \sigma_\varepsilon^2(t) \; \forall i$), for any strictly positive weighting scheme, we have that:

$$E\left[CSV_t^{(w_t)}\right] = \sigma_\varepsilon^2(t)\left(1 - \sum_{i=1}^{N_t} w_{it}^2\right) \tag{7}$$

To analyze the variance of the CSV estimator, we further make the assumption of multivariate normal residuals $\varepsilon \sim N(0, \Sigma^\varepsilon)$, where Σ^ε denotes the variance covariance matrix of the residuals. Under this additional assumption, we obtain:

$$Var\left[CSV_t^{(w_t)}\right] = 2\sigma_\varepsilon^2(t)\left(\left(\sum_{i=1}^{N_t} w_{it}^2\right)^2 + \sum_{i=1}^{N_t} w_{it}^2 - 2\sum_{i=1}^{N_t} w_{it}^3\right) \tag{8}$$

Proof See Appendix B for a proof in the slightly more general case when the homogeneous specific variance assumption has been relaxed.

Hence the CSV is a biased estimator for idiosyncratic variance, with a bias given by the multiplicative factor $\left(1 - \sum_{i=1}^{N_t} w_{it}^2\right)$, which can be easily corrected for since it is available in explicit form. In the end, the bias and variance of the CSV appear to be minimum for the EW scheme, which corresponds to taking $w_{it} = 1/N_t$ at each date t. It is easy to see, that this bias disappears and the variance tends to zero for the equally-weighted scheme when the number of stocks grows large, as explained in the following proposition.

Proposition 3 ***(Properties of the equally-weighted CSV)***

The bias and variance of the EW CSV as an estimator for specific variance disappear in the limit of an increasingly large number of stocks:

$$E\left[CSV_t^{EW}\right] \underset{N_t \to \infty}{\longrightarrow} \sigma_\varepsilon^2\left(t\right).$$
$$Var\left(CSV_t^{EW)}\right) \underset{N_t \to \infty}{\longrightarrow} 0.$$

Proof See Appendix B for a proof in the slightly more general case when the homogeneous specific variance assumption has been relaxed..

The equally-weighted CSV thus appears to be a consistent and asymptotically efficient estimator for idiosyncratic variance. As such, it is the best estimator in the class of CSV estimators defined under any positive weighting scheme, and it dominates in particular the cap-weighted CSV as an estimator for idiosyncratic variance. If we relax the homogeneous residual variance assumption, we obtain that:

$$E\left[CSV_t^{EW}\right] \underset{N_t \to \infty}{\longrightarrow} \frac{1}{N_t}\sum_{i=1}^{N_t}\sigma_{\varepsilon_{it}}^2.$$

Hence, the assumption of homogenous residual variances comes with no loss of generality. In the general case with non-homogenous variances, the CSV simply appears to be an asymptotically unbiased estimator for the *average* idiosyncratic variance of the stocks in the universe. We also have:

$$Var\left(CSV_t^{EW}\right) < 2\bar{\sigma}_\varepsilon^4\left(t\right)\left(\frac{1}{N_t}\right) \underset{N_t \to \infty}{\longrightarrow} 0.$$

where the quantity $\bar{\sigma}_\varepsilon^2\left(t\right)$ is an upper bound for the individual idiosyncratic variances (see Appendix B).

We now discuss the impact on these results of relaxing the homogeneous beta assumption.

2.4 Relaxing the Homogeneity Assumption for Factor Loadings

Relaxing the homogenous beta assumption involves a bias that remains strictly positive even for an infinite number of stocks and an equal-weighting scheme. We characterize this bias in the next proposition in order to gauge its magnitude for given models of returns.

Proposition 4 ***Bias of CSV as an estimator for average idiosyncratic variance in the presence of heterogenous betas****: Relaxing the assumptions $\beta_{it} = \beta_t = 1 \ \forall \ i, t$ (homogeneous beta assumption) we have, for any strictly positive weighting scheme:*

$$E\left[CSV_t^{(w_t)}\right] = \sum_{i=1}^{N_t} w_{it}\sigma_{\varepsilon_i}^2(t) - \sum_{i=1}^{N_t} w_{it}^2\sigma_{\varepsilon_i}^2(t) + E\left[F_t^2 CSV_t^{\beta}\right], \tag{9}$$

where CSV_t^{β} denotes the cross-sectional variance of stock betas:

$$CSV_t^{\beta} = \sum_{i=1}^{N_t} w_{it}\left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt}\right)^2.$$

Proof See Appendix C.

The first term $\sum_{i=1}^{N_t} w_{it}\sigma_{\varepsilon_i}^2(t)$ in equation (9) represents the average idiosyncratic variance of stocks within the universe under consideration. The second term $-\sum_{i=1}^{N_t} w_{it}^2\sigma_{\varepsilon_i}^2(t)$ is the negative bias that was also present even in the presence homogenous beta assumptions. If we focus on the equally-weighted scheme, the sum of these two terms is equal to $\frac{1}{N_t}\sum_{i=1}^{N_t}\sigma_{\varepsilon_i}^2(t)\left(1 - \frac{1}{N_t}\right)$ so that the bias disappears in the limit of an increasingly large number of stocks. The third term $E\left[F_t^2 CSV_t^{\beta}\right]$ in equation (9) represents, on the other hand, an additional (positive) bias for the CSV as an estimator of average idiosyncratic variance, which is introduced by the cross-sectional dispersion in betas, and which does not disappear in the limit of a large number of stocks.

Using the explicit expression provided here, in section 3.1 we directly measure this *beta dispersion bias* using the CAPM and the Fama and French three-factor model as benchmark factor models. As we will see, although the cross-sectional dispersion of betas has a non-negligible magnitude, once it is multiplied by the square of the return of the market portfolio its relative size with respect to the level of idiosyncratic risk becomes very small. An extensive analysis of the CSV in the empirical section suggests that the homogeneous beta assumption does not represent a material problem for the CSV as an estimator of idiosyncratic variance as implied by standard asset pricing models (i.e. CAPM and Fama-French).

2.5 Competing Measures of Idiosyncratic Risk

In this section, we describe measures that have been used in the literature, and which will be used for comparison purposes in subsequent sections of the paper. The standard approach consists of considering idiosyncratic variance either relative to the CAPM and or to the Fama-French (FF) model (Fama and French (1993)):

$$r_{it} = b_{0it} + b_{1it}XMKT_t + b_{2it}SMB_t + b_{3it}HML_t + \varepsilon_{it}^{FF} \tag{10}$$

where r_{it} denotes the excess return at time t of stock i, $XMKT$ is the excess return on the market portfolio, SMB is the size factor and HML is the value factor. The idiosyncratic variance for asset i is the variance of the residuals of the regression, that is, $\sigma^2(\varepsilon_{it}^{FF})$. To obtain an estimate for average idiosyncratic variance, Bekaert et al. (2008) and Wei and Zhang (2006) use a market capitalization weighting:

$$FF_t^{CW} = \sum_{i=1}^{N_t} w_{it}\sigma^2(\varepsilon_{it}^{FF}). \tag{11}$$

For comparison purposes we also look at the equally-weighted average of FF idiosyncratic variance in what follows. An alternative approach to average (*mostly*) idiosyncratic risk estimation has been suggested by Goyal and Santa-Clara (2003), with a measure given by:

$$GS_t^{EW} = \frac{1}{N_t}\sum_{i=1}^{N_t}\left[\sum_{d=1}^{D_t} r_{id}^2 + 2\sum_{d=2}^{D_t} r_{id}r_{id-1}\right], \tag{12}$$

where r_{id} is the return on stock i in day d and D_t is the number of trading days in month t.[8]

Campbell et al. (2001) propose yet an alternative measure of average idiosyncratic variance, under a very particular setting that allows one to avoid running regressions each period.[9] However, their measure is not instantaneous since a window of data is still needed to estimate individual variances. In what follows, we do not repeat the analysis with this measure because Bekaert et al. (2008) have shown that it is very closely related to the measure obtained from standard asset pricing models. In particular, Bekaert et al. (2008) find that the measure of Campbell et al. (2001) and the FF-based one have a correlation of 98% and share most of the same structural breaks.

3 Empirical Implementation

In order to perform an empirical analysis of our measure for idiosyncratic risk, we collect daily US stock returns (common equity shares only) and their market capitalization from

[8] As in Goyal and Santa-Clara (2003), when the second term makes the estimate negative, it is ignored. This measure has been originally used in French et al. (1987).

[9] They assume that all betas are equal to one and substract industry returns in addition to market returns to control for risk.

CRSP data base. Our longest sample runs from July 1963 to December 2006. We also extract the FF factors and the one-month Treasury bill from Kenneth French web-site data library for the same sample period. Each month, we drop stocks with missing returns and with non-positive market capitalization at the beginning of the month. The number of firms varies between 377 and 7293, and remains greater than one thousand 75% of the time. The maximum number of stocks is reached during the .com bubble. Then, we estimate every month the cap-weighted idiosyncratic variance as in equation (11), as well as the equal-weighted version.[10] Similarly, we estimate the cap-weighted and equal weighted average idiosyncratic variance relative to the CAPM. We also estimate the GS average variance measure as in equation (12) and its cap-weighted version. Finally, we estimate on a daily basis the equal and cap-weighted versions of the CSV as in equations (4) and (5). In order to construct the monthly series for our cross-sectional measures, we estimate the average of the daily series at the end of each month. For comparison purposes we also estimate the FF-based average idiosyncratic variance (EW and CW) on a daily basis using a rolling window sample of one month. We annualize all figures in order to compare daily and monthly measures. Following Bekaert et al. (2008), we fit a regime-switching model to the monthly and daily series in order to further compare the different measures. Last, we look at the relation between the CSV measures of idiosyncratic variance and selected economic and financial variables.

3.1 Measuring the CSV bias

Some of the previous research on idiosyncratic volatility has been conducted under the assumption of homogeneous betas across stocks (see Campbell et al. (2001) and Goyal and Santa-Clara (2003) in particular). As illustrated in Proposition 4 and discussed in Appendix C, the presence of non-homogeneous betas introduces a positive bias on the CSV as an estimator for average idiosyncratic variance, which is given by the first term in equation (9). We now measure the impact of this bias with respect to the CAPM as a benchmark model.

First, we compute the bias $E\left[F_t^2 CSV_t^\beta\right]$ for every month in the sample using beta estimates for each stock with both the equal-weighted and the cap-weighted market returns. To gauge its importance, we divide it by the average idiosyncratic variance, also measured with respect to the CAPM.[11]

Table 1 presents a summary of the distribution of the time series of cross-sectional dispersion of betas, its product with the squared return of the market portfolio (hence the

[10] We use previous period market capitalization and assume it is constant within the month.

[11] This is measured as in equation (11) with just the market returns with both weighting schemes.

bias itself) and the proportion of this bias with respect to the average idiosyncratic variance at the end of every month. Although the cross-sectional dispersion of betas is sizable, once it is multiplied by the squared return of the market portfolio, the size of the bias remains relatively small. The median of the distribution of $\frac{F_t^2 CSV_t^\beta}{\sigma^2_{\varepsilon_t}}$, is 0.348% for the equal-weighted scheme and 0.351% for the cap-weighted measure, computed over the whole sample (July 1963 to December 2006). The 97.5 quantiles are 3.24 and 3.47 respectively.

On the other hand, the formal discussion about the properties of the CSV as a measure of idiosyncratic variance on section 2.4 also uncovered the fact that another bias (but negative in sign) coming from the CSV weighting scheme concentration is also introduced. Proposition 2 predicts two properties about this weighting bias: first, it should be negative and minimal for an equally-weighted scheme. Second, it should be very small for a high number of stocks. The beta-bias then is more likely to dominate the concentration-bias when using an equal-weight scheme.

Using the explicit expression for this bias provided in Proposition 4 we estimate the proportion of the size of this weights-concentration bias with respect to the average idiosyncratic variances implied by the CAPM.[12] In the last line of the upper and lower panels of Table 1 we report quantiles of the distribution of this bias for both weighting schemes. The corresponding medians are 0.030% and 0.426% for the EW and CW schemes respectively. Since the bias is of opposite sign to the beta cross-sectional dispersion bias, we need to assess the resulting overall bias.

We measure the total bias as the intercept of a regression of the CSV on the average idiosyncratic variance estimated with respect to the CAPM or the Fama-French three-factor model:

$$CSV_t^{w_t} = bias + \psi \sigma^2_{model}(w_t) + \zeta_t, \tag{13}$$

where w_t refers to the weighting scheme (equal-weight or market-cap) and model stands for either the CAPM or the Fama-French three-factor model.

Table 2 reports summary statistics for regression (13). The bias of the CSV measured with respect to standard asset pricing models is small in magnitude for both weighting schemes (in the order of 10^{-5}). While it remains statistically significant, we can safely consider that the impact of the bias remains immaterial for any practical purposes. Another interesting finding is the sign of the bias. For the equal-weighted quantities, the sign of the bias is positive, while it is negative for the cap-weighted ones. Therefore, the beta bias dominates the weighting bias for equal-weighted averages in both models. This is consistent with the prediction made by the theoretical analysis regarding the relative impact of the

[12] As noted earlier, it would be straightforward to remove the impact of this bias by dividing the CSV measure by the factor $\left(1 - \sum_{i=1}^{N_t} w_{it}^2\right)$, equal to $\left(1 - \frac{1}{N_t}\right)$ in the EW case.

weighting-bias for different weighting schemes. Regarding the model, the bias is larger when the idiosyncratic variance is measured with respect to the Fama-French model instead of the CAPM for both weighting schemes, as expected, but its magnitude remains negligible.

3.2 Comparison with Other Measures

In this section we compare the CSV measure to the afore-mentioned, more conventional, measures of idiosyncratic risk (i.e., FF-based, CAPM-based and GS). To obtain these other measures, we need to re-estimate the relevant factor model using a rolling window of one-month worth of daily data to allow for time-variation in beta estimates (or total-variance variation for the GS). In Table 3, we report summary statistics for the monthly time series of annualized idiosyncratic variances based on 516 observations from January 1964 to December 2006.[13]

On the monthly series, the annualized means of the equally-weighted CSV, FF-based and CAPM-based measures are 38.4%, 38.3% and 38.7%, respectively, while the EW GS variance is 34.2%. The standard deviations are 8.5%, 8.6%, 8.7% for the CSV, FF-based and CAPM-based measures and 7.0% for the GS measure. For the cap-weighted version, the CSV, FF and CAPM idiosyncratic variance measures have an annualized mean of 8.5%, 7.6%, 8.0%, respectively and the GS measure mean is 11.2%. The standard deviations are also closer for the CSV, FF and CAPM measures than for GS. Although GS argue that their measure fundamentally constitutes a measure of idiosyncratic risk, with the idiosyncratic component accounting for about 85% of the total EW average measure, it is strictly speaking an average of total stock variance. Our CSV measure is very close to idiosyncratic variance measures derived from traditional asset pricing models, confirming that the assumption about beta homogeneity is not a major problem.

The cross-correlation matrix reported in Table 3 provides further evidence on the closeness of the CSV to the other model-based measures. Correlations are very high between CSV^{EW} and $CAPM^{EW}$ (99.93%) and FF^{EW} (99.75%, as well as between CSV^{CW} and $CAPM^{CW}$ (99.48%) and CSV^{CW} and FF^{CW} (98.56%). The high correlations between the CAPM and the FF measures (99.88% and 99.18% for EW and CW respectively) also indicate that adding factors does not drastically affect the estimation of idiosyncratic variance. Correlations between the GS measures and the other measures are always smaller but remain close to 90% when considering the same weighting scheme. Correlations between measures for different weighting schemes are much lower, irrespective of the estimation method, indicating that the choice over the weighting scheme is fundamentally important

[13] In this section of the paper, we start the sample period in January 1964 to allow for direct comparison with Bekaert et al. (2008). In the predictability section, we instead start the sample in July 1963.

for estimating idiosyncratic variance, as stressed in our theoretical analysis in section 2.

Table 4 provides mean and standard-deviation estimates for the daily average idiosyncratic variance measures. The mean of the EW CSV is 38.4%, practically equal to the mean of EW idiosyncratic variance based on the FF model. For the cap-weighted measures, the CSV has a slightly higher mean than the FF-based one. For the CSV daily series, the standard deviation is higher than for the FF-based measure for both weighting schemes. This is due to the different nature of the two series. The CSV only includes information from the cross-section of realized returns, while the FF idiosyncratic variance is a persistent, overlapping, rolling-window estimate. Each daily estimate of idiosyncratic variance for the FF model differs from the previous one by only two observations out of the approximately 21 trading days included in a month (the first and last days).

The smoothness of the idiosyncratic variance estimates obtained with the rolling-window methodology is illustrated in Figures 1 and 2, which plot daily CSV and FF idiosyncratic variances for each weighting scheme respectively. It should also be noted that the estimation of the FF-measure is computationally much more expensive than for the CSV measure, which is based on observable quantities.

The lower panel of Table 4 presents cross-correlations for the daily series of idiosyncratic variance measures. Although the coefficients are smaller than for the monthly series, the relationship remains strong provided the comparison is done for the same weighting scheme: 82.6% and 73.9% for EW and CW measures respectively. The difference with the monthly series correlations may again be explained by the presence of the smoothed estimation procedure inherent to the FF-based measure. Overall, it appears that the CSV measure is extremely close to CAPM or FF-based measures at the monthly frequency, when the latter measures suffer from no particular bias, and that the CSV measure appears to be a good and instantaneous proxy for idiosyncratic variance at the daily frequency, when the standard measures are subject to artificial smoothing due to overlapping data.

3.3 Extracting Regimes in Idiosyncratic Risk

Bekaert et al. (2008) fit a Markov regime-switching model with a first-order autocorrelation structure (see Hamilton (1989)) for the monthly series of idiosyncratic variance based on the FF model. In this section, we want to estimate this model with our CSV measure both at the monthly and daily frequencies. While we expect that the fit will be close to Bekaert et al. (2008) for the monthly series given our previous results on the similarity of the series, we want to verify whether such a model provides a similar fit for the daily series.

In this model, two regimes are indexed by a discrete state variable, s_t, which follows a Markov-chain process with constant transition probabilities. Let the current regime be indexed by i and the past regime by j and x_t be the original idiosyncratic variance. In this parsimonious model, x_t follows an AR(1) model:

$$x_t - \mu_i = \phi(x_{t-1} - \mu_j) + \sigma_i e_t, \; i, j \in \{1, 2\} \tag{14}$$

The transition probabilities are denoted by $p = P[s_t = 1|s_{t-1} = 1]$ and $q = P[s_t = 2|s_{t-1} = 2]$). The model involves a total of 7 parameters, $\{\mu_1, \mu_2, \sigma_1, \sigma_2, \phi, p, q\}$.

We first verify that the CSV and the FF-based measures give the same results for the monthly series. The estimation results for the monthly series of the $FF^{CW}, CSV^{CW}, FF^{EW}$ and CSV^{EW} are reported in the upper panel of Table 5. For corresponding weighting schemes, the parameters in both regimes are similar between the two measures. For both measures the low-mean, low-variance regime presents a higher probability of remaining in the same state.

We then fit the same model to the daily time series and present the parameter estimates in the lower panel of Table 5. It should be stressed that for our CSV measure, the parameter values of the average level of idiosyncratic variance μ in both regimes are found to be quite close to the values obtained with the monthly series. This result suggests that the process observed at the daily frequency is not just a noisy series, but actually captures the same underlying process observed at the monthly frequency. This stands in sharp contrast with the FF-based measure, for which the maximum-likelihood estimation procedure could not recognize two regimes when daily data is used, as evidenced by the fact that the parameter values for the mean level of idiosyncratic variance are basically the same for the two regimes. This problem, combined with an autocorrelation parameter very close to one, is likely caused by the overlapping data problem present in the daily FF measure, which corresponds to the *smoothing* effect mentioned in the previous section.

In Figures 3 and 4 we plot the filtered probabilities (conditional on information up to time t) of remaining in state 1 (high-mean and high variance regime), as well as the monthly CSV and FF average idiosyncratic variance time series for the CW and EW weighting schemes, respectively.[14] At the monthly frequency, our measure and the FF-based measure appear to be remarkably close for both the equal-weighted and cap-weighted schemes. Also, we find that the dates of regime changes, marked by the filtered probabilities, are the same most of the times for the cap-weighted and the equal-weighted measures.[15] We also find that periods in the higher-mean and higher-variance regime are more persistent for the equally-weighted measure compared to the cap-weighted measure (except during the tech bubble period).

[14]These are estimates of the transition probabilities conditional to information up to time t given all sample data.

[15]One notable exception is the regime change of 1980 : 05, which is present for the cap-weighted measure and absent for the equally-weighted one.

Overall, our filtered probability series resembles closely the one presented in Bekaert et al. (2008) for the cap-weighted FF and Campbell et al. (2001) measures.[16]

The shaded areas in Figures 3 and 4, which time stamp the NBER recession periods, indicate that the peaks in the probability of remaining in the high-mean high-variance regime coincide most of the times with the contraction periods. Therefore, the CSV measure is counter-cyclical, the dispersion of returns being high and quite variable when economic growth subsides. In the next section, we want to analyze further the relation between the CSV and other economic and financial variables.

3.4 CSV Relation with Economic and Financial Variables

To put this analysis in the proper context, we should go back to the very nature of idiosyncratic risk. In an asset pricing model, it represents the risk that belongs specifically to an individual firm, after accounting for the sources of risk that are common to all firms. In the previous sections, we have shown that the cross-sectional variance of returns provides a very good measure of this idiosyncratic risk, even if it ignores the risk exposures to the usual common risk factors such as the market return or the Fama-French factors. Yet we concluded our time series analysis of CSV by stressing its strong counter-cyclical behavior. To pursue this analysis further we need therefore to rely on equilibrium models that link returns to economic fundamentals. Recently, Bansal and Yaron (2004) have revived consumption-based asset pricing models by showing that two sources of long-run risk — expected consumption growth and consumption volatility as a measure of economic uncertainty — determine asset returns. Further, Tédongap (2010) provides strong evidence that consumption volatility risk explains a high percentage of the cross-sectional dispersion in average stock returns for the usual set of size and book-to-market portfolios that have been used in tests of asset pricing models. Another strand of literature based on the intertemporal CAPM or the conditional CAPM has linked the cross-section of expected returns to other economic or financial variables such as the term spread, default spread, implied or realized measures of aggregate returns variance, and many others.

While our CSV measure is based on the cross-sectional dispersion of realized returns over the whole universe of traded stocks, as opposed to the cross-sectional dispersion of average returns of a limited number of size and book-to-market portfolios, the same theoretical implications should prevail. Therefore, we present below a simple correlation and graphical analysis of the relation between the CSV and some of these key variables. For the economic

[16]The small difference might come from the fact that Bekaert et al. (2008) fit a model with two different autocorrelation coefficients (one for each regime) as opposed to one. However, they find the two coefficients to be fundamentally equal in both regimes, which supports using a more parsimonious model.

variables, we chose consumption-growth volatility as a measure of economic uncertainty. Following Bansal and Yaron (2004) and Tédongap (2010), we filter consumption-growth volatility with a GARCH model. For consumption, we used FRED's personal consumption expenditures of non-durables and services monthly series, divided by the consumer price index and the population values to obtain a per-capita real consumption series. We then compute its growth rate from July 1963 to 2006.[17] The second economic variable we consider is inflation volatility, which we filter also with a GARCH process.[18] For the financial variables we use Welch and Goyal (2008)'s data for corporate bond yields on BAA and AAA-rated bonds, long-term government bond yield and 3-months T-bill rate to estimate the credit spread and term spread (as the difference between the first and the second rate in both cases).[19] In Table 6 we report the correlations between the equally-weighted and cap-weighted measures of cross-sectional variance and the five economic and financial variables during the 1990-2006 period. We also explore some potential asymmetries by computing the CSV^{EW} for the positive and negative returns.

The highest correlation (0.401) is obtained between consumption growth volatility and the equally-weighted measure CSV^{EW}. In Figure 5 we plot the two series for the period 1990 to 2006. While the CSV series is much noisier than consumption-growth volatility, the coincident movements between the two series are quite remarkable. After a high volatile period just before 2000, both series show a marked downward trend after the turn of the century. A reasonable explanation for this strong correlation is to think about a common factor (aggregate economic uncertainty) affecting the idiosyncratic variance of each security. Aggregating over all securities will make the CSV a function of economic uncertainty. In intertemporal asset pricing models of Bansal and Yaron (2004), Bollerslev et al. (2009) and Bollerslev et al. (2009), economic uncertainty is a priced risk factor that affects returns, therefore providing a fundamental rationale for the observed correlation between CSV and consumption growth volatility. This suggests that CSV should appear to be priced when a Fama-MacBeth procedure is applied to a set of portfolios. We explore this issue in Section 5. The correlation of the cap-weighted CSV with consumption growth volatility is not as high (0.241) since it puts more weight on large cap securities, which are in general less

[17]The series IDs at the FRED's webpage are, PCEND and PCES for "Personal Consumption Expenditures: Nondurable Goods" and "Personal Consumption Expenditures: Services", CPIAUCNS for "Consumer Price Index for All Urban Consumers: All Items" and POP for "Total Population: All Ages including Armed Forces Overseas". Bansal and Yaron (2004) used the Bureau of Economic Analysis data available at www.bea.gov/national/consumer_spending.htm on real per-capita annual consumption growth of non-durables and services for the period 1929 to 1998. The series is longer but is available only at annual and quarterly frequencies.

[18]For space considerations, we do not report parameter estimates for the two AR(1)-Garch(1,1) we estimate. They are available upon request from the authors.

[19]Data available at Amit Goyal's webpage: http://www.bus.emory.edu/AGoyal/Research.html

affected by economic uncertainty. Looking at the split between CSV^{EW+} and CSV^{EW-}, we see that the correlation is higher for the CSV when conditioning on the negative returns (0.346). This suggests that return dispersion in bear periods is relatively more affected by economic uncertainty.

The next most highly negatively correlated variable is inflation volatility (-0.367). Since 1998, inflation volatility seems to have been on an upward trend, while the cross-sectional variance of returns has been sharply declining. This is clearly apparent in Figure 6. In presence of higher inflation uncertainty, investors will move towards allocating more to stocks relative to bonds in their portfolios, generating a general increase in stock returns that reduces their cross-sectional variance. The T-bill rate is also relatively highly correlated with CSV^{EW} (0.302). In the type of equilibrium models we have referred to, the risk-free rate, proxied here by the T-bill, will be a function of consumption growth volatility, hence its positive relation with the cross-sectional variance.

For the financial variables (credit spread and term spread), it is interesting to note that the higher correlations are with the cap-weighted measures of the cross-sectional variance. The signs are intuitive. Credit risk affects differently individual firm returns and therefore tends to increase CSV, while a pervasive term spread risk will reduce dispersion by being common to many securities due to a move of investors away from bonds into the stock market.

Given that the cross-sectional variance is significantly linked with economic and financial factors that have been shown to predict returns, we explore in the next section the predictive power of CSV for aggregate returns at various frequencies, especially at daily frequencies, since our measure of idiosyncratic variance allows us to measure CSV at any frequency without any artificial smoothing effect. This is a main advantage over other methods of recovering this idiosyncratic variance.

4 New Evidence on the Predictability of the Market Return

There is an ongoing debate on the predictive power of average idiosyncratic variance for average (or aggregate) stock market returns. Goyal and Santa-Clara (2003) find a significantly positive relationship between the equal-weighted average idiosyncratic stock variance and the cap-weighted portfolio returns for the period 1963:07 to 1999:12. They find that their measure of average idiosyncratic (in fact total) variance has a significant relationship with next month return on the cap-weighted portfolio. The regression in GS is as follows:

$$r_{t+1}^{CW} = \alpha + \beta \nu_t^{EW} + \varepsilon_{t+1}, \tag{15}$$

where ν_t^{EW} corresponds to GS_t^{EW}. In a subsequent analysis, Bali et al. (2005) argue that this relationship disappeared for the extended sample 1963:07 to 2001:12, and attribute the relationship observed in GS to high-tech-bubble-type stocks (i.e., stocks traded on the NASDAQ) and a liquidity premium. In a similar way, Wei and Zhang (2005) find that the significance of the relationship found by GS disappeared for their sample 1963:07 to 2002:12 and argue that the presumably temporary result of GS was driven mainly by the data in the 1990s. Wei and Zhang (2005) criticize the fact that GS looked at the relationship between an equally-weighted average stock variance and the return on a cap-weighted average stock return, as opposed to an equally-weighted portfolio return. Moreover, both Bali et al. (2005) and Wei and Zhang (2005) find no significant relationship between the cap-weighted measures and the cap-weighted portfolio return in all three sample periods (ending in 1999, 2001 and 2002, respectively).

4.1 Monthly Evidence

In this section we confirm existing results and extend them in a number of dimensions, including a longer sample period. The first panel in Table 7 presents the predictability regression of equally-weighted variance measures on the cap-weighted return as in Goyal and Santa-Clara (2003) and Bali et al. (2005) for their sample periods, as well as the extended sample up to 2006:12. The regression is as in equation 15, where ν_t^{EW} corresponds to the EW CAPM-based measure and the CSV.[20] For comparison purposes we start the sample period in this section in 1963:07, as in Goyal and Santa-Clara (2003), Bali et al. (2005) and Wei and Zhang (2005).

For the monthly series, we confirm that there is a significant positive relationship in the first sample, and also that it weakens for the subsequent extended samples.[21] The Newey and West (1987) autocorrelation corrected t-stat for 12 lags of the β coefficient of both CSV and the CAPM-based measures goes from 3.5 for the first sample period down to 0.9 for the largest sample. Consequently, the adjusted R^2 goes from 1.3% down to 0.04%. This result confirms the findings of Bali et al. (2005) and Wei and Zhang (2005) for the further extended sample. In section 4.4 we propose a possible explanation for this puzzling result.

In the second panel of Table 7 we present the results of the regression between the equally-weighted average return with the lagged equally-weighted idiosyncratic variance measure, as given by:

$$r_{t+1}^{EW} = \alpha + \beta \nu_t^{EW} + \varepsilon_{t+1} \qquad (16)$$

[20] As explained before, the monthly CSV is the average of its daily values during the month.

[21] We found a similar result using the GS measure of equally-weighed average variance. We do not present these regression results for the sake of brevity given that they generate a similar picture, which has also been confirmed in Bali et al. (2005) and Wei and Zhang (2005).

where ν_t^{EW} is taken as the CAPM-based average idiosyncratic variance or as the CSV measure. In contrast with the former regression, the relationship is found to be significantly positive for the three sample periods for both measures.[22]

In the third panel of Table 7 we present the results for the three sample periods of the one-month-ahead predictive regression of the cap-weighted market portfolio using the cap-weighted idiosyncratic variance return as a predictor. In this case, the beta of the idiosyncratic variance is not significant for all three sample periods. This result confirms the findings of Bali et al. (2005) and Wei and Zhang (2005) for the extended sample.

4.2 New Predictability Evidence at Daily Frequency

Prevailing measures used in the literature require a sample of past data to estimate additional parameters, constraining existing evidence to the monthly estimations. Fu (2009) finds that high idiosyncratic volatilities of individual stocks are contemporaneous with high returns, which tend to reverse in the following month. Huang et al. (2009) find that the negative relationship between idiosyncratic variance and expected returns at the stock level uncovered in Ang et al. (2006) and Ang et al. (2009) becomes positive after controlling for the return reversals. This provides additional motivation for looking at the predictability relation at a higher frequency than the monthly basis. Using the CSV as a proxy for aggregate idiosyncratic variance allows us to check this relationship at the aggregate (market) level in a more direct way (without having to control for reversals). Taking advantage of the instantaneous nature of the CSV, we run the same predictability regression (16) on the one-day-ahead portfolio return using the average idiosyncratic variance.

The upper panel of Table 8 shows that at a daily basis, this relationship is much stronger, with (Newey-West corrected) t-stats of coefficients for the average idiosyncratic variance across the three samples ranging between 4 and 4.7.

In the lower panel of Table 8 we report the results for the one-day-ahead predictive regression on the cap-weighted pairs (CSV and market return) for which we find the relation also to be positive and significant, but with a much more obvious deterioration of the t-stat of the cap-weighted idiosyncratic variance coefficient, going from about 5.91 in the first sample down to 1.97 for the longest sample. For this reason and for brevity, we now focus

[22] Wei and Zhang (2005) find a significantly positive relation between the equal-weighted GS measure and the equal-weighted market return for the initial sample. They also test the robustness of the relation by using an equally-weighted cross-sectional variance of monthly returns. They found a significantly positive coefficient for predicting the equal-weighted portfolio return mainly for the long samples starting in 1928 but not for the sample going from 1963 to 2002. Note that our cross-sectional measures differ. Ours is an average of the daily cross-sectional variances over the month. Theirs is the cross-sectional variance of the returns computed over the month.

on the relationship between aggregate idiosyncratic risk and the equal weighted market return.[23]

4.3 Interpretation of Predictability results

Given this evidence on the predictability of average aggregate returns by idiosyncratic risk, a natural question to ask would be: why does the relationship between the equal-weighted measure and the cap-weighted differ across different sample periods?

Wei and Zhang (2005), Bali et al. (2005) argue that the relationship between idiosyncratic risk and the market index first found by Goyal and Santa-Clara (2003) on the sample ending in 1999:12 was driven by small stocks traded in the NASDAQ and the data coming from the dot-com bubble period. Although we confirm their empirical findings for our sample period, we disagree with their conclusion that the relationship between average idiosyncratic risk and expected returns disappeared since the end of the dot-com bubble. Even though it appears clear that NASDAQ companies played an important role in the relationship of the equal-weighted average idiosyncratic variance with the average market-capitalization expected return during the end of the 1990s, which (obviously) weakened after the burst of the bubble, we find that the relationship between average idiosyncratic risk and future average market returns is robust to choices of the sample period, provided that adequate weighting schemes and horizons are chosen to test this inter-temporal relationship.

The transitory relationship between the equal-weighted average idiosyncratic variance and the cap-weighted market index observed up to the end of the 1990, can be explained by the heterogeneous and transitory nature of the omitted sources of risk captured by idiosyncratic risk and its relation with the inflated valuation of several NASDAQ companies during that period [24].

Some intuition behind the far more robust relationship between the equally-weighted average idiosyncratic variance and the equally-weighted portfolio comes precisely from the logic of standard asset pricing theory. As discussed in the introduction, there are multiple reasons for which average idiosyncratic risk should be related to average returns, due to the heterogeneous sources that may compose idiosyncratic risk. According to CAPM, only

[23] The corresponding results using a market cap-weighted scheme can be obtained from the authors upon request.

[24] The strongest omitted factors in that period (call it the irrational.com factor), partially captured by the equally weighted idiosyncratic variance, started to be increasingly represented in the market-cap index, due to the suddenly-higher market capitalization of precisely the group of companies carrying this temporarily strong omitted factor. The posterior reversal of the situation (i.e., the burst of the bubble) subsequently explains the sharp fade in the relationship between the average idiosyncratic variance and the market-cap portfolio, precisely due to the posterior sudden deterioration of the market capitalization of most stocks carrying this irrational.com factor, and hence notably reducing their representation in the market-capitalization index.

systematic risk should explain future returns. However, if during a certain period of time there exists anomalies of any kind (priced omitted risk factors) that, presumably, are not proportionally reflected in the current market capitalization of the companies carrying these factors, then the omitted sources of risk are more likely to explain the returns of a portfolio where all kinds of firms are represented in a similar manner, such as the EW as opposed to a portfolio where big companies are proportionally better represented than smaller ones.

Along these lines, Pontiff (2006) argues that idiosyncratic risk is the largest holding cost borne by rational arbitrageurs in their pursuit of mispricing opportunities. This theory implies that the current level of idiosyncratic risk should predict returns since it should measure the amount of current mispricing opportunities present in the market. Assuming that the same mispricing opportunities disappear in the long run, it appears more likely to observe this relationship between idiosyncratic variance and returns over very short horizons. Moreover, all things being equal, large-cap stocks are less likely to present misspricing and hence the predictability implied by this theory would be more likely to be present on the equal-weighted index return rather than the cap-weighted index return, as we observed in predictive regressions at daily and monthly horizons.[25] The sign of the relationship is not predicted by Pontiff's theory in general, because it depends on whether the average (equal or cap-weighted) portfolio is over- or under-priced (it predicts a positive sign for underpriced stocks and a negative sign for overpriced stocks).

4.4 Robustness Checks

In this section, we test further explore the relationship documented in the former section in several dimensions. We first want to place the return predictability by idiosyncratic variance in the context of the literature of the risk-return trade-off. Most of the literature on this topic is based on a linear regression between return and volatility. We want to see if including the return variance in the regression changes the predictability results. Second, we test the robustness of the relationship in the presence of an option implied volatility measure. Third, we further test the predictability relationship at quarterly and annual horizons. Finally, we look for the potential asymmetry in the relationship between idiosyncratic variance and future average returns, when the cross-sectional variance is split in two and is computed for returns above or below the mean. Such an asymmetry often exists for positive and negative returns in the volatility modeling of financial time series. The reported presence of asymmetries will provide us with a motivation for extending the cross-sectional dispersion

[25]It is well known that large cap stocks are more liquid than small-cap stocks, which implies a higher number of people trading them and usually a higher number of analysts looking at them. Together with less constraints to short-selling, we expect a higher price efficiency for large cap stocks.

measure to the third moment and find this measure is related with average idiosyncratic skewness and has strong predictive power of the average market return.

4.4.1 Inclusion of Return Variance

In order to check wether the relationship between the market portfolio expected return and the aggregate level of idiosyncratic variance (which we document at the monthly and daily frequency) is robust to the inclusion of the variance of the market portfolio, we run the following joint regression:

$$r_{t+1}^{EW} = \alpha + \beta CSV_t + \vartheta Var\left(r_t^{EW}\right) + \epsilon_{t+1}. \tag{17}$$

We also run the univariate regression:

$$r_{t+1}^{EW} = \alpha + \vartheta Var\left(r_t^{EW}\right) + \epsilon_{t+1}. \tag{18}$$

For the monthly estimations of $Var\left(r_t^{EW}\right)$ we use the realized sample variance over the month (from daily returns). For daily estimations we fitted an AR(1)-EGARCH(1,1) model on the overall sample.[26] In the first two panels of Table 9, we report regression results at the monthly and daily frequency of both (17) and (18). In the latter univariate regression, the variance of the equally-weighted portfolio returns does not appear to be significant in explaining the average future returns at the monthly and daily frequencies.

In the regression from equation (17), the coefficient of $Var\left(r_t^{EW}\right)$, ϑ, is negative and non-significant at the monthly frequency. At the daily frequency, the coefficient ϑ was still found to be negative and (marginally) significant. The significance of the CSV coefficient remains valid for both monthly and daily frequencies, and if anything improves slightly after the inclusion of the equally-weighted portfolio variance.

The latter two panels of Table 9 present the regression results at the monthly and daily frequency of both (17) and (18) but using the cap-weighted index and CSV equivalents. The relationship at the daily horizon becomes non significant after the inclusion of the realized variance of the market cap-weighted index. At the monthly horizon the relationship remains non significant.

In Table 10, where we report the quarterly and annual predictability with and without the market variance, we confirm that the equally-weighted cross-sectional variance does not forecast future average returns at low frequencies. However, for the cap-weighted measure of CSV, we observe predictability over the period 1963 to 2006 when it is joined with market

[26] Using the overall sample to estimate the parameters would only give the portfolio variance an advantage to predict future returns. However, from the results we see that even when using such forward-looking estimates for $Var\left(r_t^{EW}\right)$, the significance of the CSV remains strong.

variance. The sign is negative while the market variance enters with a positive sign as predicted by the benchmark risk-return trade-off[27]

One fair remark on the results of the predictability regressions is that the relationship using equal-weighted measures only holds at shorter horizons (i.e. daily and monthly). However, this result is in line with Pontiff (2006)'s interpretation of idiosyncratic risk as a barrier for arbitrageurs and with the evidence presented by Fu (2009) at the stock level, who finds that high idiosyncratic volatilities of individual stocks are contemporaneous with high returns, which tend to reverse in the following month.

4.4.2 Inclusion of Market Realized Variance and Implied Variance

Other measures of variance have been used in trying to link market returns to a measure of market risk. Implied variance (VIX^2) has been used as a forward-looking measure of market variance in addition to realized variance (the sum of squared returns at higher frequency than the targeted frequency for the measure of variance)[28]. We use these measures in Table 11 along with both CSV measures for daily and monthly predictability. We repeat the exercise in Table 12 for quarterly and annual frequencies. For these regressions we start the sample in 1990 for data availability for the implied volatility.

Results are similar to the ones in the previous section with market variance. For CSV^{EW}, we observe predictability at high-frequency but not at low frequency, while it is the opposite for CSV^{CW}. For the daily estimates with CSV^{EW}, we find a R^2 of almost 5% when we include all three measures of variance, and all coefficients are significant. But the remarkable result, undocumented until now to our knowledge, is the very high R^2 obtained at quarterly and annual frequencies for the CSV^{CW} measure. When using CSV^{CW} alone as a predictor we obtain R^2s of 4% and 26% at quarterly and annual frequencies, respectively. Adding the implied variance brings the R^2s to almost 19% and 29%. If instead one uses the realized variance instead of implied variance the R^2s are close to 11% and 34%. In all these predictability regressions, the sign of the CSV^{CW} variable is negative.

Guo and Savickas (2008) argue that average idiosyncratic volatility is negatively related to future stock market returns possibly because of its negative correlation with the aggregate book-to-market ratio.[29] If idiosyncratic volatility is measured from a CAPM model

[27] See also Guo and Savickas (2008) for similar results.

[28] For example, for the monthly variance, one will sum the daily squared returns, while for the daily variance, it is customary to use five-minute or one-minute squared returns.

[29] The argument starts by considering average idiosyncratic volatility as a proxy for changes in the opportunity set related related to technological shocks. They argue that technological innovations have two effects on the firm's stock price: they tend to increase the level of the firm's stock price because of growth options and they also tend to increase the volatility of the firm's stock price because of the uncertainty about which firms will benefit from the new opportunities. The final argument is to say that the book-to-market ratio

then it will capture the missing book-to-market factor. This explanation runs counter to our previous findings regarding the very high correlation between the measures of idiosyncratic volatility based on the CAPM and the Fama-French models. The two series were almost identical. A more appealing explanation may be to think of cross-sectional variance as a measure of dispersion of returns reflecting the dispersion of opinions among market participants. The negative sign of this relationship at quarterly and annual horizons in the presence of market variance as the second predictor (and also at monthly horizons in the presence of implied variance as the second predictor) is consistent with the model of Cao et al. (2005), in which dispersion of opinions among investors is positively related to stock market volatility but negatively related to conditional excess stock market returns. Furthermore, one may argue that differences of opinions forge themselves over a period of time and hence this effect is more likely to be present at horizons longer than a day.

More generally, we may interpret the CSV as measuring the hedging terms in an intertemporal CAPM model. In this regard, it is interesting to see that the positive risk-return trade-off at the aggregate level, i.e., the relationship between market volatility and expected returns, becomes significant only when taking into account the presence of the omitted factors as captured by the CSV. It is also interesting to note that the interactions of the CSV with the realized variance of the market take place at longer horizons (quarterly and annual), while its interactions with implied variance (VIX^2) tend to be more important at shorter horizons.

4.4.3 Asymmetry in the Cross-Sectional Distribution of Returns

We now explore for a potential asymmetry in the relationship between idiosyncratic variance and future average returns, when the cross-sectional variance is split in two and is computed for returns above or below the mean.

This asymmetry may be the result of the leverage effect put forward by Black (1976) since we are considering individual firms in the cross section. We also mentioned in an earlier section that consumption volatility risk affects differently small and large firms or value and growth firms. Therefore, we explore *i*) whether the predictability power is the same for the CSV of returns to the *left* and *right* of the center of the returns' distribution, *ii*) whether the relationship is driven by one of the sides and *iii*) whether the relationship with both sides would have the same sign on their coefficient. In order to do this, we define the CSV_t^+ as the cross-sectional variance of the returns to the right of the cross-sectional distribution (i.e., meaning the cross-section distribution that includes all stocks such that $r_{it} > r_t^{EW}$) and conversely define the CSV_t^- as the cross-sectional variance of the returns to

captures these investment opportunities

the left of the cross-sectional distribution (i.e., meaning the cross-section distribution that includes all stocks such that $r_{it} < r_t^{EW}$). Then we run the following regression:

$$r_{t+1}^{EW} = \alpha + \beta^{+} CSV_t^{+} + \beta^{-} CSV_t^{-} + \epsilon_{t+1}. \tag{19}$$

Table 13 presents the results of regression (19) for daily, monthly, quarterly and annual estimates, and shows a couple of interesting findings. First, splitting the CSV into right and left sides of the cross-sectional distribution made the adjusted R^2 of the predictive regression jump from 0.8% to 1.17% on monthly data and from 0.6% to 1.36% on daily data. Second, there is an asymmetric relationship between the CSV of the returns to the right and left of the cross-sectional distribution and the expected market return: the coefficient of the CSV_t^{+} is positive while the one of CSV_t^{-} is negative in both daily and monthly regressions. However, the coefficients (of both right and left CSVs) are significant only on the daily regression. The summary statistics of the predictive regression on the cap-weighted index using the equivalent cap-weighted CSV measures, displayed in the lower panel of Table 13, are qualitatively similar to the results on the equal-weighted measures.

These findings suggest that a measure of asymmetry of the cross-sectional distribution would be relevant in the context of exploring the relationship between market expected returns and aggregate idiosyncratic risk. Another key advantage of the CSV measure is that it can be easily extended to higher-order moments. We consider below the skewness of the cross-sectional distribution of returns and assess its predictive power for future returns. To the best of our knowledge, this additional factor, which appears as a natural extension of the CSV for measuring idiosyncratic risk[30], is entirely new in this context.[31] We follow Kim and White (2004) and use a quantile-based estimate (see Bowley (1920)), generalized by Hinkley (1975), as a robust measure of the skewness of the cross-sectional distribution of returns:[32]

$$RCS = \frac{F^{-1}(1-\alpha_1) + F^{-1}(\alpha_1) - 2Q_2}{F^{-1}(1-\alpha_1) + F^{-1}(\alpha 1)} \tag{20}$$

for any α_1 between 0 and 0.5 and $Q_2 = F^{-1}(0.5)$. The Bowley coefficient of skewness is a special case of Hinkley's coefficient when $\alpha_1 = 0.25$ and satisfies the Groeneveld and Meeden (1984)'s properties for reasonable skewness coefficients. It has upper and lower bounds $\{-1, 1\}$.

[30]We show formally in an appendix available upon request from the authors that there is a link between idiosyncratic skewness and the skewness of the cross-sectional distribution of returns.

[31]At the stock level, Kapadia (2009) uses cross-sectional skewness to explain the puzzling finding in Ang et al. (2006) that stocks with high idiosyncratic volatility have low subsequent returns.

[32]The usual non-robust skewness measure of the cross-section of returns is highly noisy compared to the proposed robust measure, especially at the daily frequency.

In Table 14, we report the results of predictive regressions at the daily and monthly frequencies where we add the robust measure of the cross-sectional skewness to the equally-weighted CSV. The first observation is that the CSV coefficients are very close to the values estimated with the CSV as the only regressor (0.4 for the daily frequency and 0.25 for the monthly one). The t-stats are also almost identical to the ones found in the CSV regressions. However, skewness appears to be a major contributor to the predictability of returns since the R^2 increases significantly compared to the regressions with CSV alone. At the daily frequency, the adjusted R^2 increases to a value of 5.8%. At the monthly frequency, it is still 4.6%. This large increase in predictability when adding skewness suggests that macroeconomic or aggregate financial shocks affect asymmetrically the distribution of returns.

5 Is Average Idiosyncratic Risk Priced?

According to Merton's ICAPM, a factor that predicts stock returns in the cross section should also predict aggregate market returns (see Campbell (1993)). By the reverse argument, motivated by the predictability power of (equal-weighted) cross-sectional variance on the average return in the market, we explore in this section whether the CSV^{EW}, interpreted as a risk factor, is rewarded and commands a premium in the cross-section.

5.1 CSV Quintiles' Premium

Using daily excess returns every month we run the following regression for each stock i:[33]

$$r_{it} = \alpha + \beta_{i,csv} CSV_t^{EW}. \tag{21}$$

At the end of every month in the sample, we sort stocks using the CSV^{EW} factor loading, β_{csv}, and form equally-weighted and cap-weighted quintile portfolios. We calculate the average return during the overall period for each quintile and the average return difference (i.e., premium) between the first quintile and each of the other four quintiles.

The results for the equally-weighted quintile portfolios are displayed in the upper panel of Table 15 and in the lower panel for the cap-weighted quintiles. As we can see from this table, all premia are significantly different from zero and economically meaningful. The difference between the first quintile (the one with higher sensitivity) and the second, third and fourth quintile, is around an annualized 30%, while the difference with the fifth quintile is around 15%. This result suggests that the relationship of the CSV and stock returns

[33]We use stocks with non missing values during the current month.

might not be best described in the simple linear form, which is in line with the asymmetric effect found in section 4.4, with the quantities CSV^+ and CSV^-.

5.2 Fama-MacBeth Procedure

In order to use the standard set of assets in the asset pricing literature, we extract daily returns data from Kenneth French data library on their 100 (10x10) and 25 (5x5) size/book-to-market portfolios for the period July 1963 to December 2006. Then we run every calendar month the following regression for each portfolio:[34]

$$r_{it} = \alpha + \beta_{i,xmkt} XMKT_t + \beta_{i,smb} SMB_t + \beta_{i,hml} HML_t + \beta_{i,csv} CSV_t^{EW}. \qquad (22)$$

Using the recorded factor loading, β (monthly) time series, we run the following cross-sectional regression every month on the next month's excess returns and record the γ coefficients:

$$r_{it+1}^m = \gamma_0 + \gamma_{xmkt}\beta_{i,xmkt}(t) + \gamma_{smb}\beta_{i,smb}(t) + \gamma_{hml}\beta_{i,hml}(t) + \gamma_{csv}\beta_{i,csv}(t). \qquad (23)$$

We finally test whether the average γ coefficients are statistically different from zero. In order to take into account possible serial correlation in the coefficients, we compute the t-statistic using Newey and West (1987) standard errors with 4 lags (same number of lags as in Ang et al. (2009)).

We use four sets of assets: 100 (10x10) size/book-to-market equally-weighted portfolios and cap-weighted weighted portfolios, and 25 (5x5) size/book-to-market equally-weighted and cap-weighted portfolios. For each of them, we use the CSV^{EW} as the fourth risk factor. The first two panels of Table 16 present the corresponding Fama-MacBeth regression results. The table displays the annualized coefficients and standard errors (multiplied by 12 from the original monthly values), as well as their corresponding autocorrelation-corrected t-stat and the average R^2. We find the γ coefficient for CSV^{EW} to be positive and significant when we use the 100 and 25 size/book-to-market Fama-French equally-weighted portfolios. However, it is not significant when we use the 25 market cap-weighted portfolios and marginally significant for the 100 market cap-weighted portfolios (although positive in both cases). This later result, again, is not entirely surprising considering that the cross-sectional variation in returns is reduced through the market-capitalization adjustment.

[34]As before, XMKT stands for excess market return, SMB and HML are the size and book to market Fama-French factors, also directly extracted from Kenneth French data library.

6 Conclusion

In this paper we formally introduce an *instantaneous* cross-sectional dispersion measure as a proxy for aggregate idiosyncratic risk that has the distinct advantage of being readily computable at any frequency, with no need to estimate other parameters. It is therefore a model-free measure of idiosyncratic risk. We extensively show how this measure is related to previous proxies of idiosyncratic variance, such as the Goyal and Santa-Clara (2003) measure and measures relative to the classic Fama and French (1993) and CAPM models, which have been previously shown to be very close to the Campbell et al. (2001) proxy as well. We confirm previous findings of Goyal and Santa-Clara (2003), Bali et al. (2005) and Wei and Zhang (2005) on the monthly predictability regressions for the extended sample period using our cross-sectional measure and more standard measures of idiosyncratic variance. We find that the results are robust across these measures. Thanks to the instantaneous nature of our measure, we are able to extend to daily data the evidence on the predictability power of idiosyncratic variance on the future market portfolio return. We provide a statistical argument to support the choice of an equally-weighted measure of average idiosyncratic variance as opposed to a market-cap weighted and explain why both empirically and theoretically such a measure should forecast better the equal-weighted market return. We also showed that this cross-sectional measure displays a sizable correlation with economic uncertainty, as measured by consumption growth volatility, and with several economic and financial variables. One additional advantage of our measure is that it generalizes in a straightforward manner to higher moments and we showed that the asymmetry of the cross-sectional distribution is a very good predictor for future returns. We leave for further research an exhaustive analysis of the properties of the skewness of cross-sectional return distribution as a measure of average idiosyncratic skewness. We also leave for further research an empirical analysis of the CSV measure using international data.

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A Proof of Proposition 1

Consider the factor model decomposition

$$r_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it}\beta_{it}F_t + \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}$$

and

$$r_{it} - r_t^{(w_t)} = \left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt}\right) F_t + \varepsilon_{it} - \sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt}$$

Under the homogeneous betas assumption, we have

$$r_{it} - r_t^{(w_t)} = \varepsilon_{it} - \sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt} \tag{24}$$

and therefore

$$\left[r_{it} - r_t^{(w_t)}\right]^2 = \varepsilon_{it}^2 + \left(\sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt}\right)^2 - 2\varepsilon_{it}\sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt}$$

so that

$$\begin{aligned} CSV_t^{(w_t)} &= \sum_{i=1}^{N_t} w_{it}\left(r_{it} - r_t^{(w_t)}\right)^2 \\ &= \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 + \left(\sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt}\right)^2 - 2\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_{jt}w_{it}\varepsilon_{it}\varepsilon_{jt} \end{aligned}$$

Noting that

$$\left(\sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt}\right)^2 = \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_{jt}w_{it}\varepsilon_{it}\varepsilon_{jt}$$

we finally have:

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 - \left(\sum_{i=1}^{N_t} w_{jt}\varepsilon_{jt}\right)^2$$

We now argue that the term $\sum_{i=1}^{N_t} w_{jt}\varepsilon_{jt}$ converges to 0 for increasingly large numbers of stocks. To show this, we need to use a recent result by Cuzick (1995) regarding the Marcinkiewcz−Zygmund strong law of large numbers for weighted sums of i.i.d. variables:

$$\frac{1}{N}\sum_{i=1}^{N} a_{Ni}X_i \longrightarrow 0 \text{ almost surely} \tag{25}$$

when $\{X, X_N, N \geq 1\}$ is a sequence of i.i.d. random variables with $E(X) = 0$, $E|X| < \infty$ and $\{a_{Ni}, 1 \leq i \leq N, N \geq 1\}$ is an array of constants uniformly bounded satisfying[35]

$$\sup |a_{Nit}| < \infty. \tag{26}$$

Here we take $a_{Nit} \equiv N_t w_{it}$ and $X_i \equiv \varepsilon_i$. For the result (25) to hold a_{Nit} needs to be uniformly bounded and to satisfy condition (26). We therefore restrict our attention to non-trivial weighting schemes, ruling out the situation such that the index is composed by a single stock. Please note that this condition together with the fact that $\sum_i w_{it} = 1$ implies $N_t > 1$ and also restrict the weights to be (strictly) positive at every given point in time. Hence, a weighting scheme (w_t), is defined as a vector process which satisfies $0 < w_{it} < 1 \; \forall \; i, t$. This condition seams reasonable since our focus is to measure idiosyncratic risk in the market.

By definition, the weighting scheme w_{it} and a_{Nit} is uniformly bounded by N_t and the following condition holds,

$$0 < w_{it} < 1 \; \forall \; i, t \tag{27}$$

Multiplying by N_t, we get

$$\begin{aligned} 0 &< N_t w_{it} < N_t \\ 0 &< N_t w_{it} < \infty \\ 0 &< a_{Nit} < \infty \\ |a_{Nit}| &< \infty \; \forall \; i, t \end{aligned}$$

which implies that condition (26) holds. Thus, for a positive weighting scheme from the strong law of large numbers for weighted sums of i.i.d. variables, it follows that:

$$\sum_{i=1}^{N_t} w_{it}\varepsilon_{it} \underset{N_t \to \infty}{\longrightarrow} 0 \text{ a.s.},$$

Using similar arguments, and the homogeneous idiosyncratic second moment assumption, $E[\varepsilon_{it}^2] \equiv \sigma_\varepsilon^2(t)$, we obtain that for a strictly positive weighting scheme, w_t, and i.i.d. ε_i,

$$\sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 \underset{N_t \to \infty}{\longrightarrow} \sigma_\varepsilon^2(t) \text{ almost surely}$$

Using these results, we finally have that:

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it}\left(r_{it} - r_t^{(w_t)}\right)^2 \underset{N_t \to \infty}{\longrightarrow} \sigma_\varepsilon^2(t) \text{ almost surely.}$$

[35]See Theorem 1.1, particular case of Cuzick (1995).

B Properties of the CSV Estimator

B.1 Bias of the CSV Estimator

Under the factor model decomposition (1) and equation (2) and using the homogeneous beta assumption, we have:

$$r_{it} - r_t^{(w_t)} = \left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt}\right) F_t + \varepsilon_{it} - \sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt} = \varepsilon_{it} - \sum_{j=1}^{N_t} w_{jt}\varepsilon_{jt} \tag{28}$$

Replacing result (28) in equation (3) we have as before:

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 - \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_{jt}w_{it}\varepsilon_{it}\varepsilon_{jt} \tag{29}$$

By definition of a strict factor model, $E\left[\varepsilon_{it}\varepsilon_{jt}\right] = 0$ for $i \neq j$, and $E(\varepsilon_{it}^2) = \sigma_{\varepsilon_i}^2$. Applying the expectation operator in equation (29) we get:

$$E\left[CSV_t^{(w_t)}\right] = \sum_{i=1}^{N_t} w_{it}\sigma_{\varepsilon_i}^2(t) - \sum_{i=1}^{N_t} w_{it}^2\sigma_{\varepsilon_i}^2(t) \tag{30}$$

The second term in (30) implies that the CSV would tend to underestimate the average idiosyncratic variance. Considering the equal-weighted scheme where $w_{it} = 1/N_t \ \forall i$, (30) simplifies into

$$E\left[CSV_t^{EW}\right] = \left(1 - \frac{1}{N_t}\right)\frac{1}{N_t}\sum_{i=1}^{N_t}\sigma_{\varepsilon_i}^2(t)$$

and we obtain:

$$E\left[CSV_t^{EW}\right] \underset{N_t\to\infty}{\longrightarrow} \frac{1}{N_t}\sum_{i=1}^{N_t}\sigma_{\varepsilon_{it}}^2.$$

B.2 Variance of the CSV Estimator

Let w_t and ε_t be column vectors of the weighting scheme and residuals respectively and $\Omega_t = w_t w_t'$, $\Lambda_t = diag\,(w_t)$, $N_t \times N_t$ matrices, and denote Σ^ε the variance covariance matrix of the residuals, which is diagonal for a strict factor model.

For a finite number of stocks in the case where $F_t \neq r^{(w_t)}$, we have from equation (29):

$$CSV_t^{(w_t)} = \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 - \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_{jt}w_{it}\varepsilon_{it}\varepsilon_{jt}$$

.

Letting $Q_t = \Lambda_t - \Omega_t$, CSV_t can be written in matrix form, as follows:

$$CSV_t^{(w_t)} = \varepsilon_t' Q_t \varepsilon_t. \tag{31}$$

Using the quadratic structure of the CSV and assuming normal residuals, we have (see for instance Kachman (1999))[36]:

$$Var\left(\varepsilon_t' Q_t \varepsilon_t\right) = 2tr\left(Q_t \Sigma_t^{\varepsilon} Q_t \Sigma_t^{\varepsilon}\right) \tag{32}$$

Under the assumption of a strict factor model, i.e. $\rho_{ijt}^{\varepsilon} = 0 \; \forall i \neq j$, equation (32) simplifies to:

$$Var\left(CSV_t^{(w_t)}\right) = 2\sum_{i=1}^{N_t} \sigma_{\varepsilon_{it}}^4 w_{it}^2 (1-w_{it})^2 + 2\sum_{i=1}^{N_t}\sum_{j\neq i}^{N_t} w_{it}^2 w_{jt}^2 \sigma_{\varepsilon_{it}}^2 \sigma_{\varepsilon_{jt}}^2 \tag{33}$$

Assuming an upper bound for the individual idiosyncratic variances, denoted as $\hat{\sigma}_{\varepsilon_t}$ equation (33) yields to the following inequality (replacing each variance for its upper bound)

$$Var\left(CSV_t^{(w_t)}\right) < 2\hat{\sigma}_{\varepsilon_t}^4 \left(\left(\sum_{i=1}^{N_t} w_{it}^2\right)^2 + \sum_{i=1}^{N_t} w_{it}^2 - 2\sum_{i=1}^{N_t} w_{it}^3\right). \tag{34}$$

When $w_t = 1/N_t$, equation (34) simplifies to

$$Var\left(CSV_t^{(w_t)}\right) < 2\hat{\sigma}_{\varepsilon_t}^4 \left(\frac{N_t - 1}{N_t^2}\right) < 2\hat{\sigma}_{\varepsilon_t}^4 \left(\frac{1}{N_t}\right). \tag{35}$$

For a large number of stocks,

$$Var\left(CSV_t^{(w_t)}\right) < 2\hat{\sigma}_{\varepsilon_t}^4 \left(\frac{1}{N_t}\right) \longrightarrow 0. \tag{36}$$

C Relaxing the Assumption of Homogenous Betas

The assumption that $\beta_{it} = \beta_t$ for all i is obviously a simplistic one and is done only for exposure purposes. Starting with the single factor decomposition on the definition of the

[36] The operator tr stands for the trace of a matrix, which is the sum of the diagonal terms.

CSV we have:

$$
\begin{aligned}
CSV_t^{(w_t)} &= \sum_{i=1}^{N_t} w_{it} \left(r_{it} - r_t^{(w_t)} \right)^2 \\
&= \sum_{i=1}^{N_t} w_{it} \left[\left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt} \right) F_t + \varepsilon_{it} - \sum_{i=1}^{N_t} w_{jt}\varepsilon_{jt} \right]^2 \\
&= F_t^2 \sum_{i=1}^{N_t} w_{it} \left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt} \right)^2 + \sum_{i=1}^{N_t} w_{it} \left(\varepsilon_{it} - \sum_{i=1}^{N_t} w_{it}\varepsilon_{jt} \right)^2 + \\
& 2F_t \sum_{i=1}^{N_t} w_{it} \left(\beta_i - \sum_{j=1}^{N_t} w_{jt}\beta_{jt} \right) \left(\varepsilon_{it} - \sum_{i=1}^{N_t} w_{jt}\varepsilon_{jt} \right)
\end{aligned}
$$

After simple rearrangement of terms we get:

$$
\begin{aligned}
CSV_t^{(w_t)} = F_t^2 \sum_{i=1}^{N_t} w_{it} \left(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt} \right)^2 + \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}^2 - \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_{jt}w_{it}\varepsilon_{it}\varepsilon_{jt} \\
+ 2F_t \sum_{i=1}^{N_t} w_{it}\varepsilon_{it}(\beta_{it} - \sum_{j=1}^{N_t} w_{jt}\beta_{jt})
\end{aligned}
$$

Applying the expectation operator and assuming a strict factor model, the last expression simplifies so as to yield:

$$
E\left[CSV_t^{(w_t)}\right] = E\left[F_t^2 CSV_t^{\beta}\right] + \sum_{i=1}^{N_t} w_{it}\sigma_{\varepsilon_i t}^2 - \sum_{i=1}^{N_t} w_{it}^2\sigma_{\varepsilon_i t}^2
$$

Under an equal-weighting scheme, we finally have:

$$
E\left[CSV_t^{EW}\right] = E\left[F_t^2 CSV_t^{\beta}\right] + \left(1 - \frac{1}{N_t}\right) \frac{1}{N_t} \sum_{i=1}^{N_t} \sigma_{\varepsilon_i t}^2
$$

D Tables and Figures

Table 1: **Estimates of the biases due to the cross-sectional dispersion of betas and weight concentration**: This table contains a summary of the distribution of the following time series: the cross-sectional dispersion of betas CSV_t^{β}, estimated with respect to the CAPM at the end of every month using daily returns; the average idiosyncratic variance $\sigma^2_{\varepsilon_t}$ with respect to the CAPM; the product of the average return of the market portfolio squared, F_t^2, and the beta dispersion, CSV_t^{β}; the proportion of the product $F_t^2 CSV_t^{\beta}$ to $\sigma^2_{\varepsilon_t}$ and the proportion of $\sum w_{it}^2 \sigma^2_{\varepsilon_{it}}$ to $\sigma^2_{\varepsilon_t}$. The upper panel corresponds to the equal-weight scheme (CSV^{EW}) and the lower panel to the market-cap weighting (CSV^{CW}). All figures are daily. The period is July 1963 to December 2006.

Equal-Weighted	$Q_{2.5}$	Q_{25}	Q_{50}	Q_{75}	$Q_{97.5}$
CSV_t^{β}	0.282	0.970	1.563	3.022	11.437
$\sigma^2_{\varepsilon_t}(\%)$	0.043	0.065	0.103	0.241	0.485
$F_t^2 CSV_t^{\beta}(\%)$	6.57e-07	6.92e-05	3.84e-04	0.001	0.005
$\frac{F_t^2 CSV^{\beta}}{\sigma^2_{\varepsilon_t}}(\%)$	0.001	0.078	0.348	0.890	3.240
$\frac{\sum w_{it}^2 \sigma^2_{\varepsilon_{it}}}{\sigma^2_{\varepsilon_t}}(\%)$	0.014	0.020	0.030	0.054	0.154

Cap-Weighted	$Q_{2.5}$	Q_{25}	Q_{50}	Q_{75}	$Q_{97.5}$
CSV_t^{β}	0.075	0.309	0.451	0.704	3.079
$\sigma^2_{\varepsilon_t}(\%)$	0.009	0.020	0.030	0.042	0.153
$F_t^2 CSV_t^{\beta}(\%)$	1.83e-07	2.30e-05	1.09e-04	2.77e-04	0.001
$\frac{F_t^2 CSV^{\beta}}{\sigma^2_{\varepsilon_t}}(\%)$	4.85e-04	0.080	0.351	0.930	3.472
$\frac{\sum w_{it}^2 \sigma^2_{\varepsilon_{it}}}{\sigma^2_{\varepsilon_t}}(\%)$	0.173	0.281	0.426	0.637	1.463

Table 2: **Total bias associated with CSV**: This table reports the output summary of the regression $CSV_t^{w_t} = bias + \psi\sigma^2_{model}(w_t) + \zeta_t$, where $\sigma^2_{model}(w_t)$ represents monthly estimates of the weighted average idiosyncratic variance estimated using the corresponding model (either CAPM or FF). The average and the CSV are computed with either the cap-weighted scheme (CW) or the equal-weighted one (EW). The period is July 1963 to December 2006.

	$CAPM^{EW}$	FF^{EW}	$CAPM^{CW}$	FF^{CW}
Bias	1.29e-05	2.23e-05	-2.09e-05	-3.74e-05
NW t-stat	1.986	2.382	-2.849	-4.767
Std. dev.	3.05e-06	5.86e-06	2.04e-06	3.50e-06
ψ	0.983	0.988	1.125	1.242
NW t-stat	153.819	100.162	39.259	39.226
Std. dev.	0.002	0.003	0.005	0.009
$\overline{R}^2(\%)$	99.866	99.503	98.946	97.117

Table 3: **Comparison of monthly measures of idiosyncratic variance**: The upper panel of this table contains the annualized mean and standard deviation of the monthly time series for the CSV, the average idiosyncratic variance based on the CAPM and the Fama-French models and the average Goyal and Santa-Clara (2003) variance measure as in equations (4), (11) and (12) using both EW and CW weighting schemes. The lower panel presents the cross-correlation matrix among these variables. The period is January 1964 to December 2006.

	CSV^{EW}	FF^{EW}	$CAPM^{EW}$	GS^{EW}	CSV^{CW}	FF^{CW}	$CAPM^{CW}$	GS^{CW}
Mean	0.384	0.383	0.387	0.342	0.085	0.076	0.080	0.112
Std.Dev.	0.085	0.086	0.087	0.070	0.020	0.016	0.018	0.029
Correlation	CSV^{EW}	FF^{EW}	$CAPM^{EW}$	GS^{EW}	CSV^{CW}	FF^{CW}	$CAPM^{CW}$	GS^{CW}
	100.00	99.75	99.93	95.44	72.16	74.98	72.83	61.25
		100.00	99.88	93.67	68.53	72.16	69.46	56.75
			100.00	94.78	70.62	73.66	71.47	59.83
				100.00	82.38	82.88	82.36	76.60
					100.00	98.56	99.48	92.64
						100.00	99.18	88.17
							100.00	92.13
								100.00

Table 4: **Comparison of daily measures of idiosyncratic variance**: The upper panel of this table contains the annualized mean and standard deviation of the daily time series for the CSV and the average idiosyncratic variance based on the Fama-French model as in equations (4) and (11) using both weighting schemes. The lower panel presents the cross-correlation matrix among these variables. The period is January 1964 to December 2006.

	CSV^{EW}	FF^{EW}	CSV^{CW}	FF^{CW}
Mean	0.384	0.383	0.085	0.078
Std.Dev.	0.021	0.019	0.005	0.004
Correlation	CSV^{EW}	FF^{EW}	CSV^{CW}	FF^{CW}
	100.00	82.63	60.33	63.96
		100.00	52.12	72.55
			100.00	73.95
				100.00

Table 5: **Regime-Switching Parameters**: This table contains the parameter estimates of the Markov regime-switching model specified in equation 14 for the CSV and the average idiosyncratic variance based on the FF model as in equations (4) and (11) using both, equal-weighted and cap-weighted schemes. The upper panel corresponds to monthly estimates and the lower panel to daily estimates. μ_i is the average level of the variable on regime i, σ_i is the standard deviation level of the variable on regime i, ϕ is the autocorrelation coefficient, p and q are the probabilities of remaining in regimes 1 and 2 correspondingly. The period is January 1964 to December 2006.

Monthly series	CSV^{EW}	FF^{EW}	CSV^{CW}	FF^{CW}
μ_1	0.401	0.363	0.107	0.115
μ_2	0.299	0.275	0.065	0.061
σ_1	0.067	0.062	0.029	0.021
σ_2	0.010	0.009	0.004	0.003
ϕ	0.980	0.981	0.839	0.839
p	0.839	0.823	0.857	0.906
q	0.963	0.951	0.980	0.990
Daily series	CSV^{EW}	FF^{EW}	CSV^{CW}	FF^{CW}
μ_1	0.446	0.261	0.110	0.048
μ_2	0.304	0.262	0.064	0.048
σ_1	0.036	2.04e-04	0.009	4.89e-05
σ_2	0.003	0.002	0.001	3.60e-04
ϕ	0.965	1.000	0.825	1.004
p	0.695	0.962	0.778	0.870
q	0.956	0.838	0.970	0.809

Table 6: **Correlations between the monthly series of several measures of cross-sectional variance and economic variables**. The sample period is January 1990 to December 2006.

	CSV^{EW}	CSV^{CW}	CSV^{EW+}	CSV^{EW-}
Consumption-Vol	0.401	0.241	0.184	0.346
Credit-Spread	0.177	0.268	0.098	0.165
Term-Spread	-0.086	-0.135	-0.107	-0.219
Inflation-Vol	-0.367	0.019	-0.137	-0.097
T-bill Rate	0.302	-0.043	0.091	0.164

Table 7: **Predictability Regression on the CRSP broad market portfolio using the CSV and CAPM-idvol equal-weighted and cap-weighted measures**: The first panel of this table presents the results of a one-month ahead predictive regression of the excess cap-weighted monthly portfolio returns, denoted by r^{CW}, on the monthly lagged equal-weighted average idiosyncratic variance and CSV^{EW} for three sample periods. The second panel presents the results of a one-month ahead predictive regression of the excess equal-weighted monthly portfolio returns, denoted by r^{EW}, on the monthly lagged equal-weighted average idiosyncratic variance and CSV^{EW}. The third panel presents the results of a one-month ahead predictive regression of the excess cap-weighted monthly portfolio returns, denoted by r^{CW}, on the monthly lagged cap-weighted idiosyncratic variance and CSV^{CW}. $CAPM^w$ is the average idiosyncratic variance derived from CAPM, estimated using one month of daily data and CSV^w is the average of the daily cross-sectional variance over each month. The intercept, the regression coefficient of the corresponding lagged idiosyncratic variance, the standard errors denoted by Std, the Newey-West corrected t-stats and the adjusted coefficient of determination denoted by $\overline{R}^2$ are reported. The sample periods are 1963:08 - 1999:12, 1963:08 - 2001:12 and 1963:08 - 2006:12.

Monthly series	1963:08 - 1999:12		1963:08 - 2001:12		1963:08 - 2006:12	
Forecasting r^{CW}	$CAPM^{EW}$	CSV^{EW}	$CAPM^{EW}$	CSV^{EW}	$CAPM^{EW}$	CSV^{EW}
Intercept	-0.001	-0.002	0.001	0.001	0.002	0.002
NW t-stat	-0.424	-0.479	0.228	0.264	0.586	0.617
Std	0.003	0.003	0.003	0.003	0.003	0.003
Coefficient	0.241	0.250	0.123	0.120	0.090	0.087
NW t-stat	3.543	3.609	1.356	1.247	1.055	0.964
Std	0.092	0.094	0.082	0.083	0.077	0.078
$\overline{R}^2(\%)$	1.336	1.365	0.275	0.233	0.072	0.044
Forecasting r^{EW}	$CAPM^{EW}$	CSV^{EW}	$CAPM^{EW}$	CSV^{EW}	$CAPM^{EW}$	CSV^{EW}
Intercept	4.51e-04	3.31e-04	-1.67e-04	-1.06e-05	0.001	0.001
NW t-stat	0.086	0.063	-0.033	-0.002	0.241	0.270
Std	0.004	0.004	0.004	0.004	0.004	0.004
Coefficient	0.247	0.254	0.238	0.235	0.217	0.215
NW t-stat	2.175	2.189	2.395	2.329	2.331	2.271
Std	0.118	0.121	0.105	0.107	0.099	0.101
$\overline{R}^2(\%)$	0.774	0.773	0.885	0.824	0.726	0.678
Forecasting r^{CW}	$CAPM^{CW}$	CSV^{CW}	$CAPM^{CW}$	CSV^{CW}	$CAPM^{CW}$	CSV^{CW}
Intercept	0.001	0.001	0.007	0.007	0.007	0.008
NW t-stat	0.150	0.334	2.226	2.459	2.551	2.823
Std	0.005	0.004	0.003	0.003	0.003	0.003
Coefficient	0.856	0.688	-0.356	-0.373	-0.404	-0.421
NW t-stat	1.192	0.995	-0.871	-1.081	-1.080	-1.334
Std	0.685	0.632	0.390	0.344	0.372	0.328
$\overline{R}^2(\%)$	0.128	0.043	-0.037	0.038	0.035	0.123

Table 8: **Daily predictability Regression on CRSP broad market portfolio with average idiosyncratic variance measures**: The upper panel presents the results of a one-day ahead predictive regression of the excess equal-weighted daily portfolio returns, denoted by r^{EW}, on the daily lagged equal-weighted cross-sectional variance denoted as CSV^{EW} estimated as in equation (4) for three sample periods. The lower panel presents the results of the predictive regression on the cap-weighted market portfolio using the cap-weighted CSV. The intercept, the regression coefficient corresponding to the CSV, the standard error of the regression coefficients denoted by std, the Newey-West corrected t-stats (30 lags) and the adjusted coefficient of determination denoted by $\overline{R}^2$ are reported. The sample periods are 1963:07 to 1999:12, 1963:07 to 2001:12 and 1963:07 to 2006:12.

Daily series	63:07-99:12	63:07-01:12	63:07-06:12
Forecasting r^{EW}	CSV^{EW}	CSV^{EW}	CSV^{EW}
Intercept	-1.58e-04	-1.40e-04	-1.29e-05
NW t-stat	-0.785	-0.714	-0.071
Std	1.09e-04	1.10e-04	1.04e-04
Coefficient	0.544	0.483	0.411
NW t-stat	4.711	4.515	4.000
Std	0.060	0.055	0.051
$\overline{R}^2$(%)	0.883	0.788	0.573
Forecasting r^{CW}	CSV^{CW}	CSV^{CW}	CSV^{CW}
Intercept	-0.001	-1.88e-04	-1.65e-04
NW t-stat	-3.521	-0.791	-0.737
Std	1.41e-04	1.23e-04	1.19e-04
Coefficient	3.404	1.189	1.151
NW t-stat	5.919	1.948	1.966
Std	0.385	0.251	0.248
$\overline{R}^2$(%)	0.831	0.220	0.186

Table 9: **Daily and Monthly predictability with and without market volatility**: This table presents summary statistics for three predictive regressions. The first two panels correspond to daily and monthly predictions on the equal-weighted broad market index using equal-weighted CSV and the realized variance of the equal-weighted broad market index and the two lower panels their counterparts using the cap-weighted index and CSV. In each panel, the first row corresponds to the regression $r_t^w = \alpha + \beta CSV_t^w + \epsilon_t$ the second row to $r_t^w = \alpha + \vartheta Var\left(r_t^w\right) + \epsilon_t$ and the third one to $r_t^w = \alpha + \beta CSV_t^w + \vartheta Var\left(r_t^w\right) + \epsilon_t$. The sample period is July 1963 to December 2006.

Daily Estimates	Intercept	T-stat	CSV^{EW}	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	-1.29e-05	-0.071	0.411	4.000			0.573
Forecasting r^{EW}	0.001	4.928			-2.437	-0.976	0.034
Forecasting r^{EW}	1.19e-04	0.629	0.470	4.672	-5.038	-2.226	0.737
Monthly Estimates	**Intercept**	**T-stat**	CSV^{EW}	**T-stat**	$Var\left(r^{EW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{EW}	0.001	0.270	0.215	2.271			0.678
Forecasting r^{EW}	0.008	2.673			0.240	0.182	-0.185
Forecasting r^{EW}	0.001	0.324	0.226	2.206	-0.478	-0.394	0.515
Daily Estimates	**Intercept**	**T-stat**	CSV^{CW}	**T-stat**	$Var\left(r^{CW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{CW}	-1.65e-04	-0.737	1.151	1.966			0.186
Forecasting r^{CW}	2.10e-05	0.183			2.680	2.003	0.068
Forecasting r^{CW}	-1.76e-04	-0.937	1.084	1.298	0.447	0.198	0.179
Monthly Estimates	**Intercept**	**T-stat**	CSV^{CW}	**T-stat**	$Var\left(r^{CW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.008	2.823	-0.421	-1.334			0.123
Forecasting r^{CW}	0.005	2.636			-0.418	-0.569	-0.116
Forecasting r^{CW}	0.008	2.815	-0.459	-1.054	0.130	0.113	-0.065

Table 10: **Quarterly and Annual predictability with and without market variance**: This table presents summary statistics for three predictive regressions. The first two panels correspond to quarterly and annual predictions on the equal-weighted broad market index using equal-weighted CSV and the realized variance of the equal-weighted broad market index and the two lower panels their counterparts with the cap-weighted index and CSV. In each panel, the first row corresponds to the regression $r_t^w = \alpha + \beta CSV_t^w + \epsilon_t$ the second row to $r_t^w = \alpha + \vartheta Var\left(r_t^w\right) + \epsilon_t$ and the third one to $r_t^w = \alpha + \beta CSV_t^w + \vartheta Var\left(r_t^w\right) + \epsilon_t$. The sample period is July 1963 to December 2006.

Quarterly Estimates	Intercept	T-stat	CSV^{EW}	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	0.009	0.567	0.191	1.761			0.682
Forecasting r^{EW}	0.008	0.730			5.662	4.027	3.124
Forecasting r^{EW}	-0.001	-0.043	0.104	0.822	5.133	3.474	2.903
Annual Estimates	**Intercept**	**T-stat**	CSV^{EW}	**T-stat**	$Var\left(r^{EW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{EW}	0.059	0.846	0.131	1.090			-0.544
Forecasting r^{EW}	0.055	1.276			4.004	2.371	-0.063
Forecasting r^{EW}	0.027	0.358	0.099	0.731	3.292	2.087	-1.548
Quarterly Estimates	**Intercept**	**T-stat**	CSV^{CW}	**T-stat**	$Var\left(r^{CW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.021	2.063	-0.277	-0.644			-0.288
Forecasting r^{CW}	0.006	0.869			1.800	1.749	1.195
Forecasting r^{CW}	0.021	2.467	-1.141	-2.608	3.686	2.188	3.719
Annual Estimates	**Intercept**	**T-stat**	CSV^{CW}	**T-stat**	$Var\left(r^{CW}\right)$	**T-stat**	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.093	2.465	-0.416	-1.265			0.142
Forecasting r^{CW}	0.045	1.297			0.606	0.451	-2.112
Forecasting r^{CW}	0.079	2.125	-1.024	-2.186	3.314	1.726	3.719

Table 11: **Daily and Monthly predictability with and without market realized variance and implied variance** (VIX^2): The first two panels correspond to daily and monthly predictions of the equal-weighted broad market index using equal-weighted CSV, the realized variance of the equal-weighted broad market index and the squared VIX index. The two lower panels feature similar regressions with the cap-weighted counterparts. In each panel, we report regression results for each predictor individually and for combinations of the three corresponding predictors over the period July 1990 to December 2006

Daily Estimates	Intercept	T-stat	CSV^{EW}	T-stat	VIX^2	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	-8.59e-05	-0.265	0.390	3.020					0.625
Forecasting r^{EW}	0.002	8.224			-8.400	-4.417			1.697
Forecasting r^{EW}	0.001	4.036					0.860	0.228	-0.020
Forecasting r^{EW}	0.002	7.254			-13.815	-4.890	19.928	3.675	2.866
Forecasting r^{EW}	0.001	2.440	0.788	4.503	-12.706	-6.115			3.866
Forecasting r^{EW}	-2.34e-05	-0.068	0.416	3.223			-2.692	-0.767	0.634
Forecasting r^{EW}	0.001	2.245	0.763	4.512	-17.653	-6.356	18.716	3.609	4.893

Monthly Estimates	Intercept	T-stat	CSV^{EW}	T-stat	VIX^2	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	-0.008	-0.799	0.335	2.193					1.963
Forecasting r^{EW}	0.007	1.686			0.008	0.876			-0.359
Forecasting r^{EW}	0.006	1.159					2.423	0.937	0.188
Forecasting r^{EW}	0.007	1.771			-0.005	-0.265	2.821	0.730	-0.283
Forecasting r^{EW}	-0.008	-0.751	0.392	2.082	-0.011	-0.886			1.688
Forecasting r^{EW}	-0.008	-0.787	0.325	1.819			0.295	0.104	1.478
Forecasting r^{EW}	-0.007	-0.697	0.368	1.969	-0.017	-0.903	1.416	0.368	1.329

Daily Estimates	Intercept	T-stat	CSV^{CW}	T-stat	VIX^2	T-stat	$Var\left(r^{CW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{CW}	1.79e-04	0.870	0.242	0.657					-0.011
Forecasting r^{CW}	0.001	5.852			-7.438	-4.129			0.772
Forecasting r^{CW}	-1.04e-04	-0.560					4.513	2.258	0.130
Forecasting r^{CW}	0.002	5.427			-27.744	-5.884	35.151	5.504	4.116
Forecasting r^{CW}	0.001	4.578	1.327	2.651	-9.616	-4.530			1.049
Forecasting r^{CW}	-2.04e-05	-0.102	-0.397	-0.807			5.740	2.116	0.128
Forecasting r^{CW}	0.002	5.354	-0.823	-1.690	-28.073	-5.942	38.058	5.959	4.186

Monthly Estimates	Intercept	T-stat	CSV^{CW}	T-stat	VIX^2	T-stat	$Var\left(r^{CW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.013	3.498	-0.640	-1.986					0.999
Forecasting r^{CW}	0.001	0.196			0.016	1.672			0.448
Forecasting r^{CW}	0.005	1.589					0.407	0.273	-0.440
Forecasting r^{CW}	-2.58e-04	-0.062			0.034	2.451	-2.230	-1.242	0.644
Forecasting r^{CW}	0.007	1.772	-1.114	-3.618	0.033	2.965			3.534
Forecasting r^{CW}	0.013	3.814	-1.470	-2.544			3.734	1.741	3.002
Forecasting r^{CW}	0.008	1.797	-1.333	-2.290	0.024	1.773	1.579	0.601	3.258

Table 12: **Quarterly and annual predictability with and without market realized variance and implied variance (VIX^2)**: The first two panels correspond to daily and monthly predictions of the equal-weighted broad market index using equal-weighted CSV, the realized variance of the equal-weighted broad market index and the squared VIX index. The two lower panels feature similar regressions with the cap-weighted counterparts. In each panel, we report regression results for each predictor individually and for combinations of the three corresponding predictors over the period July 1990 to December 2006

Quarterly Estimates	Intercept	T-stat	CSV^{EW}	T-stat	VIX^2	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	-0.009	-0.278	0.249	1.502					0.833
Forecasting r^{EW}	-0.007	-0.450			0.039	3.548			4.178
Forecasting r^{EW}	0.024	1.551					1.880	0.826	-1.082
Forecasting r^{EW}	-0.006	-0.345			0.057	2.964	-4.664	-1.537	4.254
Forecasting r^{EW}	-0.013	-0.426	0.051	0.211	0.036	1.979			2.751
Forecasting r^{EW}	-0.009	-0.277	0.258	1.218			-0.293	-0.098	-0.708
Forecasting r^{EW}	-0.017	-0.509	0.102	0.425	0.053	2.497	-5.049	-1.571	3.010

Annual Estimates	Intercept	T-stat	CSV^{EW}	T-stat	VIX^2	T-stat	$Var\left(r^{EW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{EW}	0.058	0.825	0.142	1.157					-5.125
Forecasting r^{EW}	0.055	1.020			0.024	1.225			-1.088
Forecasting r^{EW}	0.084	1.556					4.111	1.746	-1.528
Forecasting r^{EW}	0.054	0.988			0.015	0.427	2.167	0.422	-8.041
Forecasting r^{EW}	0.054	0.847	0.002	0.008	0.024	0.913			-8.864
Forecasting r^{EW}	0.068	0.773	0.030	0.135			3.848	1.178	-9.266
Forecasting r^{EW}	0.061	0.741	-0.017	-0.075	0.016	0.436	2.224	0.413	-17.023

Quarterly Estimates	Intercept	T-stat	CSV^{CW}	T-stat	VIX^2	T-stat	$Var\left(r^{CW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.047	4.082	-0.866	-2.996					4.166
Forecasting r^{CW}	-0.007	-0.431			0.027	1.865			3.574
Forecasting r^{CW}	0.018	1.528					0.281	0.136	-1.494
Forecasting r^{CW}	-0.018	-1.100			0.087	5.163	-8.025	-4.416	12.469
Forecasting r^{CW}	0.017	1.158	-1.687	-6.670	0.054	4.670			18.666
Forecasting r^{CW}	0.046	4.149	-1.999	-4.315			5.686	2.257	11.164
Forecasting r^{CW}	0.011	0.683	-1.449	-4.175	0.066	3.589	-2.109	-0.722	17.768

Annual Estimates	Intercept	T-stat	CSV^{CW}	T-stat	VIX^2	T-stat	$Var\left(r^{CW}\right)$	T-stat	$\overline{R}^2(\%)$
Forecasting r^{CW}	0.227	6.448	-1.079	-6.158					26.614
Forecasting r^{CW}	0.115	2.665			-0.006	-0.476			-6.531
Forecasting r^{CW}	0.141	3.486					-1.946	-1.197	-1.348
Forecasting r^{CW}	0.086	1.964			0.032	1.818	-5.026	-2.138	-3.052
Forecasting r^{CW}	0.171	3.068	-1.391	-7.922	0.023	2.093			28.832
Forecasting r^{CW}	0.214	4.254	-1.905	-7.880			4.579	3.342	34.155
Forecasting r^{CW}	0.224	3.222	-1.950	-5.227	-0.005	-0.186	5.193	1.406	28.789

Table 13: **Predictability Regression on CRSP broad market index with right and left CSV measures**: The upper panel presents the results of a one-day, one-month, one quarter and one year ahead predictive regressions of the excess equal-weighted portfolio returns, denoted by r^{EW}, on the daily or monthly (correspondingly) lagged equal-weighted cross-sectional variance of the returns to the right (higher than) of the cross-sectional distribution mean (which is actually r_t^{EW}) denoted as CSV^+ and the cross-sectional variance of the returns to the left (lower than) the mean of the cross-sectional distribution r_t^{EW}, denoted as CSV^-. The lower panel presents the results of the predictive regressions on the cap-weighted market index using the cap-weighted CSV measures as predictors. The intercept, the regression coefficients corresponding to the CSV^+ and CSV^-, the standard error of the regression coefficients denoted by Std, the Newey-West corrected t-stats and the adjusted coefficient of determination denoted by $\overline{R}^2$ are reported. The sample period is 1963:07 to 2006:12.

Forecasting r^{EW}	$Daily^{EW}$	$Monthly^{EW}$	$Quarterly^{EW}$	$Annual^{EW}$
Intercept	0.001	0.003	0.014	0.050
NW t-stat	3.944	0.727	0.822	1.069
Std	1.10e-04	0.004	0.017	0.076
CSV^+	0.488	0.375	-0.042	-0.288
NW t-stat	3.360	2.400	-0.282	-0.845
Std	0.038	0.161	0.190	0.403
CSV^-	-1.200	-0.486	0.824	1.701
NW t-stat	-3.551	-1.129	0.842	1.346
Std	0.142	0.432	0.902	1.610
$\overline{R}^2(\%)$	1.552	1.114	-0.595	-1.862

Forecasting r^{CW}	$Daily^{CW}$	$Monthly^{CW}$	$Quarterly^{CW}$	$Annual^{CW}$
Intercept	-1.12e-04	0.008	0.020	0.107
NW t-stat	-0.588	2.753	2.656	2.147
Std	1.16e-04	0.003	0.010	0.043
CSV^+	4.942	0.071	-0.163	-2.354
NW t-stat	3.546	0.054	-0.254	-1.669
Std	0.528	1.983	0.914	1.824
CSV^-	-2.842	-1.302	-0.870	1.227
NW t-stat	-2.736	-0.879	-0.672	0.869
Std	0.669	2.233	1.477	2.527
$\overline{R}^2(\%)$	0.785	-0.069	-0.849	1.215

Table 14: **Daily and Monthly predictability with skewness for** r^{EW}: This table presents the results of one-day and one-month ahead predictive regressions of the excess equal-weighted daily portfolio returns, denoted by r^{EW}. The first explanatory variable is lagged estimate of the equal-weighted CSV estimated as in equation (4); The second explanatory variable is the robust estimate of skewness estimated as in equations (20). The intercept, the corresponding regression coefficients together with their Newey-West autocorrelation corrected t-stats (with 30 lags for daily and 12 lags for monthly) and standard errors are reported. $\overline{R}^2$ denotes adjusted coefficient of determination. The regression is reported for the main sample period from 1963:07 to 2006:12.

Daily horizon	Coeff.	t-stat	Std.Dev.	$\overline{R}^2$(%)
Intercept	-3.7e-005	-0.234	0.000	5.833
CSV^{EW}	0.402	4.013	0.053	
Skewness	0.004	20.190	0.000	

Monthly horizon	Coeff.	t-stat	Std.Dev.	$\overline{R}^2$(%)
Intercept	0.000	0.107	0.004	4.587
CSV^{EW}	0.250	2.518	0.102	
Skewness	0.078	4.458	0.017	

Table 15: Quintiles premium. The upper panel present the results for equal-weighted quintile portfolios and the lower panel on cap-weighted quintile portfolios. The first column presents the (arithmetic) average return annualized on quintiles formed at the end of every month on CSV^{EW}'s coefficient estimated with one month of daily returns. The second column presents the average return difference of the first quintile with every other quintile. The third column presents the p-value of the test of the difference to be significantly positive. The sample period is July 1963 to December 2006.

EW Quintiles	Quintile Return	$Q_1 - Q_i$	p-value(%)
Q_1	0.390	0.00e+00	
Q_2	0.083	0.307	0.00e+00
Q_3	0.044	0.346	0.00e+00
Q_4	0.050	0.340	0.00e+00
Q_5	0.237	0.154	0.130

CW Quintiles	Quintile Return	$Q_1 - Q_i$	p-value(%)
Q_1	0.383	0.00e+00	
Q_2	0.087	0.296	0.00e+00
Q_3	0.036	0.347	0.00e+00
Q_4	0.047	0.335	0.00e+00
Q_5	0.247	0.136	0.725

Table 16: This table displays the average values, standard errors and Newey-West corrected t-stats for the coefficients in the Fama-MacBeth regression run every month in the sample, using the 3 Fama-French factors and CSV^{EW} on 100 and 25 size/book2market Fama-French equally-weighted (first two panels) and cap-weighted (last two panels) portfolios. It also displays the average R^2 across subsamples of Fama-MacBeth regressions. The sample period is July 1963 to December 2006.

100-EW Portfolios	Intercept	XMKT	SMB	HML	CSV^{EW}	$R^2(\%)$
γ	0.223	-0.067	0.029	0.048	0.005	24.657
SE	0.024	0.016	0.012	0.012	0.002	
tstat	9.222	-4.173	2.320	3.916	2.847	

25-EW Portfolios	Intercept	XMKT	SMB	HML	CSV^{EW}	$R^2(\%)$
γ	0.278	-0.133	0.042	0.064	0.009	51.962
SE	0.028	0.021	0.017	0.016	0.003	
tstat	9.969	-6.447	2.480	3.927	2.703	

100-CW Portfolios	Intercept	XMKT	SMB	HML	CSV^{EW}	$R^2(\%)$
γ	0.155	-0.007	0.004	0.037	0.003	24.262
SE	0.023	0.017	0.012	0.013	0.002	
tstat	6.896	-0.421	0.323	2.762	1.909	

25-CW Portfolios	Intercept	XMKT	SMB	HML	CSV^{EW}	$R^2(\%)$
γ	0.175	-0.034	0.010	0.048	0.004	50.815
SE	0.024	0.021	0.016	0.016	0.003	
tstat	7.417	-1.624	0.625	2.910	1.395	

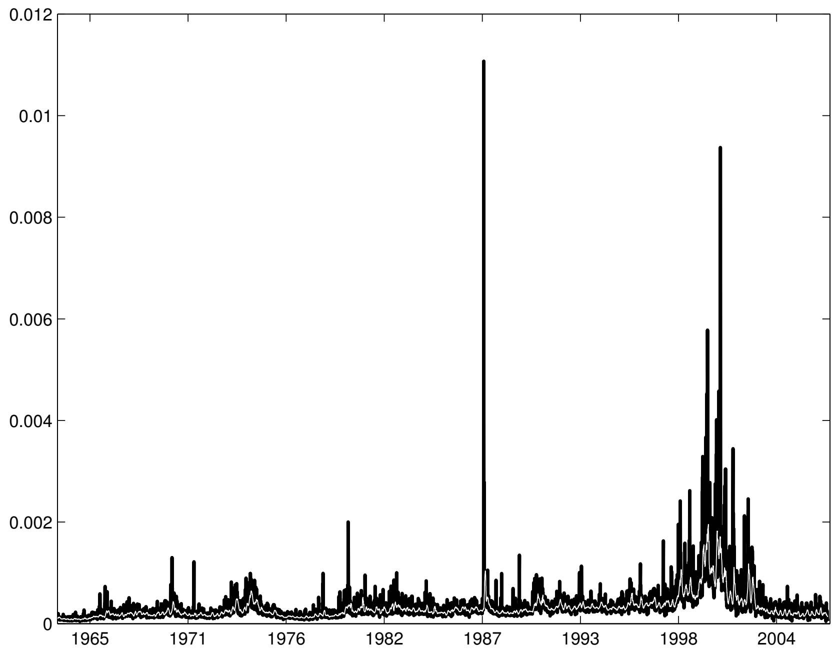

Figure 1: Cap-weighted idiosyncratic variances, daily estimation: The white line is the time series of the cap-weighted idiosyncratic variance with respect to the FF model estimated daily as in equation 11. The darker line shows the time series of the cap-weighted version of CSV estimated daily as in equation 5. The sample period is January 1964 to December 2006.

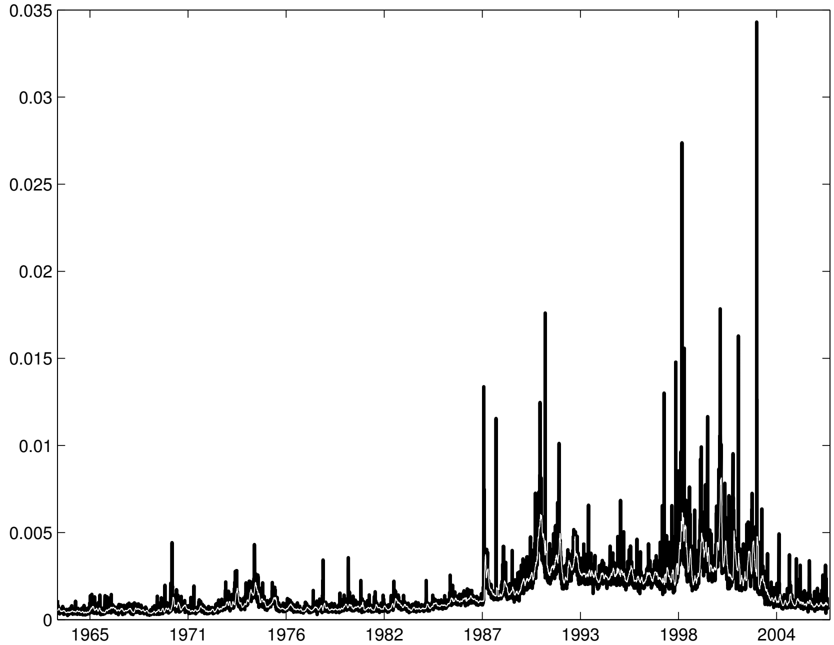

Figure 2: Equally-weighted idiosyncratic variances, daily estimation: The white line is the time series of the equal-weighted average idiosyncratic variance with respect to the FF model estimated daily similar to equation 11. The darker line shows the time series of the CSV^{EW} estimated daily as in equation 4. The sample period is January 1964 to December 2006.

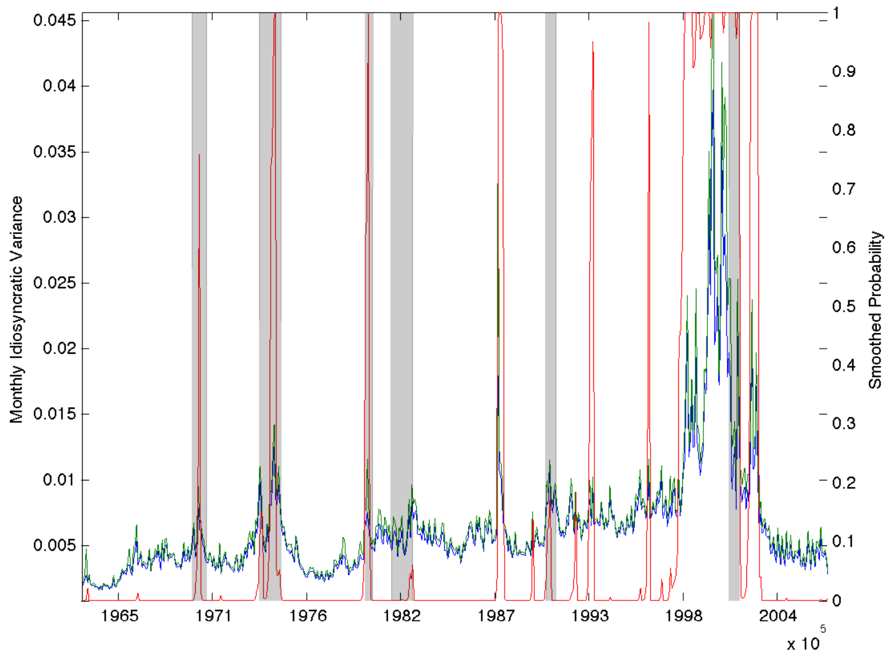

Figure 3: Filtered probabilities and cap-weighted CSV, monthly estimation: The red line plots the filtered probability of the CSV^{CW} being in the high-mean high-variance regime of a Markov regime-switching model specified in equation 14. The blue line shows the monthly time series of the CSV^{CW} estimated at the end of each month as the average of the daily estimations (as in equation 4) during the month. The shaded areas are the NBER recessions. The sample period is July 1963 to December 2006.

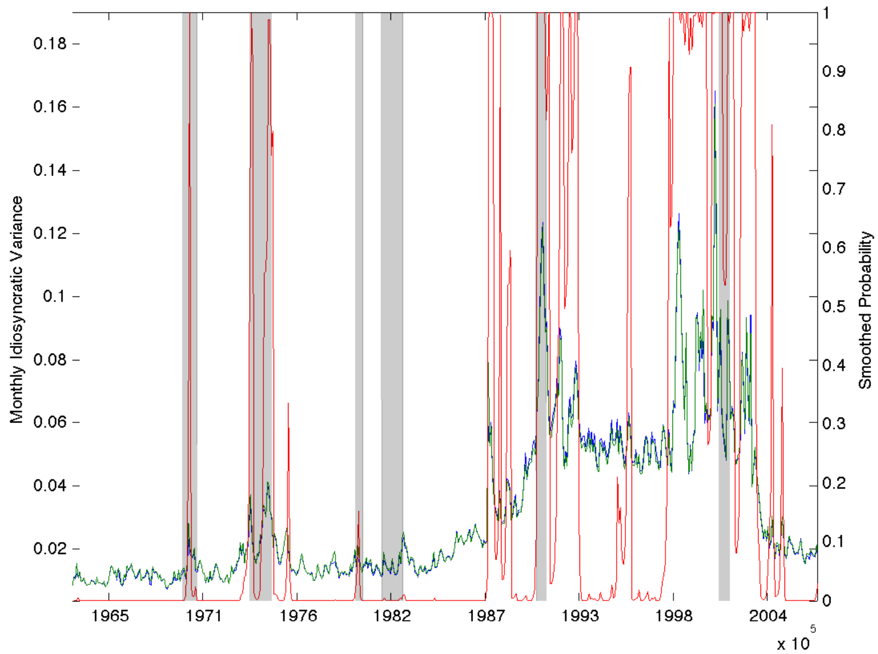

Figure 4: Filtered probabilities and equal-weighted CSV monthly estimation: The red line plots the filtered probability of the CSV^{EW} being in the high-mean high-variance regime of a Markov regime-switching model specified in equation 14. The blue line shows the monthly time series of the CSV^{EW} estimated at the end of each month as the average of the daily estimations (as in equation 4) during the month. The shaded areas are the NBER recessions. The sample period is July 1963 to December 2006.

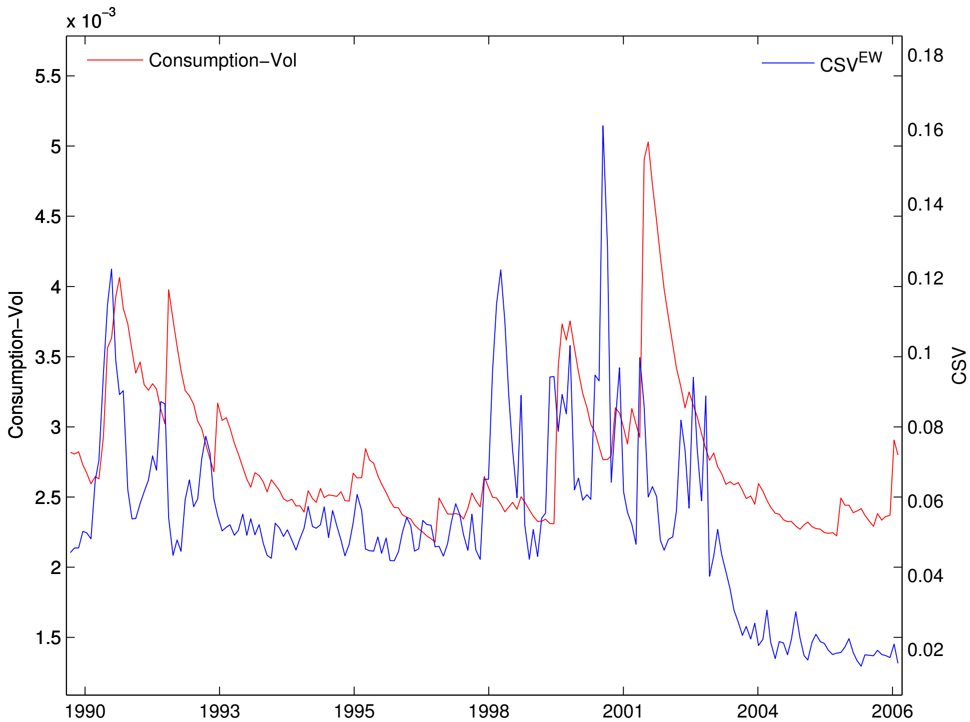

Figure 5: Monthly time series of CSV^{EW} on the right-hand axis and Consumption Volatility on the left-hand axis. The sample period is January 1990 to December 2006.

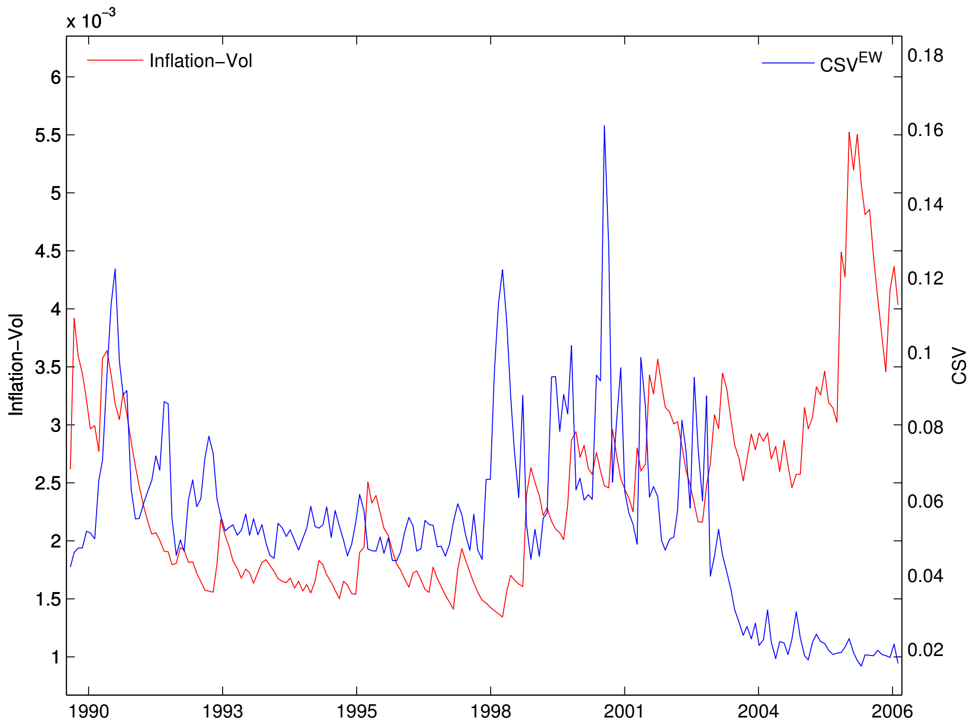

Figure 6: Monthly time series of CSV^{EW} on the right-hand axis and Inflation Volatility on the left-hand axis. The sample period is January 1990 to December 2006.