Retirement Investing: Analyzing the 'Roth' Conversion and Re-characterization Options

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Summary

The paper explores a rich example of the importance of tax options for investment management. Starting in 2010, all individuals, regardless of income level, can convert their "traditional IRAs" into "Roth IRAs" by paying ordinary income tax on the market value (at the time of conversion) of the assets being converted. In particular, even individuals with the largest tax-deferred accounts now are able to convert some or all of their tax-deferred holdings to Roth IRAs (prepaying the ordinary income tax). The U.S. tax law also permits investors to reverse or "re-characterize" these conversions at an individual account level until the final income tax filing deadline for that tax year (i.e., October 15th of the following year). The nature of the resulting opportunities is very different than for traditional tax-deferred investments. At both a positive and normative level our conclusions shed considerable light on the conversion and re-characterization decisions of investors. Our analysis also suggests a potential framework to understand the implications of the Roth tax-deferred accounts and the re-characterization option for the dynamics of government deficits and revenue.

We examine the costs and benefits of converting to a Roth IRA from a traditional tax-deferred account and analyze the optimal conversion decisions by investors under various circumstances absent the re-characterization option. We then extend our analysis to allow the investor to reverse (re-characterize) these conversions decisions and examine the implications for asset location. The insights that arise with respect to optimal conversion go beyond the conventional wisdom, such as converting if and only if the investor's current tax rate is below his anticipated future tax rate. Volatility is very valuable for positions that are subject to potential re-characterization, which is in sharp contrast to the traditional asset location advice. Re-characterization leads to substantial optionality for investments in certain tax-deferred accounts. These perspectives raise interesting challenges for understanding the stochastic and dynamic structure of government revenue.

1. Introduction

The financial market crisis has stimulated renewed appreciation of the importance of long-run savings (including a sharp increase in the savings rate) as well as a growing recognition of the tax treatment of different savings vehicles. Consequently, there has been tremendous interest concerning strategies for tax-efficient savings and renewed focus upon tax-deferred investing. The landscape for retirement investing itself has shifted in the past decade to emphasize defined-contribution (DC) rather than defined-benefit (DB) plans. This provides the beneficiary the personal responsibility and opportunity to determine his own asset location and allocation choices. The opportunities for tax-deferred investing in defined-contribution plans have become more complex with the development of Roth IRAs (and the recently introduced Roth 401k and 403b plans). These "Roth" plans are typically not subject to taxation of withdrawals, though the Roth plans do not offer an up-front tax deduction, and also do not require withdrawals during the original owner's lifetime.[1]

Under specific conditions the owners can "convert" traditional tax-deferred IRAs into Roth IRAs by paying income tax on the amount converted with the option to re-characterize or reverse the conversion until the income tax filing deadline for that tax year (e.g., October 15th of the following calendar year—the final deadline for tax filings and extensions from the prior calendar year).[2] Throughout our discussion we refer to these options as the "conversion option" and "re-characterization option." These

[1]Because withdrawals would not be required on a Roth IRA until it is inherited, in a setting with progressive tax rates that are constant over time it's plausible that the beneficiary of the inherited Roth IRA might be in a lower tax bracket than the original owner, who had accumulated a substantial fund. Of course, the statutory tax rates could change over time and indeed, there has been considerable recent speculation that tax rate schedules will rise over time.

[2]One important exception is that conversion is not allowed once an account has been inherited, except from a spouse.

potential conversions are attracting considerable attention because starting in 2010 these are no longer being limited to those taxpayers whose remaining taxable income does not exceed $100,000. Hence, many large traditional IRA accountholders now have become eligible to convert. A key source of value to conversion emerges because the income tax liability can be paid by funds outside the tax-deferred regime, effectively expanding the proportion of an individual's financial base within the tax-deferred arena.[3]

Receiving less popular attention is the re-characterization option, which itself has a dramatic impact on the "asset location" pattern of an investor, i.e., what assets to locate in various vehicles.[4] Specifically, the re-characterization option highlights the value associated with investment strategies that include very volatile assets in accounts that are subject to re-characterization. This is a new theme in asset location choice, which we explore in more detail in the paper.[5] We characterize the optimal exercise of both the conversion and re-characterization options—for example, highlighting the universal value of conversion in the presence of a re-characterization option as the investor would always be able to reverse the conversion costlessly and characterizing the optimal re-characterization boundary in a number of ways.

This change in the retirement investment opportunity raises an array of fundamental issues about asset location and allocation. For example, under what circumstances is

[3]In the context of traditional tax-deferred investing, Dammon, Spatt and Zhang (2004) highlight the potential importance of the fraction of the investor's financial resources that are within his tax-deferred account.

[4]Among the features that distinguish Roth accounts from traditional tax-deferred accounts are the (a) greater effective after-tax exposure that the Roth account represents due to the prepayment of the tax (especially when the tax liability can be paid with taxable funds (see prior footnote), (b) how the conversion and re-characterization options add significantly to the economic value of the traditional tax-deferred account and influence the optimal location patterns, and (c) how the Roth accounts do not require mandatory withdrawals during the investor's lifetime.

[5]Dammon, Spatt and Zhang (2004) show that it is optimal for an investor who doesn't face borrowing restrictions or differential borrowing (vs. lending) costs to locate the high-yielding asset in his tax-deferred account, minimizing the investor's tax liability given his risk exposure.

conversion a beneficial strategy for investors? How should converted funds be invested? When will re-characterization be optimal? What are the implications of this for the use of traditional and Roth tax-deferred investing over time? How does the removal of the income cap for conversion influence the temporal structure of income tax revenues? How does the re-characterization option influence the stochastic structure of tax revenues? These issues relate to both financial planning, but also government revenues. Though we have previously studied tax-deferred investing (Dammon, Spatt and Zhang (2004)) and its contrast with taxable investing,[6] the specific opportunities associated with the conversion and re-characterization options for Roth IRAs have not been subject to analysis from the perspectives of modern financial economics. The interest in Roth IRAs, the richness of the tax environment and the apparent differences in conclusions from those in Dammon, Spatt and Zhang (2004) make exploration of these issues very fruitful. In addition to our earlier work on asset location, the current analysis also builds upon tools utilized in our work on the tradeoffs between realizing capital gains taxes and rebalancing one's portfolio (Dammon, Spatt and Zhang (2001a, 2001b, 2003)), insights on estate planning (Dammon, Spatt and Zhang (2007)) as well as the realization of capital gains as a function of time (Dammon and Spatt (1996)).

One of the key goals of this paper is to understand the impact of Roth IRAs upon asset location and allocation and retirement planning. More specifically, we will offer fundamental insights about investing tax-deferred Roth IRA portfolios subject to the re-characterization option. We use basic finance principles to identify the inherent value of the conversion and re-characterization options. We will use arbitrage analysis to highlight the importance of possessing assets with considerable volatility in a Roth IRA portfolio still subject to re-characterization. We anticipate that the arbitrage analysis will

[6]Also, see Dammon, Poterba, Spatt and Zhang (2005).

lead to this conclusion not only relative to traditional tax-deferred accounts, but also relative to the taxable account. We later plan to undertake a series of welfare analyses to highlight the nature of the ex ante value of the conversion and re-characterization options.

We model the re-characterization decision by exploiting the basic insight that if a portfolio is re-characterized, there will be a series of subsequent conversion opportunities and re-characterization options. For example, if the Roth portfolio does not appreciate considerably (or if it depreciates) after conversion while being subject to re-characterization, then the investor will want to re-characterize because of the subsequent ability to convert again (and augmented further by additional re-characterization potential) as well as the benefit of avoiding an immediate relatively high tax exposure. The investor re-characterizes when the account value is sufficiently low. Secondarily, the threshold value such that the investor will terminate his current option and re-characterize will be somewhat dependent upon the current point in calendar time since the sooner the investor re-characterizes, the sooner he will be able to convert again (and later potentially re-characterize again) adding to the value of that option. The IRS allows subsequent conversion thirty days after re-characterization or January 1st of the year after the conversion date, whichever is later. In effect, the individual can't convert the same funds twice within the same tax year and also must wait a minimum of thirty days. Hence, there typically would not be an incentive to re-characterize prior to early December of the year of conversion. Starting at that point the threshold would decline somewhat over the year because the value of the future re-characterization

option shrinks with the passage of time during the year. This portion of our work will utilize the techniques of compound option analysis.[7]

The value of the re-characterization option could reflect not only the stochastic structure of asset prices, but uncertainty until the filing deadline associated with the evolution of the tax rate and anticipated future tax rates (typically financial models do not reflect tax rate uncertainty). Consequently, income risk and the possibility of legislative changes in tax rates (but not to the re-characterization option itself) could enhance the value of the re-characterization option as the investor's incremental tax situation would influence the positions that he chooses to re-characterize. Unlike most financial options, the re-characterization option is defined jointly over both asset price movements and tax rates.

The overall dynamic value associated with the re-characterization and conversion options is considerable, especially for an investor with a relatively long horizon over many years. The investor can divide his holdings into a number of accounts and convert them separately before or after investing these separate accounts in relatively volatile and aggressive manners, exploiting separate re-characterization and conversion options for the different accounts. For a relatively long horizon investor, it is advantageous to pay the tax from conversion (rather than re-characterize) only when the benefit is exceptionally large—highlighting the degree to which these options substantially enhance value.

Our analysis begins by considering the conversation option in various scenarios without the option to re-characterize, where conversion is a single "now or never" decision. We

[7]Such methods are illustrated by both Dammon and Spatt (1992, 1996). An early analysis of compound options is in Geske (1979).

explore the tax effects associated with Roth conversion in Section II and evaluate the conversion option in Section III. We examine some numerical examples in Section IV. The interaction between Roth conversion and estate taxes is examined in Section V. We then introduce consideration of the re-characterization option and explore its qualitative properties in Section VI. We offer concluding comments in Section VII.

II. Tax Effects of a Roth Conversion

The decision to convert a traditional IRA into a Roth IRA depends ultimately upon whether the Roth conversion leads to higher after-tax wealth for the individual at retirement. In this section, we derive the after-tax wealth at retirement for a traditional IRA and for a Roth conversion. Section III evaluates the conditions under which a Roth conversion is beneficial for the individual as a one-time option.

Strategy 1: Retain the Traditional IRA

To begin our analysis, let's first consider a traditional IRA, with current market value of V_0. Without a Roth conversion, the after-tax value of the traditional IRA at retirement is:[8]

$$\text{After-Tax Value of Traditional IRA at Retirement} = [V_0(1+r)^N](1-T_{p,N}) \qquad (1)$$

[8]Here we assume that the IRA is liquidated in its entirety at the retirement date, even though individuals are allowed to defer withdrawals until age 70½. (Any withdrawals after age 59½ are not subject to the 10% early withdrawal penalty.) However, assuming a constant tax rate during retirement, the present value of the annual tax payments is the same regardless of the timing of the withdrawals, because the government's claim is a fraction $T_{p,N}$ of the IRA account's value.

where N is the number of years until retirement, r is the ***realized*** (geometric) average annual pre-tax rate of return on the assets held in the IRA account, and $T_{p,N}$ is the individual's ordinary income tax rate during retirement. Equation (1) captures the fact that the individual earns the pre-tax rate of return, r, on assets held in the IRA account until the account is liquidated in retirement. Distributions from the traditional IRA are then taxed at the individual's ordinary income tax rate during retirement, $T_{p,N}$.

Strategy 2: Convert to a Roth IRA

Now suppose the individual converts the traditional IRA to a Roth IRA and pays the ordinary income tax on the market value of the IRA assets. There are two cases to consider: (A) individual liquidates assets held in the IRA account to pay the tax (and any penalty) and (B) individual liquidates assets held outside the IRA account to pay the tax. Each of these cases has a different effect on the after-tax value of the individual's retirement wealth.

Case A: *Finance the tax (and penalty) on the Roth conversion from the IRA assets.*

If the individual is forced to finance the tax (and penalty) on the Roth conversion from the IRA assets, the after-tax retirement wealth is:

After-Tax Value of Roth Conversion = $V_0[(1 - T_{p,0} - q)/(1 - q)](1+r)^N$ (2A)

Financed with IRA Assets

where $T_{p,0}$ is the individual's current ordinary income tax rate[9] and q is the penalty for early withdrawal when assets from the IRA are used to pay the tax (and penalty) on the conversion. Under current tax law, the penalty is q = 10% for individuals younger than 59½ and q = 0% for investors older than 59½.[10]

The after-tax retirement wealth under a Roth conversion captures the fact that the tax (and penalty) on the Roth conversion is paid immediately, with no further taxes due on the Roth distributions in the future.[11] If the individual uses IRA assets to pay the tax and penalty, then the value of the Roth IRA immediately falls to $V_0[(1 - T_{p,0} - q)/(1 - q)]$ after the conversion. The assets left in the Roth IRA then earn a tax-free return of r over the next N years.

Case B: *Finance the tax on the Roth conversion from non-IRA assets.*

If the individual makes a Roth conversion and uses non-IRA assets to finance the tax payment, then the after-tax value of the investor's wealth at retirement is:[12]

After-Tax Value of Roth Conversion =

Financed with Non-IRA Asset

[9]The current ordinary tax rate must be the individual's marginal tax rate, which would include the effect of V_0 in adjusted gross income on the individual's allowable personal exemptions and itemized deductions, the taxable portion of Social Security payments, and other tax-related items. The progressivity of marginal tax rates may make it optimal to spread the Roth conversions across multiple tax years. This issue is not addressed explicitly here.

[10] If an individual who is younger than 59½ converts to a Roth IRA and then withdraws funds from the Roth IRA within 5 years of the conversion, and before reaching age 59½, the early withdrawal penalty will apply (with some hardship exceptions).

[11]Our analysis makes the assumption that the tax on a Roth conversion is paid immediately.

[12]If the individual uses non-IRA assets to finance the tax payment on a Roth conversion, then, regardless of age, there is no penalty for early withdrawal.

$$V_0 (1 + r)^N - \left[\frac{V_0 T_{p,0}}{1 - gT_g} \right] \left[1 + r(1 - T_i) \right]^N \qquad (2B)$$

where g is the unrealized capital gain (as a percentage of current market value) on the assets sold to finance the tax payment on the Roth conversion, T_g is the capital gains tax rate, and T_i is the effective tax rate on investment income when assets are held in a taxable account (see the Appendix for a derivation of the effective tax rate).[13]

If the individual uses non-IRA assets to pay the tax, the value of the Roth IRA remains V_0 after the conversion, which then earns a tax-free return of r over the next N years. This is captured in the first term in Equation (2B). To finance the payment of the tax on the Roth conversion, however, the individual must sell non-IRA assets worth $V_0 T_{p,0}$ after capital gains taxes are paid on the sale. These non-IRA assets would have earned the individual an annual after-tax return of $r(1- T_i)$ over the next N years when held outside the IRA.[14] This is captured in the second term of Equation (2B).

II. Evaluating the Roth Conversion Option

To determine whether it is beneficial for an individual to convert an existing traditional IRA into a Roth IRA it is necessary to compare the after-tax value of retirement wealth with a Roth conversion (Equation (2A) or (2B)) to the after-tax value of retirement wealth without a Roth conversion (Equation (1)).

[13]We also don't adjust here for the difference in tax basis that results from the sale of some of the appreciated assets in the taxable account.

[14]To keep the individual's portfolio composition unchanged, we assume that the non-IRA assets that are sold to finance the tax payment are the same as those that the individual holds in the IRA account. See Dammon, Spatt, and Zhang (2004) for an analysis of the optimal allocation and location of asset classes across taxable and tax-deferred accounts. In general, it is more tax-efficient to hold heavily-taxed assets (e.g., taxable bonds) in tax-deferred accounts.

Case A: *Finance the tax (and penalty) on the Roth conversion from the IRA assets.*

For an individual who is forced to use IRA assets to pay the tax (and penalty), a Roth conversion is beneficial only if the after-tax wealth in Equation (2A) is greater than the after-tax wealth in Equation (1). After some algebra, a Roth conversion is beneficial in this case provided the following condition is satisfied:

$$\frac{T_{p,N}}{T_{p,0}} > \frac{1}{1-q} \qquad \text{(3A)}$$

Equation (3A) says that the ratio of the expected future ordinary income tax rate to the current ordinary income tax rate must be greater than 1/(1-q) in order for a Roth conversion to benefit an individual who is forced to liquidate IRA assets to pay the tax (and penalty) on the conversion. Interestingly, notice that this condition is independent of the individual's planned retirement date, N. For all individuals younger than 59½ (where q = 10%), the ratio of tax rates must be greater than 1/(1-.10) = 1.11, while for all individuals older than 59½ (where q = 0%), the ratio of tax rates must be greater than 1.0. Thus, if there is no early withdrawal penalty, the traditional IRA accountholders should convert to a Roth IRA when their anticipated ordinary income tax rate during retirement is higher than their current ordinary income tax rate. With the early withdrawal penalty, the anticipated ordinary income tax rate during retirement needs to be at least 11% higher than the current ordinary income tax rate for a traditional IRA accountholder to benefit from converting to a Roth IRA.

While much of the analysis assumes that the conversion decision is "now or never," even if Equation (3A) is satisfied currently, the individual may still be better off delaying the Roth conversion to a future year if $[1/(1-q)]T_{p,0}$ is expected to fall prior to retirement.

This implies that a Roth conversion should be delayed to a future year for (1) individuals younger than age 59½ who are currently subject to the q = 10% penalty for early withdrawal and (2) individuals older than 59½ who expect their ordinary income tax rate to decline further prior to retirement. It is also likely that delaying the Roth conversion will be beneficial if it allows the individual to accumulate sufficient non-IRA assets to pay the tax on the conversion. This is the case we consider next.

Case B: *Finance the tax on the Roth conversion from non-IRA assets.*

For an individual who can finance the tax payment using non-IRA assets, a Roth conversion is beneficial provided the after-tax wealth in Equation (2B) is greater than the after-tax wealth in Equation (1). After algebraic manipulation, the Roth conversion is beneficial in this case provided the following condition is satisfied:

$$\frac{T_{p,N}}{T_{p,0}} > \left[\frac{1}{1-\mathrm{gT_g}}\right]\left[\frac{1+\mathrm{r}(1-T_{\mathrm{i}})}{1+\mathrm{r}}\right]^{\mathrm{N}} \qquad \text{(3B)}$$

Equation (3B) says that a Roth conversion, where the tax is financed from non-IRA assets, makes sense only if the ratio of the future ordinary income tax rate to the current ordinary tax rate is above some critical level. When the embedded capital gain is g = 0%, the critical ratio of tax rates is less than 1.0. This means that the individual may find it beneficial to convert to a Roth IRA, even though the anticipated future ordinary income tax rate is less than the current tax rate (a departure from conventional wisdom). The reason a Roth conversion can be beneficial, even though the tax rate during retirement would be below the current tax rate, is because the payment of the tax from non-IRA assets is akin to allowing the individual to scale up his/her tax-deferred savings. The critical ratio of tax rates increases when an individual is forced to liquidate assets with an embedded capital gain to finance the tax payments. According to Equation (3B), the critical cutoff ratio of tax rates is likely to be lower for individuals who are younger (with

more time until retirement, i.e., a larger N), hold assets with higher expected returns and effective tax rates, i.e., a higher r and T_i, and have the liquidity to fund the tax liability without triggering an embedded capital gain.

Figure 1: Critical Tax Ratios for Roth Conversion

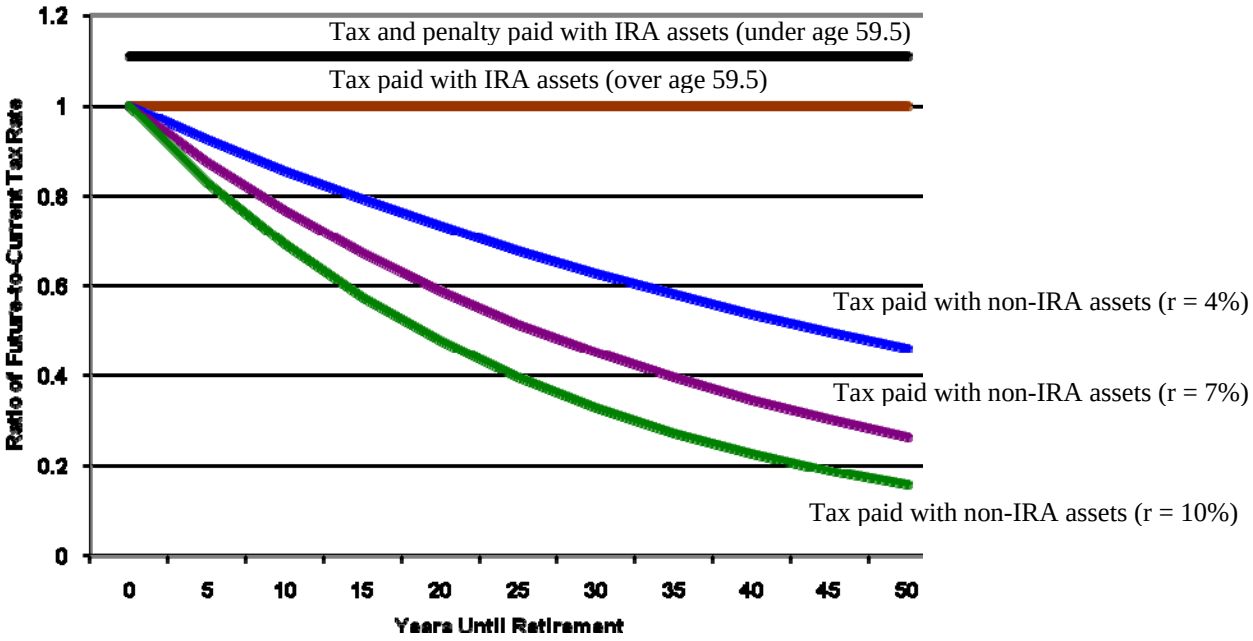

Figure 1 below shows the critical cutoff ratio of tax rates for different retirement horizons and different asset returns. When applicable, the penalty for early withdrawal is 10%. When non-IRA assets are used to fund the tax liability, it is assumed that the embedded capital gain is g = 0% and that T_i = 40% (i.e., interest income). It is beneficial for the individual to convert to a Roth IRA provided the ratio of the expected future ordinary income tax rate to the current ordinary income tax rate is above the critical ratios shown in Figure 1.

III. Examples

Example 1: *Young individual (younger than 59½) who is in the 28% tax bracket, has a traditional IRA, but has no non-IRA savings or ability to borrow.*

For this individual, a Roth conversion will generate both a tax liability and an early withdrawal penalty that is financed with the assets held in the IRA account. For this individual, the Roth conversion is optimal only if the expected ordinary tax rate during retirement is greater than $T_{p,N}$ > 28% x 1.11 = 31.11% (see Eq. (3A)). This conclusion does not depend upon the assumed pre-tax return, r, or the planned retirement date, N.

Example 2: *High-taxed individual ($T_{p,0}$ = 40%) who can pay the tax on a Roth conversion using non-IRA assets with no embedded capital gain, g = 0%. The individual holds high-yield corporate bonds (T_i = 40%) in a traditional IRA with an expected pre-tax yield of 7%. The individual has a planned retirement date in 20 years.*

According to Equation (3B), it is beneficial to convert to a Roth IRA provided the individual's expected ordinary tax rate in retirement is greater than 23.536% (i.e., .5884 x 40% = 23.536%).

Example 3: *High-taxed individual ($T_{p,0}$ = 40%) who can pay the tax on a Roth conversion by liquidating non-IRA assets with an embedded capital gain of g = 15%. The capital gains tax rate is T_g = 20%. The individual holds high-growth stocks in a traditional IRA with an expected pre-tax return of r = 10*% and an effective tax rate on investment income of T_i = 16.2%. T*he individual has a planned retirement date in 20 years.*

Once again, Equation (3B) tells us that this individual should convert to a Roth IRA as long as the expected ordinary tax rate in retirement is greater than 30.649% (i.e., .7662 x 40% = 30.649%).

V. Roth Conversion and Estate Taxes

A Roth conversion has some additional important benefits, such as potentially reducing estate taxes.

Reduction in Estate Taxes

United States estate taxes are computed on the pre-tax value of the individual's estate, including the pre-tax value of any IRAs. If the IRA is a traditional pre-tax IRA, the owner also will be required to pay income taxes on the IRA assets as they are withdrawn from the IRA account. A Roth conversion can reduce potentially the value of an individual's estate, thereby reducing the estate tax liability, by selling assets to pay the income tax on the conversion. However, the benefit that comes from reducing the estate tax is partially offset by a potential increase in income taxes.

An important, but often overlooked, deduction in the tax code allows a beneficiary to reduce the impact of estate taxes on traditional pre-tax IRA accounts. This deduction is called an ***income in respect to a decedent (IRD)*** deduction, which allows the beneficiary to deduct the estate taxes paid on IRA assets against future taxable withdrawals from the IRA account.[15] The purpose of the IRD deduction is to reduce the

[15]IRD deductions are not subject to the 2% of adjusted gross income limitation.

burden of double taxation (income and estate taxes) on IRA assets, but it can also reduce the value of a Roth conversion.

The following two examples illustrate how the IRD deduction works, how estate and income taxes interact, and how a Roth conversion affects the total tax liability. In Example 4, the individual has non-IRA assets that exceed the estate tax exclusion. In Example 5, the individual has non-IRA assets that are worth less than the estate tax exclusion, but total estate assets (non-IRA plus IRA) that exceed the estate tax exclusion. From these two examples it is clear that a Roth conversion allows an individual to reduce the impact of estate taxes only when the value of non-IRA assets is less than the estate tax exclusion, but the total value of the estate (non-IRA plus IRA) exceeds the estate tax exclusion.[16]

Example 4: Non-IRA assets exceed the estate tax exclusion.

Suppose an individual has a $6.5 million estate: $5.5 million of non-IRA assets and $1 million of traditional pre-tax IRA assets. If the estate tax exclusion is $5 million, the estate must pay estate taxes on $1.5 million.[17] With an estate tax rate of 35%, the total estate tax liability is $525,000 (35% x $1.5 million). Excluding the IRA assets, the estate tax liability would have been $175,000 (35% x $500,000). Therefore, the IRS treats the remaining $350,000 of estate taxes as having been generated by the $1 million of IRA

[16]In the two examples we assume that there is not a spousal beneficiary, as there is an unlimited marital deduction for a spousal beneficiary under the estate tax law.
[17]The estate tax exclusion had been $3.5 million in 2009 (with a maximum estate tax rate of 45% on the amount in excess of the exclusion), became unlimited in 2010 (i.e., no estate taxes), and then became $5 million in 2011 and 2012 (with a maximum estate tax rate of 35% on the amount in excess of the exclusion). The example in the text is based upon the 2011-2012 tax law parameters.

assets and allows the beneficiary a $0.35 IRD deduction on each $1 withdrawn from the IRA account (up to a total deduction of $350,000).

Assuming that the beneficiary has an ordinary income tax rate of 30%, the total tax liability (estate and income) on the inherited estate is:[18]

Total tax liability = Estate Taxes + Income Taxes

= [0.35 x ($6.5 – $5.0)] + [0.30 x ($1.0 - $0.35)]

= $525,000 + $195,000 = $720,000

Now suppose the individual converts his/her IRA assets to a Roth IRA immediately before death.[19] Assuming that the individual has the same ordinary tax rate as his/her beneficiary (i.e., 30%), the Roth conversion generates an immediate income tax liability of $300,000 (30% x $1.0 million). Assuming the individual liquidates non-IRA assets (with no embedded capital gain) to pay the income tax, the individual's remaining estate will be composed of $5.2 million of non-IRA assets and $1.0 million in a Roth IRA. The total tax liability (estate and income) paid by the individual and his/her beneficiary in this case is:

Total tax liability = Estate Taxes + Income Taxes

[18]In practice, the income tax liability and IRD deduction are realized at the time the beneficiary withdraws assets from the IRA account, or makes a Roth conversion. Here we assume that the IRD deduction is realized at the time that the beneficiary takes ownership of the estate.
[19]In practice, a non-spousal beneficiary of an inherited IRA is barred from doing a Roth conversion after death.

= [0.35 x ($6.2 – $5.0)] + [0.30 x $1.0]

= $420,000 + $300,000 = $720,000

Despite having reduced the estate tax, the total tax liability (estate and income) paid by the individual and his/her heir is the same with and without a Roth conversion.

Example 4 is based upon the assumptions that (1) the individual and his/her beneficiary have identical ordinary income tax rates and (2) the non-IRA assets that are used to finance the income tax on a pre-death Roth conversion have no embedded capital gain. More generally, when an individual's estate includes non-IRA assets that are worth more than the estate tax exclusion, a Roth conversion immediately prior to death reduces total taxes (estate and income) provided the following condition is satisfied:

$$\frac{T_b}{T_p} > \left[\frac{1}{1 - gT_g}\right] \qquad (4)$$

where T_p is the individual's ordinary income tax rate, T_b is the beneficiary's ordinary income tax rate, T_g is the capital gains tax rate, and g is the embedded capital gain (as a percentage of current market value). Note that this condition is independent of the estate tax rate.

Example 5: Non-IRA assets worth less than the estate tax exclusion, total estate (non-IRA plus IRA) worth more than estate tax exclusion.

Now consider the same set of facts as in Example 4, except that the individual's $6.5 million estate is composed of $4.0 million in non-IRA assets and $2.5 million in a traditional pre-tax IRA. At the time of the individual's death, the estate tax liability is again equal to $525,000. Absent the IRA assets, the individual's estate tax liability would have been $0, since the non-IRA assets ($4.0 million) are worth less than the $5 million estate tax exclusion. Consequently, the IRS would consider the full $525,000 as attributable to the IRA assets and would allow the beneficiary a $0.21 IRD deduction on each $1 withdrawn from the IRA account (up to a maximum deduction of $525,000).[20]

The total tax liability (estate and income) paid by the beneficiary is:

Total tax liability = Estate Taxes + Income Taxes

= [0.35 x ($6.5 – $5.0)] + [0.30 x ($2.5 - $0.525)]

= $525,000 + $592,500 = $1,117,500

If the individual converts his/her IRA assets to a Roth IRA immediately before death, then the income tax liability is $750,000 (30% x $2.5 million). Assuming the individual liquidates non-IRA assets (with no embedded capital gain) to pay the tax, the individual's remaining estate will be composed of $3.25 million of non-IRA assets and $2.5 million in a Roth IRA. The total tax liability (estate and income) paid by the individual and his/her beneficiary in this case is:

[20]The $0.21 IRD deduction for each $1 withdrawn from the IRA account is determined by dividing the total estate tax attributable to the IRA ($525,000) by the total dollar value of the IRA ($2.5 million).

Total tax liability = Estate Taxes + Income Taxes

= [0.35 x ($5.75 – $3.5)] + [0.30 x $2.5]

= $262,500 + $750,000 = $1,012,500

In this case, the total tax liability (estate and income) is reduced by making a Roth conversion before death, even when the ordinary income tax rate is the same for the individual and his/her beneficiary. In contrast to Example 4, the individual's non-IRA assets are worth less than the estate tax exclusion. This reduces the amount of the IRD deduction per dollar of IRA assets and makes a pre-death Roth conversion more attractive.[21]

More generally, when the value of non-IRA assets is less than the estate tax exclusion, but the total estate is worth more than the estate tax exclusion, the condition under which a Roth conversion immediately prior to death will reduce the total tax payments (estate and income) for the individual and his/her beneficiary is:

$$T_b\left[1-\frac{(V-X)T_e}{V_R}\right]+\left[\frac{(V-X)T_e}{V_R}\right]>\left[\frac{T_p}{1-gT_g}\right] \qquad (5)$$

where T_p and T_b are the ordinary income tax rates for the individual and his/her beneficiary, respectively, T_e is the estate tax rate, T_g is the capital gains tax rate, V is the total value of the estate (non-IRA plus IRA assets), and V_R is the value of the IRA

[21]Our analysis makes the simplifying assumption that the IRD deduction is received at the time the IRA is inherited, instead of over time as the assets in the IRA are withdrawn. Consequently, in practice the IRD deduction is somewhat less valuable than in our illustrations.

assets. The right-hand side of Equation (5) is the total tax paid per dollar converted to a Roth IRA prior to death, where the ordinary income tax payment is financed by selling non-IRA assets with an embedded capital gain of g. The left-hand side is the total tax paid (estate and income) by the beneficiary if the Roth conversion is not made prior to death. The first term captures the ordinary income taxes paid on the inherited IRA (net of the IRD deduction) and the second term captures the estate tax payment per dollar of IRA assets.

VI. The Re-characterization Option

One important benefit of a Roth conversion is the ability to reverse the transaction without penalty any time before October 15th of the year following the conversion. This is called *re-characterization*. The rationale for re-characterization reflects the nature of the underlying uncertainty. After a Roth conversion there is great uncertainty about the value that will be left in the Roth account when the taxes and tax return are due, even whether the taxpayer would have the resources to pay the tax liability. Because the taxable income and tax liability created by the conversion is potentially quite large, there is considerable uncertainty about the effective tax rate facing the taxpayer on his conversion.

Re-characterization can be particularly valuable when asset values decline following the original conversion. Recall that the tax on the Roth conversion depends upon the value of the assets held in the IRA account at the time of the conversion. If asset values subsequently decline, the individual may wish to reverse the transaction and execute (after the mandatory waiting period) a new Roth conversion using the lower asset values

to compute the tax liability.[22] Since the tax liability is computed based upon the conversion value, it can be optimal to reverse the conversion if the account value has declined substantially (or even risen only slightly) prior to October 15th in the next year. The cap on the tax liability for this creates an option as the investor can eliminate the liability by re-characterizing his conversion back to the traditional tax-deferred account. This also suggests that until the re-characterization option expires (or otherwise has little value) it can be valuable to locate volatile assets (and especially a highly concentrated portfolio) in a tax-deferred account, unlike the focus of the location analysis in Dammon, Spatt and Zhang (2004) in generic tax-deferred settings.[23]

The conversion and re-characterization options have become especially important starting in 2010 as 2006 tax legislation repealed the $100,000 income restriction for individuals and couples to be eligible to convert.[24] This facilitates conversion by many of those with especially large tax-deferred accounts.[25] We can use arbitrage methods (see

[22]Another discussion of the conversion and re-characterization provisions for traditional and Roth IRAs is in Lange (2006), but the import of the tools of financial economics is not developed there. The re-characterization option was independently identified by both Lange (2006) and earlier versions of our work on estate planning (current version is Dammon, Spatt and Zhang (2007)).

[23]The special tax treatment for "net unrealized appreciation" for employer stock held in 401k plans also alters the conventional asset location advice, but will not be the focus of our analysis.

[24]The ability to convert to a Roth can lead even to the optimality of "after-tax" annual IRA contributions by those not satisfying the income test for annual traditional or Roth IRA contributions. The tax liability of converting after-tax contributions to a Roth IRA is solely the tax on the growth of the after-tax IRA account when the individual does not possess a traditional IRA. This tax likely would be modest if the after-tax contributions were made only a few years before. However, the attractiveness of after-tax contributions in this scenario can be limited due to "blending" rules on withdrawals if the individual already has a substantial traditional IRA accumulation. Under these rules the after-tax funds can only be withdrawn proportionate to the traditional IRA funds of the investor. Consequently, much of the advantage for marginal contributions of after-tax IRA funds of only being taxed on the growth of the funds would not be available in some situations in which only partial conversion would be optimal.

[25]Because of progressivity of the income tax schedule full conversion may not be optimal in a single year. Instead, it may be optimal to push to the end of an interval before which a particular marginal tax rate will jump up, assuming a stepped and progressive tax schedule. Alternatively, for taxpayers subject to the Alternative Minimum Tax (AMT) conversion of an amount consistent with eliminating the AMT liability (and therefore, an effective tax rate of 28% in many situations) would be advantageous in some circumstances. The formal boundary analysis of the optimal exercise of the re-characterization option would be modified slightly for the case in which

Dammon, Spatt and Zhang (2004)) to show that it is efficient to locate very volatile positions in the tax-deferred accounts still subject to the potential for re-characterization. We anticipate that this conclusion would obtain even relative to locating volatile positions in the taxable account (where the ability to benefit from losses is limited and gains liability is deferred rather than eliminated), but have not proved that formally.

Similarly, eligible participants can re-characterize a newly funded traditional IRA as a Roth IRA and a Roth IRA as a traditional IRA until October 15th of the year after the contribution. To maximize the option value it is advantageous to separate these accounts by their respective expiration dates. In effect, accounts in which the option to re-characterize expires on October 15th of the current year, those that expire on October 15th of the following year, and those for which the option is no longer available should not be mixed together.

A fundamental component of our paper is to characterize the optimal re-characterization decisions subsequent to conversion from the traditional tax-deferred account to the Roth account. To illustrate in its simplest form why re-characterization could be optimal, suppose that the investor converts $75,000 from his traditional tax-deferred account into his Roth IRA. If the investor is in a combined federal and state tax rate of 40%, then the tax liability would be $30,000. Suppose that the value subsequently declined by half prior to the re-characterization. The tax liability would be 80% of the reduced value of $37,500. The total account wealth on an after-tax basis would be $22,500 after re-characterization versus $7,500 without re-characterization—suggesting a huge incentive

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conversion occurs over a number of years. While the context of progressive taxes and the Alternative Minimum Tax are somewhat different, the underlying principle in the examples is similar because in both situations we focus on the possibility that the marginal effective tax rates are about to jump upward. Of course, the stochastic aspect of the re-characterization option also would be reflected in a formal analysis.

to re-characterize. In fact, the incentive to re-characterize is reinforced strongly because it preserves the future options to convert and re-characterize.

Because of the value of the future options, in a setting with constant parameters (including tax rates), re-characterization won't be optimal only when the realization is sufficiently high. As an illustration, note that if we modified the above example so that the value of the converted account doubled to $150,000, the tax liability still would be $30,000, which highlights the substantial option value to the potential option about whether or not to reverse the conversion. In general, re-characterization will be optimal only if the account performance is below a critical value. In fact, given the substantial option value associated with re-characterization the investor will find it optimal to re-characterize only if he is sufficiently compensated for foregoing the subsequent series of options to convert and re-characterize on those funds. Because the investor who does not re-characterize foregoes a series of future conversion and re-characterization options rather than just a single pair, the dynamic nature of these options makes it much more valuable to re-characterize and potentially raises the threshold far above the critical value when there would be just a single pair of conversion and re-characterization options. The above example indirectly highlights that the length of the investor's horizon (the number of re-characterization opportunities) will influence the current re-characterization boundary.

The investor will re-characterize unless the performance in the relevant account is sufficiently strong. In effect, we are solving for the cutoff below which exercise of the re-characterization option is optimal.

Proposition 1 (Re-characterization cutoff): The investor will optimally re-characterize his conversion (at the expiration of the re-characterization option) unless the value of his account is above a critical value. [26]

The precise cutoff level for exercising the re-characterization option itself across accounts would depend upon the performance of the securities positions and the specifics of the marginal tax rate function. The overall idea is a robust one that does not rely upon a constant tax environment over time or a flat tax rate in each period. Instead, the result is consistent with a progressive tax schedule and tax rates that change over time. However, under the assumptions of a flat tax in each period, a constant tax rate over time and an infinite horizon, the optimal critical value will be time independent.

Building upon the proposition, we can identify several observations about optimal conversion and re-characterization. Of course, if the investor can costlessly re-characterize and reverse the conversion, then the set of situations in which initial conversion is optimal is greatly enhanced. In fact, the set of situations in which initial conversion is optimal is very general.

Proposition 2 (Optimality of conversion): In the presence of a costless re-characterization option, initial conversion is always optimal because the initial conversion can always be reversed.

This perspective just exploits the observation that the conversion option always has value due to the option to re-characterize the conversion.[27] The benefits associated with

[26]To avoid numerical complications associated with the correlation in the universe of assets, it may be useful numerically to limit the number of potential conversion dates per year, such as to once per month.

this strategy reflect not only the stochastic character of asset prices, but also the effect of statutory changes to tax rates and the investor's financial situation, which also can influence effective marginal tax rates. The only scenario that would appear to invalidate this conclusion is the possibility that the re-characterization option is repealed without the investor possessing the opportunity to reverse his decision.

We also can point to a variety of comparative statics. For example, if future tax rates unexpectedly increased after the conversion but current tax rates were unchanged, then the willingness to re-characterize should decline to avoid the impact of the higher anticipated future tax rates. Arguably such an effect is especially strong at present due to the combination of an anticipated increase in future tax rates and the uncertainty about future tax rates.

Furthermore, the movements in asset prices and even movements in their variability during the last several years highlight the huge potential value to the re-characterization option associated with conversion. For example, the future ability to re-characterize is particularly valuable in a setting in which future asset prices are relatively volatile or where volatility itself is especially volatile and so the current re-characterization option becomes more beneficial. Formally,

Proposition 3 (Volatility and cutoff): An increase in exogenous asset price volatility increases the value of the future re-characterization option, expanding the set of states of the world in which the re-characterization of current conversion options is beneficial.

[27]This is true even if tax rates are expected to decline by the deadline because re-characterization allows the conversion to be costlessly reversed.

This emphasizes the value of appropriate option strategies to fully benefit from the re-characterization option. An interesting institutional aspect concerning how re-characterization works is that the investor can choose which specific accounts to re-characterize. More specifically, the investor can re-characterize some, but not all, accounts towards the end of the relevant tax filing period, i.e., the re-characterization is not an all-or-none decision. Using this institutional observation and the idea underlying a central result in Merton (1973) that a portfolio of options is more valuable than an option on the underlying portfolio implies:

Proposition 4 (Portfolio of options): By structuring each distinct investment position or strategy and converting into a separate Roth IRA account, one maximizes the value of the portfolio of re-characterization options.

Given the structure of investments, this would allow the maximal option value at the stage of potential re-characterization as the account holder could “cherry pick” specific components to re-characterize. At the expiration of a portfolio of re-characterization options the investor faces choices as to which of these options to exercise and reverse the conversion and for which to leave the original conversion standing (which is why the individual account conversions would be beneficial—each would then face a separate re-characterization option).

Of course, the strategies or investments themselves need not be viewed as fixed. In particular, the discussion highlights the contribution of the volatility of the underlying individual positions in this context and that the volatility of the underlying strategies should be endogenous. The combined conversion and re-characterization option over time makes volatility especially valuable. The investor could use tax-deferred accounts

not subject to re-characterization (or even his taxable holdings) to help meet his diversification goals and/or to diversify across the basket of volatile positions and accounts. Consequently, the location choices between such accounts and the taxable account are very different than in the traditional tax-deferred and taxable accounts. In contrast, Dammon, Spatt and Zhang (2004) highlight the theme in the traditional tax-deferred investment context that the tax-deferred account should focus on the high yielding assets. In fact, the same analytics and comparative statics as in Dammon, Spatt and Zhang (2004) would apply to a Roth IRA account as well as the traditional tax-deferred account, absent the re-characterization option and also once the re-characterization option has expired (and conversion has been confirmed in the long-term dynamic).

Proposition 5 (Volatility and value): The re-characterization option increases the value of locating highly volatile assets in tax-deferred accounts in which the re-characterization option has not expired.

However, in the spirit of *Proposition 4* the importance of volatility is within individual component accounts subject to re-characterization rather than necessarily for the overall tax-deferred Roth portfolio. Substantial idiosyncratic risk and strategies that are not highly correlated with one another (or ideally strongly negatively correlated) in individual component accounts would be especially helpful in this context. In fact, appropriately structured option-based strategies can potentially eliminate a substantial amount of the tax liability and reduce the effective tax rate associated with the tax-deferred investment. The ability to obtain separate re-characterization options across accounts and the ability to have significant volatility underlying these points to the substantial value of the options, even for a single stage re-characterization option.

As we've already discussed, the underlying dynamics significantly enhances the option value. To understand the import for an investor, it's helpful to consider what happens to the cutoff level as the investor ages. Several factors immediately point to non-stationarity in the underlying problem in practice:

a) The investor will be required to start mandatory withdrawals from the traditional tax-deferred account at age 70½, but not for the Roth IRA;
b) Conversion is not allowed after the investor's death (e.g., by an inherited IRA non-spousal beneficiary) and the investor's mortality likelihood increases over time.
c) The owner loses the potential advantage of additional effective tax-deferred assets in the interim by delaying conversion.

These factors in isolation point to the cost of delaying the conversion, but that has to be traded off against the substantial value of the re-characterization option as well as the potential option in at least some situations for lower tax rates (given the progressivity and uncertainty in rates, etc.) in the future.

Proposition 6 (Horizon effect): As a result of increasing mortality and the required withdrawal from the traditional tax-deferred account starting at age 70½, the investor will re-characterize in fewer states as he ages.

The effect of the required withdrawal starting at age 70½ may influence the cutoff at somewhat earlier ages because of the possibility that re-characterization is optimal in the future. More broadly, the horizon or aging effect that we point to shows that the cutoff

level is time-varying, reflecting the impact of aging on required withdrawals and mortality and the dynamic structure of the underlying options.

Now we turn to the implication of uncertainty or lack of knowledge of future tax rates.

Proposition 7 (Option on tax rates): The value of the re-characterization option is enhanced by the lack of predictability in both tax rates and the investor's specific tax circumstances.

This is an unusual aspect of the re-characterization option (at least assuming that the re-characterization option itself is not called into question) as most tax planning devices benefit from predictability in the tax environment. In effect, the re-characterization option provides the investor a partial hedge against changes in the tax rules and expectations of changes in future tax rules through October 15th of the following year.

This still leaves open the question of how does this cutoff level or boundary change over time prior to the expiration of the re-characterization option in a given year. First, we should note that ex ante a longer re-characterization option will be more valuable. Hence, early exercise of a prior re-characterization option can be optimal when that will lead to a longer subsequent re-characterization option. Otherwise, the original option still will expire at the final tax filing deadline during the year following conversion (in mid-October).[28] This implies the following result:

[28]Once the investor exercises his current re-characterization option, he would be able to exercise another conversion option and obtain another re-characterization option only after thirty days (or January 1st of the year following conversion, if later). In the formal analysis it could be helpful to abstract from aspects of the tax year structure.

Proposition 8 (Cutoff boundary): The optimal exercise boundary at each point in time will be characterized by a critical value (the investor only considers re-characterizing the lowest performing accounts). The critical value of the exercise boundary on the current re-characterization option will increase monotonically over time (so that the set of states in which re-characterization is optimal increases over the year).

The solution to the exercise boundary is crucial to helping investors understand when to terminate the re-characterization option. The ranking of decisions across accounts about re-characterization before expiration of the option will be ranked by past performance. The volatilities of the various accounts as well as the persistence of the account returns will not influence the decisions about which accounts to re-characterize and reverse the conversion, because absent trading costs these volatilities and correlations can be altered costlessly or reset on a prospective basis within tax-deferred accounts.[29] In many situations it will be optimal to delay exercising the re-characterization options until they are about to expire in order to decide which accounts to re-characterize. Analogously, it is often optimal to defer the exercise of the option to realize short-term losses until the end of the short-term region (see Dammon and Spatt [1996]).

VII. Concluding Remarks

The benefits of a Roth conversion depend upon a variety of factors. This paper attempts to identify the most important factors that influence the tradeoff between the immediate tax cost and future tax benefits of a Roth conversion. In some situations a Roth conversion can be beneficial even if the ordinary income tax rate in retirement is

[29]Of course, the effects associated with re-characterization prior to the end of the re-characterization period are likely to be secondary compared to exploiting the option at its expiration.

expected to be less than the current ordinary income tax rate. The reason is that the payment of the tax on the Roth conversion using non-IRA assets is equivalent to allowing the individual to scale up his/her IRA savings, which has the advantage of earning pre-tax returns.[30]

A Roth conversion also has the potential to reduce the impact of estate taxes for some individuals. However, the benefit of a Roth conversion for estate tax purposes is offset, at least in part, by an increase in ordinary income taxes. The paper extends existing wisdom about the role of different types of tax-deferred accounts as well as the taxable account, including both traditional and Roth tax-deferred accounts and accounts in which the possibility of re-characterization remains open. The use of arbitrage methods will provide fundamental insight about optimal asset location patterns and welfare analyses that focus upon the utility value or shadow price of wealth in different accounts will offer valuable guidance on the choice of optimal distribution decisions of these accounts (what to use for expenditures). The analysis of the re-characterization boundary will help guide decisions as to when the accountholder should re-characterize his conversion and later attempt to again convert. By varying our model assumptions and/or parameter values we would be able to undertake a range of sensitivity analyses.[31]

Our focus upon tax-deferred investing and the Roth IRAs also will potentially facilitate evaluation of contributions to "after-tax" IRAs that would be undertaken by individuals not

[30]The advantage of using non-IRA assets to pay these tax obligations is especially great for a wealthy investor, who typically has only a limited portion of his financial wealth available in tax-deferred vehicles (as reflected in the utility value to an investor of having a larger fraction of their wealth on a tax-deferred basis, when little of the investor's wealth is tax deferred—see Dammon, Spatt and Zhang (2004)).

[31]Numerical solution is helpful because of the time dependence of the re-characterization boundary prior to expiration as well as the repeated nature of the option to convert and re-characterize until the investor decides not to re-characterize.

permitted to make current deductible contributions to their traditional IRA (and not permitted to make Roth IRA contributions). The removal of the income cap for Roth IRA conversions has made it advantageous under some conditions to contribute to these "after-tax" IRAs because conversion to a Roth would subject the holder only to tax on the growth above the contributed non-deductible principal. We anticipate that the analysis will enhance our understanding of the potential efficacy of "after-tax" IRAs (however, the blending rules on withdrawals between traditional and "after-tax" IRAs would limit the effectiveness of this strategy in many circumstances).

While much of our analysis so far has approached the investor's decisions about conversion and re-characterization from a valuation perspective, it also will be interesting to explore further the time profile of government revenues, using results about investor strategies and the stochastic structure of asset returns.

To the extent that conversion and especially re-characterization provide useful choices to investors, obviously those options come at the expense of the government's revenue (increasing the investor's *present value* and reducing the government's). It's also interesting to consider the collateral effects on the time profile of the government's revenue. Roth conversions could generate considerable revenue initially, especially due to the *pent-up* demand in the first years that they are allowed for high income individuals. These revenues are likely to not be sustained and will taper off over time as existing tax-deferred wealth has access to the opportunity for several years. The revenue effect may be especially strong in the first two years because the tax liability from initial year conversions will be spread over two years. Of course, the revenues from conversion will come at the expense of reduced withdrawals later from tax-deferred accounts, *displacing* future revenue. It is also important to recognize that the time profile of the revenue

effects is *stochastic* because the decisions about whether to exercise the re-characterization options depend upon the price movements.

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Appendix: Derivation of the Effective Tax Rate on Investment Income

The effective tax rate on investment income, T_i, is a weighted average of the tax rates on interest income, dividend income, and capital gain income, where the weights depend upon the proportion of asset returns that come from each source. Since the expected return on taxable bonds is composed entirely of interest income, the effective tax rate on taxable bonds is the ordinary tax rate.

For equities, the effective tax rate is affected by the proportion of dividends and *realized* capital gains. In particular, the effective tax rate on equity returns is equal to the following:

$$T_i = \frac{dT_d + g(\alpha T_g)}{r} \qquad \text{(A1)}$$

where d is the dividend yield, T_d is the tax rate on dividends, g is the expected capital gain return, T_g is the tax rate on dividend income, r = d+g is the total pre-tax return, and α is a present value adjustment to take into account the fact that capital gains taxes can be deferred.

The present value adjustment, α, depends upon the individual's average holding period, h. The value of α satisfies the following condition:

$$(1+g)^h - T_g\left[(1+g)^h - 1\right] = \left[1+g(1-\alpha T_g)\right]^h \qquad \text{(A2)}$$

The left-hand side of Equation (A2) is the after-tax value of the individual's equity holding after h periods. Notice that the left-hand side takes into account that capital gains taxes are paid only at the end of h periods. The right-hand side of Equation (A2) is the value of the individual's equity holding after h periods assuming that capital gains taxes are paid annually at the effective tax rate of αT_g. The value of αT_g that satisfies Equation (A2) is the effective capital gains tax rate that is needed in Equation (A1).

Solving the Equation (A2) for αT_g yields:

$$\alpha T_g = \frac{(1+g) - [(1+g)^h (1-T_g) + T_g]^{1/h}}{g} \qquad \text{(A3)}$$

To illustrate, suppose an asset pays a 2% dividend yield and an average capital gain of 8% for a total pre-tax return of r = 10%. Assume that the tax rate on both dividends and capital gains is 20%. The average holding period for the individual is h = 10 years. In this case, Equation (A3) gives the effective capital gains tax rate as:

$$\alpha T_g = \frac{(1.08) - [(1.08)^{10}(1-.20) + .20]^{1/10}}{.08} = 15.246\% \qquad \text{(A4)}$$

The effective tax rate on total equity returns, T_i, can now be determined using Equation (A1):

$$T_i = \frac{.02(.2) + .08(.15246)}{.10} = 16.2\% \qquad \text{(A5)}$$