Sources of Liquidity and Liquidity Shortages

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Abstract

This paper develops a model of liquidity provision in which a financial sector interacts with a liquidity-providing "household sector." Banks have varying need for liquid assets; they can use their own resources, borrow from other banks or households, or sell illiquid assets to fulfill these needs. Demand for liquidity imposes systemic externalities on other banks, but the nature of the externality depends on the degree of elasticity of the supply of liquidity from the household sector. If supply is liquid, then there is an unambiguous bias towards excessively illiquid holdings by banks. When the banking sector is large enough to affect the price of liquidity provided by households, then the direction of the inefficiency in bank liquidity holdings depends on the relative costs of raising external liquidity in anticipation of future shortages and after a liquidity shortage has already hit. We also use our model to analyze whether the banking sector produces an efficient degree of opacity, and if not, whether banks have incentives to invest in assets that are too opaque from a welfare perspective. We find that the answer again depends on the elasticity of liquidity supply from households and in addition on the source of asset opacity.

1 Introduction

The events of recent years have demonstrated that availability or shortages of liquidity can have dramatic effects on financial markets. This has led to increased awareness of the importance of developing models of liquidity provision in these markets.

When trying to understand the effect of liquidity provision in a financial market, it is important to examine the behavior at the boundaries. Models of liquidity and liquidity crises provide very different predictions depending on what is posited about backstop provision of liquidity. In some models, particularly, but not exclusively, those billed as "short term" models, the sole source of emergency liquidity to the banking system is the central bank; absent its intervention, liquidity crises are due to a fixed supply of liquidity suddenly being inadequate for the needs of the financial system. On the other extreme some models postulate the existence of a sea of "liquidity providers" to the economy, with deep pockets of liquid assets, serving as an important backup in the resolution or even prevention of liquidity crises.

The price of liquidity and the elasticity of its supply are central to the model's predictions. But to determine them requires closing the model by taking a stronger stand on the characteristics of these liquidity sources: their objectives, their resources and their comparative advantages relative to the financial institutions with whom they trade liquidity.

In this paper we develop a model of liquidity provision in which a banking sector interacts with a "household" sector[1], sometimes supplying, sometimes demanding liquidity. The banking sector has a comparative advantage in finding valuable but opaque projects. But it has limited liquidity: that is, it has limited capacity for raising funds from the uninformed households—or what is ultimately related, limited ability to trade its own assets to uninformed households. When we examine the interactions between these households and intermediaries we will be able to address the following questions: to what extent is liquidity supplied optimally to the financial sector by the households? What are the determinants of the net supply or demand of liquidity? What effect does securitization or recapitalization (the ability to sell parts of the asset structure, or the ability to sell additional equity) have on the levels of liquidity ex ante and ex post during a liquidity crisis?

In our model, banks at an initial date decide the level of funds they wish to raise and

[1]This sector can also more widely be interpreted as the non-bank sector, thus including hedge funds, private equity and insurance companies.

on how to split their funds between liquid and illiquid assets. At an intermediate date the banks will have varying need for liquid assets. Banks can use their own liquidity to provide these resources. If those are not sufficient, banks have various ways to raise funds: by borrowing from other banks, by borrowing from households, or by selling assets to other banks. A bank that faces a liquidity deficit prefers to first raise funds from the interbank market. If overall liquidity demands by banks are large, borrowing by deficit banks may exhaust the pool of liquidity at surplus banks. Banks then turn to households for raising liquidity. Crucially, the ability of banks to raise funds is still limited by the fact that only the bank itself can make full use of the assets, and that the bank cannot credibly promise the full return on the assets to outsiders. Banks may thus have to resort to asset sales for raising additional liquidity, a relatively costly way of liquidity generation.[2]

As we show, banks' demand for liquidity imposes externalities on other banks in the system. The nature of these externalities depends on the degree of elasticity of the supply of liquidity from the household sector.[3]

The simplest case arises when the banking sector is small, so that household liquidity supply is elastic. In this case, once banks reach their borrowing capacity, they sell assets to other banks in order to raise more liquidity. When banks are limited in their abilities to borrow by the value of their collateralizable assets, they will in general fail to take account of the impact of their liquidity needs on interbank asset prices and the resultant effect on other banks' liquidity needs. The outcome is an unambiguous bias towards excessively illiquid holdings by banks and institutions underprovide liquid reserves as buffers for one another.[4]

It should be noted that this tendency goes beyond the familiar arguments for overin-

[2]In an extension we consider an additional source of generating liquidity: physical liquidation (discontinuation) of assets at even greater costs than asset sales.

[3]In standard banking models (e.g., Diamond and Dybvig, 1983) this elasticity does not play a role since these models (implicitly) assume that it is ex-ante optimal to move all resources to the financial system. As a result, banks cannot raise funds from households in a crisis. There are, however, various channels through which banks generate funds from the household sector in a crisis, for example by raising new equity (as in the crisis of 2008-2009) or through deposit inflows (see for example Gatev and Strahan, 2006).

[4]In the main portion of the paper we demonstrate this with an extreme version of the model, in which when the liquidity demands at deficit banks are very large, the asset sales required to finance their assets exceed the liquidity of the banking system. At that point, some bank assets can no longer be continued at all and become worthless. In an appendix we extend the model to show that the same phenomenon can also arise with a smooth variability in the liquidation value of assets.

vestment in risky assets by institutions subject to limited liability. It is instead a systemic effect: when a bank invests more in risky assets (and finances this by raising funds from households at the initial date), the bank will tend to have higher requirements for additional resources at the interim date. This makes it more likely that the liquid resources in the banking sector will be exhausted, causing either an welfare-reducing transfer of assets across banks or forcing asset discontinuations. Notably, this result obtains even though banks can also use the risky asset to raise liquidity at the interim date.

When the banking sector is large, so that liquidity demand has an impact on the price of liquidity provided by households, additional factors come into consideration. In particular, an essential role is played by the relative costs of moving liquid assets to the banking sector ex post (that is, after a liquidity shortage has already hit) and ex ante (that is, in anticipation of, or protection against, future liquidity shortages). If the costs of moving funds ex post is relatively high, then liquidity should be held in the banking sector as a precaution. If the costs of moving the funds ex ante is high, then liquidity should only be transferred if the need actually arises. We show that the externalities in the market work against complete adjustments in either direction. Banks may then either raise too much or too little liquidity from an ex-ante perspective. This is because they neither fully perceive the social benefits from having more liquidity in the banking sector, nor the social costs from having less liquidity available outside the banking sector that is ready to provide funds to banks. We show that the direction of the inefficiency depends on the relative costs of raising household liquidity ex ante and ex post; in other words, when it is efficient to transfer large funds in advance of liquidity crises, the system transfers too little, and when it is efficient to retain most funds in the hands of households until needed, the system transfers too much.

In sum, our analysis emphasizes that once one takes a more complete stance on the various sources of generating liquidity, it is no longer obvious that the familiar result that banks tend to underprovide liquidity (e.g. Bhattacharya and Gale, 1987) holds. Our analysis also suggests that for the policy implications two factors, which so far have received little attention, play a crucial role: the elasticity of the liquidity supply by the non-bank sector and the relative costs of raising liquidity ex-ante versus ex-post.

We can also use our framework to address a relatively new question, that is, the question whether the banking system produces the "correct" amount of opacity. The ongoing crisis has to some extent being blamed on the complexity of financial products created by banks, which made it difficult for outsiders to evaluate banks but also directly contributed to illiquidity in the crisis. An extension studies whether banks, if given a menu of assets that differ with respect to their opacity (that is, the difficulty for outsiders to evaluate the true value of the assets) and their expected returns, choose the right mix of assets. We find that the answer is generally no. On the one hand, if banks choose more opaque assets, they will find it harder to raise liquidity from outside the financial system in times of crisis. Individual banks do not fully internalize the social costs associated with this. This is because they then tend to become dependent on liquidity from other banks in times of crisis, implying that there is a greater risk that the pool of bank liquidity will be exhausted. Because of this effect, banks have an incentive to choose more opaque assets than socially desirable. On the other hand, a bank will only forego the benefits from having a transparent asset if it is compensated for this through a higher expected return on opaque asset. This in turn provides positive value for the economy, since it means that banks find it less likely that they hit borrowing constraints in times of crisis. This effect suggests that banks may end up with a lower degree of opacity than optimal. We also study the net-effect coming from these two opposing forces and find that it again depends on the elasticity of household liquidity and also on the source of opacity.

The remainder of the paper is organized as follows. The next section relates the paper's setting to the literature. The following section presents the basic model. Sections 4 and 5 analyze the model under two different parameter regions: one in which the banking sector is small relative to the household sector, and one in which it is large. In each case the equilibrium outcomes within the economy are contrasted with efficient outcomes. Section 6 extends the analysis to asset opacity. The final section concludes.

2 Related Literature

Our paper is connected to several strands of literature. Allen and Gale have in various contributions developed the concept of *cash-in-the-market* pricing (e.g. Allen and Gale, 1994, Allen and Gale, 2004): when the supply of liquidity in a crisis is inelastic, the available amount of cash in the market determines asset prices. While in most papers liquidity supply is from within the banking system, Acharya and Yorulmazer (2004), and Acharya Shin and Yorulmazer (2007) have extended the analysis by considering external liquidity supply. In particular, Acharya, Shin and Yorulmazer (2007) analyze the role of foreign investors in supplying liquidity. They provide a model in which in a financial crisis asset supply by domestic firms is so high that it exhausts the demand by domestic investors. Foreign investors then step in, however at an efficiency loss as they are not best users of the assets of the firms.

While our model features both internal and external liquidity, a difference in our analysis is in addition that also the supply of external liquidity (household liquidity in our case) is endogenous and determined from an ex-ante perspective. In particular, we show that there is a trade-off for banks between raising liquidity ex-ante and ex-post. This also contrasts with the setup in Diamond and Dybvig (1983). In their setup, which forms the basis of many banking models, it is implicitly assumed that all resources are channeled to the banking sector at the initial date. In our setup this assumption is relaxed, and it turns out that this has important consequences. This is because maintaining ready resources outside the banking sector can constitute an important source of liquidity in crisis times, making the allocation between both sectors a non-trivial one. Relaxing this assumption also matters crucially for the efficiency implications. While the standard result in banking is that banks have a tendency for underinvesting in liquidity, we show that this results depends on the elasticity of liquidity supply from outside the financial system. In the same way as banks may not internalize the benefits from raising liquidity ex-ante for the financial system (e.g. Bhattacharya and Gale, 1987), they also may not perceive the negative effects of raising liquidity for ex-post liquidity provision from outside the financial system. Depending on the relative efficiency losses from raising liquidity ex-ante versus ex-post, the standard results may then, as we show, be overturned.[5]

While illiquidity in our paper arises from limited pledgeability of assets, the literature has emphasized various other sources of illiquidity and liquidity (see, e.g., Holmström and Tirole, 1998; Holmström and Tirole, 2001; Gorton and Huang (2004); Eisfeldt, 2004; Brunnermeier and Pederson, 2005; Brunnermeier and Pedersen, 2009). In particular, while our setting emphasizes the importance of ex-ante decisions for the amount of liquidity that is available in a crisis, several papers have focused on the "dynamics" of liquidity supply in times of crises. Brunnermeier and Pederson (2005) present a model where forced liquidation at an institution is worsened by predatory traders. These traders initially

[5] The results in our paper also contrasts with Acharya, Shin and Yorulmazer (2007) even when the supply of external liquidity is exogenous. While in this case liquidity in our setting is always underprovided, in Acharya et al. liquidity can also be overprovided. This is because in their paper the risky asset can also be used as a collateral for raising liquidity ex-post. This channel is also present in our paper, however, we also assume that purchasers of bank assets in liquidation cannot extract the full value of these assets (due to loss of information in the process). Hence buyers do not perceive the full social value of liquidity, resulting in liquidity underprovision.

withdraw liquidity in order to create price swings, from which they can profit later by buying assets. Brunnermeier and Pedersen (2007) show that funding liquidity (the ease with which traders can finance themselves in the face of collateral constraints) and market liquidity (the ease with which assets can be sold) can be mutually reinforcing in creating illiquidity. Furthermore, Diamond and Rajan (2009) argue that a fear of future asset sales by troubled banks reduces the willingness of potential buyers to purchase assets. In a similar vein, Acharya, Gromb and Yorulmazer (2009) suggest that surplus banks can exploit their monopoly power by waiting for the failure of deficit banks, rather than purchasing their asset right away.

Our paper also relates to the growing literature on asset opacity. While the traditional banking literature has emphasized that banks have a choice between investing in risky illiquid and liquid safe assets, the crises has highlighted as another source of instability in the financial system the inability of outsiders to value assets correctly (see, e.g., Gorton, 2008). Pagano and Volpin (2008) have shown that issuers of securitizations may rationally release only limited information about their products, as the resultant opacity reduces adverse selection problems in primary markets. This, however, may result in social inefficiencies as there is an (assumed) negative externality from asset opacity.[6] Our paper models such an externality, showing that it can arise because more opaque assets make it more difficult to raise liquidity from outside the banking sector in times of crises, thus making it more likely that the liquidity of the financial system itself is exhausted. Interestingly we also show that the externality can go the other way as banks themselves only invest in more opaque assets if they are compensated for this by higher returns, and choosing assets with higher returns tends to generate positive externalities.

3 The Model

There are three dates: 0, 1, 2. All agents are risk neutral and consume at date 2. There are two types of agents, distinguished by their endowments. Households come from a continuum of mass h and are each endowed with one unit of a liquid asset at date 0. Project owners come from a continuum of mass one. They have no endowment of liquid assets but possess a set of potential illiquid (bank) projects. The projects require one unit

[6] While our paper takes opacity of an asset as given (but lets banks choose between assets of different degrees of opacity), Pagano and Volpin show how a bank can modify the opacity of a given asset pool by choosing how much information to release about it.

of liquid assets in period 0 as input and possibly return R in period 2 (more on this later). There is also a storage technology in the economy (available to everyone), which turns one unit of liquid assets in a period into a one unit of liquid asset in the next period.

At date 0 each project owner sets up a bank (which we index with i). He decides on how much liquid funding to raise from households, y_i, and how much of these funds to invest in illiquid projects, x_i. The remaining part, $y_i - x_i$, is stored as liquidity in the bank. The funds y_i are raised by selling stakes in the bank to households. Providing liquidity to the banking sector is not costless; in any period t transferring liquid assets from the household sector to banks incurs a proportional transfer cost of δ_t.

Households have to decide whether to become bank owners (by transferring liquidity to the bank in return for equity) or to hold on to their liquidity. In the former case, a household only invests in a single bank, which can be justified by a (small) fixed cost of investment. After date 0 there is no difference between the initial owner of projects and households that bought a stake in his bank; they all jointly own and manage the bank.

In period 1, half of the banks will turn out to need a liquidity injection for their illiquid projects. The amount of liquidity needed is l per project, common across all banks in the economy whose projects need a liquidity injection. The variable l is distributed according to a distribution function $\phi(l)$ with full support on $[0, \infty)$. The identities of banks needing liquidity and the size of l are both learned in period 1. Projects with liquidity needs are worthless if the liquidity injection is not provided, otherwise they return the liquidity injection in period 2 (in addition to the normal return of R).

Illiquid projects have the following additional characteristics. If a project is transferred to a household in period 1, it becomes worthless. If transferred to another bank in period 1, its value shrinks by factor β. Projects are not completely attachable: if a bank owes funds to a creditor in period 2, creditors can at most extract $(1 - \alpha)R$ from bank owners, even through bankruptcy.[7] We assume $\alpha > \beta$, that is the capacity to generate funds through asset sales is higher than through borrowing against the assets, even though borrowing may incur lower deadweight losses.

A bank which learns at date 1 that its projects need a liquidity injection can first use stored liquidity to provide the injection. It can also borrow funds from other banks or

[7]Note that we assume, for simplicity, that the liquidity injection does not change the attachability of the project. Otherwise the total pledgeable returns from the banking sector would vary with the liquidity injection, making the calculations more difficult. The main results should continue to hold as long as the liquidity injection does not increase attachability one-for-one.

households.[8] Transferring liquidity from households incurs a deadweight loss of δ_1 per unit of funds. The maximum repayment a bank can promise debtors is constrained by the limited attachability of its assets. Banks can also sell assets to other banks to raise liquidity.

At date 2 all projects that have not been discontinued at date 1 return R or $(1-\beta)R$, depending on whether they are continued at the originating bank or not. Projects with a liquidity injection additionally return l. These returns, together with any liquidity holdings are first used to repay creditors. The residual goes to the bank owners.

Two different constellations arise in this economy, depending on the relative size of the banking sector. We first examine the case where the banking sector is small and household liquidity is in elastic supply. For simplicity we will assume in this case that $\delta_1 = 0$. We will return to the case of $\delta_1 > 0$ when we turn to the examination of a large banking sector.

4 The Banking Sector is Small

We first consider a situation where the resources held in the household sector after period 0 are relatively large to the banking sector. This will be the case, for example, if R is not very large, and hence it is not optimal to invest many funds in banks. In this case, banks effectively face a fully elastic supply of household liquidity at all dates, and households' required return on liquidity will be equal to the return on storage.

Consider the possible situations at date 1. At this date half of the banks have to provide a liquidity injection of l for their projects (deficit banks), while the other half has no liquidity needs for their projects and hence may have spare liquidity (surplus banks). The liquidity a deficit bank requires to continue all its projects is $x_i l$, while its available liquidity is $y_i - x_i$. The liquidity of a surplus banks is $y_i - x_i$, while household liquidity is effectively unlimited.

Depending on the size of l, the following situations can arise:

1. Liquidity requirements are low. In this case, the bank can use its own liquidity or is able to raise sufficient funds from other banks and households to continue all projects. Since there is an excess of liquidity on the level of the economy, the required repayment by creditors (households or other banks) is one per unit of liquidity raised. A deficit bank's total amount of borrowing is constrained by the limited attachability of its assets. The

[8] We preclude raising equity in the interim period.

condition for a deficit bank being able to borrow all required liquidity is

$$x_i l - (y_i - x_i) \leq (1-\alpha)x_i R, \tag{1}$$

that is, the bank's net liquidity needs, $x_i l - (y_i - x_i)$, do not exceed its total attachable returns when all projects are continued, $(1-\alpha)x_i R$.

A deficit bank's date 2 return in this case is simply $x_i R + (y_i - x_i)$ since borrowing does not induce higher costs than storage and the liquidity injection is only a transfer of returns across periods.[9] The return for surplus banks is likewise $x_i R + (y_i - x_i)$ since the price of liquidity at date 1 is one and hence they do not make any extra return on their liquidity holdings. The household sector's return on liquidity holdings is $h - y(1+\delta_0)$, again, because lending to deficit banks at date 1 does not generate a larger return than storage.

2. Liquidity requirements are intermediate. Due to the limited attachability of its assets a deficit bank is now longer able to generate the required liquidity through borrowing:

$$x_i l - (y_i - x_i) > (1-\alpha)x_i R. \tag{2}$$

However, the liquidity requirements are still low enough such that the debt capacity of the banking sector in its entirety (that is, deficit and surplus banks together) can generate enough liquidity to continue all projects. Since household liquidity is not constrained, borrowing from households will still take place at zero interest rates. The condition for the debt capacity of the banking sector being sufficient hence may be stated as:

$$\frac{xl}{2} - (y - x) \leq (1-\alpha)Rx, \tag{3}$$

where we have expressed averages of bank variables by omitting the bank index (e.g., $x = \int_0^1 x_i di$). The left-hand side of (3) gives the total liquidity needs in the banking sector. Those consist of its total liquidity requirements (stemming from deficit banks), $\frac{xl}{2}$, minus the combined liquidity holdings of surplus and deficit banks, $y - x$. The right-hand side gives the total feasible repayments that can be promised to creditors when all projects in the banking sector are continued.[10] Since interest rates are zero, this equals the total amount that can be borrowed at date 0 by the banking sector. Note that condition (3) is an aggregate condition, in contrast to condition (1) which is on the bank level.

[9] A deficit bank's borrowing is $b_i = x_i l - (y_i - x_i)$, its return at date 2 (after paying off creditors) is thus $x_i(R+l) - b_i = x_i R + (y_i - x_i)$.

[10] Note that surplus banks can also pledge their newly acquired assets. Note also that the asset transfer does not reduce attachable part of assets.

Since by condition (2) deficit banks themselves cannot borrow the amounts required to continue all their assets, they have to sell assets to surplus banks. Surplus banks can then use their own liquidity to continue these assets (as a matter of accounting convention, we assume that assets are always sold with no liquidity injected) but can also borrow against their own assets and the newly acquired assets. Since surplus banks are not liquidity-constrained and can borrow from households at zero interest rates, competition among these banks will ensure that the price of assets in the interbank market is $(1-\beta)R$, that is, equal to the value the surplus banks can extract from the acquired assets.

We denote the amount of assets continued at a deficit bank by c_i^M, the amount of assets transferred to surplus banks by t_i^M and its amount of borrowing by b_i^M. A bank's total liquidity after borrowing and asset sales is $y_i - x_i + t_i^M(1-\beta)R + b_i^M$, hence it can continue an amount $c_i^M = \frac{y_i - x_i + t_i^M(1-\beta)R + b_i^M}{l}$ of assets. Since its assets are either continued or transferred we have that the amount of sold assets has to equal the total amount of assets minus the amount of assets the bank continues itself ($t_i^M = x_i - c_i^M$), or

$$t_i^M = x_i - \frac{y_i - x_i + t_i^M(1-\beta)R + b_i^M}{l}. \tag{4}$$

Since deficit bank's are borrowing-constrained, they always borrows up to their attachable assets. This condition may be stated as

$$b_i^M = \frac{y_i - x_i + t_i^M(1-\beta)R + b_i^M}{l} R(1-\alpha). \tag{5}$$

Note that both asset sales and borrowing itself modify a bank's debt capacity since they change the amount of assets a bank can continue and hence the amount it can promise to debtors in date 1. Combining equations (4) and (5) we obtain for a bank's asset sales

$$t_i^M = \frac{x(l - R(1-\alpha)) - (y_i - x_i)}{l - R(\beta - \alpha)}. \tag{6}$$

The returns at date 2 in this case, expressed relative to the case where there are no liquidity needs for projects in the economy ($l = 0$), are as follows: Deficit banks do not make any losses on the assets they can continue since they can finance them at zero interest rates. However, they incur losses from the fact that sold assets only yielded $(1-\beta)R$, while their worth if continued at the bank is R. Given that the total amount of assets sold is t_i^M, a deficit bank's loss is then $t_i^M \beta R$. Surplus banks neither gain nor lose since competition among them ensures that the price at which they acquire assets fully reflects their value to them. Households in turn also do not lose or gain since interest rates from lending to banks are zero.

3. Liquidity requirements are high. The third case arises when liquidity requirements are so high that the combined liquidity the banking sector can generate is no longer sufficient to continue all assets. Analogous to equation (3), this condition may be stated as

$$\frac{xl}{2} - (y - x) > (1 - \alpha)Rx. \tag{7}$$

As before, deficit banks will borrow from households and sell assets to surplus banks, who in turn will raise liquidity from households. However, this no longer enables the banking sector to generate sufficient liquidity. Some projects have to be discontinued and become worthless. Competition among deficit banks then drives the price of assets in the interbank market to zero. This implies that the only way deficit banks can raise liquidity is through borrowing from households. Given that total liquidity after borrowing is $y_i - x_i + b_i^H$, the total return that can be promised to debtors is hence $\frac{y_i - x_i + b_i^H}{l} R(1 - \alpha)$. Since deficit banks find it optimal to borrow up to this limit, we have

$$b_i^H = \frac{y_i - x_i + b_i^H}{l} R(1 - \alpha). \tag{8}$$

Surplus banks effectively face a fully elastic supply of projects at zero costs (through the asset sale market) and take up as much as they can continue. Their attachable assets consist of their originated assets, x_i, plus the newly acquired ones that can be continued. Given borrowing from households (denoted bb_i^H) and own liquidity holdings, $y_i - x_i$, the amount of new projects continued will be $\frac{y_i - x_i + bb_i^H}{l}$. Surplus banks will also borrow up to their limit, implying that their borrowings are given by

$$bb_i^H = (x_i + \frac{y_i - x_i + bb_i^H}{l})R(1 - \alpha). \tag{9}$$

We have for the amount of (deficit bank) assets that can be continued at deficit banks (c_i^H) and surplus banks (denoted cc_i^H)

$$c_i^H = \frac{y_i - x_i + b_i^H}{l} \tag{10}$$

$$cc_i^H = \frac{y_i - x_i + bb_i^H}{l}. \tag{11}$$

Combining equations (8)-(11) we obtain

$$c_i^H = \frac{y_i - x_i}{l - R(1 - \alpha)} \tag{12}$$

$$cc_i^H = \frac{y_i - x_i + x_i l}{l - R(1 - \alpha)} - x_i. \tag{13}$$

The returns at date 2 are then as follows. Deficit banks can continue c_i^H assets at zero costs. However, they fully lose the return on the remaining part of their assets, either because they are discontinued or sold at a price of zero to other banks. A deficit bank thus loses $(x_i - c_i^H)R$ in this case. Surplus banks gain because they can acquire projects for free from deficit banks. Their total gain equals the return from all additional projects they can finance, keeping in mind that they cannot extract their full return. Hence they gain $cc_i^H(1-\beta)R$. As in the other cases households do not make any extra gains.

Having considered the various cases that can arise at date 1, we can now derive the date-0 value of a bank. For this we denote with $\underline{l}_i$ the liquidity shock at which a bank's borrowing capacity becomes binding, and with $\bar{l}$ the liquidity shock at which the banking sector's borrowing capacity becomes binding. At these shocks equation (1) and (3), respectively, hold with equality. Rearranging we get

$$\underline{l}_i(x_i, y_i) = (\frac{y_i}{x_i} - 1) + (1-\alpha)R \tag{14}$$

$$\bar{l}(x, y) = 2(\frac{y}{x} - 1) + 2(1-\alpha)R. \tag{15}$$

We can see that the critical shocks only depend on the (inverse) share of illiquid assets in a banks' portfolios ($\frac{y_i}{x_i}$ or $\frac{y}{x}$) and is hence invariant to a proportional scaling of the banking sector. Note that we have $\bar{l}(x,y) = 2\underline{l}_i(x,y)$ since in a symmetric allocation the borrowing capacity of the banking sector is twice the borrowing capacity of deficit banks.

The expected value generated by bank i is derived as follows. In absence of any liquidity injections at date 1 ($l = 0$) a bank returns $Rx_i + y_i - x_i$ at date 2. Since household liquidity is unconstrained at date 0, their required rate of return on bank equity is one. The unit costs of equity are hence solely determined by the deadweight losses from raising funds, and given by $1+\delta_0$. The value generated by a bank is then simply $Rx_i + y_i - x_i - (1+\delta_0)y_i$, which simplifies to $(R-1)x_i - \delta_0 y_i$.

When liquidity requirements are low ($l \leq \underline{l}_i$), a bank does not make any gains or losses relative to this pay-off, as previously discussed. In the case of a intermediate requirements ($\underline{l}_i < l \leq \bar{l}$), a deficit bank loses an amount $t_i^M \beta R$ due to the value loss from transferring assets. In the case of high liquidity requirements ($l > \bar{l}$), deficit banks lose $(x_i - c_i^H)R$, that is the return on all assets they cannot continue themselves. Surplus banks gain $cc_i^H(1-\beta)R$, which is the return they obtain from all newly-acquired assets. Summing up, we have that the value of bank i is

$$E[\pi_i] = (R-1)x_i - \delta_0 y_i - \frac{1}{2}\int_{\underline{l}_i}^{\bar{l}} t_i^M \beta R\phi(l)dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty}(x_i - c_i^H)R\phi(l)dl + \frac{1}{2}\int_{\bar{l}}^{\infty} cc_i^H(1-\beta)R\phi(l)dl. \tag{16}$$

Welfare in the economy consists of the combined expected consumption of the initial project owners and households. A project owner's expected consumption is equal to the expected profits of the bank since when selling stakes at date 0 he retains the surplus of the venture.[11] Their total expected consumption can hence be obtained by integrating $E[\pi_i]$ over all i. Households' expected consumption is simply h since the return on holding liquidity is one and the expected return on bank investment is one well. Presuming symmetric allocations in the banking sector we obtain

$$W = h + (R-1)x - \delta_0 y - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} t^M \beta R\phi(l)dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty}(x - c^H - (1-\beta)t^H)R\phi(l)dl, \tag{17}$$

where the (aggregate) functions c^H, t^M, t^H and $\underline{l}$ are obtained by replacing the arguments in the respective bank-specific functions with bank-sector averages (e.g., $c^H(x,y) = c_i^H(\int_0^1 x_i di, \int_0^1 y_i di\ ))$ and noting that $t^H = c^H$ (i.e., the total amount of assets transferred by deficit banks equals the total amount of new assets continued by surplus banks).

4.1 Efficient Allocations in the Economy

From (17) we have for the first-order condition for y

$$\frac{\partial W}{\partial y} = -\delta_0 - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\partial t^M}{\partial y}\beta R\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty}(-\frac{\partial c^H}{\partial y} - (1-\beta)\frac{\partial t^H}{\partial y})R\phi(l)dl$$
$$+\frac{1}{2}(t^H(l = \bar{l}) - t^M(l=\bar{l}))\beta R\phi(\bar{l})\frac{\partial \bar{l}}{\partial y} = 0 \tag{18}$$

The first term is the deadweight loss from raising funds for banks at the initial date. The second term arises because higher liquidity holdings lower the amount of assets that have to be transferred across banks when liquidity requirements are intermediate (this terms is positive since $\frac{\partial t^M}{\partial y} < 0$). The next term is due to the fact that when liquidity requirements

[11] Since the expected date 2 return of the bank is $E[\pi_i] + y_i$, he has to sell a stake of $r_i = \frac{y_i}{E[\pi_i]+y_i}$ in the bank in order to raise funds of y_i at date 0. His pay-off at date 1 is $(1-r_i)(\pi_i + y_i)$, which in expected terms is $E[\pi_i]$.

are large, more assets can be continued at their originating bank instead of sold ($\frac{\partial c^H}{\partial y} > 0$) and more assets can be transferred instead of being discontinued ($\frac{\partial t^H}{\partial y} > 0$). The final term arises because higher liquidity holdings reduce the likelihood of the banking sector in its entirety being borrowing-constrained by increasing the critical shock at which the constraint binds ($\frac{\partial \bar{l}}{\partial y} > 0$). This is beneficial because a binding constraint is costly - even if marginally so - because it lowers interbank asset prices by a discrete amount. This in turn means that deficit banks can raise less liquidity and hence have to sell more assets,[12] thus implying larger losses due to asset transfers for the economy.

Note that even though the marginal costs of raising funds from households (δ_0) is constant, an interior solution for y (for given x) can exist. This is because as y becomes larger, the likelihood high liquidity shortages falls and the likelihood of only low requirements increases (both $\underline{l}$ and $\bar{l}$ decline). This, in turn, implies that the marginal benefits from holding liquidity decline. In the limit, as can be easily verified from (18), the benefits from holding liquidity approach zero for arbitrarily large y, thus permitting interior solutions.

We next derive some comparative static results for the efficient choice of y.

Proposition 1 *The efficient amount of bank liquidity y in the economy (at a given amount of investment in the illiquid asset x) is: a) decreasing in the deadweight losses from raising funds from households δ_0; b) increasing in the productivity of illiquid assets R, the amount invested in risky assets x, the deadweight losses from transferring assets across banks β and the non-attachability of assets α.*

We next consider the efficient amount of investment in the risky asset. The first order condition for x is

$$\frac{\partial W}{\partial x} = R - 1 - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\partial t^M}{\partial x}\beta R\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty}(1 - \frac{\partial c^H(x)}{\partial x} - (1-\beta)\frac{\partial t^H(x)}{\partial x})R\phi(l)dl$$
$$+\frac{1}{2}(t^H(l=\bar{l}) - t^M(l=\bar{l}))\beta R\phi(\bar{l})\frac{\partial \bar{l}}{\partial x} = 0. \quad (19)$$

The first term in (19) represents the benefits from more investment in the illiquid asset in the absence of liquidity injections, arising due to its excess return over storage, $R-1$. The other terms are similar to the ones from holding more liquidity but run in the opposite

[12] It can be easily verified that at the marginal shortage we have $t^H(l = \bar{l}) - t^M(l = \bar{l}) = \frac{x\bar{l}}{2}\frac{R(1-\beta)}{(l-R(1-a))(l-R(\beta-\alpha))} > 0$.

direction, that is higher investment in the risky asset increases the losses in the economy when there are sufficiently large liquidity shocks.

Note that for given y an interior solution for x can exist even though the marginal productivity on bank assets is constant. This is because as x increases, the likelihood of intermediate and high liquidity requirements increases, which in turn increases the marginal costs of investment in x.[13]

The next proposition summarizes the comparative static results for x:

Proposition 2 *The optimal amount of investment in the illiquid asset x (for given liquidity holdings y) is: a) decreasing in: the deadweight losses from raising funds from households δ_0, the deadweight losses from transferring assets across banks β and the non-attachability of assets α; b) increasing in: the productivity of illiquid assets R and the amount invested in liquid assets y.*

4.2 Equilibrium Allocations and Efficiency

Project owners chose y_i and x_i so as to maximize the expected profit of their respective banks. Noting that $W = E[\pi_i] + E[\pi_{-i}] + h$, we have for the marginal impact of y_i on $E[\pi_i]$ that $\frac{\partial E[\pi_i]}{\partial y_i} = \frac{\partial W}{\partial y_i} - \frac{\partial (E[\pi_{-i}]}{\partial y_i} - \frac{\partial h}{\partial y_i}$. We focus on symmetric equilibria. Setting $y_i = y$ in the derivatives, we thus obtain $\frac{\partial E[\pi_i]}{\partial y_i} = \frac{\partial W}{\partial y} - \frac{\partial (E[\pi_{-i}]}{\partial y} - \frac{\partial h}{\partial y}$. Using the expression for $\frac{\partial W}{\partial y}$ from (18) and using

$$\frac{\partial E[\pi_{-i}]}{\partial y} = \frac{1}{2}(t^H(l = \bar{l}) - t^M(l = \bar{l}))\beta R\phi(\bar{l})\frac{\partial \bar{l}}{\partial y_i} \tag{20}$$

and $\frac{\partial h}{\partial y} = 0$, we get for the first order condition

$$\frac{\partial E[\pi_i]}{\partial y_i} = -\delta_0 - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\partial t^M}{\partial y}\beta R\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty}(-\frac{\partial c^H}{\partial y} - (1-\beta)\frac{\partial t^H}{\partial y})R\phi(l)dl = 0. \tag{21}$$

Similarly, we have $\frac{\partial E[\pi_i]}{\partial x_i} = \frac{\partial W}{\partial x} - \frac{\partial (E[\pi_{-i}]}{\partial x} - \frac{\partial h}{\partial x}$ and

$$\frac{\partial E[\pi_{-i}]}{\partial x} = \frac{1}{2}(t^H(l = \bar{l}) - t^M(l = \bar{l}))\beta R\phi(\bar{l})\frac{\partial \bar{l}}{\partial x}, \tag{22}$$

[13] Since everything is proportional in the banking sector, the overall optimal size is indeterminate. We could close the model in various ways—for instance by having a maximum number of assets banks can invest in, or by having increasing costs of marginal funds at date 0, or by declining marginal returns on investment.

and $\frac{\partial h}{\partial x} = 0$. Using equation (19) we can then obtain the first order condition for x_i

$$\frac{\partial E[\pi_i]}{\partial x_i} = R - 1 - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\partial t^M}{\partial x}\beta R\phi(l)dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty}(1 - \frac{\partial c^H(x)}{\partial x} + -(1-\beta)\frac{\partial t^M(x)}{\partial x})R\phi(l)dl = 0 \qquad (23)$$

Comparing these conditions to the ones for efficiency (18 and 19) we can see that they differ in that individual banks do not perceive the impact of their choices on $\bar{l}$, and thus the likelihood of a liquidity shortage in the banking sector due to binding borrowing constraints. Since a marginal liquidity shortage is costly, and more liquidity and less investment in the illiquid asset lower the likelihood of aggregate borrowing constraints becoming binding (we have $\frac{\partial \bar{l}}{\partial y} > 0$ and $\frac{\partial \bar{l}}{\partial x} < 0$), there is a positive externality associated with y and a negative with x. The size of these externalities are given by equations (20) and (22). We conclude

Proposition 3 *In equilibrium banks a) invest more in the risky asset than is efficient, b) raise less liquidity from households than is efficient.*

Part a) of the proposition is the familiar result that banks tend to invest too much in risky assets. It arises here, however, not due to an externality on bank debtors but in a systemic context. The systemic externality is due to the fact that a bank does not internalize that when it invests more in risky assets, its net liquidity needs at the intermediate date increase. This is socially costly because it means that the pool of liquidity in the banking sector is exhausted more quickly, making liquidity shortages more likely. Part b) of the proposition is novel in that it suggests that the equilibrium distribution of liquidity between households and banks is inefficient and tilted towards households. The reason why this result obtains is that household liquidity does not have public good character since it is effectively unlimited when the banking sector is relatively small, while bank liquidity entails positive externalities (for the same reason why bank investment entails negative externalities). Since neither households nor banks internalize these effects, too much liquidity remains in the household sector.

Externalities arise in our model only at the marginal liquidity shortage and are due to a discrete change in interbank asset prices. This is the simplest form of an externality that works through the total pool of liquidity at date 1. In the appendix we extend the model slightly by adding liquidation opportunities in period 1 and thereby show that results are not sensitive to the discrete nature of the externality.

5 The Banking Sector is Large

We now consider the case where banks raise an amount of funds that makes household's provision of liquidity no longer fully elastic at all times. Such situations arise when the banking sector is sizeable relative to the amount of funds held by the household sector. In particular, we presume that there are situations where at date 1 the banking sector's liquidity needs exhaust the available household liquidity. Households then possibly earn rents on their liquidity. This will have, in turn, implications for the price of liquidity at the initial date, and hence also for banks' incentives to raises liquidity from the household sector to start with.

For the analysis of this case we now also introduce the deadweight costs δ_1 of raising household liquidity at date 1. We assume for these costs that

$$\delta_1 < (1-\beta)R/l, \tag{24}$$

that is, it is still worthwhile for surplus banks to finance acquired assets by borrowing household liquidity when interest rates are zero.

Due to these deadweight losses deficit banks now strictly prefer to borrow from other banks at date 1. They do this until the liquidity in the banking sector is exhausted. After that, they additionally borrow from households until the liquidity in the household sector is exhausted. Following this they have to physically liquidate assets. At this point, the price of household liquidity will also increase. This also undermines the borrowing capacity of deficit banks, and they hence may also have to engage in asset sales in order to improve the borrowing capacity of the banking sector.

1. Liquidity requirements are low. In this case, the liquidity needs of deficit banks can be met by borrowing from surplus banks. The condition for this is

$$\frac{xl}{2} - (y-x) \leq 0, \tag{25}$$

that is the total liquidity requirements in the banking system, $\frac{xl}{2}$, should not exceed the liquidity available in the banking system, $y-x$. Since there is an excess of liquidity in the banking sector, the interest rate on interbank lending will be zero. It follows that neither deficit nor surplus banks gain or lose in this case, relative to a situation without liquidity needs.

2. Liquidity requirements are intermediate. When

$$\frac{xl}{2} - (y-x) > 0, \tag{26}$$

the liquidity in the banking sector no longer is sufficient to continue all projects. Deficit banks then have to turn to households for liquidity. Since liquidity requirements are not large they can obtain all the required liquidity in this way. Since the liquidity held by households is $h - (1 + \delta_0)y$ and raising funds at date 1 incurs deadweight losses of δ_1, the condition for this may be stated as

$$(1 + \delta_1)(\frac{xl}{2} - (y - x)) \leq h - (1 + \delta_0)y. \tag{27}$$

Since there is no shortage of liquidity in the household sector, households' required return is one. A bank's effective costs of household liquidity, however, is $1+\delta_1$ due to the deadweight loss. Competition for scarce liquidity from surplus banks then drives up its price to $1 + \delta_1$, which makes deficit banks indifferent between either form of borrowing.

Since deficit banks have to borrow

$$b_i^M = x_i l - (y_i - x_i) \tag{28}$$

in order to continue all their assets, they incur losses $\delta_1 b_i^M$. Surplus banks gain δ_1 per unit of liquidity they have available, thus in total $\delta_1(y_i - x_i)$. Households do not earn an extra return on their liquidity holdings.

3. Liquidity requirements are high. The combined liquidity of banks and households is now no longer sufficient to continue all projects. The condition for this being the case is

$$(1 + \delta_1)(\frac{xl}{2} - (y - x)) > h - (1 + \delta_0)y. \tag{29}$$

Since some assets have to be discontinued, competition among deficit banks drives their price in the interbank market (if operative) to zero. Surplus banks, facing an elastic supply of assets, compete for household liquidity to finance acquired projects. Since household liquidity is scarce, not all projects can be continued in this way. The price of household liquidity in equilibrium hence has to raise by at least an amount such that surplus banks cannot increase their profits by financing newly acquired projects with household borrowing. We assume that at this interest rate deficit banks become borrowing-constrained. They hence have to sell assets to surplus banks to increase borrowing capacity.[14]

Since deficit banks will be borrowing-constrained, the price of household liquidity will then be determined by arbitrage considerations on the side of the surplus banks. The

[14] If deficit banks are not borrowing-constrained at this point, no assets will have to be transferred across banks at the marginal aggregate shortage. In this case banks' choices should be efficient. For our overall efficiency implications to hold we thus need that there is a positive likelihood of states of the world where individual borrowing constraints become binding at the marginal aggregate shortage.

price of liquidity raises hence exactly to the level that makes surplus banks indifferent to financing newly acquired assets with household borrowing. The condition for this is $(1-\beta)R+l-(1+r_1)(1+\delta_1)l=0$. Rearranging we thus get for the interest rate at date 1

$$r_1 = \frac{(1-\beta)R/l - \delta_1}{1+\delta_1}. \tag{30}$$

Note that we have $r_1 > 0$ by assumption (24). The condition that deficit banks are indeed constrained by the limited attachability of their assets at this interest rate is

$$(1+r_1)(1+\delta_1)(y_i - x_i l) > (1-\alpha)Rx_i, \tag{31}$$

where $y_i - x_i l$ are the bank's borrowing needs and $(1-\alpha)Rx_i$ is the total amount that can be pledged when all assets are continued. Note that since at a marginal aggregate liquidity shortage the price of liquidity increases by a discrete amount, there are always parameter values for which the attachability of assets indeed becomes binding precisely at the marginal liquidity shortage.

The maximum amount of borrowing a deficit bank can undertake is given by

$$(1+r_1)(1+\delta_1)b_i^H = \frac{y_i - x_i + b_i^H}{l}(1-\alpha)R, \tag{32}$$

where the left-hand side gives us the repayment the bank has to promise at date 2 if its want to raise b_i^H units at date 1 and the right-hand side gives the bank's total attachable assets when all borrowing is used to continue projects (recalling that since the price of assets is zero deficit banks do not generate any liquidity through asset sales). The amount of assets the bank can continue are given by

$$c_i^H = \frac{y_i - x_i + b_i^H}{l}. \tag{33}$$

Combining (32) and (33) we get

$$b_i^H = \frac{(y_i - x_i)(1-\alpha)Rl}{(1+r_1)(1+\delta_1)l - (1-\alpha)R} \tag{34}$$

$$c_i^H = \frac{(y_i - x_i)(1+r_1)(1+\delta_1)}{(1+r_1)(1+\delta_1)l - (1-\alpha)R}. \tag{35}$$

Surplus banks borrow the remaining household liquidity, which is $(2(h-(1+\delta_0)y)-(1+\delta_1)b_i^H)$ per surplus bank, and use it together with their own liquidity to continue projects acquired from deficit banks. Given deadweight losses δ_1 of raising funds, surplus banks have in total $y - x + 2\frac{h-(1+\delta_0)y}{1+\delta_1} - b_i^H$ of liquidity at their disposal. The total amount of projects that can be continued at surplus banks is hence

$$cc^H = \frac{y - x + 2\frac{h-(1+\delta_0)y}{1+\delta_1} - \frac{(y-x)(1-\alpha)Rl}{(1+r_1)(1+\delta_1)l-(1-\alpha)R}}{l}. \tag{36}$$

Note that this is an aggregate condition since for an individual surplus bank it is not uniquely determined how many assets it continues (since surplus banks are indifferent between asset continuations and lending out liquidity to other banks).

Deficit banks lose due to the projects they cannot continue, due to higher interest rates on borrowing and due to the deadweight loss associated with borrowing. The loss per unit of borrowing is $(1 + r_1)(1 + \delta_1) - 1 = r_1 + \delta_1 + r_1\delta_1$, hence their total losses are $(x_i - c_i^H)R + b_i^H(r_1 + \delta_1 + r_1\delta_1)$. Surplus banks only make gains due to the usage of their liquidity, either through lending or by acquiring and continuing assets. They do not gain by borrowing and continuing acquired assets as the interest rate has adjusted such that they are indifferent to this action. A surplus bank's gains are hence $(y_i - x_i)(1 - \beta)R/l$, or $(y_i - x_i)(r_1 + \delta_1 + r_1\delta_1)$ (from 30). The gains for the household sector are r_1 $(= \frac{(1-\beta)R/l-\delta_1}{1+\delta_1})$ per unit of liquidity, thus in total $r_1(h - (1 + \delta_0)y)$.

We denote the liquidity levels at which the liquidity in the banking system and at which the liquidity in the entire economy are exhausted by $\underline{l}$ and $\bar{l}$. From (25) and (27) these shocks are given by

$$\underline{l} = 2(\frac{y}{x} - 1), \tag{37}$$

$$\bar{l} = 2(\frac{y}{x} - 1) + 2\frac{h - (1 + \delta_0)y}{(1 + \delta_1)x} \tag{38}$$

Note that in contrast to the case where household liquidity was elastically supplied both shocks are now pinned down by aggregate conditions (previously, losses were also triggered by individual borrowing constraints).

A bank's expected profit can be derived as follows. When liquidity requirements are low ($l \leq \underline{l}$), neither banks nor households make any gains or losses relative to a situation without liquidity needs. When liquidity needs are intermediate ($\underline{l} < l \leq \bar{l}$), deficit banks suffer losses of $\delta_1(x_i l - (y_i - x_i))$ due to borrowing costs, while surplus banks gain $\delta_1(y_i - x_i)$ from lending out liquidity. When liquidity requirements are high ($l > \bar{l}$), deficit banks lose $(x_i - c_i^H)R + b_i^M(r_1 + \delta_1 + r_1\delta_1)$, while surplus banks gain $(y_i - x_i)(r_1 + \delta_1 + r_1\delta_1)$.

Denoting the required (excess) return on household funds at date 0 with r_0, we have that the expected profit at bank i is

$$E[\pi_i] = (R-1)x_i - (\delta_0 + r_0 + \delta_0 r_0)y_i - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \delta_1(x_i l - (y_i - x_i))\phi(l)dl + \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \delta_1(y_i - x_i)\phi(l)dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty} (x_i - c_i^H)R + b_i^H(r_1 + \delta_1 + r_1\delta_1)\phi(l)dl + \frac{1}{2}\int_{\bar{l}}^{\infty} (y_i - x_i)(r_1 + \delta_1 + r_1\delta_1)\phi(l)dl. \tag{39}$$

Households make an extra gain of r_1 per unit of utility when $l > \bar{l}$, thus in expected terms $\int_{\bar{l}}^{\infty} r_1\phi(l)dl$. It follows that their required return at date 0 is

$$r_0 = \int_{\bar{l}}^{\infty} r_1(l)\phi(l)dl. \tag{40}$$

Household's total expected returns sum to (obviously)

$$y(1+\delta_0)(1+r_0) + (h - y(1+\delta_0))(1 + \int_{\bar{l}}^{\infty} r_1(l)\phi(l)dl) = y(1+r_0). \tag{41}$$

As before, we obtain for welfare in the economy by integrating over bank profits and adding households' returns:

$$W = h + (R-1)x - \delta_0 y - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \delta_1(xl - 2(y-x))\phi(l)dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty} ((x - c^H - (1-\beta)t^4)R + 2(h - y(1-\delta_0))\frac{\delta_1}{1+\delta_1})\phi(l)dl, \tag{42}$$

where we have made use of the relationship

$$\frac{1}{2}\int_{\bar{l}}^{\infty} (x_i - c_i^H)R + b_i^H(r_1 + \delta_1 + r_1\delta_1)\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty} (y_i - x_i)(r_1 + \delta_1 + r_1\delta_1)\phi(l)dl$$
$$= \frac{1}{2}\int_{\bar{l}}^{\infty} ((x - c^H - (1-\beta)t^4)R + 2(h - y(1-\delta_0))\frac{\delta_1}{1+\delta_1})\phi(l)dl - r_0(h - y(1-\delta_0)).$$

5.1 Efficient Allocations

From (42) we have that the first-order condition for y is given by

$$\frac{\partial W}{\partial y} = -\delta_0 + \frac{1}{2}\int_{\underline{l}}^{\bar{l}} 2\delta_1\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty} ((-\frac{\partial c^H}{\partial y} - (1-\beta)\frac{\partial t^4}{\partial y})R - \frac{2(1-\delta_0)\delta_1}{1-\delta_1})\phi(l)dl$$
$$+\frac{1}{2}t^4(l = \bar{l})\frac{\partial \bar{l}}{\partial y} = 0. \tag{43}$$

Efficient bank liquidity thus trades off the deadweight losses from raising funds at the initial date δ_0 (first term) with the benefits that arise because less liquidity has to be raised from households at the intermediate date when liquidity needs are intermediate or high (first integral and last term of second integral) and because more assets can be continued and transferred (instead of liquidated) when liquidity requirements are high (first and second term in the second integral). Additionally, more bank liquidity also affects welfare by modifying the likelihood of aggregate shortages in which assets have to be transferred across banks resulting (last term of the equation). For the impact of y on $\bar{l}$ we have (from 38)

$$\frac{\partial \bar{l}}{\partial y} = \frac{2}{x}\frac{\delta_1 - \delta_0}{1 + \delta_1}. \tag{44}$$

This expression is positive when $\delta_1 > \delta_0$, and negative otherwise. Thus, when $\delta_1 > \delta_0$, bank liquidity brings about benefits by reducing the likelihood of shortages, while when $\delta_1 < \delta_0$ it entails losses because it increases the likelihood of shortages.

Proposition 4 *Comparative statics for efficient level of bank liquidity: as in Proposition 1. Additionally we should have that the optimal level of bank liquidity is increasing in δ_1, the deadweight losses of raising funds at date 1.*

An interesting exercise is to consider an increase in the costs of raising funds at date 1 and 2 by an equal amount. Differentiating (43) with respect to δ_0 and δ_1, we have that this unambiguously reduces the benefits from holding liquidity at banks. The reason is that when funds are held at banks, the additional costs have to be incurred with certainty, while otherwise they only have to be incurred in a crisis. This suggests, for example, that banks that are plagued by a higher degree of informational problems and hence face higher costs of raising funds, should hold less liquidity.

We next derive the first-order condition for x is

$$\frac{\partial W}{\partial x} = R - 1 - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \delta_1(l+2)\phi(l)dl - \frac{1}{2}\int_{\bar{l}}^{\infty} ((1 - \frac{\partial c^H}{\partial x} - (1-\beta)\frac{\partial t^4}{\partial x})R)\phi(l)dl$$
$$+\frac{1}{2}t^4(l = \bar{l})\frac{\partial \bar{l}}{\partial x} = 0. \tag{45}$$

Optimal investment thus trades-off the excess returns from illiquidity, $R - 1$, with higher costs at date 1, which are of opposite nature than the benefits from liquidity holdings described above. The effect that comes through the likelihood of aggregate liquidity shortages (last term in 45) is, however, now unambiguous and negative ($\frac{\partial \bar{l}}{\partial x} < 0$).

Proposition 5 *Comparative statics for efficient level of investment in the risky asset: as in Proposition 2. Additionally we should have that optimal level of investment in the illiquid asset is decreasing in δ_1, the deadweight losses of raising funds at date 1.*

5.2 Equilibrium Allocations and Efficiency

We use again that $\frac{\partial E[\pi_i]}{\partial y_i} = \frac{\partial W}{\partial y} - \frac{\partial (E[\pi_{-i}]}{\partial y} - \frac{\partial h}{\partial y}$. We have from (42) that

$$\frac{\partial (E[\pi_{-i}]}{\partial y} = \frac{1}{2} t^4(l = \bar{l}) \frac{\partial \bar{l}}{\partial y} \tag{46}$$

and we also have from (41) that $\frac{\partial h}{\partial y} = 0$. Using these expressions we then obtain from (43) after imposing symmetry

$$\frac{\partial E[\pi_i]}{\partial y_i} = -\delta_0 + \frac{1}{2} \int_{\underline{l}}^{\bar{l}} 2\delta_1 \phi(l) dl$$

$$-\frac{1}{2} \int_{\bar{l}}^{\infty} ((-\frac{\partial c^H}{\partial y} - (1-\beta) \frac{\partial t^4}{\partial y} - \frac{2(1-\delta_0)\delta_1}{1-\delta_1}) R \phi(l) dl = 0 \tag{47}$$

Similarly, we have

$$\frac{\partial (E[\pi_{-i}]}{\partial x} = \frac{1}{2} t^4(l = \bar{l}) \frac{\partial \bar{l}}{\partial x}, \tag{48}$$

and $\frac{\partial h}{\partial x} = 0$, and the first order condition for the bank's choice of x_i is

$$\frac{\partial E[\pi_i]}{\partial x_i} = R - 1 - \frac{1}{2} \int_{\underline{l}}^{\bar{l}} \delta_1 (l+2) \phi(l) dl - \frac{1}{2} \int_{\bar{l}}^{\infty} ((1 - \frac{\partial c^H}{\partial x} - (1-\beta) \frac{\partial t^4}{\partial x}) R) \phi(l) dl = 0 \tag{49}$$

We note that for both y and x the private and social effects differ because a bank does not internalize the impact of its actions on $\bar{l}$. Since an increase in x unambiguously lowers $\bar{l}$, we have that investment in the illiquid assets is associated with a negative externality, as in the first case. The impact of y on $\bar{l}$ now depends on the relation between δ_0 and δ_1: when $\delta_1 > \delta_0$ bank liquidity increases $\bar{l}$ but reduces it otherwise. Hence there is a positive externality associated with bank liquidity when $\delta_1 > \delta_0$, and a negative externality one otherwise. Note that in contrast to the determinants of the net benefits from raising liquidity, the probability of ending up with high liquidity requirements does not play a role for the efficiency results.[15]

Proposition 6 *In equilibrium banks a) raise less (more) liquidity from households than is sufficient if $\delta_1 > \delta_0$ ($\delta_1 < \delta_0$); b) invest more in the risky asset than is efficient.*

The relationship between δ_0 and δ_1 is a priori ambiguous. On the one hand one may, for example, argue that we should have $\delta_1 < \delta_0$ because when funds are already raised at date 0, banks have control over them for a total of two periods as opposed to one, and hence overall agency problems may be more intense. On the other hand, it can also be argued that raising liquidity in times of crisis is likely to be associated with higher informational problems, and hence we may have $\delta_1 > \delta_0$.

[15] By contrast, in the equation for $\partial W / \partial y$ (equation 43) δ_1 affected the benefits from bank liquidity only when $l > \underline{l}$.

6 The Degree of Opacity in the Banking Sector

We now consider whether banks' decisions to invest in opaque (as opposed to transparent) assets is socially efficient, and if not so, in what direction the bias goes. Asset opacity has two dimensions in our setup: the degree of non-attachability, α, and the value loss to outsiders, β.

Consider first the choice of α. In order to study efficiency, we have to analyze whether there are externalities from the bank's choice of α. We have learned from the previous analysis that any externalities arise through the critical shock at which aggregate shortages occur, $\bar{l}$.

When the banking sector is small, a higher α is associated with a reduction in the critical shock (equation 15). The reason for this is that a lower attachability of assets forces a deficit bank to switch earlier from household borrowing to interbank asset sales in order to alleviate its borrowing constraint. Thus, the pool of bank liquidity is more quickly exhausted. For a surplus bank there is a similar effect since it can now borrow less and hence create less bank liquidity at date 1.

Asset opacity in the form of α is thus associated with a negative externality. However, this does not imply that banks choose overly opaque assets since asset opacity is also costly for banks itself since for more opaque assets its individual borrowing capacity is more easily exhausted (it can be easily verified from (16) that $\partial E[\pi_i]/\partial\alpha < 0$). Thus, banks only invest in more opaque assets when they are also compensated for this through a higher return, R. A higher return, however, is associated with a positive externality since it relaxes the aggregate borrowing constraint by increasing the bank's pledgable returns (we have $\partial\bar{l}/\partial R > 0$). Hence there are potentially offsetting externalities and it is not clear whether (and how) the opacity chosen by banks differs from the efficient one.

In order to address this question we have to study whether the bank gets the trade-off between returns and opacity right. From the equation for $\bar{l}$ we have that the critical shock is not affected by the asset choice if $(1-\alpha)R$ remains constant. We thus consider that a bank can choose from a menu of assets that all differ with respect to their R and α but for which $(1-\alpha)R$ is constant.[16] In particular we assume that the menu is given by a function $\alpha(R)$ ($\alpha'(R) > 0$) with $a(R)$ defined through $(1-\alpha(R))R = \bar{k}$. Rearranging gives

$$\alpha(R) = 1 - \frac{\bar{k}}{R} \tag{50}$$

[16]This menu may also be the result of banks choosing how much information to release about existing asset pools, see Pagano and Volpin (2008).

If the bank is indifferent between these assets, it faces the same trade-off as society, and hence its choice will be efficient. If the bank prefers more opaque assets from this menu, it will tend to invest too much in opacity, and if it prefers the more transparent ones, it will invest too little.

The bank's profits (16) depend directly on R, and indirectly on both R and α through t_i^M, c_i^H and cc_i^H. Using (50) to substitute for α in t_i^M, c_i^H and cc_i^H we obtain

$$t_i^M = \frac{x(l-\overline{k})-(y_i-x_i)}{l-\overline{k}+R(\alpha)(1-\beta)} \tag{51}$$

$$c_i^H = \frac{y_i-x_i}{l-\overline{k}} \tag{52}$$

$$cc_i^H = \frac{y_i-x_i+x_il}{l-\overline{k}}-x_i. \tag{53}$$

From the first equation we see that when the bank chooses a higher α (and thus higher R) it benefits from this because less assets have to be transferred when liquidity requirements are intermediate (t_i^M falls). When liquidity requirements are high, it turns out that the number of assets that can be continued by the bank are not affected by the asset choice (c_i^H and cc_i^H do not depend on $R(\alpha)$). Thus, the indirect effect on banks profits is positive. Since the direct effect (coming from a higher R) on the bank's profits has to be positive as well, it follows that the bank strictly prefers assets with a higher α from the menu. It follows in turn that the bank invests more in opaque assets than socially desirable.

We consider next the choice of β. The critical shock $\overline{l}$ does not depend on β. Thus a higher β-opacity does not entail any externalities. However, a higher β reduces a bank's expected profits by increasing the costs from asset sales (we have $\partial E[\pi_i]/\partial\beta < 0$). Thus, again, a bank only invests in more opaque assets if it is compensated through a higher return R. Since a higher return is associated with a positive externality (since it reduces $\overline{l}$) but opacity itself is not associated with an externality, a bank facing a menu of assets with a return-opacity trade-off would choose assets that are not "opaque enough" from a social perspective (because low-opacity assets are also associated with lower returns and the bank does not internalize the negative externality arising from this). We thus conclude

Proposition 7 *When the banking sector is small, banks tend to invest in assets with a higher α than socially optimal but also in assets with a lower β than socially optimal.*

In the case of a large banking sector the critical shock $\overline{l}$ neither depends on α nor on β (equation 38). Thus, the same arguments as for β-opacity in the case of the a small banking sector can be applied. It follows

Proposition 8 *When the banking sector is large, banks tend to invest in assets with a lower α and a lower β than socially optimal.*

7 Summary

This paper has developed a model of liquidity provision in which a financial sector can obtain necessary liquidity at a cost from external sources, as well as reallocate liquidity among its institutions through interbank borrowing or through sale of assets. The demand for liquidity imposes externalities on other banks, but the nature of the externality depends on the degree of elasticity of the supply of external liquidity from the household sector. If supply is liquid, then there is an unambiguous bias towards excessively illiquid holdings by banks. When the banking sector is large enough to affect the price of liquidity provided by households, then the direction of the inefficiency in bank liquidity holdings depends on the relative costs of raising external liquidity in anticipation of future shortages and after a liquidity shortage has already hit. A priori, it is not clear which of these two costs should be greater: on the one hand, the costs of acquiring liquidity under stress in mid-crisis are likely to be great, particularly because of informational limitations likely to arise in such circumstances. On the other hand, moving otherwise unneeded liquid assets onto a financial institution's balance sheet in anticipation of a low-probability event is also likely to be expensive, not least because of agency problems at banks, leading managers to use free cash inefficiently. But the fact that this calculation can go either way shows that it is not an easy matter to determine the proper regulatory response to potential liquidity shortages.

Nonetheless, our analysis suggests several dimension along which public policy can improve welfare in the financial system. First, it may improve upon the mix of risk and liquidity that banks would choose themselves. Second, it may induce a more efficient allocation of resources between the financial system and the remainder of the economy. Third, it may avoid banks' exacerbating liquidity problems by investing in overly opaque assets. Fourth, from an ex-post perspective, regulators may improve welfare by inducing a more efficient allocation of liquidity between the household sector, surplus and deficit banks. For example, part of the inefficiency losses in a crisis can be avoided by channelling funds from surplus to deficit banks in order to avoid an inefficient transfer of assets within the financial system. However, these and other ex-post policy measures also have important ex-ante affects on the allocation of resources in the economy. Thus, constrained-efficient policies both have to take into account both ex-ante and ex-post effects.

A Appendix: Small Banking Sector and Smoothed Externality

Externalities arise in our model only at the marginal liquidity shortage and are due to a discrete change in interbank asset prices. This is the simplest form of an externality that works through the total pool of liquidity at date 1. In the following we show that results are not sensitive to the discrete nature of the externality. For this we relax the assumption that assets cannot be physically liquidated at date 1. This will introduce additional externalities that arise at any level of aggregate liquidity shortages and are, moreover, of smooth nature.

We modify the baseline model as follows. We now allow banks to liquidate their projects at date 1 at a loss. The marginal liquidation loss of a project is $\gamma(k) = \beta + \gamma_0 k$ ($\gamma_0 > 0$), where k is the amount of projects the bank has already liquidated. The loss from liquidating a small amount of projects is thus not higher than the value loss from transferring projects to other banks.[17] Furthermore, the marginal costs are increasing in the amount of projects liquidated. The interpretation of this is that a bank has projects that differ with respect to their liquidation value. Since a bank will first liquidate the projects that induce lower losses, average liquidation losses increase then with the amount of projects liquidated.

As long as there is no aggregate liquidity shortage, the possibility for banks to (physically) liquidate their assets does not affect the economy as the loss from selling assets to surplus banks is lower than the one from physical liquidation ($\beta < \gamma$ if $k > 0$).

Consider now a situation with an aggregate liquidity shortage ($\frac{xl}{2} - (y-x) > (1-\alpha)Rx$ from equation (7)). As then not all assets can be continued, some assets have to be liquidated. At a given interbank asset price p, deficit banks will liquidate assets until they become indifferent between asset sales and more liquidation. Recalling that c_i denotes the amount of assets continued at a deficit bank and t_i the amount of assets it transfers this condition may be stated as

$$(1 - \beta - \gamma_0(x_i - c_i - t_i))R = p. \tag{54}$$

It follows that in equilibrium we have $p < (1 - \beta)R$ since due to an aggregate shortage some banks are liquidating and hence we have $x_i - c_i - t_i > 0$. Thus, surplus banks make a gain from acquiring assets and will hence demand assets until they have acquired the maximum number of assets they can continue.

[17] If $\gamma(0) > \beta$ we would again obtain a discrete externality at $l = \bar{l}$ as asset prices would then drop by a discrete amount at the marginal shortage.

We next derive the borrowing constraint of deficit banks. Compared the baseline model, deficit banks now have additional sources of liquidity due to physical liquidation and because interbank asset prices no longer are zero. From condition (54) we obtain for the optimal amount of liquidation:

$$k_i = x_i - c_i - t_i = \frac{1 - \beta - p/R}{\gamma_0}, \tag{55}$$

which is declining in the interbank asset price p. The total proceeds from disposing of assets consist of the proceeds from physical liquidations, $\int_0^{k_i}(1-\gamma(x)Rdx$, plus the revenues from selling assets, pt_i. Solving the integral (using equation (55) to substitute l_i) and adding the sales revenues we obtain for the total proceeds: $p(x_i - c_i) + d(p)$, where

$$d(p) = \frac{\frac{p^2}{R} - (1-\beta)(2p - (1-\beta)R)}{2\gamma_0}. \tag{56}$$

The proceeds are thus equal to the revenues from selling all assets that cannot be continued, $x_i - c_i$, plus an additional term that depends on p. The reason for this additional term is that the marginal liquidation costs are increasing, and hence the average liquidation proceeds are higher than the proceeds on the last liquidated unit (which are equal to p). This can be verified by noting that $d(p) = 0$ for $p = (1-\beta)R$ and that $d(p)$ is decreasing in p (follows from taking derivative with respect to p and using that $p < (1-\beta)R$).

A deficit bank's borrowing limit is thus given by

$$b_i^H = \frac{y_i - x_i + b_i^H + p(x_i - c_i) + d(p)}{l} R(1-\alpha), \tag{57}$$

where the difference to borrowing constraint of the baseline case (equation (8)) is the revenue from disposing of assets, $p(x_i - c_i) + d(p)$. Since liquidation of either type is costly for the bank, we must have that a deficit bank generates just a sufficient amount of liquidity that allows to continue its remaining assets. Noting that the amount of assets a bank can continue equals its liquidity, $y_i - x_i + b_i^H + p(x_i - c_i) + d(p)$, divided by l, this condition may be stated as:

$$c_i = \frac{y_i - x_i + b_i^H + p(x_i - c_i) + d(p)}{l}. \tag{58}$$

We now turn to the surplus banks. A surplus bank takes on at given price p ($< R$) as much projects as it can continue. Its liquidity after borrowing an amount bb_i and purchasing cc_i units of assets is given by $y_i - x_i + bb_i - p \cdot cc_i$.[18] The maximum number of assets it can

[18] A surplus bank may also (physically) liquidate own projects in order to generate liquidity for purchasing assets from deficit banks. Given that the proceed on the first unit of liquidation is $(1-\beta)R$, a surplus

continue is thus given by

$$c_i = \frac{y_i - x_i + bb_i - p \cdot cc_i}{l}. \tag{59}$$

Its borrowing limit is given by

$$bb_i^H = (x_i + cc_i)R(1-\alpha), \tag{60}$$

and includes the attachable part of newly acquired assets.

Combining equations (57)-(60) to eliminate the amount of borrowing, we obtain for the number of assets that are continued at deficit and at surplus banks:

$$c_i = \frac{y_i - x_i + px_i + d(p)}{l - R(1-\alpha) + p} \tag{61}$$

$$cc_i = \frac{y_i - x_i + x_i R(1-\alpha)}{l + p - R(1-\alpha)}. \tag{62}$$

We next derive an expression for the interbank price p. Equation (62) gives us the asset demand per surplus bank at a given price p. Aggregating over surplus banks we obtain

$$cc(p) = \frac{y - x + xR(1-\alpha)}{l + p - R(1-\alpha)}. \tag{63}$$

Demand is downward sloping because when asset prices are higher, purchases by surplus banks more quickly exhaust their liquidity. This reduces the amount of newly acquired assets they can finance, and hence reduces their demand.

Deficit banks supply assets. For each individual deficit bank its supply is implicitly determined by the fact that it physically liquidates assets until the marginal loss has reached the loss from selling at the market. Solving equation (55) for t_i and using equation (61) to substitute for c_i we obtain for aggregate supply after aggregating over deficit banks

$$t(p) = x_i - \frac{y - x + px}{l - R(1-\alpha) + p} - \frac{1 - \beta - p/R}{\gamma_0}. \tag{64}$$

Supply may be either up- or downward sloping. The former is because when interbank market prices are higher, banks find it optimal to liquidate less physically and hence supply a larger amount of assets. This corresponds to the last term, $-\frac{1-\beta-p/R}{\gamma_0}$, which increases in p. The latter is because a higher price for assets means that banks have to liquidate less in order to satisfy their liquidity needs. This relates to the term $-\frac{y-x+px}{l-R(1-\alpha)+p}$, for which it can be verified that it is decreasing in p when there is an aggregate shortage.

bank would need to liquidate $\frac{p+l}{(1-\beta)R}$ of its own assets in order to acquire and continue one asset from deficit banks. The impact on its date 2 output would then be $-\frac{p+l}{(1-\beta)R}R + (1-\beta)R + l$. However, this expression is negative if $p > (1-\beta)^2 R - \beta l$ (and in particular if $p = (1-\beta)R$), hence unless interbank market prices are very low, this is not optimal.

Setting demand equal to supply ($cc(p) = t(p)$) and rearranging we obtain:

$$\frac{2(y-x) + 2xR(1-\alpha) - lx}{l + p - R(1-\alpha)} + \frac{1 - \beta - p/R}{\gamma_0} = 0. \tag{65}$$

Note that the left hand side of this expression is declining in p. The left hand side is also increasing in y, from which it follows that we must have $p'(y) > 0$. Taking the derivative of the left hand side with respect to x ($= \frac{-2+2R(1-\alpha)-l}{l+p-R(1-\alpha)}$) and imposing the condition for an aggregate shortage ($\frac{xl}{2} - (y-x) > (1-\alpha)Rx$) we find that the left hand side declines when x increases. We hence have that $p'(x) < 0$. Summarizing we thus have that a higher supply of liquidity by banks (or lower investment in the risky asset) increases the interbank price p.

The value of a bank is identical to the baseline case (equation (16)), except for the cases of aggregate shortages (occurring when $l > \bar{l}$). When having a liquidity deficit a bank no longer loses the entire return on assets it cannot continue ($x_i - c_i^H$) but only an amount $R - p$ per asset. It additionally gains $d(p)$ due to convex liquidation costs. Furthermore, when being a surplus bank assets no longer can be acquired for free, reducing the benefits per acquired asset by p. We thus have for the expected value of bank i

$$E[\pi_i] = (R-1)x_i - \delta_0 y_i - \frac{1}{2}\int_{\underline{l}_i}^{\bar{l}} t_i^M \beta R \phi(l) dl$$
$$-\frac{1}{2}\int_{\bar{l}}^{\infty} ((R-p)(x_i - c_i^H) - d(p))\phi(l)dl + \frac{1}{2}\int_{\bar{l}}^{\infty} cc_i^H((1-\beta)R - p)\phi(l)dl, \tag{66}$$

where c_i^H and c_i^{HH} are now given by equations (61) and (62). We obtain welfare in the economy by integrating over all banks (presuming symmetric allocations):

$$\begin{aligned} W &= h + (R-1)x - \delta_0 y - \frac{1}{2}\int_{\underline{l}}^{\bar{l}} t^M \beta R \phi(l) dl \\ &\quad -\frac{1}{2}\int_{\bar{l}}^{\infty} \left((R-p)(x - c^H - cc^H) - d(p) + \beta cc^H R\right) \phi(l) dl. \end{aligned} \tag{67}$$

Welfare differs from the baseline model in that the value loss from physical liquidation is only $(R-p)(x - c^H - cc^H) - d(p)$ instead of previously $R(x - c^H - cc^H)$. As in the baseline model we can identify externalities by considering the impact of a bank's individual choice on the value of other banks, $E[\pi_{-i}]$. From equation (67) we obtain for the liquidity externality

$$\frac{\partial E[\pi_{-i}]}{\partial y_i} = -\frac{1}{2}p'(y_i)\int_{\bar{l}}^{\infty} \frac{\partial\left((R-p)(x - c^H(p) - cc^H(p)) - d(p) + \beta cc^H(p)R\right)}{\partial p}\phi(l)dl. \tag{68}$$

Note that there is no longer an externality at $l = \bar{l}$ since the amount of liquidated assets $(x - c^H - cc^H)$ is zero at $\bar{l}$ (and hence also $d = 0$) and we additionally have that $t^M(\bar{l}) = cc^H(\bar{l})$, that it the total amount of assets transferred by deficit banks equals the total amount continued at surplus banks.

As shown previously we have $p'(y_i) > 0$, that is a higher supply of liquidity increases prices. The term in the integral is negative. First, an increase in p reduces the losses from assets that cannot be transferred: $\frac{\partial(R-p)}{\partial p}(x - c^H(p) - cc^H(p)) < 0$. Second, it reduces the amount of assets that are liquidated, additionally lowering liquidation costs: $(R - p)\frac{\partial(x-c^H-cc^H)}{\partial p} - \frac{\partial d(p)}{p} < 0$ (this can be verified from taking derivatives in equations (56), (61) and (62) and and imposing the condition for an aggregate shortage, equation (7)). Third, it reduces the occurrence of asset transfers: $\frac{\partial cc^H}{\partial p} < 0$ (follows directly from (62). We can thus conclude that investing more in liquidity constitutes a positive externality.

Likewise, we obtain for the externality from investment in the risky asset:

$$\frac{\partial E[\pi_{-i}]}{\partial x_i} = -\frac{1}{2}p'(x_i)\int_{\bar{l}}^{\infty} \frac{\partial \left((R-p)(x - c^H(p) - cc^H(p)) - d(p) + \beta cc^H(p)R\right)}{\partial p}. \tag{69}$$

Since $p'(x_i) < 0$ and observing that the integral is the same as in the expression for liquidity, we have that this externality is a negative one. We can thus conclude that, as in the baseline model, banks will hold in equilibrium too few liquidity and invest too much in the risky asset from a welfare perspective. Note that in contrast to the baseline model the externality works at all $l > \bar{l}$ by (smoothly) changing asset prices.

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