# Self-fulfilling Liquidity Dry-ups

February 15, 2011

**Abstract**

Secondary markets for long-term assets might be illiquid due to adverse selection. In this paper, I show that: (1) when agents expect a liquidity dry-up on such markets, they optimally choose to hoard non-productive but liquid assets; (2) this hoarding behavior worsens adverse selection and dries up market liquidity; (3) such liquidity dry-ups are Pareto inefficient equilibria; (4) the government can rule them out. Additionally, I show that idiosyncratic liquidity shocks *à la* Diamond and Dybvig have stabilizing effects since they improve market liquidity. Requiring agents to self-insure against such shocks can only reduce welfare; imposing financial institutions to hold liquidity buffers may thus have unintended adverse consequences.

# 1 Introduction

Liquidity can be understood as the ability to transform long-term assets into current consumption goods. In that sense, secondary markets play a crucial role for liquidity provision. However, we know from Akerlof (1970) that they might be subject to a *lemons problem*. What I present here is a model in which the fear of a market breakdown due to adverse selection might induce agents to adopt hoarding behaviors that would actually cause such a breakdown. In that case, it becomes extremely costly to transform long-term assets into current consumption goods, which is the rationale for calling such self-fulfilling episode a liquidity dry-up.

The crucial mechanism is that the proportion of *lemons* in the market increases when sellers exhibit hoarding behavior. The intuition is best apprehended from a buyer point of view: the more a seller tends to hoard liquidity, the less likely it is that he trades because he needs cash. Why would he want to sell then? The answer is striking: the asset must be a *lemon*! This mechanism resonates with the recent financial crisis, which has seen massive cash hoarding behavior and the dry-up of markets such as those for asset-backed securities. Moreover, the fact that these assets have been dubbed "toxic" strongly suggests that fears of adverse selection have been at play (Tirole, 2010, Morris and Shin, 2010).

The key ingredients of the model are that agents want to smooth consumption and that they face a potential *lemons problem* in the secondary markets for long term assets.

Firstly, I show how hoarding behavior due to the fear of future market illiquidity worsens adverse selection and may lead to self-fulfilling liquidity dry-ups. The unveiling of this feedback relationship between adverse selection and hoarding is the main contribution of the paper. Then, as a secondary contribution, I introduce heterogeneity in preferences and I discuss the interactions between idiosyncratic illiquidity shocks, market liquidity, and risk sharing. One of the main implications being that requiring agents to self-insure against idiosyncratic illiquidity shocks reduces welfare in such a set-up. A policy imposing banks to hold liquidity buffers may thus have adverse unintended consequences.

The ex-ante identical agents of the model live for three dates and desire to smooth consumption. They may invest in long-term risky projects and they also have access to a risk-less one-period storage technology. If successful, long-term projects yield a better return than storage, and conversely if they fail. Agents privately observe the quality (success or failure) of their projects at the interim date. At this point, they might want to realize a share of their long-term projects either to take advantage of their private information or to provide for current consumption needs. Because of adverse selection, the price they can get on the secondary market is determined by the average seller motive for trading. There is therefore a return-liquidity trade-off: long-term investment is on average more productive but liquidation on the secondary market might be endogenously costly because of adverse selection.[1]

One of the main point of the paper is to show that, in such a situation, investment decisions (the extent to which agents expose themselves to maturity mismatch) present strong strategic complementarities. To illustrate this, consider the two polar cases:

If agents hoard, they will *not need* to participate in the secondary market to provide for interim consumption. Therefore, assets can only trade at the *lemons* price and the market will be illiquid, which justifies the initial hoarding decision.

Conversely, if agents decide to be fully invested in the long-term technology, they will *need* to participate in the secondary market to provide for interim consumption. This is true irrespective of their asset quality. Of course, there will still be *lemons* in the market, but there will also be good assets, and if the mixture is good enough, the market price might be higher than the return to storage. In that case, the market is liquid and there is no reason to hoard, which justifies the initial decision to be fully invested in the long term technology.

Whereas the decision to self-insure (that is, to hoard) is individually optimal in a low-liquidity world, it is socially costly in two respects. First because it wastes resources on the storage technology (long-run investment is on average more productive) and, second, because ex-ante self-insurance hinders ex-post risk sharing since it prevents agents from providing the positive externalities associated with the issuance of claims to high-return projects.

Hoarding behavior imposes thus negative externalities on other agents and this leads to multiple equilibria for a wide range of parameter sets. In this case, even if a "high liquidity" equilibrium is possible, when agents expect the interim market to be illiquid, they optimally choose to hoard. Such a response reduces ex-post market participation which worsens adverse selection and dries up market liquidity. This is how a liquidity dry-up can be self-fulfilling.

It is typical to find multiple equilibria in games of strategic complementarities. Still, relaxing the assumption that agents share a common knowledge of the economic environment is likely to imply equilibrium uniqueness. Interestingly, this need not be the case here because investment decisions are actually strategic *substitutes* up to the level at which they become *complements*, which precludes the use of standard global games techniques (Carlsson and van Damme, 1993; Morris and Shin, 1998; Frankel, Morris, and Pauzner, 2003; Vives, 2005). Furthermore, the *single-crossing conditions* for uniqueness required by other approaches (Goldstein and Pauzner (2005); Mason and Valentinyi (2010); Bueno de Mesquita (2011)) are also violated,

[1]This concept of adverse-selection-driven endogenous liquidity in asset markets is introduced by Eisfeldt (2004). In her model, current needs for resources depend on past decisions and on information about future income. Thus, there can be reasons (e.g. consumption smoothing in the case of a negative income shock) to sell high quality claims, even at a discount. The higher the discount one concedes to sell a good asset, the lower its liquidity. Similarly, the higher the proportion of agents trading for consumption-smoothing or risk-sharing purposes (instead of private information about payoffs), the lower the adverse selection and the higher the liquidity of the market.

which suggests that multiplicity is quite robust. Indeed, such feature of the model is not driven by specific modeling choices but, instead, is intimately related to the endogenous nature of liquidity. A metaphoric way to put it would be that agents only take actions that will contribute to future market liquidity when they are truly optimistic about future liquidity conditions. In the case they are pessimistic, a decrease in pessimism would only induce actions that contribute to the crowding out of future liquidity.

The idea that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998), and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment is present in Eisfeldt (2004). However, that a liquidity dry-up can endogenously arise for the very reason that investors self-insure against it is a new result.

Heider, Hoerova, and Holthausen (2010) present a model of adverse selection in the interbank market. Besides focusing on another market, my model is different on several important dimensions, the main one being that the price at which long-term assets can be sold is endogenously determined. Their results are also different as they do not find a clear relationship between hoarding and adverse selection. This is due to the fact that they only consider a single and *exogenous* reason to borrow in the interbank market: a liquidity shock. Hoarding can therefore not have an impact on the average reason to participate in the market. Parlour and Plantin (2008) propose a model in which banks issue loans to firms and may sell them on a secondary market. Banks might then be tempted not to monitor the firm properly and therefore sell *lemons* on the market. The authors analyze the optimal contract between firms and banks and show that the existence of a liquid secondary market might be inefficient from an ex-ante point of view. They focus on the implication of the existence of a secondary market on the ex-ante optimal contract. While my model abstracts from this dimension, their modeling approach also rules out the channel by which hoarding feeds back into adverse selection.[2]

The fact that long-term investment crucially depends on expectations on future market liquidity leads to the first policy implication of the model. Notwithstanding moral-hazard concerns, this "expected liquidity channel" should indeed not be overlooked when considering public intervention in the case of a financial crisis. In the model, a public liquidity insurance prevents self-fulfilling liquidity dry-up because the prospect of a market bailout suppresses the return-liquidity trade-off. This prevents wasteful self-insurance and boosts long-term investment and ex-post market participation, which then has a positive feedback effect on liquidity.

To expose the secondary contribution of the paper, I consider heterogeneous preferences: as in the banking literature, agents face idiosyncratic illiquidity shocks.

When the realization of such shocks is private information, I find that market liquidity improves. Because *early* (those that are hit by the shocks) agents issue claims irrespective of the underlying project return, it

[2] See subsection 3.2 on robustness for more details.

indeed reduces adverse selection. As market liquidity improves risk sharing, this suggests that idiosyncratic liquidity shocks need not be socially a bad thing. Moreover, requiring investors to self-insure against them, which might be desirable due to considerations out of the scope of this paper, decreases market liquidity in the good equilibrium and shrinks the parameter sets consistent with its existence. Hence, such a policy can only decrease welfare in this set-up and may even provoke a liquidity dry-up. It should therefore not be taken for granted that liquidity buffers only have positive externalities. This is the second policy implication of the paper. Finally, when there exists a technology that enables them to disclose their liquidity position, ex-post *early* agents are better-off under disclosure. It isolates them from the negative externalities exerted by other lemons owners (those that are not hit by the shock; I dub them *normal* agents). However, these ex-post *normal* agents incur a larger liquidity discount because the probability that they try to sell a lemon, conditionally to have a good liquidity position, increases. Therefore the effect on ex-ante risk sharing is ambiguous. This last result illustrates the ambiguity of transparency on liquidity suggested by Holmström (2008).

The relationship I show between market liquidity, hoarding and risk sharing complements the literature on the competing role of banks and markets for the provision of liquidity. In Diamond and Dybvig (1983), there is no market and banks can generally provide liquidity and improve on the autarky allocation. Jacklin (1987) shows that this is not the case if there exists a secondary market because the bank demand deposit contract is no longer incentive compatible. Diamond (1997) generalizes these results with a model of exogenous limited market participation. He finds that the lower the participation in the market, the greater the role of the banking sector. In that respect, besides the addition of a *lemons problem*, the key differences of my model are that limited market participation is endogenous and investors are needed to run the initial phase of the project. Indeed, I assume that projects could not be run mutually in the first period and that there is no means by which agents could credibly commit to invest. Otherwise, agents could form a coalition in order to pool resources and diversify idiosyncratic risk away. This coalition would correspond to the bank[3] in Diamond and Dybvig (1983) and, as there is no aggregate shock to the fundamentals in my model, it could implement the first-best allocation. These assumptions are strong, as real banks do pool individual resources. However, there certainly are frictions preventing them to ex-ante pool resources among themselves and achieve full risk-sharing.

Section 2 presents the model, section 3 studies liquidity dry-ups, section 4 relates the model to the 2007-2009 financial crisis, section 5 considers the impact of idiosyncratic liquidity shocks and section 6 concludes.

[3] To avoid confusion, such a bank would correspond to a coalition of investors (or of small banks, seen as investors) in my model.

## 2 The model

### Technology

There are three dates ($t = 0, 1, 2$) with a unique consumption good that is also the unit of account. At dates 0 and 1, investor have access to a risk-free one-period storage technology that yields an exogenous rate of return $r$. At date 0, they have also access to a risky long-run technology. Such projects are undertaken at date 0 and only pay off at date 2. They succeed with probability $q > 0$, in which case they yield a return $R_H$ per unit invested. In case of failure, which occurs with probability $1 - q$, they yield $R_L$, with $0 \leq R_L < R_H$. Investors are needed to initiate their own projects and have no means to credibly commit to properly invest. Projects can therefore not be sold at date 0. What I want to capture with this is that moral hazard concerns make date-2 output not pledgeable at date 0, which of course restricts ex-ante risk-sharing[4]. It is equivalent to assuming that output is only pledgeable *after* the project has been properly initiated. A way to justify that would be to assume that proper initiation is observable but not verifiable[5]. Also, storage is neither observable nor verifiable by outsiders. This prevents agent to ex-ante write contracts contingent on that variable.

Projects cannot be physically liquidated at date 1. However, at that date, investors may issue claims to their projects in a competitive and anonymous[6] secondary market. For simplicity, the output of the underlying project will be verifiable at date 2. Therefore, I abstract from moral hazard problems once the project has been properly initiated[7]. I do not allow for short-selling and there are thus no other way to borrow against future income than to issue claims to ongoing projects.

To make the analysis interesting, I assume $qR_H + (1-q)R_L > 1 + r$ and $R_L < 1 + r$: on average, long-term projects are more productive than storage, but they yield less than storage in case of failure.

At the beginning of date 1, investors observe their project's quality. This is private information, and quality is common to all the projects of a given investor. We can thus think of each investor owning only one project of variable size. However, quality is independent across agents, and average quality is thus deterministic.

[4]There is a consequent literature on the role of banks for such purpose when contracts are available to pool resources ex ante. See for instance Diamond (1984). What I study here is an economy in which such pooling is not possible.

[5]This distinction is borrowed from the incomplete contracts literature (see for instance Tirole, 1999). Non verifiable means that it cannot be proved in court. A reason for this could be that "proper initiation" is a too complex and subjective notion to be accurately described in juridical terms. See Malherbe (2010) for further discussion.

[6]The anonymity assumption simplifies the analysis. It is actually not required to derive the main result.

[7]Moral hazard does of course play a crucial role in the funding of risky projects (see for instance Stiglitz and Weiss (1981), Hart and Moore (1994), Kiyotaki and Moore (1997), Holmstrom and Tirole (1997) and (1998)). However, the present paper focuses on adverse selection. It makes thus sense to contain the moral hazard channel.

#### Investors

There is a measure one of ex-ante (at $t = 0$) identical *investors*[8] maximizing expected utility, which they derive from consumption at date 1 and 2. Their period utility function $u(\cdot)$ is increasing, strictly concave, and twice continuously differentiable. At date 0, they are endowed with one unit of the consumption good, which they allocate between the long-term risky investment and short-term storage.

These agents *may* face idiosyncratic illiquidity shocks: at date 1, they learn whether they are *normal* or *early* consumers[9]. These two kinds of agents differ by the subjective factor $\beta$ they use to discount date-2 utility: $\beta \in \{0,1\}$ with $Prob(\beta = 1) = p$ and $0 < p \leq 1$. In the spirit of Diamond and Dybvig (1983), and in line with Eisfeldt (2004), *early* agents can also be viewed as agents incurring a need for liquidity due to either a current income shock or a very good investment opportunity[10]. Patience is independent across agents and is also independent of project returns. Ex-ante probabilities are common knowledge and aggregate investment is observable once it has been committed. There is thus no aggregate uncertainty in the fundamentals of this model. Ex-interim (at date 1) agents *may* thus differ in two dimensions: project quality and patience, which make four potential types of agents from that point in time. From an ex-ante point of view, these are four individual states of nature.

The presence or not of *early* consumers (that is whether $p < 1$ or $p = 1$) does not matter for the main contribution of the paper. For this reason, and for the sake of simplicity, I will assume them away until section (5) where I present the secondary results.

#### The time line

At date 0, investors

- Form anticipations about date-1 secondary market price.
- Choose $\lambda$ the share of endowment they invest in the long-term technology, the remaining being stored.

At date 1, they

- Learn their true type: project return and patience.

[8] One can for instance think of these investors as entrepreneurs undertaking real projects or as banks issuing loans. I use the word "investors" because I model their problem as a portfolio choice.

[9] Most authors that use such preference shocks use the terminology *early* and *late* to distinguish between consumer types. I do not stick to that terminology because whereas the typical *late* consumers do only care about date 2 consumption, the *normal* agents of my model are standard consumption smoother.

[10] In Eisfeldt (2004), liquidity shocks are endogenous as they take the form of current income shocks and information about future income shocks that both depend on past investment decisions.

- Choose how much unit claims to issue out of their long-term investment and how much to consume at date 1 and store until date 2.
- Take $P$, the unit price at which they may issue claims on their projects as given.

At date 2:

- Projects pay off and output is distributed to claimants.
- Agents consume their remaining resources and die.

**Demand for claims and market price**

There is also a measure one of "deep-pocket" agents which have resources available but do not have access to the long-term technology. They only have access to storage and to the market for claims to ongoing projects. For simplicity, they are risk-neutral[11], and hence they are ready to buy any asset at the expected discounted value of the underlying payoffs. I assume that they have, on aggregate, enough resources to clear the competitive secondary market at that price.

This implies a perfectly elastic demand for claim, which is of course not realistic, at least in the short run. Nevertheless, this assumption is highly desirable and permits to focus on adverse selection issues, because it neutralizes cash-in-the-market (CITM) pricing mechanisms, which are well-known to generate other kind of dry-ups[12]. Actually, the specification of the demand side of the market does not play an important role in the model[13], and adverse-selection-driven liquidity dry-ups could also obtain with a more general downward sloping demand curve[14], but then it would be potentially complicated to disentangle between adverse-selection and CITM driven liquidity dry-ups.

When project quality is private information, a key variable to determine asset prices on secondary market is average quality (Akerlof (1970)). As the deep-pocket agents have access to the storage technology, a simple no-arbitrage argument suffices to establish the market unit price $P$ of an asset, given average quality on that market:

$$P(\eta) = \frac{R_L + \eta(R_H - R_L)}{1+r} \tag{1}$$

[11]Eisfeldt (2004)proposes an alternative formalization: she assumes that such agents are risk-averse and that their endowment streams and utility function are such that they want to save, for instance for precautionary saving motive. Then, she assumes perfect divisibility of claims and costless diversification. This generates the same perfectly elastic demand function.

[12]See section 4 for a description of the CITM models and how it generates liquidity dry-ups

[13]See the discussion of robustness in the next section.

[14]See the appendix for a sketch of such a version of the model.

Where $\eta$ denotes the proportion of good quality claims in the secondary market. Note that I do not need $\eta$ to be directly observable since it could easily be inferred in equilibrium. Under the anonymous market assumption, it is without loss of generality that I only consider a single unit price. In general, a unit price that is a decreasing function of quantities could be used by the sellers of claims to high-return projects as a signaling device. However, in an anonymous market, sellers can always split their sales, which would render signaling impossible. This simplifies the analysis and does not affect the main results[15].

**Endogenous market liquidity**

When there is at least one low-return claim on the market, claims to high-return projects are sold (if any) at a discount with respect to $R_H/(1+r)$, the price that would prevail absent asymmetry of information. It is in that sense that adverse selection makes high-quality claims "illiquid".

For it is a direct measure of the proportion of agents trading for other reasons than private information about future payoffs and because it determines the illiquidity discount, $\eta$ embodies market liquidity in this model.

**Equilibrium definition**

A triple $\gamma \equiv (P^*, \lambda^*, \eta^*)$ is an equilibrium for this economy if and only if:

$$\begin{cases} P^* = P(\eta^*) \\ \lambda^* \in \lambda(P^*) \\ \eta^* = \eta(P^*, \lambda^*) \end{cases} \tag{2}$$

That is, $P^*$ is the unit price buyers are ready to pay for the average quality implied by $\eta^*$; $\lambda^*$ is an optimal investment decision given $P^*$; and $\eta^*$ is the proportion of high-return claims in the market implied by optimal liquidation behavior at the level of investment $\lambda^*$ and at price $P^*$.

There always exists at least one such equilibrium[16]. Uniqueness, however, depends on parameter values. Roughly speaking, it at least requires that average project return $E[R]$ is low enough or sufficiently high. For all the intermediate cases, there are multiple equilibria.

[15] See the end of subsection 3.1 for a discussion.

[16] Kakutani's theorem ensures that there exists a price $P^{'}$ such that: $P^{'} \in \eta\left(P^{'}, \lambda(P^{'})\right) R_H/(1+r) + \left(1 - \eta\left(P^{'}, \lambda(P^{'})\right)\right) R_L/(1+r)$. Such a price pins down an equilibrium.

# 3 Liquidity dry-ups

In this section, I solve the model backward and I show how the negative externalities linked to hoarding may lead to robust multiple equilibria that can be Pareto ranked according to their respective level of market liquidity. To illustrate the crucial role of self-insurance on liquidity and risk sharing, I also give a numerical example and compare equilibria with a benchmark first-best allocation. Finally, I show how the government can implement the second-best by the mean of a public liquidity insurance scheme and I suggest policy implications.

For the ease of exposition, I solve here a very simple version of the model. All the results extend to the general model (a generalization and a proof of the main proposition are provided in appendix B).

Throughout this section, I make thus the following assumption:

*Assumption 1*

- Return to storage is normalized: $r = 0$.
- All agents are *normal*: $p = 1$
- Projects succeed and fail with equal probabilities: $q = 0.5$
- Period utility is logarithmic: $u(C_t) = \ln C_t$

## 3.1 Equilibria

A first implication of *assumption 1* is that there are only two types of date-1 agents and, by the law of large number, there is a measure one half of each type. I therefore consider two date-1 representative agents named after their type $j \in (H, L)$. Where $H$ stands for agents with high-return projects and $L$ for low-return.

### The problem of the agent

Long term investment is risky: on the one hand, it pays well in the case of success but, on the other hand, it might yield a relatively low return in case of failure or early liquidation. Conversely, storage yields the same amount in each state of nature and is liquid; it can be consumed 1 to 1 at each date. There is thus a *return-liquidity* trade-off and risk-averse agents might use storage to self-insure against the risk they face.

Formally, at date 0, agents solve:

$$\max_{\lambda, L_j, S_j} U_0 = E_0 \left[ \ln(C_1) + \ln(C_2) \right] \tag{3}$$

$$s.t. \begin{cases} C_{1j} + S_j = 1 - \lambda + L_j P \\ C_{2j} = (\lambda - L_j) R_j + S_j \\ 0 \leq L_j \leq \lambda \leq 1 \\ prob(j = H) = 0.5 \end{cases}$$

Where $E_t[.]$ is the conditional (upon information available at date $t$) expectation operator, $\lambda$ is the share of endowment invested in the long-term technology, and the following variables are contingent on being in state of nature $j$: $C_{tj}$ is consumption at date $t$ , $S_j \geq 0$ is storage between dates 1 and 2 , $L_j$ is the number of claims (to unit projects) issued at date 1.

The budget constraints state the following: date-1 resources consist of storage $(1-\lambda)$ from date 0 plus the revenue from claim issuance ($L_j$) at the market price ($P$). These resources can be consumed ($C_{1j}$) or transferred to date 2 through storage ($S_j$). At date 2, resources available for consumption consist of the output from long-term investment that has not been liquidated $(\lambda - L_j)R_j$ plus storage from date 1.

To determine optimal behavior with respect to this trade-off, I solve the problem backward.

#### Date-1 optimal liquidation policy

Let $L_j(P,\lambda)$ denote the optimal liquidation correspondence that solves the date-1 problem for agent $j$ for each couple $(P,\lambda)$. I restrict my analysis to prices that are consistent with (1): $P \in [R_L, R_H]$.

Agent $L$ knows he owns lemons and he sells off any project he holds as soon as $P > R_L$. In the case $P = R_L$, optimal liquidation is undetermined. I assume for simplicity that he sells off any project he holds too. Accordingly:

$$L_L(P,\lambda) = \lambda \tag{4}$$

The problem of $H$, the agent with high-return projects, is given by:

$$\max_{L_H, S_H} U_1 = \ln(C_1) + \ln(C_2)$$

$$s.t. \begin{cases} C_{1H} + S_H = 1 - \lambda + L_H P \\ C_{2H} = (\lambda - L_H) R_H + S_H \\ 0 \leq L_H \leq \lambda \end{cases}$$

From the first order conditions, I have:

$$L_H(P,\lambda) = \max\left\{0; \frac{P\lambda - 1 + \lambda}{2P}\right\} \tag{5}$$

The intuition is the following. The optimal liquidation of agent $H$ is weakly increasing in $P$ and in $\lambda$. If both are high enough, $\frac{P\lambda-1+\lambda}{2P}$ is positive because the resources available at date 1 (before liquidation) are smaller than the share of wealth he wants to dedicate to consumption at that period. Conversely, if $\frac{P\lambda-1+\lambda}{2P}$ is negative, the agent would like to "create" ongoing projects. This is of course ruled out by the definition of the long-term technology. In that case, agent $H$ does not participate in the market and $L_H(P,\lambda) = 0$. Note that if $\lambda$ is really small, he might roll part of its storage over to date 2.

**Date-0 optimal investment policy**

Agents choose investment according to expected utility maximization.

PROPOSITION 1 (**self-insurance**)

*Let* $\lambda(P) \equiv \arg\max_{\lambda} U_0(\lambda, P)$ *be the set of solutions for a given P to the date* 0 *problem (3), then:*

$$\lambda(P) = \begin{cases} \left\{\tilde{\lambda}\right\} & ; P < 1 \\ \{1\} & ; P > 1 \\ \left\{\left[\frac{1}{2}, 1\right]\right\} & ; P = 1 \end{cases} \tag{6}$$

*With* $0 \leq \tilde{\lambda} < \frac{1}{2}$.

Proof: see Appendix A.

When $P$ is smaller than the gross return to storage, it hurts to liquidate. In that case, agent $L$ (which ends up with lemons), wishes he had invested nothing. Agent $H$, however, wishes he had invested $1/2$ (which is a well known property of logarithmic utility functions). Consequently, expected utility maximization implies a choice of $\lambda$ strictly lower than $1/2$. Self-insurance is thus crowding out productive investment.

Conversely, if $P$ is to be high, investment dominates storage -even in the case of early liquidation- and makes thus the agent better-off irrespective of its project quality. Agents therefore choose to invest as much as possible.

If $P = 1$, investment is rather high, though undetermined over the range $\left[\frac{1}{2}, 1\right]$: whereas agent $L$ is indifferent over the whole range of admissible values $[0,1]$, agent $H$ is indifferent over this specific range and strictly prefers it to any lower value. This is the reason why the optimal investment policy has not a functional form at $P = 1$.

**Supply for claims and average quality**

I can now evaluate the optimal liquidation functions (4) and (5) at the optimal investment level given price $P$ (*proposition 1*):

$$L_L\left(P,\lambda\left(P\right)\right)=\lambda$$

$$L_H\left(P,\lambda\left(P\right)\right)=\begin{cases}0 & ;P<1\\ \frac{1}{2} & ;P>1\\ \in\left[0,\frac{1}{2}\right] & ;P=1\end{cases}$$

And I can define $\eta\left(P\right)$, the proportion of claims to high-return projects for a given $P$:

$$\eta\left(P\right)\equiv\frac{L_H\left(P,\lambda\left(P\right)\right)}{L_L\left(P,\lambda\left(P\right)\right)+L_H\left(P,\lambda\left(P\right)\right)}=\begin{cases}\eta_{illiq}=0 & ;P<1\\ \eta_{liq}=\frac{1}{3} & ;P>1\\ \eta_1\in\left[\eta_{illiq},\eta_{liq}\right] & ;P=1\end{cases} \tag{7}$$

If the price is anticipated to be low, relative to return on storage, market participation is anticipated to be limited: there will only be lemons in the market. However, in the case of a high price, there is full participation and therefore there is a higher proportion of claims to high-return projects in the market. The latter case implies a smaller discount and thus greater market liquidity (recall that $\eta$ is a direct measure of liquidity in this model).

**Equilibria and liquidity dry-ups**

In this economy, the same fundamentals ($R_H, R_L, r=0, p=1, q=0{,}5$) might lead to multiple equilibria that primarily differ by their level of liquidity. Accordingly, I interpret equilibria with the lowest level of liquidity as a liquidity dry-ups.

To find the equilibria of this economy, I define the implied price correspondence:

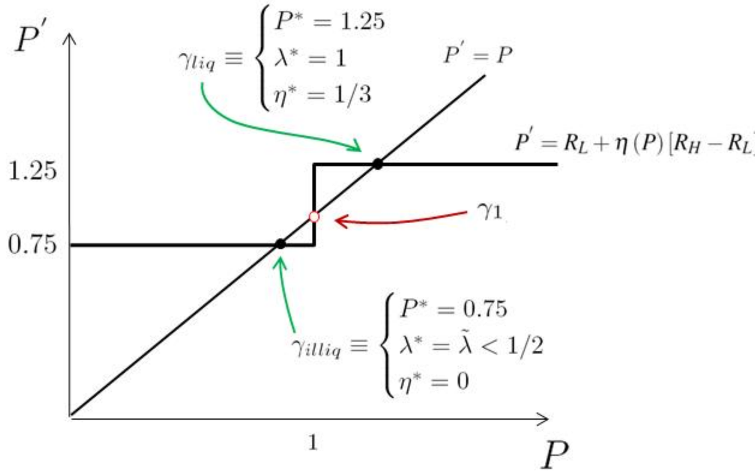
$$P^{'}=R_L+\eta\left(P\right)\left[R_H-R_L\right]$$

$P^{'}$ is the market price corresponding to a proportion of high-return projects $\eta\left(P\right)$. Therefore, a fixed point $P^{'}=P$ pins down an equilibrium price (call it $P^{*}$) for the economy. The corresponding values of $\lambda^{*}$

and $\eta^*$ are given by (6) and (7) respectively.

Figure (1) gives an example with three equilibria for a given set of parameters.

Figure 1: Multiple equilibria ($R_H = 2.25$ and $R_L = 0.75$)




In a first equilibrium, which I denote $\gamma_{illiq}$, agents anticipate a liquidity dry-up. They take as given that the price will be low ($P < 1$) and, according to *proposition 1,* they choose $\lambda(P) = \tilde{\lambda} < 1/2$. This in turn imply that agents with high-return projects will not enter the secondary market at date 1 and that the resulting liquidity will be low ($\eta^* = 0$) which make the anticipation of a low price a self-fulfilling prophecy: the liquidity has dried up.

Similar argument applies to $\gamma_{liq}$, the self-fulfilling high-liquidity equilibrium. Expecting high liquidity, which means that liquidation does not hurt ($P \geq 1$), agents invest only in the long term technology ($\lambda^* = 1$). Given that investment is high, type-$H$ agents enter the market, and their participation increases the proportion of trade for reasons other than private information about future payoff. Hence, equilibrium liquidity and price are indeed high ($\eta^* = 1/3$ and $P^* = 1.2$ ).

Both equilibria are locally stable in the sense that agents best-response to any small perturbation to the equilibrium price would bring the price back to equilibrium. There is also an equilibrium (call it $\gamma_1$) which is unstable. As they are of low economic relevance and they are not fundamental to my argument, I do not discuss such unstable equilibria further in the text. However, an interpretation is proposed in the appendix.

One might wonder if the example depicted in figure (1) is an exception**,** but it is actually typical in such a setup. I formalize this statement in the following proposition.

PROPOSITION 2 (**multiplicity**)

*Under assumption 1, $\forall R_H > 3-2R_L$, problem (3) has at least two distinct solutions with different level of liquidity.*

This simply means that for any admissible low return ($0 \leq R_L < 1$) there is a threshold for the high return ($R_H$) from which these fundamentals lead to multiple equilibria.

Proof: Let $\Gamma(R_L,R_H)$ denote the set of equilibria for these parameters. From (1), (7) and *proposition 1*, I directly get that, if $R_H > 3-2R_L$, then the following two elements belong to $\Gamma(R_L,R_H)$:

$$\gamma_{illiq}(R_L,R_H) \equiv \begin{cases} P^*_{illiq} = R_L \\ \lambda^*_{illiq} = \tilde{\lambda} < 1/2 \\ \eta^*_{illiq} = 0 \end{cases}$$

$$\gamma_{liq}(R_L,R_H) \equiv \begin{cases} P^*_{liq} = R_L + \frac{1}{3}(R_H - R_L) \\ \lambda^*_{liq} = 1 \\ \eta^*_{liq} = \frac{1}{3} \end{cases}$$

□

The low-liquidity equilibrium is a separating equilibrium which is the unique one *given that* agents are self-insured. The high-liquidity equilibrium is a pooling equilibrium (in prices, not in quantities) and is also unique *given* the date-0 decision to fully invest in the long-term technology[17].

The existence of a high-return threshold for multiple equilibria neither depends on assumption 1, which can be relaxed without altering the results, nor on the existence of deep pocket agents. (see appendix B for for a generalization and the sketch of the model without deep-pocket agents.) It instead relies on the non-convexities implied by the feedback effect between hoarding and adverse selection or, to put it another way, by the negative externalities linked to hoarding.

Before showing how these externalities affect welfare, I discuss the robustness of the results.

## 3.2 Robustness

In this subsection I first explain which of my model ingredients are crucial to the feedback relationship between hoarding behavior and adverse selection. Then, I explain why the multiple equilibria result is

[17]Note that without the anonymity assumption, uniqueness given date-0 decision remains in most of the cases in the general model. However, there exist example of utility function and parameter values under which a separating equilibrium may also exist.

particularly robust. Finally, I interpret this robustness as reflecting the strong regime change susceptibility of market liquidity.

### Hoarding and adverse selection

Starting from the assumption that the long-term technology is on average more productive than storage, the two key ingredients to obtain this feedback effect between hoarding and adverse selection is a potential *lemons problem* and an endogenous demand for liquidity at the interim date.

First, to generate the *lemons problem*, I assume private information about asset quality and I restrict the set of claims agents can trade (or contracts they can write) before learning such private information. Otherwise, if markets were complete, they could achieve the first best allocation and a *lemons problem* would not obtain. Concretely, the assumption that they cannot commit to properly invest (or that proper investment is not verifiable) prevents them to sell the project upfront at its expected value.

Second, the endogenous demand for liquidity (current consumption goods) comes from agents' desire to smooth consumption across time. Other modeling strategies, for instance an investment opportunity or a refinancing need[18], would yield such a demand as well. In fact, all that is required is a time mismatch between income and needs for resources, and these variables to be endogenous.

These two ingredients, the demand for liquidity and the potential *lemons problem,* give agents a reason to hoard. But hoarding also feeds back on adverse selection: while lemon owners trade because private information allows them to make an arbitrage, others trade according to their marginal rates of substitution, which depend on hoardings. This really is the core of the model.

Remark 1

Surprisingly, preference shocks *à la* Diamond and Dybvig are not sufficient to obtain the feedback effect[19]. The reason is that such modeling strategy is equivalent to assuming that *early* agents' marginal rate of substitution is infinite. Therefore, such agents sell because... it is assumed that they should sell. Hoarding can therefore have no impact on the proportion of good assets in the market. In that case, the severity of adverse selection at equilibrium is uniquely determined by the fraction of agents hit by the shocks, as is for instance the case in Parlour and Plantin (2008).

Remark 2

Even though they have a potential *lemons problem* in the unsecured interbank market and a demand for liquidity, Heider, Hoerova, and Holthausen (2010) do not find a clear relationship between hoarding and

[18]Which is a more standard way to model liquidity demand in the corporate finance literature (see for instance: Holmström and Tirole (1998)).

[19]Neither are they necessary, as I have already established.

adverse selection. The main reason for this is that the only motive to borrow funds at the interim date in their model is to cover a sudden liquidity need. As there is no interim investment opportunities, a "bad bank" has no incentive to borrow funds, even at a cross-subsidized interest rate. Hoarding has thus, again, no influence on the average reason to trade[20]. To obtain a feedback effect in such a set-up, one could for instance introduce an interim risky investment opportunity because "bad banks" are more likely to fail and have therefore a higher incentive to gamble for resurrection.

**Multiplicity**

In the textbook *lemons problem*, adverse selection leads to multiplicity when different prices can clear the *same* market.

The story is different in my model: *given date-0 investment decision*, there is only one price that clears the secondary market at date 1. However, the opportunity cost of not participating in the market at date 1 (the marginal utility of current consumption goods) is endogenously determined by date-0 investment decision. Therefore, different date-0 decisions imply different market-participant characteristics at date 1, which explains why the same fundamentals can lead, from a date-0 perspective, to multiple self-fulfilling date-1 equilibrium prices. The roots of multiplicity lie thus in the strong dynamic strategic complementarities underlying the feedback relationship between adverse selection and hoarding.

Important also is the restriction that contracts contingent on storage (liquidity hoarding) are not enforceable. This would provide agents a mean to coordinate on the high-liquidity equilibrium. Given the difficulties a firm, or a bank, would have to prove the extent to which their balance sheets are illiquid, this restriction seems however reasonable (Tirole, 2011).

It is typical to find multiple equilibria in games of strategic complementarities. Still, relaxing the assumption that agents share a common knowledge of the economic environment is likely to imply equilibrium uniqueness. Interestingly, this need not be the case here because investment decisions are actually strategic *substitutes* up to the level at which they become *complements*, which precludes the use of standard global games techniques**.** This is because *global strategic complementarities* are required to apply equilibrium selection techniques based on the iterative deletion of dominated strategies (Carlsson and van Damme, 1993; Morris and Shin, 2004; Frankel, Morris, and Pauzner, 2003; Vives, 2005). Furthermore, the *single-crossing conditions* for uniqueness required by other approaches (Goldstein and Pauzner (2005); Mason and Valentinyi (2010); Bueno de Mesquita (2011)) are also violated[21], which suggests that multiplicity is quite

[20] What determines whether or not there is *lemons problems* in their model is the price at which long term assets can be sold since it is exogenously higher for "good banks" than for "bad banks".

[21] See Appendix B for further discussion

robust. Indeed, such feature of the model is not driven by specific modeling choices but, instead, is intimately related to the endogenous nature of liquidity.

To understand why, one might consider the case in which at least one investor choose not to self-insure. In such a case, the decision for an agent to increase investment from a low level has *negative* externalities, as long as the considered increase is not large enough to ensure future market participation (that is, $\lambda < 1/2$). The intuition is the following: a self-insured agent does not participate in the market if he ends-up with good assets, and, therefore, increasing his investment only increases the quantity of potential *lemons* he would sell. Hence, it is only if he decides to rely on market liquidity provision that he may contribute to it too and thus provide positive externalities. In a metaphoric way, agents only take actions that contribute to future market liquidity when they are truly optimistic about future liquidity conditions. In the case they are pessimistic, a decrease in pessimism only induce actions that contribute further to the crowding out of future liquidity.

Finally, if one were to reduce the model to a binary decision game[22] (imposing $\lambda \in \left\{\frac{1}{4}; 1\right\}$ for instance), it would then be solvable in global games. Such simplification might prove useful to the study of some specific questions, but it should not occult the fact that it is only a particular case. All in all, I interpret this relative robustness as reflecting the strong regime change susceptibility of market liquidity, and as therefore suggesting that multiplicity of equilibria might be a natural outcome of endogenous liquidity models.

Remark 3:

Other papers study adverse-selection-driven illiquidity in the same context but do not find multiple equilibria, which deserves a brief discussion. Eisfeldt (2004) looks at the relationship between productivity and liquidity and focuses on economies that *seem* not to admit multiple equilibria. Actually, she does not prove uniqueness and selects parameter values for which her solution algorithm does not lead to multiplicity, up to numerical precision. My guess is that the jumps in productivity she considers are too large relative the parameter multiplicity region, she jumps therefore from low-liquidity-equilibrium-only to high-liquidity-equilibrium-only economies[23]. Kurlat (2009) proposes another dynamic model in which productivity interacts with adverse selection, and where downturns can cause market shutdowns. However, he does not allow for storage in the economy, which rules out the effect hoarding might have on the average reason to sell and, therefore, on the severity of the *lemons problem*.

[22]Holmström and Tirole (2011) discuss the results of the present present on the basis of a binary decision version of the model.

[23]This intuition finds support in figure 4 in her paper. Consider the mapping and the fixed point corresponding to her benchmark economy. It is clear that the mapping slope is greater than 1 for higher initial values than the fixed point. As an increase in productivity shifts mappings upward, this strongly suggests the existence of multiple equilibria for an economy with slightly higher productivity.

## 3.3 Externalities, self-insurance and welfare

This subsection studies the mechanisms by which endogenous liquidity dry-ups affects welfare. The key starting point of this exercise is that, in this economy, the market may fail to allocate resources efficiently. I first state it formally, then give the main intuition and propose a numerical illustration based on the same parameter values as those used for figure 1.

PROPOSITION 3 (**market failure**)

*A liquidity dry-up is a Pareto-dominated equilibrium, both from an ex-ante and an ex-post point of view.*

Proof: see appendix A.

Ex-post inefficiency means that conditionally on his type, no agent ends up better-off in a liquidity dry-up than in the corresponding high-liquidity equilibrium, and at least one of them is strictly worse-off[24]. Ex-post inefficiency of course implies ex-ante inefficiency: expected utility is lower in the case of a dry-up.

The two reasons why the low-liquidity equilibrium is inefficient are he following:

- First, because resources are wasted in the storage technology (long-run investment is on average more productive).
- Second, because self-insurance has negative externalities (hoarding decreases ex-post market participation which hinders risk sharing).

When an agent optimally chooses to issue a claim on a high-return project, it increases average quality and all claims can be sold for a better price. The social benefit is thus higher than the individual cost. However, in a liquidity dry-up, since the market does not provide liquidity insurance, agents optimally decide to self-insure, which prevents them ex post to issue claims on high-return projects. The social cost of self-insurance is therefore higher than the private benefit.

In order to illustrate this, it is useful to identify the first best allocation of resources as a benchmark.

**The first-best allocation**

To help compare risk sharing across allocations, I define ex-interim and ex-post wealth as follows:

$$\begin{cases} W_j \equiv 1-\lambda+\lambda R_j \\ W_j^* \equiv 1-\lambda+L_jP+(\lambda-L_j)R_j \end{cases} \tag{8}$$

[24] In fact, in this example, all types of agents are strictly worse-off in a dry-up.

They are, respectively, the date-2 wealth of agent $j$ computed at the *beginning* of date 1 (that is, before the liquidation decision) and the date-2 wealth of agent $j$ computed at the *end* of date 1 (that is, after liquidation decision).

At the first-best allocation, aggregate output is maximized and agents are perfectly insured. Output maximization requires that all agents invest their full endowment in the risky technology ($\lambda = 1$). As all ex-post agents have the same preferences, full insurance implies that they receive the same share of the pie.

The maximum per capita resources as of date 1 in the economy considered in figure 1 is given by $0.5\,(W_H + W_L) = E\,[R] = 1.5$, which allow for 0.75 unit of consumption good available for consumption at each date for each agent. Of course, with this allocation, marginal utility is equal across periods and states of nature, and ex-post wealth is hence equal across agents ($W_H^* = W_L^*$). The corresponding date-0 expected utility is $U_0^{FB} = -1.15$.

Table 1 displays such allocation[25].

Table 1: The first-best allocation ($R_H = 2.25$ and $R_L = 0.75$)

| $C_{tj}$ | Date 1 | Date 2 | $W_j$ | $W_j^*$ |
|---|---|---|---|---|
| State H | 0.75 | 0.75 | $R_H = 2.25$ | 1.5 |
| State L | 0.75 | 0.75 | $R_L = 0.75$ | 1.5 |

In this allocation, per capita resources are maximal: $0.5\,(W_H + W_L) = E\,[R]$, consumption is equal across states and date: $C_{tj} = E[R]/2, \forall t, j$ and the reached level of expected utility is $U_0^{FB} = -1.15$.

This allocation maximizes date-0 expected utility but is not ex-interim incentive compatible under the assumption of private information on project return. Indeed, if a planner were to bundle all assets and sell them in the secondary market[26], the corresponding unit price would be $P_{FB} = E\,[R]$. However, as $R_L < P_{FB} < R_H$ incentives to sell are distorted: agents with high-return projects have an incentive to retain part of their projects as it is a more efficient way to provide for date-2 consumption needs. Consequently $P_{FB}$ cannot be a competitive equilibrium price.

**The competitive allocations**

As depicted in figure 1, there are two stable competitive allocations when $R_H = 2.25$ and $R_L = 0.75$.

In $\gamma_{liq}$, all resources are invested in the long run. From a date-1 perspective, and because investment has been determined at date 0, it is as if agent $H$ had a date-2 endowment of $R_H$. Of course, this agent wants to smooth consumption and wishes to transfer resources across period. Because of adverse selection, he cannot

[25] It is easy to check that this allocation maximizes date-0 expected utility subject to the following aggregate resources constraint only $(0.5\,(C_{1L} + C_{1H} + C_{2L} + C_{2H}) \leq E\,[R])$.

[26] A way to implement this allocation would be to do so and to share the proceeds equally.

do it 1 to 1: the rate of transformation is 1 to $\frac{P^*}{R_H} = 0.56$. This means that his investment is relatively illiquid and that issuing claims on it is costly. However, such issuance has a positive externality on agent $L$, the lemon owner: he does an arbitrage and transfers resources at the rate 1 to $\frac{P^*}{R_L} = 1.67$. Still, it is *ex-ante* and *ex-post* incentive-compatible for agent $H$ to issue claims on his projects. *Ex ante* because liquidation still dominates storage and *ex post* because it is, in this case, the only way to obtain consumption goods at date 1. The latter justification is crucial and illustrate the fact that long-term investment can be seen has a secondary-market-participation commitment technology. So, in $\gamma_{liq}$, thanks to their decision to *ex ante* tie their hands, agents are *ex post* pretty well insured: they still face the risks of project failure, but in that case they take advantage of adverse selection and get compensated by a relatively high price on the market. Furthermore, they do not face the risk to liquidate good assets in an illiquid market[27]. Table 2 displays this allocation.

Table 2: The high-liquidity competitive allocation ($\gamma_{liq}$)

| $C_{tj}$ | Date 1 | Date 2 |
|---|---|---|
| State H | 0.625 | 1.125 |
| State L | 0.625 | 0.625 |

| $W_j$ | $W_j^*$ |
|---|---|
| $R_H = 2.25$ | 1.75 |
| $R_L = 0.75$ | 1.25 |

In this allocation, the size of the pie is still maximum ($0.5\,(W_H + W_L) = 1.5$) but the market fails to share it equally ($W_H^* > W_L^*$). As a consequence, expected utility is lower: $U_0^{\gamma_{liq}} = -1.29$. However, the market still provides some insurance as there is a positive transfer of resources from the "lucky" state $H$ to the "unlucky" state $L$ ($W_H - W_H^* = W_L^* - W_L = 0.5$).

On the other hand, in $\gamma_{illiq}$, agents anticipate that the secondary market will be illiquid and will provide poor insurance. Accordingly, they optimally choose to compensate with self-insurance: they use storage to hoard liquidity. This is a coordination failure: as no agent ties his hands, there is individually no incentive to do so, and no one provides externalities in this low-liquidity equilibrium. Thus, paradoxically, when agents choose to self-insure, they end-up rather poorly insured. They are no longer able to transfer resources from state $H$ to state $L$ ($W_j = W_j^*$). Even worse, as long-term investment dominates storage, self-insurance is wasteful and the size of the pie decreases. ($0.5\,(W_H + W_L) = 1.22 < 1.5$). As a result of these two combined effects, expected utility drops further $U_0^{\gamma_{illiq}} = -2.21$. The low-liquidity competitive allocation is shown in table 3.

Table 3: The low-liquidity competitive allocation ($\gamma_{illiq}$)

| $C_{tj}$ | Date 1 | Date 2 |
|---|---|---|
| State H | 0.57 | 0.97 |
| State L | 0.45 | 0.45 |

| $W_j$ | $W_j^*$ |
|---|---|
| 1.53 | 1.53 |
| 0.9 | 0.9 |

In this allocation, there is no transfer across states ($W_j = W_j^*$) and the size of the pie is not maximized ($0.5\,(W_H + W_L) = 1.22 < 1.5$). As a result of these two combined effects, expected utility drops further $U_0^{\gamma_{illiq}} = -2.21$.

[27] There is empirical support for the pricing of such risk in financial markets. See Acharya and Pedersen (2005).

The fact that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998) and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment is present in Eisfeldt (2004). However, that liquidity dry-ups can endogenously arise for the very reason that investors self-insure against it is a new result.

### 3.4 The government

Implementing the second-best allocation in the presence of positive externalities is a standard and rather simple problem[28]. I propose in this subsection a public liquidity insurance scheme that enables the government to achieve such a goal. For simplicity, I do this under *assumption 1*, but it can readily be extended to the general case. Also, I assume that the fundamentals are such that a market failure is possible[29].

#### The public liquidity insurance

The idea for the insurance is extremely simple. The bad equilibrium is a coordination failure, which happens when investors fear to sell claims in an illiquid market. Therefore, if the government pledges to compensate them for the loss (with respect to storage) in such a case, the incentive to self-insure vanishes and the only possible outcome is the high-liquidity equilibrium. Of course, this result relies on the possibility for the government to levy a break-even lump-sum tax after observing aggregate behavior. This is the reason why a private agent could not do it (the reader might want to check that the tax scheme is not incentive compatible). This public liquidity insurance is actually very similar to Diamond and Dybvig (1983) demand deposit insurance with the same assumptions about the fiscal ability of the government

PROPOSITION 4 **(public insurance)**

*The public liquidity insurance implements the second-best.*

Proof: see Appendix A.

Here is the intuition. Under this public liquidity insurance, the ex-ante trade-off between return and liquidity (with respect to storage) disappears and the date-0 first order condition for $\lambda = 1$ always holds:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

Whatever the anticipated date-1 market price, $\lambda^* = 1$ maximizes expected utility. Hence, it is a dominant strategy to fully invest in the long-term technology. Under this scheme, the only one equilibrium corresponds

[28] See Dybvig and Spatt (1983).

[29] That is: $R_H > 3 - 2R_L$, see *proposition 2*.

to $\gamma_{liq}(R_L, R_H)$ and the insurance is never claimed. As in Dybvig and Spatt (1983), such an insurance is thus free.

The Pareto improvement through public liquidity provision *absent aggregate shocks* departs from the literature. In Diamond and Dybvig (1983) and Holmström and Tirole (1998), there is a role for the government only in the case of aggregate uncertainty about the fundamentals. This discrepancy might be understood when related to Allen and Gale (2004). They show that under complete markets for aggregate risk, if intermediaries can offer complete contracts, the equilibrium is incentive-efficient (corresponds to the first-best allocation). When contracts are incomplete (they focus on demand-deposit contracts) the equilibrium is constrained efficient (second-best). Here, I go one step further, as demand-deposit contracts are exogenously ruled-out and the market can fail, even in the absence of aggregate shocks.

**Implementation**

There are several ways to implement the public liquidity insurance. For instance, the government may pledge to buy any claim at a price of 1[30]. Of course, sellers would only claim this insurance in the case of a dry-up. To break even, the government needs to levy the following lump-sum tax:

$$\tau(P) = \begin{cases} (1-P)\sum_j \frac{L_j}{2} & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

Where $\tau$ is the per capita lump-sum tax needed to fund the insurance.

The net effect of such a scheme is thus a transfer from agents that liquidate few to agents that liquidate more:

$$transfer_j = \begin{cases} (1-P)\left[L_j - \frac{\sum_j L_j}{2}\right] & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

**Robustness**

Such a transfer is always feasible. As the incentive constraints are circumvented thanks to the government regalian power to raise lump-sum taxes, the only remaining issue is the one of binding resource constraints. However, they will never be binding. First, from an aggregate point of view, because the value of aggregate resources cannot decrease over time. Second, from an individual point of view, because a highly negative

[30]The following subsidy to liquidation (and a break-even lump-sum tax) would lead to an equivalent outcome: $Subs(L) = \min\{(1-P), 0)\}$

transfer to agent $H$ would force him to liquidate part of its portfolio. This would trigger positive externalities and increase the average value of traded claims which, in turn, would decrease the size of the needed transfer and relax the government budget constraint. Consequently, agent $H$ could never run out of resources because of this scheme[31].

Whether the government would still be willing to impose such a transfer in the (out-of-equilibrium) low-liquidity case raises the question of credibility. Feasibility does not mean that it is always ex-post desirable. One could state this as follows: under which condition is it credible for the government to intervene in case of a dry-up? As it implies a transfer from rich (or lucky) to poor (or unlucky) agents, an utilitarian government would for instance be willing to do so[32]. Still, if there is a doubt about government commitment, one can no longer rule out the bad equilibrium (this could be formalized in a model with uncertainty about the government true type).

**Policy implications**

The fact that long-term investment crucially depends on expectations on future market liquidity leads to the first policy implication of the model. Notwithstanding moral-hazard and credibility concerns, this "expected liquidity channel" should indeed not be overlooked when considering public intervention in the case of a financial crisis.

During financial crises, the fear of a credit crunch[33] might lead to various public interventions such as liquidity injection, bank recapitalization or even nationalization. To assess them in the light of self-fulfilling liquidity dry-up mechanisms, it is first important to note that the public liquidity insurance is only effective ex ante. Assume the government did not commit to bail out the market and agents coordinated on the low-liquidity equilibrium. It is still feasible to implement the scheme, but it would no longer be a Pareto improvement. As all agents would be self-insured, there would be mostly lemons in the market, the public intervention would not restore liquidity[34], and the tax-payers would pay the burden of the operation.

Since only the promise of a future market bailout could restore liquidity in such a world, it raises the question of moral hazard. This is indeed well known that public intervention might induce investors to take on too much risk in the future[35]. This is the reason why the policy debate focuses on the costs of a

[31] Were agent $H$ to liquidate its whole portfolio, the equilibrium price would be $E[R] > 1$ which is not consistent with agents selling claims to the government.

[32] Still, such a government might be tempted to go further with the subsidy and try to implement the first best. However, this would distort incentive to launch project properly (recall that proper project initiation is observable but not verifiable).

[33] See for instance Bernanke and Gertler (1989) for the magnifying effect on business fluctuations of a drop in net worth.

[34] While public interventions such as liquidity injections (through TARP for instance), eased the short-term funding of financial institution in the fall 2008, one might easily argue that they did not restore liquidity in the securitized markets.

[35] See for instance Diamond (1984) and Freixas, Rochet, and Parigi (2004).

credit crunch versus the hazard to sow the seeds of the subsequent crisis. However, my results suggest the existence of an additional term to that trade-off. Indeed, a public liquidity insurance improves expectations about market liquidity, shifts investors horizon towards the long-run and actually avoids liquidity dry-ups. So, the present model advocates for policies that insure investors against sudden generalized price drops but let them with enough long-run risk. If the floor price is sufficiently penalizing with respect to average long-run returns, the moral hazard problem might be contained. In a sense, the government should design a policy that reduces enough adverse selection so that it restores self-fulfilling market liquidity but not too much. Otherwise, agents would find it profitable to game the system and invest in -or simply roll over- bad projects. All in all, the underlying moral hazard problem might be the price to pay to exit a flight-to-liquidity trap and avoid a credit crunch.

# 4 Liquidity dry-ups and the *2007-2009* financial crisis

The literature on financial crises is abundant and is considerably expanding due to the 2007-2009 crisis[36]. In this section, I explain how the model can contribute to the joint understanding of cash hoarding behavior and persistent market disruptions during the 2007-2009 financial crisis.

During this crisis the functioning of many markets has been seriously impaired. Together with the lack of financial muscle due to massive deleveraging (Brunnermeier, 2009; Adrian and Shin, 2008), the fear to buy an adverse selection of assets has probably played a role in these breakdowns (Tirole, 2010; Morris and Shin, 2010). Also, the potential presence of such "toxic" assets on balance sheets fostered the fear of counterparty risk and contributed to the freeze of the interbank market.

Unusual hoarding behaviors have also been widely observed: from August 2007, UK banks have increased their liquidity buffers by 30% (Acharya and Merrouche, 2009); from September 2008, there has been a dramatic increase in the excess reserves of European banks (Heider, Hoerova, and Holthausen, 2010), and of US major deposit institutions (by a factor larger than 18[37]); and, last but not least, despite the huge surge in supply (net of Fed open-market operations), T-Bills prices have been high during the whole period. Therefore, the demand for short term safe assets should have been very high too.

A classic explanation for the lack of financial muscle is the cash-in-the-market (CITM) theory (Allen and Gale, 1994). The idea is the following: there is an opportunity cost to hoard cash if the alternative, a productive long-term asset, has a higher expected return. This cost should be compensated by gains in some states of the world. These gains are realized when there are "many" sellers of the long-term asset. In this

[36] See Brunnermeier (2009) for a chronology of the crisis and Allen and Carletti (2008) for a focus on liquidity issues.

[37] See http://www.federalreserve.gov/econresdata/releases/statisticsdata.htm for publicly available data.

case, as the demand is limited by the amount of cash hoarded, the price might drop below the fundamental value. This mechanism is particularly appealing to explain sudden price drops and can also be extended to generate downward price spirals[38]. Clearly, there were episodes of the crisis that had such a flavor.

CITM models seem very good at capturing the unfolding of liquidity crises. Indeed, they are, by construction, designed to capture short-term phenomena. The assumption that there is no access to any external funding is indeed an application of the limits of arbitrage put forward by Shleifer and Vishny (1997). However, such limits are likely to fade away with time and the assumption would be less reasonable in a model that would seek to explain persistent breakdowns, especially if hoarding behavior is simultaneously observed.

The story my model tells us is thus complementary to the CITM hypothesis. It seems indeed reasonable to think that a severe CITM pricing episode would trigger concerns about future market liquidity. Then, as I have shown, the fear of illiquidity would lead to hoarding, which would feed illiquidity as it would make more likely that future sales will be motivated by private information rather than a need for cash. The self-fulfilling prophecy might therefore help to explain why some markets, asset backed securities (ABS) markets for instance, did not recover when cash became available again.

Despite being complex, ABS used to be quite liquid before the crisis. They were also widely used as collateral in repurchase agreements (repo's). As noted by Gorton and Pennacchi (1995), debt claims are, in normal times, not sensitive to the value of the underlying assets. In that case, asymmetry of information or heterogeneity in investor sophistication should not really matter. A slightly different view is that better informed or more sophisticated investor are actually able to extract a rent but that, in normal times, the gains from trade are large enough to fund this rent (Morris and Shin, 2010).

However, following bad news, debt claim prices can become information sensitive, which raises adverse selection concerns. This is particularly true of Collateralized Debt Obligations backed by Mortgage Backed Securities because their payoffs are highly skewed (Coval, Jurek, and Stafford, 2009). Furthermore, this can be reinforced by their lack of transparency (Pagano and Volpin, 2008). Arguably, the burst of the US housing-market bubble hit sufficiently hard the value of their underlying claims so as to make their value sensitive to their specific composition (Morris and Shin, 2010), making them potentially "toxic" and laying the ground to the *lemons problem*.

By way of illustration, according to the computations of the Bank of England (2008), the Spring 2008

[38] Cash-in-the-market and similar mechanisms have been widely used in the literature to "generate" liquidity dry-ups and related events. In Bolton, Santos, and Scheinkman (2009), adverse selection is combined with cash-in-the-market pricing to deliver multiple equilibria with prices below fundamentals. In Brunnermeier and Pedersen (2009), arbitrageur have limited resources and face margin calls from their funding institutions. When these margin calls increase with volatility, they create a liquidity spiral and assets might be traded below fundamental value. Morris and Shin (2004) and Gennotte and Leland (1990) can also be understood in a similar way: there is a given downward sloping demand curve and arbitrageurs face resource constraints that are functions of price.

prices of the ABX.HE index[39] implied an expected loss of 38% on subprime mortgages, which could correspond to a 76% foreclosure rate with a 50% recovery value. Both being not really credible, according to that report. In June 2009, the ABX.HE prices were even lower and the foreclosure rate on subprime mortgages had "only" hiked to around 20%. If fundamental value seem much higher than the price of traded assets it might be due to adverse selection: retained assets might be on average better than traded ones.

It is of course fair to say that my model is highly stylized and that the claims traded in the secondary market look rather like equity than CDOs. However, the insight that hoarding behavior has a feedback effect on adverse selection applies to the trade of any asset whose price is information sensitive. Why would one sell an asset in an illiquid market, if he does not need cash? Most likely to get rid of a *lemon*!

# 5 When illiquidity shocks improve... liquidity

In this section, I consider the case where investors face idiosyncratic preference shocks. At date 1, they learn whether they are *normal* or *early* consumers, the latter deriving utility from consumption at date 1 only. While the presence of such *early* agents is not necessary to the existence of liquidity dry-ups, it still has an impact on market liquidity and hence risk sharing. In the classic banking literature, such preference shocks are interpreted as illiquidity shocks and incentive compatibility problems arise when patient agents pretend they are impatient (see for instance Diamond and Dybvig (1983) and Jacklin (1987)). This might generate bank runs and hinder the ability of deposit contracts to improve risk sharing.

When ex-ante pooling of resources is ruled out and when private information about future payoffs is already a source of adverse selection, I find that private information about idiosyncratic liquidity shocks enhances market liquidity. Adverse selection is indeed reduced because *early* agents issue claims irrespective of the underlying project return. As market liquidity improves risk sharing, this suggests that idiosyncratic liquidity shocks need not be socially a bad thing. This as the implication that requiring banks to self-insure against such shocks (a suggestion that has been widely heard recently[40]) may have unintended but serious adverse consequences.

Introducing illiquidity shocks also permits to study the question of transparency[41]. I show in subsection 5.2 that transparency about liquidity position has two effects. First, it decreases the liquidity discount for *early* agents[42]. This can be counter-intuitive, but it explains why sellers on a secondary market often pretend

[39] http://www.markit.com/en/products/data/indices/structured-finance-indices/abx/documentation.page?#. See Gorton (2008) for a description.

[40] See for instance Perotti and Suarez (2010) or the June 2009 Bank of England Financial Stability Report.

[41] This is relevant because the lack of transparency as been widely blamed for the current crisis.

[42] I thank Jean Tirole for pointing out this relationship between liquidity position disclosure and market liquidity discount, as well as for the example below.

they sell for reasons exogenous to the asset's quality[43]. Second, transparency increases the liquidity discount for the others investors: as they did not incur a liquidity shock, it is more likely that they sell because of adverse selection. As a consequence, the effect of transparency on ex-ante risk sharing is ambiguous, as suggested by Holmström (2008).

Formally, in this extension, agents differ by the subjective factor $\beta$ they use to discount date-2 utility: $\beta \in \{0,1\}$ with $Prob(\beta = 1) = p$ and $0 < p < 1$. Also, for the sake of simplicity, I keep the assumption that period utility is logarithmic and that projects succeed or fail with equal probabilities.

The machinery of the model is basically the same, the major difference is that there are now *four* types of agents as of date 1. Agents indeed differ on two dimensions: project return $R_j$ and patience $\beta_i$. From here onward, I name agents after their type $ij$ where $j$ still reflects projects return and $i = e, n$ accounts for *early* ($\beta_e = 0$) and *normal* ($\beta_n = 1$) agents respectively. Indices on date-1 decision variables are modified accordingly.

## 5.1 Equilibrium with illiquidity shocks

Investors are still ex-ante identical. They solve:

$$\max_{\lambda, L_{ij}, S_{ij}} U_0 = E_0 \left[ \ln(C_1) + \beta_i \ln(C_2) \right] \tag{9}$$

$$s.t. \begin{cases} C_{1ij} + S_{ij} = 1 - \lambda + L_{ij} P \\ C_{2ij} = (\lambda - L_{ij}) R_j + S_{ij} \\ 0 \leq L_{ij} \leq \lambda \leq 1 \end{cases}$$

Where $i \in \{e, n\}$with $Prob(i = n) = p > 0$, $\beta_e = 0$ and $\beta_n = 1$, and $j \in \{L, H\}$with $Prob(j = H) = 0.5$.

While the liquidation decision of *normal* agents ($nL$ and $nH$) are still given by (4) and (5), those of *early* agents ($eL$ and $eH$) are now determined by their first order condition for $L_{ej} = \lambda$, which always holds:

$$\frac{P}{C_{1ej}} \geq 0 \quad ; \quad j = H, L$$

As they only care about utility of consumption at date 1, they sell off any project they hold, whatever the quality[44]:

[43] For instance, in their ads for second-hand cars, students often mention that they want to sell because they are graduating and moving out of town.

[44] If $P = 0$ they are indifferent, in which case I assume for simplicity that they liquidate their whole position.

$$L_{eH}(P,\lambda) = L_{eL}(P,\lambda) = \lambda$$

Accordingly, the proportion of claim to high-return projects is:

$$\eta(P) = \begin{cases} \eta_{illiq} = \frac{1-p}{2-p} & ; P < 1 \\ \eta_{liq} = \frac{2-p}{4-p} & ; P > 1 \\ \eta_1 \in \left[\eta_{illiq}, \eta_{liq}\right] & ; P = 1 \end{cases} \tag{10}$$

Which implies that equilibrium market liquidity depends on $p$. Equilibrium prices are still given by the fixed point $P^{'} = P$, with:

$$P^{'} = R_L + \eta(P)(R_H - R_L)$$

It follows that for any admissible value of $p$ and $R_L$ there is a range for $R_H$ that implies multiple equilibria (see appendix B for the proof). And the two possible levels of liquidity in stable equilibria depend on $p$:

$$\begin{cases} \eta^*_{illiq} = \frac{1-p}{2-p} & ; P < 1 \\ \eta^*_{liq} = \frac{2-p}{4-p} & ; P > 1 \end{cases}$$

**Effect on equilibrium market liquidity**

I present here a short comparative statics exercise.

PROPOSITION 5 (**market liquidity**)

*For any admissible value of $R_L$*, as *the probability $(1-p)$ of being hit by an illiquidity shock increases:*

1. *Equilibrium market liquidity increases:* $\frac{\partial \eta^*}{\partial(1-p)} > 0$
2. *The range for a high-liquidity equilibrium increases*
3. *The range for a low equilibrium decreases*

Proof: straightforward.

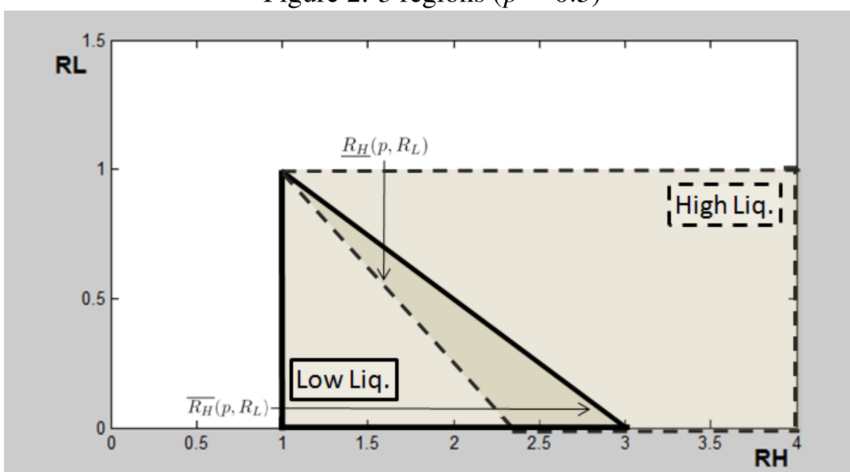
First, it might appear counter-intuitive but illiquidity shocks enhance market liquidity. In fact, investors hit by such shocks liquidate assets irrespective of their quality[45]. As a consequence, average quality increases with $1-p$, which is also the proportion of *early* agents. Second, when this proportion increases, a lower $R_H$

[45] They are in fact very similar to noise traders in the market micro-structure literature.

is needed for a high-liquidity equilibrium to exist. As *early* agent's liquidation behavior imply more units of claims to high-return projects, a lower return par unit is needed to bring the price up to 1. Third, as soon as $p < 1$, there are always claims to high-return projects that are traded. So, if $R_H$ is sufficiently high for the price at the low level of market liquidity to be higher than 1, it cannot be an equilibrium. Obviously, the higher the proportion of *early* agents, the lower that upper bound.

The lower bound for a high-liquidity equilibrium is $\underline{R_H}(p,R_L) \equiv R_L + \frac{4-p}{2-p}(1-R_L)$. The upper bound for a low-liquidity equilibrium is $\overline{R_H}(p,R_L) \equiv R_L + \frac{2-p}{1-p}(1-R_L)$. For a given $p$, they define three regions in the space of admissible parameter values for $R_H$ and $R_L$. Figure 2 illustrates this for $p = 0.5$.

Figure 2: 3 regions ($p = 0.5$)




This figure presents the three regions that define multiplicity of equilibrium. As $(1-p)$ increases, the low-liquidity-equilibrium region (delimited by the solid lines) shrinks and the high-liquidity-equilibrium one (dashed lines) widens. The overlap is the region with multiple equilibria.

**Liquidity buffer requirement**

In the aftermath of the recent crisis, it has been suggested that banks should self-insure against idiosyncratic illiquidity shocks, for instance through the holding of liquidity buffers[46]. The main rationale being to prevent fire sales externalities.

I present here a very simple exercise that illustrates the unintended consequences such policies might have. To be clear, my model has nothing to say about fire sales externalities and I do certainly not intend to challenge the view that they should be avoided. Rather, what I claim is that the policy maker should not overlook the *negative* externalities of hoardings either.

I consider a liquidity hoarding requirement imposed by the government to the investors of my model:

[46] See for instance the June 2009 Bank of England Financial Stability Report, and more recently the recommendations of the Basel Committee on Banking Supervision (BIS, 2009).

$$1-\lambda \geq \alpha,$$

which simply means that a fraction $\alpha$ of the initial endowment should be kept in liquid assets (i.e. should be stored). To make things interesting, I assume that $\alpha$ is lower than the amount of liquidity that is hoarded in a low-liquidity equilibrium.

There is of course no rationale for such a requirement in the model but, as I am only interested here in the unintended consequences of a policy motivated by other legitimate reasons, such concern is irrelevant.

In this set-up, the direct effect of such a policy is that, in a high liquidity equilibrium, agents are less exposed to maturity mismatch, hence they issue less high-quality claims:

$$L^*_{nH}(\lambda = 1-\alpha, P) = \frac{1}{2} - \alpha\frac{(P+1)}{2} < \frac{1}{2} = L^*_{nH}(\lambda = 1, P) \tag{11}$$

and adverse selection is more severe. Therefore:

- The range of parameter sets consistent with a high-liquidity equilibrium shrinks;
- If a high-liquidity equilibrium exists, fewer positive externalities are provided.

This, together with the fact that resources are wasted in storage leads to the following conclusion:

PROPOSITION 6 (**unintended consequences**)

*A liquidity buffer requirement can only reduce welfare, and may even cause a liquidity dry-up.*

Proof:

First, note that the requirement has no impact on a low-liquidity equilibrium.

Second, assume that a high-liquidity equilibrium exist. Then, $\forall\lambda$,

$$\frac{\partial U_0}{\partial \lambda} |_{P\geq 1} \geq 0$$

Hence, since a liquidity requirement imposes a cap on $\lambda$, it can only decrease welfare in such an equilibrium.

Also, from (11), one can derive $\eta^*(P)$ and show that it is decreasing in $\alpha$ and in $P$. The fixed point that pins down the equilibrium, call it $P^*_{liq}(\alpha)$, should therefore be decreasing in $\alpha$. If $P^*_{liq}(\alpha) < 1$, a high liquidity equilibrium no longer exist, and the requirement dries up liquidity.

## 5.2 Transparency and liquidity

Here, I assume that there exists a technology that enables agents to credibly disclose their patience parameter $\beta_i$. What I have in mind is for instance a bank that could credibly disclose that it has been hit by a liquidity shock (a wave of unexpected deposit withdrawal for instance). In that case, buyers are able to classify sellers in two categories and they update their priors about average quality accordingly. There are thus two separated markets.

PROPOSITION 7 (**transparency**)

*Early agents are better-off disclosing their liquidity position*

Proof (for the high-liquidity equilibrium case): Assume disclosure and let $\eta_e^*$ and $P_e^*$ denote equilibrium liquidity and price, conditionally on the seller to be an *early* agent. Such agents liquidate any project they hold: $L_{eH} = L_{eL} = \lambda = 1$. Thus $\eta_e^* = \frac{1}{2}$ and $P_e^* = E[R] > P_{liq}^*$ with $P_{liq}^* = R_L + \eta_{liq}^*(R_H - R_L)$ being the equilibrium price without disclosure. Therefore, as these agents get a better price, their ex-post wealth increase, they are better-off and they have no incentive to deviate (that is, to issue claims without liquidity position disclosure).

However, the price *normal* agents can get is back to the level without illiquidity shocks ($P_n^* = P_{liq}^*(p = 1) < P_{liq}^*(p < 1)$) and they are thus worse-off under disclosure. In that sense, transparency modify ex-ante risk-sharing opportunities and might deteriorate liquidity. The view that it is not the lack of transparency but rather the asymmetry of information that can cause illiquidity is expressed by Gorton (2008) and Holmström (2008). An extreme way to show this in my model is the following: imagine for a moment that agents only learn the quality of their projects between date 1 and date 2. Under such an assumption, agents are less informed but adverse selection is no longer an issue and the first best allocation is the only equilibrium of the model.

# 6 Conclusion

In this paper, I have shown that adverse-selection-driven market breakdowns are endogenous to past balance sheet decisions.

When they face a return-liquidity trade-off and when secondary markets for long term assets are subject to adverse selection, agent investment decisions, and in particular the extent to which they expose themselves to maturity mismatch, present strong strategic complementarities. In other words, hoarding has strong negative externalities: when agents choose hoarding as a mean to accommodate short-term needs, it becomes more likely that they subsequently trade for private information motive. There are thus proportionally more *lemons* in the market and it becomes more costly to transform long-term investment into current consumption goods, which justify the decision to hoard. This is a self-fulfilling liquidity dry-up.

This feedback relationship between hoarding and adverse selection is intuitive and helps to jointly explains the persistent market breakdowns and the unusual hoarding behavior that took place during the 2007-2009 financial crisis.

Furthermore, this results shows that long-term investment crucially depends on expectations on future market liquidity, which seems an important issue for policy: notwithstanding moral-hazard and credibility concerns, this "expected liquidity channel" should indeed not be overlooked when considering public intervention in the case of a financial crisis. Finally, I have shown that the policy-maker should not take for granted that liquidity buffers only have positive externalities. Therefore, requiring banks to hold liquidity buffers as a self-insurance against idiosyncratic liquidity shocks, a proposition widely heard recently, might have adverse unintended consequences.

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# A Additional Proofs

## A.1 Proof of PROPOSITION 1 (self-insurance)

1) The case where $P > 1$ is trivial as storage is a dominated mean to transfer resource to date 1 and date 2. Thus $\lambda(P > 1) = 1$.
2) When $P = 1$, storage is equivalent to invest and then liquidate. The optimal investment policy is therefore indetermined. To see this, let $B_j \equiv S_j - L_j$ be the net storage from date 1 to date 2 for agent $j$. The date-0 program is:

$$\max_{C_{1j},C_{2j}} E\left[\ln(C_{1j})+\ln(C_{2j})\right]$$

$$\begin{cases} C_{1j} = & 1-\lambda-B_j \\ C_{2j} = & \lambda R_j + B_j \\ \lambda \in [0,1] \end{cases}$$

It is straightforward to show that the optimal consumption plan is $C^*_{2H} = R_H/2$ and $C^*_{1H} = C^*_{1L} = C^*_{2L} = 1/2$, which can be implemented with any $\lambda \in \left[\frac{1}{2},1\right]$.
3) Consider now $P < 1$ and let $U^{'}_j \equiv \left[\frac{\partial U_0}{\partial \lambda}\,|j\right]$ be the marginal utility of $\lambda$ conditionally of being in the state $j$.

I first have:

$$U^{'}_L < 0, \forall \lambda \in [0,1] \tag{12}$$

Which simply comes from the fact that, in state $L$, any resource invested in a project gives a negative return whether it is liquidated ($P < 1$) or it is held to maturity ($R_L < 1$). When markets are illiquid, investors that realize that their project will fail wish they had invested nothing.

From the date-1 first order conditions, I have: $L^*_H(P,\lambda) = \max\left\{0; \frac{P\lambda-1+\lambda}{2P}\right\}$ and $S^*_H(P,\lambda) = \max\left\{0; \frac{1-\lambda-\lambda R_H}{2}\right\}$.

Thus, I can construct the conditional utility function:

$$U_H(\lambda,P) \equiv \begin{cases} 2\ln(\frac{1+(R-1)}{2}), & \lambda \leq \frac{1}{1+R_H} \\ \ln(1-\lambda)+\ln(\lambda R_H), & \frac{1}{1+R_H} \leq \lambda \leq \frac{1}{1+P} \\ 2\ln(\frac{1+(P-1)}{2}), & \lambda \leq \frac{1}{1+P} \end{cases}$$

And evaluate the derivative with respect to $\lambda$:

$$U^{'}_L = \begin{cases} U^{'}_H < 0, & \lambda < \frac{1}{2} \\ U^{'}_H = 0, & \lambda = \frac{1}{2} \\ U^{'}_H > 0, & \lambda > \frac{1}{2} \end{cases}$$

Expected utility maximization implies thus that $\lambda(P<1) < 1/2$, which completes the proof of proposition 1.

## A.2 Proof of PROPOSITION 3 (market failure)

Let $R_H > 3 - 2R_L$. By *proposition 2*, I have:

$$\left\{\gamma_{illiq}(R_L, R_H) = \left(R_L, \tilde{\lambda}_L, 0\right); \gamma_{liq}(R_L, R_H) = \left(R_L + \frac{(R_H - R_L)}{3}, 1, \frac{1}{3}\right)\right\} \in \Gamma(R_L, R_H)$$

If both agents are better-off in $\gamma_{liq}$, it ex-post Pareto dominates $\gamma_{illiq}$. Denote $C_{tj}^{\gamma}$ the optimal consumption of agent $j$ at date $t$ in equilibrium $\gamma$.

i) Agent $L$ is better-off

Obvious since $C_{1L}^{\gamma_{illiq}} = C_{2L}^{\gamma_{illiq}} = \frac{1-\lambda(R_L-1)}{2} < \frac{R_L + \frac{(R_H - R_L)}{3}}{2} = C_{1L}^{\gamma_{liq}} = C_{2L}^{\gamma_{liq}}$

ii) Agent $H$ is better-off

If $S_1^*$denotes the optimal level of savings of this agent at date 1, I have:

$$\begin{cases} C_{1H}^{\gamma_{illiq}} = 1 - \tilde{\lambda} - S_1^* \\ C_{2H}^{\gamma_{illiq}} = \tilde{\lambda} R_H + S_1^* \end{cases}$$

In $\gamma_{liq}$, the budget constraints of this agent are:

$$\begin{cases} C_{1H}^{\gamma_{liq}} \leq P_{liq} L \\ C_{2H}^{\gamma_{liq}} \leq (1-L) R_H \end{cases}$$

Assume he sets: $L = \frac{C_{1H}^{\gamma_L}}{P_{liq}}$, then:

$$\begin{cases} C_{1ns}^{\gamma_{liq}} = C_{1ns}^{\gamma_{illiq}} \\ C_{2ns}^{\gamma_{liq}} = \left(1 - C_{1ns}^{\gamma_{illiq}}\right) R_H \end{cases}$$

and $C_{2H}^{\gamma_{liq}} > C_{2H}^{\gamma_{illiq}}$ as $\tilde{\lambda} < 1$ and $S_1^* \geq 0$. So, the optimal choice of this agent in $\gamma_{illiq}$ is still feasible and lets some spare resources. He can thus do strictly better than in $\gamma_{illiq}$. If utility is strictly higher in all states of the world ($L$ and $H$), $\gamma_{liq}$ Pareto dominates $\gamma_{illiq}$ , both ex-ante and ex-post.

## A.3 Proof of PROPOSITION 4 (public liquidity insurance)

Under this liquidity insurance, date-1 budget constraints are then contingent to $P$:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + L \max(P, 1) - \tau(P) \\ C_2 = (\lambda - L) R_j + S_1 \end{cases}$$

Where:

$$\tau(P) = \begin{cases} (1-P)\sum_j \frac{L_j}{2} & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

It simply states that if the market liquidation price is low, agents will have to pay $\tau(P)$ but they will also be compensated for the loss of value with respect to the opportunity cost - the return on storage.

Of course, I still have: $L_L^*(P) = \lambda^*(P)$. Such a subsidy will not decrease the willing to liquidate of these agents. Thus:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + \lambda \max(P, 1) - \tau(P) \\ C_2 = (\lambda - L)R_j + S_1 \end{cases}$$

Conditionally on being $L$, the date-0 first order condition of problem (9) for ($\lambda = 1$) always holds:

$$E_0 \left[ \max(P-1, 0) \ln^{'}(C_1) + \ln^{'}(C_2) R_j \,| L \right] > 0 \tag{13}$$

The return-liquidity trade-off has well disappeared. As it is true irrespective to the competitive market price, $\lambda = 1$ is a dominant strategy.

Conditionally on $H$, there are two cases depending on $P$:

If $P \geq 1$, the budget constraints are:

$$\begin{cases} C_1 = 1 - \lambda + \max(\lambda - 1/2; 0)P \\ C_2 = (\lambda - \max(\lambda - 1/2; 0))R_H \end{cases}$$

From this, I can consider the first order condition of problem (9) with respect to $\lambda$:

- if $\lambda > \frac{1}{1+P}$ => $L_H = \frac{P\lambda - 1 + \lambda}{2P}$ => $\frac{(P-1)}{2} \ln^{'}(C_1) > 0$
- if $\lambda \leq \frac{1}{1+P}$ => $L_H = 0$ => $-\ln^{'}(1-\lambda) + \ln^{'}(\lambda R_H) R_H \geq 0$

If $P < 1$, the budget constraints are:

$$\begin{cases} C_1 = 1 - \lambda + L - \tau \\ C_2 = (\lambda - L)R_H \end{cases}$$

Which implies the following on the first order condition of problem (9) with respect to $\lambda$:

- if $\lambda > \frac{1-\tau}{2}$ => $L_{ns} = \lambda - \frac{1-\tau}{2}$ => $-\ln^{'}(1-\lambda-\tau)+\ln^{'}(\lambda R_H)R_H = 0$
- if $\lambda < \frac{1-\tau}{2}$ => $L_{ns} = 0$ => $-\ln^{'}(1-\lambda-\tau)+\ln^{'}(\lambda R_H)R_H > 0$

Hence $\frac{\partial U_H(\lambda,P)}{\partial \lambda} \geq 0$, which together with (13) give: $\frac{\partial U_0}{\partial \lambda} > 0$, which concludes the proof.

# B Extensions and additional material

## B.1 Robustness: multiple equilibria in the general model

Consider the general problem:

$$\max_{\lambda,L_{ij},S_{ij}} U_0 = E_0\left[u(C_1)+\beta_i u(C_2)\right]$$

$$s.t. \begin{cases} C_{1ij}+S_{ij} = 1-\lambda+L_{ij}P \\ C_{2ij} = (\lambda-L)R_j+S_{ij}(1+r) \\ 0 \leq L_{ij} \leq \lambda \leq 1 \end{cases}$$

Where $i \in \{e,n\}$with $\beta_e = 0$, $\beta_n = 1$ and $Prob(i=n) = p > 0$, and $j \in \{L,H\}$with $Prob(j=H) = q$ and $0 < q < 1$.

PROPOSITION 1bis (**self insurance generalized**)

*Let $\lambda(P) \equiv \arg\max_{\lambda} U_0(\lambda,P)$ be the set of optimal $\lambda$ to the problem above, given P and the date-1 optimal liquidity and saving policies ($L^*_{ij}, S^*_{ij}$), then:*

$$\lambda(P) \begin{cases} < \lambda^*_{nH}(P,R_H) & ; P < 1+r \\ = 1 & ; P > 1+r \\ \in \{[\lambda^*_{nH}(P,R_H),1]\} & ; P = 1 \end{cases} \tag{14}$$

*With $\lambda^*_{nH}(P,R_H) \equiv \arg\max_{\lambda} u((1-\lambda)P)+u(\lambda R_H)$* being the optimal endowment share invested in the long-run technology, for a given $P$ and conditionally on being of type $nH$.

The proof of proposition 1bis follows exactly the same logic as that of proposition 1. It is therefore omitted here.

This proposition states that when the market is expected to be illiquid, the optimal investment decision is strictly inferior to the ex-post optimal level for agent $nH$. Since, by definition of $\lambda^*_{nH}(P,R_H)$, I have: $u^{'}(1-\lambda^*_{nH}(P,R_H))P - u^{'}(1-\lambda^*_{nH}(P,R_H))R_H = 0$, it must be the case that:

$$\frac{\partial U_0}{\partial L}\,|_{\lambda<\lambda^*_{nH}(P,R_H),L=0}< 0$$

Therefore, agent $nH$ does not participate in the market when $P<1+r$. I can thus define the generalized implied price correspondence $P'(P,R_H):\left\{\left[\frac{R_L}{1+r},\frac{R_H}{(1+r)}\right],R_H\right\}\rightarrow\left[\frac{R_L}{1+r},\frac{R_H}{(1+r)}\right]$:

$$P'(P,R_H)=\begin{cases}P'_{illiq}(P,R_H)=\frac{R_L}{1+r}+\eta_{illiq}(P,R_H)\frac{R_H}{(1+r)} & ;P\leq 1+r\\ P'_{liq}(P,R_H)=\frac{R_L}{1+r}+\eta_{liq}(P,R_H)\frac{R_H}{(1+r)} & ;P\geq 1+r\\ P'(1+r,R_H)\in\left[P'_L(P,R_H),P'_H(P,R_H)\right] & ;P=1+r\end{cases}$$

With $\eta_{illiq}(P,R_H)=\frac{q-pq}{1-pq}\equiv\eta_{illiq}$, $\eta_{liq}(P,R_H)=\frac{q-pq\lambda^*_{nH}(P,R_H)}{1-pq\lambda^*_{nH}(P,R_H)}$, which comes from proposition 1bis and the fact that all agents other than $nH$ sell of their assets: $L_{nL}(P,\lambda)=L_{eL}(P,\lambda)=L_{eH}(P,\lambda)=\lambda$.

PROPOSITION 2bis (**dry-ups generalized**)

*Let $\bar{\Omega}$ be the range of admissible values[47] for parameters $\{R_L,p,q,r\}$. Let $\Gamma(\Omega,R_H)=\{(P^*,\lambda^*,\eta^*)\}$ denote the set of stable equilibria defined by (2) for a vector $(\Omega\in\bar{\Omega},R_H)$ of parameters.*

*$\forall\Omega\in\bar{\Omega}$, $\exists\underline{R_H}<\overline{R_H}$ such that $\forall R_H\in]\underline{R_H};\overline{R_H}[$, $\Gamma(\Omega,R_H)$ has at least two distinct elements corresponding to equilibria with different level of liquidity.*

Before turning to the core of the proof, I establish the continuity of $P'_{liq}(P,R_H)$ and the fact that liquidity is always higher when agent $H$ participates in the market.

**Condition** 1: $u'(0)>u'(R_H)\frac{R_H}{1+r}$

This (mild) condition will ensure $C^*_{1,nH}>0$, that is I rule out the case where agent $nH$ optimally chooses not to consume at date 1.

LEMMA 1 (liquidity dominance)

*Under condition 1,*

$$\eta_{illiq}<\eta_{liq}(P,R_H)<q\,,$$

$\forall P\geq 1+r$.

Proof:

[47] $0<p\leq 1, 0<q<1, 0\leq R_L<1+r, r>-1$

First, from $\lambda^{*}_{nH} \in [0,1]$, I have:

$$\eta_{illiq} \leq \eta_{liq}(P,R_H) \leq q$$

Then, *Condition 1* implies that: $u^{'}(0) > u^{'}(R_H)\frac{R_H}{P_{liq}(P,R_H)}, \forall P \geq 1+r$. This, in turn, implies that $\lambda^{*}_{nH}(P,R_H) > 0$ and the first inequality is strict. The second strict inequality comes from: $\frac{\partial U_0}{\partial L} \mid_{L=1} < 0$, which is always true since $P \leq R_H$.

LEMMA 2 (continuity)

*$P^{'}_{liq}(P,R_H)$ is continuous in $P$ and $R_H$.*

Proof:

As $\lambda^{*}_{nH} \in ]0,1[$, $P^{'}_{liq}$ is a continuous function of $\lambda^{*}_{nH}(P,R_H)$. The implicit functions theorem applied to the first order condition for an interior $\lambda^{*}_{nH}$ ensures that $\lambda^{*}_{nH}(P,R_H)$ is continuous in both its arguments which implies *Lemma 2*.

Proof (of proposition 2bis):

As I only consider stable equilibria, I am not interested in the vertical locus for $P'(P,R_H)$ which corresponds to $P = 1+r$ . I will consider separately the two functions $P^{'}_{illiq}(P,R_H)$ and $P^{'}_{liq}(P,R_H)$ defined respectively on the sets $\left\{\left[\frac{R_L}{(1+r)}, 1+r\right], R_H\right\}$ and $\left\{\left[1+r, \frac{R_H}{(1+r)}\right], R_H\right\}$ and show that there exists a range of $R_H$ that generates at least an equilibrium for both functions:

**The upper bound for a low-liquidity equilibrium** Brouwer's fixed point theorem gives the necessary and sufficient condition on $R_H$ for a unique fixed point $P^{'}_{L}(P,R_H) = P$:

$$R_H \leq R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$$

There exists thus a unique low-liquidity equilibrium if and only if $R_H$ is low enough, and I can thus set:

$$\overline{R_H} \equiv R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$$

**The lower bound for a high-liquidity equilibrium:** In order to derive a sufficient condition for the existence of a fixed point $P^{'}_{liq}(P,R_H) = P$, I construct a function $G(P,R_H) \equiv P - P^{'}_{liq}(P,R_H)$ , defined on the interval for $P$: $\left[1+r, \frac{R_H}{(1+r)}\right]$. Clearly, given the continuity of $P^{'}_{liq}(P,R_H)$ (*Lemma 2*), if $G(P,R_H)$ changes sign on its domain, there is a fixed point for $P^{'}_{H}(P,R_H)$.

Since $P'_{liq}(P,R_H) = \frac{R_L}{1+r} + \eta_{liq}(P,R_H)\frac{R_H}{(1+r)}$ and $\eta_{liq}(P,R_H)$ is bounded above by $q<1$, I have: $P'_{liq}(P,R_H) < \frac{R_H}{(1+r)}$ and thus $P'_{liq}\left(\frac{R_H}{(1+r)},R_H\right) < \frac{R_H}{(1+r)}$. It implies:

$$G\left(\frac{R_H}{(1+r)},R_H\right) > 0 \tag{15}$$

For any $R_H$, given (15), a sufficient condition for the existence of a high-liquidity equilibrium is thus:

$$G((1+r),R_H) \leq 0$$

Which is equivalent to:

$$P'_{liq}(1+r,R_H) \geq (1+r)$$

And thus to:

$$R_H \geq R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r,R_H)} \tag{16}$$

As $\eta_{liq}(P,R_H)$ is bounded, there will always exist a $R_H$ high enough such that condition (16) is satisfied. Yet, I am interested in a lower bound on $R_H$ for this condition to hold. That is, a $\underline{R_H}$ such that:

$$\forall R_H \geq \underline{R_H},\, R_H \geq R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r,R_H)}$$

In order to find $\underline{R_H}$, I construct the function $R'_H(R_H) \equiv R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Given *lemma 1*, it is bounded below by $R_L + \frac{(1+r)^2-R_L}{q}$ and above by $R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}$. Also, given *Lemma* 2, it is continuous over the range corresponding to these bounds. It admits thus at least a fixed point: $R'_H = R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R'_H)}$.

For a wide class of utility functions[48], $\eta_{liq}(P,R_H)$ is monotonic in $R_H$. It implies that there is a unique fixed point; call it $R'^*_H$. *Lemma 1* ($\eta_{illiq} < \eta_{liq}(P,R_H)$) implies that $R'_H\left(R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}\right) < R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}$ and thus $\forall R_H \geq R'^*_H$, $R_H > R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Hence, this unique fixed point is a lower bound on $R_H$ for the existence of a high-liquidity equilibrium. I can thus choose $\underline{R_H} \equiv R'^*_H$. If there are multiple fixed points, the correct lower bound is the highest valued fixed point:

$$\underline{R_H} \equiv \max\left\{R'_H : R'_H = R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r,R'_H)}\right\}$$

[48]Including CARA, CRRA and quadratic utility functions.

**The range for multiple equilibria** *Lemma 1* implies $\underline{R_H} < \overline{R_H}$ which concludes the proof: $\forall R_H \in ]\underline{R_H}; \overline{R_H}[$ there exists at least two equilibria.

## B.2 Robustness: multiplicity under incomplete information

I sketch here a global game version of the model in which agents map a privately observed signal $R_{Hi}$ about the true parameter $R_H$ of the model[49], into an initial investment decision $\lambda(R_{Hi})$, and I show that the best-response to non-decreasing strategies can be non-monotonic and violates the *single-crossing condition* for uniqueness of equilibrium used in the literature.

The intuition for this can be apprehended thanks to an analogy to the binary action game (*to run* or *not to run* the bank) studied by Goldstein and Pauzner (2005). In their game, for a given level of fundamental ($\theta$) the expected utility differential $v(\theta,n)$ of choosing *to run* is increasing in $n$, the proportion of agents that run, *whenever v is negative* (that is for any $n$ such that *not to run* is preferable). Accordingly, for any level of fundamental ($\theta$), there can be at most one $n$ such that $v(\theta,n) = 0$ and an agent is indifferent between *to run* and *not to run* (there is a single crossing between $v(\theta,n)$ and the *x-axis*). This is the key condition to ensure uniqueness in their model.

Here, if one were to (abusively) summarize all other agents investment decision by their average, call it $\bar{\lambda}$, the example I show below corresponds to a situation in which the secondary market price $P$ is non-monotonic in $\bar{\lambda}$. Therefore, as the marginal utility of $\lambda$ is strictly increasing in $P$, it is decreasing in $\bar{\lambda}$ over a certain range, and the best-response is non-monotonic. Crucially, it crosses more than once the level at which externalities become positive, which lets the door to multiple equilibria open (see Bueno de Mesquita, 2011, for binary action games with multiple equilibria when the single-crossing condition is not satisfied, and Mason and Valentinyi, 2010 where single-crossing is one of the sufficient conditions for uniqueness in games with a continuum of actions).

Sketch of the model:

- $R_L = 0$, $q = 0.5$, $r = 0$ (without loss of generality)
- $p = 0.5$ (if $p = 1$, that is if there are no early agents, one can actually prove that multiplicity remains, see Bueno de Mesquita (2011)).
- $R_H$ is drawn from the a uniform distribution with support: $]0+\varepsilon;\infty[$
- Each agent observes a private signal $R_{Hi}$ from a uniformed distribution centered on the true value of $R_H$: $[R_H - \sigma; R_H + \sigma]$, with $0 < \sigma < \varepsilon$ and maps it into an initial investment decision $\lambda(R_{Hi})$.

[49]This parameter could equally be $R_L$, $p$, $q$, or $r$.

First, note that there exist multiple equilibria of the complete information game when $7/3 \leq R \leq 3$. Then, consider the following (non-decreasing) strategies:

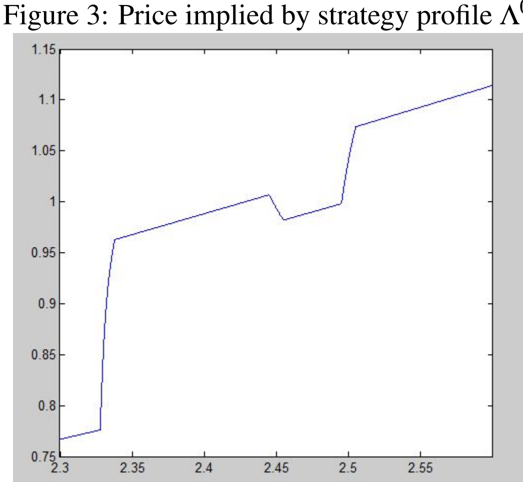
$$\lambda_1(R_{Hj}) \equiv \begin{cases} 1/4 & ; R_{Hj} < 2.45 \\ 1/2 & ; 2.45 \leq R_{Hj} \leq 2.5 \\ 1 & ; 2.5 < R_{Hj} \end{cases} \qquad \lambda_2(R_{Hj}) \equiv \begin{cases} 1/4 & ; R_{Hj} < 7/3 \\ 1 & ; 7/3 \leq R_{Hj} \end{cases}$$

Figure 3 displays the price that would result for any realization of $R_H$ if half agents play $\lambda_1(R_{Hj})$ and the other half play $\lambda_2(R_{Hj})$ (call this strategy profile $\Lambda^0$).

$$P(R_H, \Lambda^0) \equiv \eta(\Lambda^0) R_H,$$

with $\eta(\Lambda^0)$ being the analogous to equation (7).

Figure 3: Price implied by strategy profile $\Lambda^0$



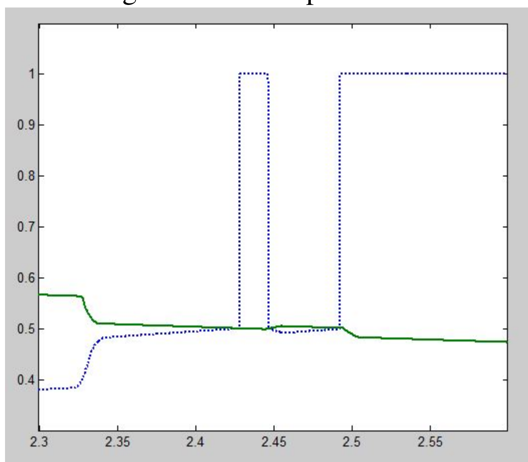

This graph displays the price (y-axis) implied by $\Lambda^0$ for the possible realization of $R_H$ (x-axis).

From this, one can compute the best-response to $\Lambda^0$ of an agent that observes a signal $R_{Hi}$:

$$\lambda^{BR}(\Lambda^0, R_{Hi}) = \arg\max_{\lambda} U_0\left(\lambda, P(R_H, \Lambda^0) \mid R_{Hi}\right)$$

Figure 4 displays such a best-response, which shows that best-responses to monotonic strategies need not be monotonic, and that, crucially, it crosses several times the locus of the joint values of $P$ and $\lambda$ from which externalities become positive, that is ($\lambda = 1/2$).

Figure 4: Best response to $\Lambda^0$




The dotted line is the best-response ($\lambda^{BR}(\Lambda^0, R_{Hi})$) of an agent (y-axis) to $\Lambda^0$, that observes a private signal $R_{Hi}$ (x-axis). Above the solid line is the region corresponding to $L_H(\lambda, P) > 0$, which implies that agents exert positive externalities.

### B.3 Unstable equilibria

There are several ways to interpret this equilibrium: for instance, it can be seen as a Nash equilibrium in pure or mixed strategy. In this equilibrium, agents expect $P = 1$. They are therefore indifferent with respect to investment choice over a wide range of value: $\lambda(P = 1) = \left[\frac{1}{2}, 1\right]$. At date 1, I have $L_L(1, \lambda) = \lambda$ and $L_H(1, \lambda) = \lambda - 0.5$. To indeed have $P = 1$ as an outcome, the actual investment density function $f(x)$ defined over the interval $\left[\frac{1}{2}, 1\right]$ should be such that: $R_L + \eta_{f(x)}(R_H - R_L) = 1$. With $\eta_{f(x)}$ being the level of liquidity implied by distribution $f(x)$, that is:

$$\eta_{f(x)} = \int \frac{x - 0.5}{2x - 0.5} f(x) dx$$

In the example of figure 1, one could for instance consider the degenerate distribution:

$$f(x) = \begin{cases} 1 & ; x = 5/8 \\ 0 & ; x \neq 5/8 \end{cases}$$

Which gives $\eta_{f(x)} = 1/6$ and indeed $P = 1$.

Obviously, in such cases, any small perturbation (to the expected price for instance) would switch the best response to either $\lambda^*(P > 1) = 1$ or $\lambda^*(P < 1) < \bar{\lambda}_1^*$ and best response iteration would never bring the economy back to $\gamma_1$.

## B.4 An hybrid model: Self-fulfilling Liquidity Dry-ups with CITM pricing

Consider the basic model presented in section 2, but without deep-pocket agents: any consumption good used at date 1 to buy long-term asset should have been carried over from date 0 (this is what will generate CITM pricing). To make it simple, I also drop the anonymity assumption to allow for a pooling equilibrium in the secondary market in which lemon owners sale the same fraction of they long-term project as good project owners.

The logic of the model is the same, but now the price is no longer the present discounted value of average future payoffs, but the one that equates supply and (limited) demand. At date 0, agents anticipate this price, which depends again on date-0 investment decision.

For ranges of parameter values, there exist multiple equilibria that differ by their level of liquidity. Under assumption 1, for instance, with $R_L = 0$ and $R_H = 12$ there exist:

- A low-liquidity equilibrium: $P = R_L = 0$, $\eta = 0$, and $\lambda = 0.2$
- A high-liquidity equilibrium: $P = 3.33 < R_H/2$, $\eta = 1/2$, and $\lambda = 0.33$

In the low equilibrium, agents anticipate a full market breakdown and therefore invest less in the long-term technology; they self-insure against the event to be stuck with lemons in an illiquid market. In the high equilibrium, agents expect a liquid market, with the participation of peach owners. They also expect cash-in-the-market pricing, so that the price is lower than the present discounted (at the rate of storage) value of future payoffs ($P < R_H/2$) but as long as $P > 1$ such an equilibrium is sustainable.