Contingent convertibles. Solving or seeding the next banking crisis?

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Abstract

A recent proposal to enhance banking stability recommends the use of contingent convertibles (CoCos). Since these hybrid securities are mandatorily converted into equity when banks are in need of a recapitalization, they are credited for reducing banks' likelihood of financial distress. In this paper, we show within a continuous-time framework that this allegedly beneficial impact hinges critically on the assumption of complete contracts. We show that although CoCos always distort risk taking incentives, CoCos can create wealth for all involved investors. Our main contribution is to demonstrate that a higher investors' wealth from CoCo financing can cause a higher bank's probability of financial distress so that the banking system as a whole will be destabilized. Thus, individually rational decisions can have systemically undesirable outcomes. Further results indicate that CoCos should be used only in conjunction with devices to mitigate risk shifting incentives. This objective can be accomplished by a stricter regulation and a higher conversion ratio.

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1 Introduction

In the aftermath of the financial market crisis of 2008/09, several culprits have been named that contributed to the near melt-down of the financial system. While still subject to ongoing debate, there seems to be consensus that one major problem was related to the high leverage ratios of banks, which left them with severe problems of recapitalization when market conditions worsened abruptly. One proposal, that has received much attention in recent times to help alleviating the problem of excessive leverage ratios is to induce banks to issue so-called contingent convertible (CoCo) bonds, also known as enhanced capital notes or contingent capital. The key feature of these hybrid securities is that they pay coupons like normal bonds but are automatically converted into ordinary shares once the equity ratio falls below a predetermined threshold. This proposal has gained momentum in November 2009 when Lloyds Banking Group launched a capital raising which included the issuance of £7.5 billion in contingent convertibles.[1] Governments as well as regulators seem to put much hope in these new securities.[2] Only recently, the Bank for International Settlement announced that as part of their reform package, they will review the role of contingent capital within the regulatory capital framework,[3] and in Switzerland, an expert group recommended that banks will have to hold 19% capital, where nine percent are required in contingent capital.[4]

Advocates of contingent convertibles consider these hybrid securities as a transparent, efficient and less costly resolution mechanism for distressed banks, because

[1] The case of Lloyds has gained attention for two reasons. First, it was the biggest issuance of CoCo bonds sofar and second, the British government has a 43% stake in Lloyds. See e.g. the *Financial Times* "Lloyds to offer sweeteners to bondholders", November 1, 2009, and *Financial Times* "UK experiment raises prospects of new asset class", November 5, 2009.

[2] As documented in speeches by Ben Bernanke, chairman of the US Federal Reserve, Paul Tucker, deputy governor of the Bank of England and Lord Turner, chairman of the Financial Services Authority. See e.g. the speech by Ben Bernanke to the US House Financial Services Committee on October 1, 2009, the *Financial Times* "The sweet fix of CoCos?", November 12, 2009 and *The Wall Street Journal* "Policy Makers Discuss Bank Capitalization", November 17, 2009.

[3] See the press release at January 11, 2010.

[4] See e.g. *Financial Times* "Capital proposal targets UBS and Credit Suisse", October 4, 2010.

they provide an increase of the equity ratio at pre-committed terms when the bank is in a difficult situation due to a severe loss of their asset value. Therefore, this feature reduces the danger of a costly default or, alternatively, supervisory intervention. At first glance, the idea seems impeccable. During good times, the bank takes advantage of the benefits of debt financing, such as e.g. exploiting financing and/or tax advantages, while in bad times, when debt obligations impose the risk of financial distress, these securities automatically convert to equity and mitigate the default risk.

In some sense, contingent convertibles seem to be the cake, one likes to have and eat it too. However, a fundamental doubt that CoCo bonds do the trick, comes from theoretical considerations about the optimality of debt financing. A large body of literature argues that within an incomplete contracts setting, debt is an optimal financing arrangement because it offers fixed payments in good states, while in bad states it stipulates transfer of ownership. The threat of losing ownership, or more generally control rights, exerts a disciplining effect on the decision-makers of the firm. Hart and Moore (1998) underscore the important distinction between cash flow rights and control rights for the theoretical explanation of debt contracts. However, by construction, CoCo bonds postpone the transfer of complete control rights. Thus, contingent convertibles may distort decision-makers' incentives.

This paper is, to the best of our knowledge the first theoretical contribution that tries to shed light on potential drawbacks of contingent convertibles due to distorted risk incentives. Within a continuous-time structural model, we consider a commercial bank engaged in the deposit taking business which satisfies additional financing needs by accessing capital markets. In line with the practical evidence from Lloyds, we analyze the impact of exchanging straight bonds with contingent convertible bonds. In our analysis, we distinguish between two cases: A complete contract and an incomplete contract setting. In the former, we assume the investment policy of the bank to be given (or to be contractible), while in the latter case bank managers have discretion over the choice of the bank's investment risk. From our analysis, we find that CoCo bonds are unambiguously beneficial if the bank cannot change its business risk. However, we also find that results change dramatically if we consider incomplete contracts. We show that

CoCo bonds always distort risk taking incentives and induces decision-makers to act less prudent. As a main result, we demonstrate that although CoCo bonds are optimal in the sense that they are fairly priced and firm value maximizing, the distorted risk-taking incentives can actually increase the bank's probability of financial distress as well as the expected distress costs substantially. Contrary to the initial intention, CoCo bonds can create negative externalities for the economy in the sense that individually rational decisions may have systemically undesirable outcomes.

While normal convertible bonds have been analyzed extensively in the literature (see e.g. Brennan and Schwartz, 1977; Ingersoll, 1977; Brennan and Schwartz, 1980; Brennan and Kraus, 1987; Nyborg, 1995), there are only few recent academic contributions dealing specifically with *contingent* convertibility provisions. Flannery (2005, 2009) argues that contingent convertibles are an effective mechanism to exert market discipline, because by issuing CoCo bonds shareholders internalize, i.e. bear the full cost of their risk taking decisions rather than rely on the (costless) regulatory bail-out option. In a related analysis, Landier and Ueda (2009) discuss several options for a bank restructuring, among which they mention convertible debt. In line with the reasoning by Flannery (2009), they conclude that convertibles can decrease the probability of default (see Landier and Ueda (2009), p. 25). Acharya et al. (2009) simply state in their recent contribution that assesses the strengths and weaknesses of the US financial reform legislation: "*Contingent capital is clearly a good idea*".[5]
More recently, Glasserman and Nouri (2010), Pennacchi (2010), and Sundaresan and Wang (2010) put forward theoretical models of CoCo bonds. Glasserman and Nouri (2010) and Pennacchi (2010) both analyze the pricing of CoCo bonds when conversion can take place continuously (Glasserman and Nouri, 2010) or when the bank's asset value follows a jump-diffusion process and interest rates are stochastic (Pennacchi, 2010). Sundaresan and Wang (2010) focus on the design of the conversion trigger, and argue that for a wide class of trigger mechanism there does not exist a unique equilibrium for equity and bond prices, thus opening up the possibility of price manipulation.

[5] Acharya et al. (2009), p. 43.

The work by Flannery (2005, 2009) has received considerable attention and CoCo bonds have been mentioned favorably in policy recommendations by e.g. Stein (2004), Kashyap et al. (2008), Kaplan (2009), and Duffie (2009). While contingent convertibles are welcomed by pointing out their ability to overcome problems of high leverage, Hart and Zingales (2009) recognize that the proposal by Flannery (2005) eliminates some of the disciplinary effects of debt.[6] However, since their focus is on the implementation of a new capital regulation for systemically important banks, they do not explore that issue in detail. As a further policy recommendation, the so-called Squam Lake Working Group on Financial Regulation advocates the use of contingent capital as a transparent, efficient and less costly resolution mechanism for distressed banks.[7]

One of the few concerns that have been raised against CoCo bonds so far points towards a potentially destabilizing effect which might occur when large institutional investors, who are not allowed to hold shares, are forced to sell their converted bond position. This effect might exacerbate the share price decline and spur doubts about the bank's stability.[8] With respect to a potential moral hazard problem, the academic literature so far seems to consider risk-shifting problems of CoCo bonds to be marginal at most. Flannery (2005, 2009) concludes from verbal reasoning rather than from a formal model-based approach that risk-taking incentives can be controlled through the internalization of risk-shifting costs. Pennacchi (2010) admits the presence of risk-shifting incentives but argues that they are less pronounced as with subordinated debt. Glasserman and Nouri (2010) analyze how higher risk impacts the yield spread, but do not explore the stability concerns of CoCo bonds.

Thus, to the best of our knowledge, our contribution is the first that focuses on potentially adverse implications of CoCo bonds from distorted risk-taking incentives. In the light of the recent banking crisis, it need not be stressed that the analysis of banks' risk taking behavior is of crucial importance. Our results are

[6] See Hart and Zingales (2009), p. 5.

[7] The Squam Lake Working Group resembles fifteen distinguished academics as e.g. Darrell Duffie, Douglas Diamond, John Cochrane, Robert Shiller and Raghuram Rajan. See Squam Lake Working Group (2010).

[8] See *Financial Times* "Stability concerns over CoCo bonds", November 5, 2009, and *Financial Times* "Report warns on CoCo bonds", November 10, 2009.

an important warning signal that CoCo bonds may create negative externalities, in the sense that the (destabilizing) risk-shifting problem induced by CoCo bonds may overcompensate the (stabilizing) effect of providing a pre-committed recapitalization to banks.

The article is organized as follows. The next section sets up the general model framework and determines the optimal financing behavior with straight bonds and CoCo bonds. In section 3, we analyze the impact of CoCo financing if complete contracts can be written, i.e. when the bank has no discretion over the risk technology. Section 4 analyzes the case where contracts are incomplete, i.e. the bank can change the investment risk. Section 5 concludes.

2 General Model Framework

2.1 Initial Bank

In line with Bhattacharya et al. (2002), Decamps et al. (2004) and Rochet (2004), we consider a bank with assets in place that continuously generate instantaneous cash flows before taxes equal to x_t, which are assumed to be driven by a Geometric Brownian Motion[9]

$$\mathrm{d}x_t = \mu x_t \,\mathrm{d}t + \sigma x_t \,\mathrm{d}z_t, \tag{1}$$

where μ and σ are constant drift and diffusion parameters, and z_t denotes a standard Wiener process. We will apply the arbitrage-free valuation principle where (for pricing purposes) the process in (1) is considered under the risk-neutral measure $\mathcal{Q}$. In the following sections, we will analyze both, the case where the process of the cash flows governed by μ and σ is exogenously given, and the case where bank managers have discretion over the cash flow process and can switch to a

[9] While being a standard assumption in the literature, the assumption of Geometric Brownian Motion implies that EBIT is always non-negative. However, this is no major restriction since it does not change the general behavior of a bank as EBT (i.e. EBIT after interest payments) can still become negative, which then requires outside financing. Furthermore, in practice, the EBIT of banks is virtually never observed to be negative.

different risk parameter σ. An increase in the bank's cash flow risk can be interpreted either as a relaxation of monitoring activities in the sense of less careful risk management activities (see e.g. Decamps et al., 2004), or as a shift in the investment policy towards more risky securities as in the classic asset substitution problem (see e.g. Jensen and Meckling, 1976; Green, 1984; Leland, 1998).

As a second salient characteristic, we adopt the frequently made assumption in the financial intermediation literature that banks' assets are sold at a significant discount when the bank is closed. The discount may reflect the illiquidity of banks' assets due to specialized human capital (as e.g. argued in Diamond and Rajan, 2000, 2001) or adverse selection costs due to opacity, i.e. information asymmetry.[10] In this sense, at the time of closure $\mathcal{T}$, banks' assets are assumed to have a liquidation value of $\Lambda = \lambda \frac{x_{\mathcal{T}}}{r-\mu}$, where r denotes the risk-free interest rate.[11] We consider a corporate tax rate equal to τ and ignore additional taxes on the private level.[12] Since the liquidation value cannot exceed the after-tax riskless value of a perpetual cash-flow stream, λ has to lie between zero and $(1-\tau)$. This formulation of λ comprises as a special case the frequently employed all equity value after taxes and bankruptcy costs which is well-known as $\lambda \frac{x_t}{r-\mu}$ with $\lambda = (1-\alpha) \cdot (1-\tau)$ for proportionate bankruptcy costs α.[13]

On the liabilities side, the main characteristic of commercial banks is their ability to take deposits. We assume the bank to have a given volume of deposits that require an aggregate continuous, instantaneous interest payment of d. We take d to be exogenous. The reasoning behind the fact that in our model, the bank cannot arbitrarily choose the deposit volume is as follows. On the one hand, the available amount of deposits is limited so that a given maximum amount can hardly be exceeded without offering unreasonable high deposit rates. On the other hand, deposits are a possibility to create a close relationship to customers

[10] See e.g. Stein (1998) for an adverse selection model, and Morgan (2002) for evidence on the opacity of banks' assets.

[11] As it is standard, r is assumed to be constant and $\mu < r$ to ensure finite solutions.

[12] This is only for reasons of parsimony and does not change subsequent results as long as there is a tax advantage to debt financing after corporate and personal taxes. Furthermore, we assume that losses cause immediate (negative) tax payments, which contrasts with a more realistic asymmetric tax-regime, but which is imposed for tractability reasons.

[13] See e.g. Leland (1994), Goldstein et al. (2001), and Morellec (2004).

and can therefore be understood as an option to future valuable deals (e.g. after investing into deposits, a customer might want to finance the acquisition of a costly good by a loan from the bank). Thus, the bank must keep the deposits on a given level to avoid an inefficient reduction of the future business. As a result, our model accounts for the fact that banks have a positive deposit volume with an exogenous nature.

As long as the bank is solvent, depositors receive their contractual interest payment d. The residual cash flow $(x_t - d)(1 - \tau)$ is paid out as dividends to the bank's equity holders. Since x_t can be considered as the earnings before interest and taxes (EBIT), the difference $x_t - d$ denotes EBT, and thus $(x_t - d)(1 - \tau)$ denotes the after tax dividend payment. We assume that total revenues are entirely paid out to investors. Moreover, we do not allow for adjustments of the debt structure. With this static view, we account for the fact that especially during times of a banking crisis, banks only have very limited possibilities to adjust the capital structure. Given that the capital structure could be arbitrarily adjusted at no costs, a default could be prevented in our model (see e.g. Koziol and Lawrenz, 2009).

A typical approach within this model class from the corporate finance literature (see e.g. Fischer et al., 1989; Leland, 1994; Goldstein et al., 2001) is to assume perfect markets and unregulated firms. Therefore, equity holders are willing to inject capital as long as they consider it to be a worthwhile investment, and default of a firm can only occur for reasons of indebtedness. The default event is a result of the optimizing behavior of owners and therefore endogenous. While this may be a reasonable approach for modeling the default decision of standard non-financial firms, it may be less appropriate for banks for basically two reasons. First, banks are regulated and not at least due to mandatory minimum capital requirements, the default decision is subject to an exogenous constraint. This is consistent with e.g. Decamps et al. (2004). Second, as witnessed during the recent banking crisis, banks in financial distress face various difficulties or costs in raising capital. Thus, severe financial frictions in times of stress also impose exogenous constraints on the financing of banks, and can result in default for reasons of illiquidity. The importance of the inability to raise capital in times of stress need not to be emphasized in light of the recent turmoil on international financial markets and is the very reason that underlies the proposal for contingent convertibles.

For these reasons, we consider financial distress as being triggered by an *exogenous constraint* that may be interpreted in both ways. Either as regulatory intervention, or as the inability to raise further capital. In technical terms, financial distress is modeled as a stopping time, which is defined as the first time, the cash flow process x_t hits the boundary ξ from above, i.e. $\mathcal{T}_\xi = \inf\{t; x_t \leq \xi\}$.

It is reasonable to expect that the financial constraint, i.e. the boundary ξ will depend on the extent of the bank's debt liabilities. In terms of a regulatory intervention threshold, it is consistent with a minimum capital requirement. Therefore, we consider the exogenous boundary to be related to the bank's debt liabilities in a linear way:

$$\xi(\pi) = \phi\,\pi, \tag{2}$$

where we use π to generally denote the bank's instantaneous payments to all debt holders, and $\phi > 0$ as a constant that determines the severity of financial constraints.

Note that in order to be binding, the exogenous threshold $\xi(\pi)$ must be at least as high as the endogenous default threshold $\xi^*(\pi)$ that would be optimal for the shareholders without any exogenous restrictions.

Under the assumptions of the model, it is standard to show that the valuation of equity holders' claim, denoted as S_t, is given by[14]

$$\begin{aligned} S_t &= \mathbb{E}^Q\left[\int_t^{\mathcal{T}_\xi} e^{-r(s-t)}\,(1-\tau)(x_s-\pi)\,\mathrm{d}s\right] \\ &= (1-\tau)\left(\left(\frac{x_t}{r-\mu}-\frac{\pi}{r}\right)-\left(\frac{\xi}{r-\mu}-\frac{\pi}{r}\right)\left(\frac{x_t}{\xi}\right)^\beta\right), \end{aligned} \tag{3}$$

Note that $\left(\frac{x_t}{\xi}\right)^\beta$ is to be interpreted as a probability-weighted discount factor, or as the state price of one unit of account conditional on the event that x_t hits ξ.[15]

[14] See e.g. Fischer et al. (1989), Mella-Barral (1999) and Decamps et al. (2004).

[15] To ease notation, we will be sloppy sometimes, and write ξ without argument.

β is the (negative) solution to the quadratic equation $\frac{\sigma^2}{2}\beta(\beta-1)+\mu\beta-r=0$, given by $\beta=-(\mu-\sigma^2/2+\sqrt{2\,r\,\sigma^2+(\mu-\sigma^2/2)^2})/\sigma^2$.

For ease of notation, we use the following convention. Write $\mathcal{V}(y,\pi)=(1-\tau)\left(\frac{y}{r-\mu}-\frac{\pi}{r}\right)$, and $\mathcal{D}(y,y')=\left(\frac{y}{y'}\right)^{\beta}$, then (3) is more compactly expressed as

$$S_t=\mathcal{V}(x_t,\pi)-\mathcal{V}(\xi,\pi)\mathcal{D}(x_t,\xi). \tag{4}$$

If the bank has only deposits, the total interest payment to debt holders, π equals d. We assume deposits to be insured, where the bank has to pay a fair deposit insurance premium. Since the work of Merton (1977), it is well known that the value of the deposit insurance protection corresponds to the value of a corresponding put option, which we denote by I.[16] For a given deposit volume d, the insurance premium is found to be

$$I_t=\max\left\{\frac{\lambda}{r-\mu}\,\xi-\frac{d}{r},\,0\right\}\cdot\mathcal{D}(x_t,\xi). \tag{5}$$

Obviously, insurance makes deposits riskless. If depositors discount future interest payments at the risk-free rate r, the aggregate deposit value, denoted as D, equals $D=d/r$. Therefore, since the bank pays I_t for protection, the value of deposits for the bank is $D-I_t$.

2.2 Bank With Bond Financing

While deposits are given, the bank can access capital markets to satisfy additional financing needs. In the following, we will compare two types of bonds, a straight bond and bonds exhibiting the mandatory convertibility feature. For tractability, we assume a straight bond to be a fixed-coupon consol bond that promises a continuous instantaneous coupon payment b to bondholders. The total interest payment π of the bank is therefore: $\pi=d+b$. In general, the higher debt liabilities will raise the default threshold ξ, since due to the higher interest payment obligation, the bank runs into financial difficulties already at higher cash flow levels. In case of default, most banking systems stipulate seniority among debt claims for depositors with bondholders being subordinated. According to

[16] An alternative consideration of 'cheap' deposits would not crucially affect our analysis because the important relationship between the equity value and risk still remains.

this priority structure, the available recovery value should be split in a way that bondholders obtain part of the liquidation value only if depositors have been compensated in full. In practice, absolute priority cannot always be enforced, and the actual split-up is often the result of a bargaining process. It would be straightforward to explicitly model the sharing rule as a (Nash) bargaining solution following e.g. Fan and Sundaresan (2000) and Mella-Barral and Perraudin (1997). However, for our purposes, the precise sharing rule is not crucially important for the following reason. As long as equity holders' claim in default is not affected by the sharing rule,[17] the split-up among debt holders is irrelevant to equity holders and therefore does not affect any decisions made by the bank owners. We take this fact as justification to pursue a reduced-form modeling approach, and to assume that a certain fraction θ of the liquidation value $\lambda\xi$ goes to bondholders. Similar to (3), the value of the straight bond at time t, denoted as B_t, can be expressed as

$$B_t = \frac{b}{r} + \left(\theta\lambda\xi_b - \frac{b}{r}\right)\mathcal{D}(x_t, \xi_b).$$

Note that the knowledge of θ is not required for our analysis, because only the total debt value $B_0 + D - I_0$ net of insurance costs will be required, which is independent of the distribution of the bank between the depositors and bondholders in the case of a default. The equity value can still be expressed as in (4), with $\pi = d + b$, and a presumably higher ξ_b which we distinguish with the superscript b to indicate the straight bond case.

While we take the deposit volume as exogenous, we assume the bank to optimally adapt their remaining financing needs by issuing a corresponding amount of bonds (proxied by the aggregate coupon payment). Therefore, b is a choice variable, and the optimal bond coupon b^* is the result of the maximization of the value of all claims net of the value of the deposit insurance premium

$$b^* = \arg\max_b \{S_t^b + B_t + D - I_t\}$$

As usual, the maximization of the bank value $S_t^b + B_t + D - I_t$ is consistent with the maximization of the equity holders' wealth $S_t^b + B_t - I_t$. This is a result of

[17] This condition is most obviously satisfied if priority between debt and equity holders is enforced, and equity holders' claim in default is zero.

the notion that equity holders keep the stocks, pay the deposit insurance I_t with cash injections and receive a special dividend equal to the bond value B_t. (Since the deposit value is insured, D equals d/r and is therefore independent of the coupon b.)

With the general constraint $\xi_b = \phi \cdot (d + b)$, the first derivative of the bank value $V_t^b = S_t^b + B_t + D - I_t$ with respect to b is

$$\frac{\partial V_t^b}{\partial b} = \frac{\tau}{r} - \frac{\zeta}{r}\mathcal{D}(x_t, \xi_b), \tag{6}$$

where $\zeta = \frac{(1-\beta)((r-\mu)\tau + \phi r((1-\tau)-\lambda)}{r-\mu}$. From the properties $\beta < 0$, $\phi > 0$ and $0 \leq \lambda \leq (1-\tau) < 1$, it follows that $\zeta > \tau > 0$ which implies a positive derivative for $\mathcal{D}(x_t, \xi_b) \to 0$, while the derivative is negative for $\mathcal{D}(x_t, \xi_b) \to 1$. The fact that $\mathcal{D}(x_t, \xi_b)$ increases monotonically in ξ_b (i.e. in the debt level) confirms the existence of an optimal debt level. Solving the first order condition yields b^* as

$$b^* = \frac{x_t}{\phi}\left(\frac{\tau}{\zeta}\right)^{-1/\beta} - d. \tag{7}$$

The first term on the right-hand side of (7) determines the debt capacity of the bank which depends upon its growth potential, risk, current cash flow level, the liquidation costs λ and the strength of financial constraints ϕ. As it can be verified from (7), high liquidation costs (small λ) as well as a strong financial constraint (high ϕ) curtail the bank's debt capacity. Note that in the optimum, a bank issues less bonds if it has already a high volume of deposits. We assume that the exogenously given deposit volume is such that the bank has not exhausted its debt capacity, i.e. $b^* > 0$.

2.3 Bank With CoCo Financing

As an alternative to raise capital with a straight bond, the bank can issue contingent convertible (CoCo) bonds. Before conversion, CoCo bonds are equivalent to ordinary straight bonds in that they pay a fixed coupon rate. We denote the continuous instantaneous coupon payment of CoCo bonds as c. The main difference to straight bonds is the conversion feature. The contract of CoCo bonds stipulates that conversion will take place once the (core) capital ratio of the bank falls below a certain threshold. Upon conversion the former bondholders will

hold an equity claim, i.e. they receive newly issued shares, which entitles them to a fraction of the bank's profit. In our model setup this is neatly reflected by introducing the parameter γ, which denotes the fraction of the bank's after-tax profits that goes to former CoCo bondholders. If the number of shares (after conversion) is normalized to 1, γ is also neatly interpreted as the number of shares going to former CoCo bondholders. The fact that former equity holders receive the remaining fraction $1-\gamma$, which is less than before conversion, is consistent with the dilution effect of a seasoned new issue. Obviously, former bondholders also receive proportionate voting rights.
To formalize the conversion feature, we define the cash flow barrier χ as the threshold at which the bond is converted into equity. Since after conversion, the debt only consists of deposits, the bank has a smaller interest payment obligation. The reduced debt obligations lower the default threshold ξ_c, where we add the superscript c to indicate the case of the CoCo bond. By applying the general threshold definition in (2), we have $\chi = \phi \cdot (d+c)$ and $\xi_c = \phi \cdot d$. This choice of χ ensures that a conversion takes place at a time when the bank would face financial distress if it had not issued bonds with a mandatory convertibility feature. Furthermore, the choice of χ also ensures that the highest volume of tax shields for given interest payments $d+c$ can be generated.[18] The valuation for CoCo bonds at time t, denoted by C_t, and the value of initial (old) equity holders is given by

$$C_t = \frac{c}{r}\left(1-\mathcal{D}(x_t,\chi)\right) + \gamma\,\mathcal{D}(x_t,\chi)\,\left(\mathcal{V}(\chi,d) - \mathcal{V}(\xi_c,d)\,\mathcal{D}(\chi,\xi_c)\right) \tag{8}$$

$$S_t^c = \mathcal{V}(x_t,d+c) - \mathcal{V}(\chi,d+c)\,\mathcal{D}(x_t,\chi) + (1-\gamma)\,\mathcal{D}(x_t,\chi)\,\left(\mathcal{V}(\chi,d) - \mathcal{V}(\xi_c,d)\,\mathcal{D}(\chi,\xi_c)\right) \tag{9}$$

As $x_t \to \infty$, $\mathcal{D}(x_t,\chi) \to 0$, and C_t approaches c/r, i.e. for very high cash flow levels, the value of the CoCo bond approaches its risk-free value. At $x_t = \chi$, $\mathcal{D}(x_t,\chi) = 1$, and CoCo bondholders are holding an equity claim worth $\gamma\,\left(\mathcal{V}(\chi,d) - \mathcal{V}(\xi,d)\,\mathcal{D}(\chi,\xi_c)\right)$, i.e. a fraction γ of total equity.

Eventually, we are interested in evaluating if a CoCo bond is an attractive alternative for the bank. At first glance, it seems meaningful to compare the

[18] We explore the implications of different choices for the conversion threshold in section 4.3.

bank's equity value when it has a straight bond outstanding to the case when it raises the *same* amount of debt with a CoCo bond.[19] However, such an approach ignores the fact that a CoCo bond issue can change the overall debt capacity of the bank. Therefore, we need to solve a related maximization problem as in the previous section, i.e. the optimal CoCo bond coupon c^* has to solve $\max_c V_t^c$, where $V_t^c = S_t^c + C_t + D - I_t$ is the bank value (including the deposit insurance obligation) under CoCo financing. Plugging in from (8) and (9), the first derivative of V_t^c with respect to c is

$$\frac{\partial V_t^c}{\partial c} = \frac{\tau}{r} - \frac{\zeta^c}{r} \mathcal{D}(x_t, \chi), \tag{10}$$

where $\zeta^c = \frac{\tau(\pi - c\beta)}{\pi}$. Again, $\zeta^c > \tau > 0$ so that as in the previous section, it can be verified that for $\mathcal{D}(x_t, \chi) \to 0$, the derivative is positive, while it changes sign as $\mathcal{D}(x_t, \chi)$ tends to one. Thus, a maximum in the coupon c exists. Although c^* cannot be determined analytically, we will show in the next section, that the optimal coupon of a CoCo bond will be higher than the optimal coupon of a straight bond.

3 CoCos as Solution Against Banking Crises – Complete Contracts

With the framework set up in the previous section, we can now address the issue whether a CoCo bond is an attractive alternative for banks and has the potential to mitigate future banking crises. We analyze this issue along two dimensions. First, we consider whether the use of CoCo bonds is worthwhile for bank owners. Second, we analyze whether the use of CoCo bonds is beneficial to the economy in the sense that it decreases the risk that a bank runs into financial distress measured by both the default probability and the costs of distress.

[19] For the analysis of rating-trigger step-up bonds, Bhanot and Mello (2006) take such an approach. For a discussion, see also **?**.

3.1 Incentive Compatibility

As a first result, we show that for a given deposit volume, the bank has a higher debt capacity if it issues CoCo bonds relative to straight bonds. In other words, the optimal coupon of a straight bond issue is lower than the optimal coupon of a CoCo bond. To derive this result, we establish the following claim.

Lemma 1 *For a given cash flow level x_0, deposit volume d, all admissible parameter values (i.e. $0 < \mu < r$, $\beta < 0$, $0 < \phi$, $0 < \tau < 1$, and $\lambda < (1 - \tau)$), and any given coupon level k such that $\xi = \phi \cdot \pi < x_t$, it holds*

$$\left.\frac{\partial V_t^b}{\partial b}\right|_{b=k} < \left.\frac{\partial V_t^c}{\partial c}\right|_{c=k}.$$

The proof is in appendix A.1. From lemma 1, two important conclusions can be drawn. Since the slope $\frac{\partial V_t^c}{\partial c}$ of the bank value with CoCo bond financing is positive for a given coupon k for which the corresponding slope $\frac{\partial V_t^b}{\partial b} = 0$ of the bank value under straight bond financing is zero, it immediately follows that the optimal coupon c^* of the CoCo bond must be larger than b^* of the straight bond. Second, the bank's firm value under the optimal coupon choice is higher for CoCo bonds than for straight bonds, i.e.

$$V^b(b^*) < V^c(c^*).$$

This inequality is a result of the fact that for $c = 0$, V_t^b must equal V_t^c since $\chi = \xi$. Thus, both firm value functions originate in the same point. By integrating, we obtain $V^b(k) < V^c(k)$ from lemma 1 for any given coupon k and therefore it must hold $V^b(b^*) < V^c(c^*)$ due to $c^* > b^*$.

Economically, the higher bank value is a result of the fact that bonds with a higher coupon can be issued that create additional tax shields. For the case of a CoCo bond, this advantage of debt is not countered by the typical disadvantage of higher expected distress costs, because due to the convertibility feature, a conversion takes place before the bank will run into financial distress. As a result, additional tax shields are created by CoCos without increasing the default probability. It is worth noting, that the same qualitative result will also hold, if an alternative trade-off model for debt financing is considered. Thus, the assumption of a tax shield is not crucial, and could be substituted by some other advantage of debt financing.

3.2 Severity of Financial Distress

From the systemic perspective that a regulator will take on, two aspects of financial distress are interesting. On the one hand, the severity of financial distress can be evaluated as the expected (present value of) distress costs. On the other hand, it is also important to quantify the likelihood that a distress event will occur within a given time period. Formally, the bank enters financial distress in our setup when the cash flow x_t hits the distress threshold, which is ξ_b in the case of straight bond financing and ξ_c for CoCo bond financing. Since interest payments after conversion are reduced to $\pi = d$, the default threshold $\xi_c = \phi \cdot d$ is lower than $\xi_b = \phi \cdot (d + b)$. The probability that the bank enters financial distress within a given time horizon T is equivalent within our framework to the probability that the cash flow process x_t hits the boundary ξ from above given a current level of x_0. From standard results,[20] this can be calculated as

$$P_{\xi,T} = \text{Prob}\{\mathcal{T}_\xi < T\} = N\left(\frac{\bar{z} - \hat{\mu}\,T}{\sigma\sqrt{T}}\right) + \exp\left\{\frac{2\,\hat{\mu}\,\bar{z}}{\sigma^2}\right\} N\left(\frac{\bar{z} + \hat{\mu}\,T}{\sigma\sqrt{T}}\right), \qquad (11)$$

where $\bar{z} = \log(\xi/x_0)$, $\hat{\mu} = \mu - \sigma^2/2$, and $N(\cdot)$ denotes the standard normal cumulative distribution function. Note that $\bar{z}$ increases in ξ, from which it is immediately obvious that $P_{\xi,T}$ is monotonically increasing in the threshold ξ. Therefore, it follows for any time horizon T that the default probability with CoCo bond financing is smaller than the corresponding probability under straight bond financing

$$P_{\xi_c,T} < P_{\xi_b,T}.$$

Apparently, the inequality holds for any arbitrary drift rate μ so that we do not need to distinguish between the physical and risk-neutral drift rate.

As a second measure of the severity of financial distress, we can compare the expected distress costs between straight bond and CoCo bond financing. We

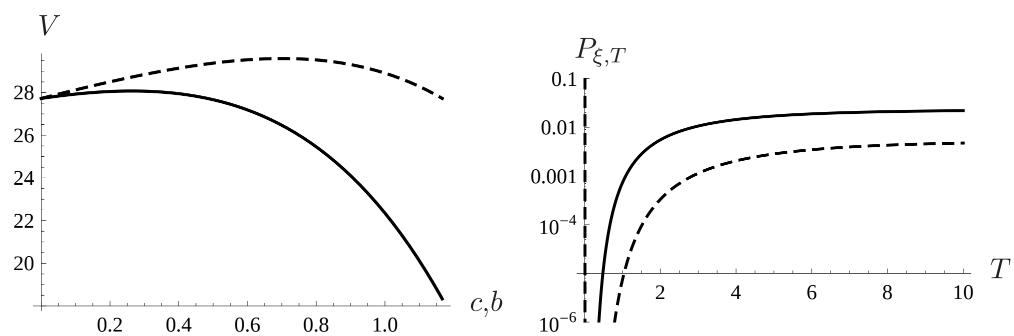
[20] See e.g. Björk (1998), Ch. 13.

find the same unambiguous positive effect of CoCo bonds and report details in Appendix A.2. We summarize the findings in this section in

Proposition 1 *(Contingent convertibles in a complete contract setting) Under the possibility to write complete contracts, CoCo bonds are incentive-compatible in the sense that an optimally designed bond increases the bank's firm value and mitigates the severity of financial distress documented by a lower probability of financial distress as well as lower present value of distress costs.*

Figure 1: Bank value and distress probabilities.

The left panel plots the firm value for a bank with straight bonds, V_t^b (solid line) and with CoCo bonds, V_t^c (dashed line) as function of the coupon rate c and b. The right panel plots the probability of financial distress $P_{\xi,T}$ (on log-scale) for straight bonds (solid line) and CoCo bonds (dashed line) for increasing time horizons T. Note that $P_{\xi,T}$ is plotted under the physical measure, and thus represents the actual probability of default. It is calculated by exploiting the relationship $\mu = \mu^P - \psi\sigma$, and assuming a market price of risk equal to $\psi = 0.5$. Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$.



As an illustration of the findings in proposition 1, figure 1 reports numerical values for the firm value (left panel) of a bank having issued straight bonds (solid line) and the value of a bank using CoCo bonds (dashed line). The graph reveals that with CoCo bonds the debt capacity of the bank is substantially higher which in turn leads to an increased firm value at the optimal coupon rate. The right panel plots the probability for financial distress (on log-scale) for a bank with an optimal amount of straight bonds (solid line) and for a bank with an optimal CoCo bond (dashed line). Again, the numerical example confirms the general finding that for any time horizon T, the probability of financial distress is lower for banks having issued CoCo bonds.

4 CoCos as Origin of Banking Crises – Incomplete Contracts

The previous section has demonstrated that CoCo bonds have substantial beneficial effects on banks as well as on the banking system. Not only do CoCo bonds raise the bank's debt capacity which increases firm value, it also decreases the probability of running into costly financial distress because of the automatic implicit recapitalization mechanism. A folk wisdom in financial economics is the saying that there is no such thing as a free lunch. So, one may wonder whether there are really only advantages to CoCo bonds, or if there is a hidden drawback. On an abstract level, a large body of theoretical literature has stressed the role of debt contracts as being a disciplining device on the decision making of managers and owners. Jensen (1986)'s free cash-flow hypothesis or the role of short-term debt as commitment device stressed by Calomiris and Kahn (1991) and Diamond and Rajan (2000, 2001) are prominent examples. From a contract-theoretical perspective, in particular Innes (1990) and Hart and Moore (1998) are important contributions which establish the optimality of a debt contract. They show that under the condition of incomplete contracts, a financing arrangement is optimal where the financier receives a fixed payment in good states of the world, while being handed over control rights in bad states of the world. This financing arrangement resembles exactly a debt contract. The crucial insight is the fact that managers (or the entrepreneur) is disciplined by the threat of being expropriated in bad states. Note that CoCo bonds undermine precisely this disciplining feature of debt contracts. In bad states of the world, former owners do not loose their control rights immediately. On the contrary, former bondholders are obliged to convert their debt claim into an equity claim. Arguably, absent the threat of losing control rights, owners face different incentives.

In this section, we analyze this doubt formally by assuming contracts to be incomplete in the sense that manager-owners can change the investment policy, or more precisely that the choice of investment policy is observable, but not contractible. The issuance of CoCo bonds instead of straight bonds has arguably an important implication for the risk taking behavior of bank manager-owners. Intuitively, since the bank's equity holders enjoy the full benefits from a cash flow increase but are protected against a default in a less favorable state due to the mandatory conversion, they might prefer a more risky strategy compared to the case with straight bond financing. For this reason, we introduce the problem of risk-shifting to the bank's financing decision and evaluate both the impact on equity holder's wealth and the severity of running into financial distress.

4.1 Risk Taking Incentives

In this section, we assume that bank managers have an unique option to increase the riskiness of the bank's operating profit. In particular, we can think about the risk-shifting option as the possibility of managers to choose different technologies such as the bank's possibility to relax their monitoring effort or to put less resources into an effective risk management system (shirking) as e.g. in Decamps et al. (2004). While these actions avoid costs and could therefore raise (expected) profits, unmonitored creditors have moral hazard incentives which increases the riskiness of returns. Alternatively, with respect to their (proprietary) trading activities, the risk-shifting option is to be interpreted as the investment in more risky securities which increases the overall asset value risk in the sense of the classic asset substitution problem as in Jensen and Meckling (1976), Green (1984) or Leland (1998).

In formal terms, we assume that the current cash flow risk is σ_l and that bank managers have an irreversible option to increase risk to σ_h, while the (for pricing purposes relevant risk-neutral) drift μ remains unaffected.[21] To compute the default probability under the physical (true) probability measure, knowledge of the physical drift μ^P is also required. Since the risk-neutral drift μ is related to the physical drift via $\mu = \mu^P - \psi\,\sigma$, the physical growth rate μ^P rises with the risk σ for a given positive market price of risk $\psi > 0$. The assumption that an increase in risk is compensated by a corresponding increase in the (physical) drift rate is precisely consistent with the notion of shirking.[22]

We abstract from manager-owner conflicts which would only add another layer of agency conflicts and assume that decisions are made in the interest of the owners.

[21] This modeling approach is standard within the asset substitution literature as e.g. in Leland (1998), Ericsson (2000), and **?**.

[22] See e.g. Decamps et al. (2004).

In this section, we analyze the incentives of equity holders to shirk, i.e. to choose the high risk investment program. We first show that the financial constraints are crucial to understanding risk taking incentives.

Once the firm has issued the bond, equity holders have an incentive to choose the high risk technology if this increases the value of their claim. In formal terms, the risk preference can be evaluated by determining the sign of the first derivative of the equity claim with respect to the risk parameter σ. Taking the first partial derivative of the equity value S_t^b for straight bond financing w.r.t. σ yields,

$$\begin{aligned} \frac{\partial S_t}{\partial \sigma} &= -\mathcal{V}(\xi, \pi) \frac{\partial}{\partial \sigma} \mathcal{D}(x_t, \xi, \sigma) \\ &= -\mathcal{V}(\xi, \pi) \mathcal{D}(x_t, \xi, \sigma) \cdot \log\left(\frac{x_t}{\xi}\right) \cdot \frac{\partial \beta(\sigma)}{\partial \sigma}. \end{aligned} \tag{12}$$

While it is immediately obvious that $\mathcal{D}(x_t, \xi, \sigma) > 0$ and $\log\left(\frac{x_t}{\xi}\right) > 0$, it is also straightforward to show that $\frac{\partial \beta(\sigma)}{\partial \sigma} > 0$. Therefore, the sign of the derivative depends on the first term $\mathcal{V}(\xi, \pi)$, which was defined as

$$\mathcal{V}(\xi, \pi) = (1 - \tau)\left(\frac{\xi}{r - \mu} - \frac{\pi}{r}\right). \tag{13}$$

Note that $\mathcal{V}(\xi, \pi)$ is neatly interpreted as the present value from a riskless after-tax perpetual net income stream given a current earnings level of ξ. For a given interest obligation π, $\mathcal{V}(\xi, \pi)$ is negative as long as ξ is sufficiently small, while for sufficiently high levels of ξ, $\mathcal{V}(\xi, \pi)$ turns out to be positive.

Economically, with stricter financial constraints (i.e. with a higher boundary ξ), equity holders loose a more valuable cash flow stream relative to the given debt liabilities if default occurs, which makes them reluctant to the risk of running into financial distress. Thus, the earlier a bank is forced into financial reorganization, the more severe is the negative effect for the equity value. As a result, stricter financial constraints are an incentive for the bank to act more prudent and to avoid excessive risks taking. We highlight the finding as

Lemma 2 *In the case of straight bond financing, equity holders' risk preferences depend on the exogenous constraints ξ. If constraints are sufficiently weak in the sense that the threshold ξ is low, equity holders have incentives to increase risk, i.e. $\partial S_t / \partial \sigma > 0$. If constraints are strong in the sense that ξ is sufficiently high, equity holders have incentives to avoid risks, i.e. $\partial S_t / \partial \sigma < 0$.*

From (13) we can immediately derive the critical threshold where risk preferences switch, i.e. where the derivative is zero, $\frac{\partial S_t}{\partial \sigma} = 0$, for any $x_t > \xi$. This critical threshold $\hat{\xi}$ directly follows from (2) and satisfies

$$\hat{\xi}(\pi) = \pi \, \frac{r - \mu}{r}. \tag{14}$$

For any ξ above $\hat{\xi}$, owners dislike higher risks, while for any ξ below $\hat{\xi}$, equity holders have incentives to increase risk. At $\hat{\xi}$, equity holders are indifferent with respect to the risk strategy. In other words, the equity claim is convex in x_t for $\xi < \hat{\xi}$, while being concave for $\hat{\xi} < \xi$ and (piecewise) linear for $\xi = \hat{\xi}$.
Note that if the bank does not face any exogenous constraints, the optimal endogenous threshold can be shown to be $\xi^*(\pi) = \pi \, \frac{r-\mu}{r} \, \frac{\beta}{\beta-1}$.[23] Since the last fraction is always smaller than one, the endogenous threshold is always below $\hat{\xi}$ which shows that absent any financial constraints, equity holders always have the usual call option-like incentive to increase risk.
The threshold $\hat{\xi}$ displays two intuitively reasonable characteristics. First, the threshold depends on the growth potential of the bank. For higher μ, $\hat{\xi}$ is lower, thus, a bank with good growth prospects is able to raise capital for cash flow levels where a comparable bank with a lower growth rate already faces financial difficulties. Second, it increases with the interest payment π, which is intuitive since with higher debt levels the bank is supposed to run into financial difficulties earlier.

To gauge the impact of a CoCo bond on the risk taking behavior of banks, we analyze how risk preferences change when the bank replaces the optimal straight bond by an optimal CoCo bond. Optimal bond volumes refer to those levels, b^* and c^*, respectively, that maximize the bank value, which need not necessarily coincide with the first-best solution that could be obtained in a complete contract case. From the previous section, we know that given the constraint $\hat{\xi}$, the bank owners have no incentives to increase risk, i.e. to shirk when having issued a straight bond. In what follows, we consider the critical level $\xi(\pi) = \hat{\xi}(\pi)$ at which the bank for straight bond financing is indifferent about its risk preferences as the financial constraint $\xi(\pi)$ for the bank in order to have a meaningful comparison for CoCo bond financing.

[23] See e.g. Dixit and Pindyck (1994), Mella-Barral (1999), and Decamps et al. (2004).

Again, we need to calculate the first derivative of the equity value S_t^c with respect to σ. The basic idea of CoCo bonds is to provide banks with a precommitted recapitalization at a time where they face difficulties in raising external financing. In line with this reasoning, we assume the conversion to occur at the time when the external constraint would be binding if there were no convertibility option, i.e. at the threshold $\chi = \hat{\xi}(d + c)$. The conversion avoids immediate financial distress because it lowers debt obligations. However, if operating profits continue to decline, the bank eventually may face financial difficulties when cash flows deteriorate further to the lower threshold $\xi_c = \hat{\xi}(d)$.
From the definition of χ and ξ_c, the terms $\mathcal{V}(\chi, d+c)$ and $\mathcal{V}(\xi_c, d)$ in (9) are zero, and by differentiating S_t^c with respect to σ, we are left with

$$\begin{aligned} \frac{\partial S_t^c}{\partial \sigma} &= (1-\gamma)\mathcal{V}(\chi, d) \frac{\partial}{\partial \sigma}\mathcal{D}(x_t, \chi) \\ &= (1-\gamma)\mathcal{V}(\chi, d)\mathcal{D}(x_t, \chi) \cdot \log\left(\frac{x_t}{\chi}\right) \cdot \frac{\partial \beta(\sigma)}{\partial \sigma}. \end{aligned} \tag{15}$$

Again, the sign of the derivative depends on $\mathcal{V}(\chi, d)$, since $\log\left(\frac{x_t}{\chi}\right) > 0$ and $\frac{\partial \beta(\sigma)}{\partial \sigma} > 0$. By the definition of $\mathcal{V}$ and χ, this equals $\mathcal{V}(\chi, d) = (1-\tau)\left(\frac{\chi}{r-\mu} - \frac{d}{r}\right) = (1-\tau)\left(\frac{c}{r}\right) > 0$ which is always positive for $\gamma < 1$. Therefore, we derive the important result that, as long as initial equity holders retain a positive fraction of cash flow rights ($\gamma < 1$), $\frac{\partial S_t^c}{\partial \sigma}$ is always positive, which means that equity holders always have an incentive to engage in risk-shifting activities. We highlight this result as

Proposition 2 *(Risk taking in incomplete contract setting) Suppose a bank has issued straight bonds and faces financial constraints ($\chi = \hat{\xi}(d + c)$ and $\xi_c = \hat{\xi}(d)$), which leaves manager-owners indifferent with respect to the risk strategy, i.e. $\frac{\partial S_t^b}{\partial \sigma} = 0$. If that bank replaces the straight bond by an optimally designed CoCo bond where initial equity holders retain a positive fraction of cash-flow rights, i.e. $\gamma < 1$, manager-owners always have an incentive to increase risk, i.e. $\frac{\partial S_t^c}{\partial \sigma} > 0$.*

The proposition shows that for the case where exogenous financial constraints are such that $\hat{\xi}(\pi) = \bar{\phi} \cdot \pi$ with $\bar{\phi} = (r - \mu)/r$, a CoCo bond for straight bond swap leads to risk-shifting incentives. Although we consider the case, where the bank initially has no incentives to change its business risk as the most relevant situation,[24] this particular choice does not limit the generality of the economic insight in our main result in proposition 2. The next subsection shows that the results can be generalized to the case of arbitrarily strict (or weak) financial constraints.

Consider the general exogenous threshold $\xi = \phi \cdot \pi$ for an arbitrary $\phi > 0$. We first show that if the CoCo bond for straight bond swap is structured in such a way that the coupon payment is kept constant, i.e $b = c$, then a risk-shift increases the bank's equity value more strongly under CoCo bond financing than under straight bond financing as indicated by the following lemma:

Lemma 3 *For any arbitrary thresholds $\chi = \xi_b$ and ξ_c with $\xi_c < \chi$ and for CoCo bonds that replace straight bonds with the same coupon, i.e. $b = c$, a CoCo bond increases risk taking incentives of manager-owners, in the sense that*

$$\frac{\partial S_t^b}{\partial \sigma} < \frac{\partial S_t^c}{\partial \sigma}.$$

The proof of lemma 3 is in the appendix. From lemma 3 together with lemma 1, we can derive the following more general proposition about risk taking incentives for an arbitrary constraint and optimal debt level choices.

Proposition 3 *(Generalization of risk taking incentives) If financial constraints are weak, then the bank will always prefer the higher risk with both optimal straight bonds as well as optimal CoCo bonds. For strong financial constraints, the firm will always prefer the low risk for optimal straight bond financing, while it might still prefer the higher risk under optimal CoCo bond financing.*

In technical terms, for arbitrary constraints $\xi = \phi \cdot \pi$ and optimal choice of the debt level, if $\xi < \hat{\xi}$, then $0 < \frac{\partial S_t^b}{\partial \sigma}\big|_{b=b^*} < \frac{\partial S_t^c}{\partial \sigma}\big|_{c=c^*}$. If $\xi > \hat{\xi}$, then $\frac{\partial S_t^b}{\partial \sigma}\big|_{b=b^*} < 0$ and $\frac{\partial S_t^c}{\partial \sigma}\big|_{c=c^*} \gtreqless 0$

The proof is in the appendix A.4. Proposition 3 generalizes the findings from proposition 2 and lemma 3 to the case of arbitrary financial constraints and optimal bond choices. Importantly, the result shows that it will never be the case

[24] The argument is that the initial bank, that has discretion over its risk policy, has already adapted its desired risk level and is in 'equilibrium' with respect to the choice over σ.

that the bank has preferences for the high risk under straight bond financing, but preferences for the low risk under CoCo bond financing.

The intuitive reason for why straight bonds impose lower risk-shifting incentives on the equity holders than CoCo bonds is due to the wealth effect for the equity holders in particularly poor states. In the case of CoCo bonds outstanding, there are poor states in which conversion of the CoCo bonds takes place and the (old) equity holders still keep their valuable equity claims. In the case of straight bond financing, however, the same state leads to default, in which case the equity holders are left with nothing according to absolute priority rules. As a result, banks with CoCo bonds outstanding have a lower incentive to prevent poor states so they are willing to accept a higher cash flow risk than the equity holders for straight bond financing would be willing to accept.[25]

This more general finding shows that the distortion of risk shifting incentives which are induced by a CoCo bond for straight bond swap is robust with respect to the assumption on the exogenous financial constraints. This leads to an important implication for the case of a bank that benefits from an implicit bail-out option by the government. If a bank is considered to be too big to fail, and the bank manager-owners anticipate the bail-out commitment, they will already face incentives to engage in risk shifting activities. Our result in proposition 3 show that in such a case, the risk taking incentives for the bank with a CoCo bond can even be increased.

Taken together, proposition 2 and 3 confirm the intuition that in incomplete contract settings, debt serves as a disciplining device, where the disciplining effect stems from the threat of losing complete control rights in a bankruptcy process. If, as in the case of CoCo bonds, equity holders only lose cash-flow rights but not complete control rights, the disciplining impact is mitigated, and manager-owners face distorted risk incentives. Hence, our analysis contradicts results from

[25] The result is a neat analogy to the case of regular convertible bonds. As first stated in Green (1984), regular convertible bonds can mitigate the classic asset substitution problem (see e.g. Jensen and Meckling, 1976), because conversion takes place at the discretion of bondholders in good states. Contingent convertible bonds are the precise polar case, where conversion takes place mandatorily in poor states, and therefore it is intuitively reasonable to expect them to aggravate the asset substitution problem.

Flannery (2005) who finds that shareholders confront undistorted risk-bearing incentives.[26]

Proposition 3 implies that we can distinguish three cases. First, the case where banks with straight bonds already have risk-shifting incentives (e.g. due to an implicit bail-out commitment) so that switching to a CoCo bond financing reinforces risk incentives. Second, the case where banks with straight bonds have an aversion against increasing risks, and for which the switch to a CoCo bond does not result in a preference for higher risks. Third, the case where banks with straight bonds have a preference for the low risk, but where the switch to CoCo bonds reverses risk preferences, i.e. where banks will engage in more risky activities once having issued CoCo bonds.

4.2 Impact of Distorted Risk Taking Incentives

In this section, we analyze the impact of risk-shifting incentives on the net-wealth of the bank, incentive compatibility conditions and the severity of financial distress. In particular, we focus on the last case identified at the end of the previous subsection, where CoCo bonds change the risk taking behavior.

Since we consider an incomplete contracts setting, the choice of the risk-strategy is observable but not contractible. Thus, in such a full-information setup, investors can rationally anticipate the risk choices of bank owners and will demand a corresponding compensation. This means that the bond will be priced by taking into account the expected choice of the risk parameter σ. Thus, a straight bond is priced by assuming σ_l, while a CoCo bond will be priced by taking into account the risk-shifting incentives, i.e. by assuming σ_h. In general, it is a well known result within this model class that a higher risk reduces the optimal firm value,[27] because within a trade-off model, the bank benefits from advantages of debt financing as long as it remains solvent but incurs losses when it suffers from a default. A higher business risk increases the likelihood of a default, which increases the present value of the losses in the case of a default without providing

[26] See Flannery (2005), p. 12.

[27] This finding has already been discussed in the basic model by Leland (1994).

any additional advantages.

In formal terms, it is straightforward to show that the firm value V_t^b depends negatively on σ, i.e.

$$\frac{\partial V_t^b}{\partial \beta} = -\pi \frac{\tau}{r} \left(\frac{x}{\xi_b}\right)^{\beta} \log\left(\frac{x}{\xi_b}\right) < 0, \tag{16}$$

where the claim follows from the repeatedly used fact that β depends positively on σ. A little more algebra shows that this finding carries over to the case of CoCo bonds. Taking the derivative yields,

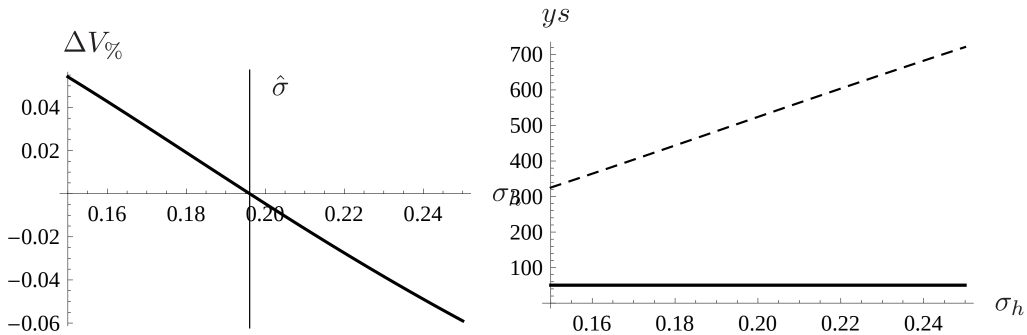
$$\begin{aligned} \frac{\partial V_t^c}{\partial \beta} &= \frac{d\tau(\mu - r) + \xi_c r(\lambda - (1-\tau))}{r(r-\mu)} \left(\frac{x}{\xi_c}\right)^{\beta} \log\left(\frac{x}{\xi_c}\right) - \\ & \frac{c\tau}{r} \left(\frac{x}{\chi}\right)^{\beta} \log\left(\frac{x}{\chi}\right) < 0. \end{aligned} \tag{17}$$

As λ is bounded above by $(1-\tau)$, and $\mu < r$, the derivative is always negative.

In section 3.1, we have established from lemma 1 that the optimal firm value with CoCo bonds is always higher than with straight bonds, i.e. $V^b(b^*) < V^c(c^*)$, if the risk-strategy is contractible. As the bank shifts towards a more risky strategy, the bank firm value for a CoCo bond financing declines. If the risk-shifting possibility is not severe, i.e. σ_h only slightly exceeds the initial risk σ_l, the bank value under CoCo financing will still be higher than under straight bond financing. Conversely, if the risk-shifting possibility is severe, i.e. σ_h is large, the optimal bank value will be lower than in the case of a straight bond issue. The left panel in figure 2 shows in line with (17) that the bank's firm value under CoCo financing, V_t^c (dashed line), declines monotonically as its risk-shifting option is more severe, i.e. as σ_h increases, while the bank value under straight bond financing, V_t^b (horizontal line), is obviously independent of σ_h. We define a critical value for σ_h for which V_t^c equals or exceeds V_t^b as $\hat{\sigma} = \sup\{\sigma_h | V_t^c(\sigma_h) \geq V_t^b(\sigma_l)\}$. As long as σ_h lies between the initial $\sigma_l = 0.15$ and the critical value $\hat{\sigma}$ (in the numerical base case $\hat{\sigma} = 0.196$), the bank's firm value is higher under CoCo bond financing. The right panel plots yield spreads (defined as $c^*/C_t - r$) for CoCo bonds and straight bonds. While the spread for a straight bond (depicted as horizontal dashed line) is only 50 basispoints, it is substantially higher for CoCo bonds. Even if bank managers do not engage in risk shifting, the yield spread

Figure 2: Bank value and yield spreads

The left panel plots the difference between the bank firm value with CoCo bonds, V_t^c and the bank firm value with straight bonds, V_t^b, as a percentage of V_t^b i.e. $\Delta V_{\%} = (V_t^c - V_t^b)/V_t^b$. The horizontal line indicates the critical level $\hat{\sigma}$ for which the difference is zero. The right panel plots yield spreads in basispoints for straight bonds (horizontal solid line) versus spreads of CoCo bonds (dashed line). Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$, $\gamma = 0.4$.



amounts to 325 basispoints. The spread rises with the risk shifting opportunity, and bond investors demand a spread of 524 basispoints if the bank has incentives to raise the risk to $\sigma_h = 0.20$. The substantially higher spreads are roughly in line with evidence from the industry where CoCo bonds are expected to trade up to an additional 300 basispoints interest margin.[28] The higher spreads are due to two factors. On the one hand, a bank with CoCo bonds has a higher leverage, and on the other hand, CoCo bondholders demand an additional premium for the fact that conversion mandatorily takes place at a time when equity values are low. Thus, the higher spreads partly reflect this insurance premium.

Two points about the results are noteworthy: First, despite the fact that CoCo bond financing is associated with a higher risk σ_h, the overall bank value can still be higher relative to the case of straight bond financing. The reason for the value increase lies in the relaxation of financial constraints (i.e. a lower default barrier) which increases the bank's debt capacity and enables the bank to exploit debt advantages to a larger extent. In our setup, the bank is able to generate a higher instantaneous tax shield. Note that the value increase due to higher tax shields is at the expense of the tax authority, which in turn receives less taxes.

Second, although CoCo bonds increase the bank firm value via a higher debt ca-

[28] See *Financial Times* "UK experiment raises prospects of new asset class", November 5, 2009.

pacity, bond investors demand a substantially higher spread which increases with the riskiness of the asset value. Spreads are high because by construction the conversion of CoCo bonds takes place at a time when conversion is not favorable from the perspective of investors, i.e. CoCo bond investors obtain a fraction of the equity claim precisely in states where equity values are low. Our findings are an important warning signal against arguments as e.g. mentioned in Flannery (2005, 2009) that the internalization of risk-taking costs will deter banks from taking too much risk. Our results show that although banks bear the costs of risk-taking via substantially higher spreads, they can still have an incentive to engage in risk-shifting activities.

The finding that a CoCo bond issue for $\sigma_h < \hat{\sigma}$ results in a higher overall bank firm value implies that equity holders obtain a higher wealth by issuing CoCo bonds. Our analysis demonstrates the important insight that although creditors anticipate a potential risk-shift so that they pay a fair price for the investment, owners are still better off. As a result, from an investors' perspective, CoCos are a Pareto improvement because no investor suffers and some obtain a strictly higher wealth.
However, it is important to stress that although from the perspective of the bank CoCo bonds are Pareto-optimal, it is a priori not clear whether the use of CoCo bonds is also desirable from a systemic point of view. To evaluate the systemic effect of CoCo bonds, we consider the probability of financial distress for the banking system in the following paragraph.

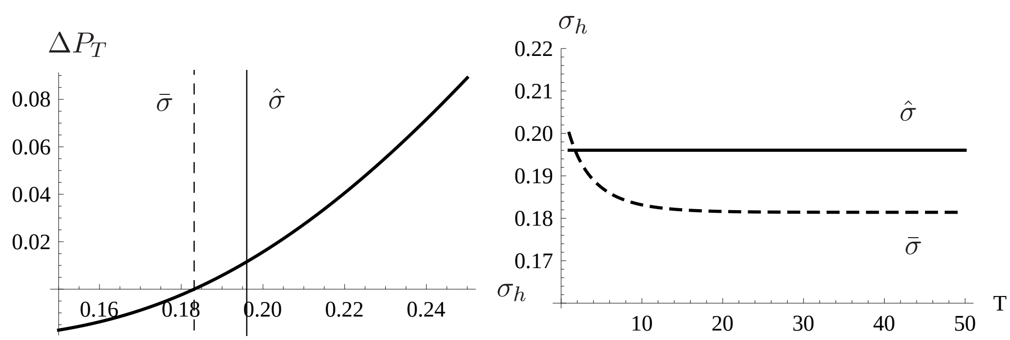
As in section 3.2, we compute the bank's probability of running into financial distress. To compare a bank with straight bonds to a bank with CoCo bonds, not only the threshold ξ differs but, contrary to the previous section also the risk parameter σ. Furthermore, from the definition of the risk-shift, we assume that the risk-neutral drift remains constant. Thus, as a result of the increasing risk, the physical drift rate increases as well. Recall from the discussion in section 4.1 that the physical drift rate μ^P is related to σ by $\mu^P = \mu - \psi\,\sigma$. Therefore, the actual increase depends on the market price of risk ψ. To account for these factors, we extend the notation and write $P_{\xi,\sigma,T}$. Further, to ease interpretation, we report the difference in distress probabilities, $\Delta P_T = P_{\xi_c,\sigma_h,T} - P_{\xi_b,\sigma_l,T}$ over a time horizon T between a bank with CoCo bonds and risk σ_h and a bank with straight bonds and risk σ_l.

Note that for $\sigma_h = \sigma_l$, we know from proposition 1 that ΔP_T must be negative. Since ΔP_T increases monotonically in σ_h to a positive value, there is a value for σ_h, for which probabilities of financial distress coincide. Therefore, we define another critical value $\bar{\sigma}$ as the smallest value of σ_h for which the difference is non-negative, i.e. $\bar{\sigma} = \inf\{\sigma_h | P_{\xi_c,\sigma_h,T} \geq P_{\xi_b,\sigma_l,T}\}$.

Numerical results for the base case scenario are reported in the left panel of figure 3.

Figure 3: Probability of financial distress.

The left panel plots the difference in distress probabilities of a bank with CoCo bonds and straight bonds, i.e. $\Delta P_T = P_{\xi_c,\sigma_h,T} - P_{\xi_b,\sigma_l,T}$, for $T = 10$ years and a market price of risk of $\psi = 0.5$. The right panel plots the critical values $\hat{\sigma}$ and $\bar{\sigma}$ for different time horizons T. Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$.



The graph confirms the general insight from section 3.2, that CoCo bonds are beneficial in terms of probability of distress *as long as* the bank has no discretion over the choice of the risk technology, i.e. for $\sigma_h = \sigma_l$. However, if contracts are incomplete, and the bank has discretion to increase the investment risk σ_h, distress probabilities increase with the severity of the risk-shift, and outweigh the initially beneficial effects. The critical value $\bar{\sigma}$ above which CoCo bonds are detrimental in terms of probability of financial distress turns out to be $\bar{\sigma} = 0.183$ (for a time horizon of $T = 10$ years), which is indicated as the vertical dashed line. For σ_h larger than $\bar{\sigma}$, a bank with CoCo bond financing will actually have a higher probability of financial distress. The graphic also shows the critical value

$\hat{\sigma}$ up to which CoCo bonds are firm value-increasing as the vertical solid line. From comparing the critical values $\bar{\sigma}$ and $\hat{\sigma}$, we obtain the important existence result, that cases are possible for which we find $\bar{\sigma}$ to be strictly smaller than $\hat{\sigma}$. In other words, there exist risk levels σ_h which the bank can choose, for which CoCo bonds are an ex ante optimal strategy for the bank in terms of value maximization, but for which the probability of financial distress is higher relative to the case of straight bond financing. The right panel in figure 3 plots the critical values $\bar{\sigma}$ and $\hat{\sigma}$ for different time horizons T and shows that the existence of such a constellation is robust over longer time horizons. Furthermore in appendix A.2, we show that we obtain the same qualitative result if we consider the expected present value of distress costs as a second measure for the severity of default.
The numerical analysis thus demonstrates the potentially adverse impact of CoCo bond financing, which we summarize as

Proposition 4 *(Wealth of investors versus banking stability) Suppose financial constraints are such that a bank with straight bonds chooses the low-risk technology σ_l (the bank monitors), while a bank with CoCo bonds has incentives to choose the high-risk strategy σ_h (the bank shirks). Suppose further that bond investors rationally anticipate distorted risk taking incentives and obtain a corresponding compensation. Then, it can be possible that a bank with CoCo bond financing obtains a higher bank value, while the probability of financial distress is higher than for optimal straight bond financing.*

The intuition for proposition 4 can be explained in terms of a trade-off. On the one hand, CoCo bonds are beneficial in the sense that they relax the financial constraints (i.e. lower the default barrier ξ_c), which allows the bank to exploit debt financing advantages to a larger extent. Therefore, the bank value increases relative to the case of straight bond financing. On the other hand, from propositions 2 and 3, we know that CoCo bond financing induces the incentive to increase the cash flow risk. Although the default barrier is lower in the case of CoCo bonds, the probability, that the more volatile cash flow process hits the default barrier, can actually increase. Intuitively, the higher cash flow volatility overcompensates the relaxation of financial constraints.

Proposition 4 highlights a potentially dangerous adverse implication of CoCo bond financing and contrasts with the main results in e.g. Flannery (2005, 2009) and Landier and Ueda (2009). The reasoning in these papers relies on a static analysis and basically resembles our results for complete contracts. Flannery (2005, 2009) and Landier and Ueda (2009) stress the potential benefits of CoCo bonds in terms of the relaxation of financial constraints, i.e. the reduction in the probability of financial distress. Although in particular Flannery (2005, 2009) also recognizes the potential risk-taking incentives, he concludes that this is not a drawback of CoCo bonds, since CoCo bond holders will anticipate distorted risk preferences and demand a higher premium. Since the bank has to internalize risk-taking costs, this will control the risk-shift. In contrast, we show that this reasoning does not need to be true in general. Our results suggest that although the bank internalizes risk-taking costs, it can still be optimal for manager-owners to increase risk. More importantly, the risk-shift can be such that it overcompensates the reduction in the default threshold, which actually increases the probability of financial distress relative to straight bond financing. Thus, contrary to the previous work on CoCo bonds, our results demonstrate that CoCo bonds can create negative externalities for the economy, and that individually rational decisions may have systemically undesirable outcomes.

Although the finding in proposition 4 is to be understood as an existence result, which issues an important warning signal against the euphoric use of CoCo bonds, we argue that it is also found to hold for a substantial range of parameter values by providing robustness results in Appendix A.5. Thereby, we focus on two crucial parameters which might have a significant impact on results: The market price of risk and the fraction of total liabilities replaced by CoCo bonds.

4.3 Incentive-compatible conversion trigger

In this section, we discuss a potential solution to regulate the moral hazard problem inherent in CoCo bonds. From the results established in lemma 2, we know that the severity of exogenous constraints controls the risk-taking incentives of manager-owners. Stricter constraints (in the sense of higher thresholds) induce manager-owners to act more prudently, since they face the risk of loosing a more valuable claim. In this subsection, we aim at designing the CoCo bond that does not provide risk-shifting incentives. The idea is that if conversion takes place at a higher threshold (i.e. at higher capital ratios), then the bank will loose valuable tax shields earlier which makes it reluctant to increase the risk of loosing these debt advantages. Recall from proposition 2 that if the conversion threshold is defined as $\chi = \bar{\phi} \cdot (d + c)$, the bank always faces risk-taking incentives in the sense that $\frac{\partial S_t^c}{\partial \sigma} > 0$. Hence, we now reverse the question and determine the conversion threshold χ such that risk preferences are unchanged. By doing so, the conversion ratio γ is taken as a constant. The notion behind this is that the (old) shareholders are not willing to accept a severe change of the majority of control rights when a conversion is triggered. For this reason, typical conversion ratios of classical convertible bonds are below 15 percent. Simplifying the condition

$$\frac{\partial S_t^c}{\partial \sigma} = 0$$

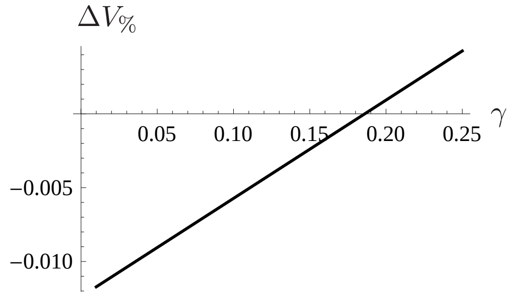
for the incentive-compatible conversion threshold χ^{IC} results in the following representation:

$$\chi^{IC} = \bar{\phi} \cdot \left(d + \frac{c}{\gamma} \right). \tag{18}$$

The incentive-compatible conversion threshold depends on the conversion ratio γ. Since $\gamma < 1$, it is immediately obvious that χ^{IC} is higher than $\chi = \bar{\phi} \cdot (d + c)$. Intuitively, the lower the conversion barrier γ is, the higher the conversion barrier χ^{IC} needs to be. This is a result of the fact that the more shares $1 - \gamma$ the (old) equity holders still keep after conversion, the less they are hurt by a conversion. Thus, a conversion needs to take place more early in order to represent a reliable threat for the equity holders against a risk increase. As a consequence of the higher conversion threshold, the CoCo bond coupon obligation c disappears earlier which ceteris paribus results in lower tax shields and therefore in a lower overall bank value. Hence, this raises the question whether CoCo bond financing is still worthwhile relative to straight bond financing from the perspective of the optimal bank value. Figure 4 reports the the difference between the firm value of a bank with incentive-compatible CoCo bonds to straight bonds as a fraction of the straight bond case, i.e. $\Delta V_{\%} = (V^c - V^b)/V^b$ for different levels of the

Figure 4: Bank value change for incentive-compatible CoCo bonds

The figure shows the difference between the firm value of a bank with incentive-compatible CoCo bonds to straight bonds as a fraction of the straight bond case, i.e. $\Delta V_{\%} = (V^c - V^b)/V^b$. Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $\psi = 0.5$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$.



conversion ratio γ. The graph reveals that as long as the conversion ratio is below approximately 0.2, the incentive-compatible CoCo bonds decrease bank value relative to the straight bond case. Only if CoCo bond holders obtain a sufficiently large part of the equity capital (i.e. if γ exceeds approximately 0.2), the incentive-compatible CoCo bonds are still value-maximizing. Note however that from a corporate governance perspective former owners will be very reluctant to issue CoCo bonds, which upon conversion give former bondholder too much voting rights. Thus, for practical applications γ is expected to be well below 0.2. Hence, we find that even though CoCo bonds can be structured in such a way that they mitigate their associated moral hazard problem by setting the conversion threshold sufficiently high, our analysis also indicates that this design will most likely result in lower bank values relative to straight bonds, because former owners will be reluctant to transfer a significant part of voting rights.

4.4 Further Concerns and Policy Implications

The existing literature considers contingent capital to be a desirable instrument for future banking regulation. One of the few concerns that have been raised so far, mentions that the conversion of CoCos may trigger an undesirable negative price spiral when large institutional investors, who are not allowed to hold equity, sell their shares.[29] The argument resembles concerns raised in Hillion and Vermaelen (2004), who have analyzed the pricing of floating-priced convertibles. It has to be stressed that the potentially adverse implications of CoCo bonds that we have shown in this contribution relies on the assumption that investors have full information, face no portfolio restrictions and act fully rational. It is not unlikely that in a situation of asymmetric information, the observation of mandatory conversion will be interpreted as a negative signal by the market, which in turn can put additional pressure on the stock price of the bank. Since we abstract from these additional market frictions, our results can even be considered as a conservative evaluation of CoCo bond financing.

The policy recommendation by the Squam Lake Working Group (2010) recognizes the important disciplining effect of debt, and recommends that conversion should be made contingent not only on the individual firms capital ratio, but also on a systemic event. However, from our analysis we cannot confirm that the inclusion of a systematic trigger entirely solves the dilemma. To see this, consider the following reasoning. Making the conversion contingent not only on the individual financial situation of the issuing bank, but also on a systemic trigger implies that if the bank runs into financial troubles, the conversion is not sure to occur. However, unless the systemic trigger is not entirely negatively correlated, there will be a positive probability that conversion occurs. Thus, although the additional systemic trigger diminishes the expected relaxation of financial constraints, the qualitative effects described in the paper are still present (even though they might be less pronounced).

Since our analysis reveals the shirking incentive implicitly contained in CoCo financing as a dangerous drawback, we strongly recommend the use of CoCo bonds together with additional devices that mitigate the risk-shifting incentive of the bank. As we have shown in section 4.3, a higher conversion threshold is one possible mechanism to restrict risk-shifting, although it may turn out to decrease overall bank value.

A second alternative refers to the simultaneous issuance of bonds with other well-known bond characteristics such as the (normal) conversion feature or rating-

[29] See *Financial Times* "Stability concerns over CoCo bonds", November 5, 2009, and *Financial Times* "Report warns on CoCo bonds", November 10, 2009.

trigger step-up feature, that usually disperse risk incentives and prevent a risk shift. However, if the inclusion of additional securities that have the potential to reverse the risk-taking incentives makes the use of CoCo bonds still incentive-compatible from the perspective of the issuing bank is an open issue and worth further research.

5 Conclusion

Contingent convertibles have received much attention in recent times as a financing arrangement that provides an automatic recapitalization mechanism for banks in times when raising new equity may be difficult. The existing literature credits CoCos for reducing the probability that the bank runs into financial distress, thereby making the banking systems more robust. Regulators seem to put much hope in these securities, as e.g. the Basel Committee on Banking Supervision has announced that it will review the role of contingent convertibles within a reform of the regulatory framework. However, to the best of our knowledge, there is not yet a comprehensive theoretical analysis of the impact of CoCo financing. Our contribution tries to fill this gap and analyzes CoCo bonds within a dynamic continuous-time framework. Our results indicate that the beneficial impact of CoCo bonds crucially hinges on the assumption if bank managers have substantial discretion over the bank's business risk. If the bank cannot change the risk technology, or in other words, if complete contracts can be written, CoCos are unambiguously beneficial. However, results change dramatically when we allow for incomplete contracts. We show that CoCo bonds always distort risk taking incentives. Therefore, equity holders have incentives to take excessive risks. We demonstrate that, although investors anticipate distorted incentives and demand a corresponding higher premium, CoCos can still be value maximizing for equity holders and therefore an optimal financing choice. More importantly, our main result shows that although CoCos can be Pareto-optimal from the perspective of the individual firm, they have the potential to substantially increase the bank's probability of financial distress as well as expected proportional distress costs. Thus, CoCos may be an example where individually rational decisions can have systemically undesirable outcomes.

Hence, the major challenge will be to find a mechanism that prevents the risk-shifting incentive imposed by CoCo bonds. In this case CoCos will be bank value and stability increasing instruments. Otherwise, they might bear the risk for seeding the next banking crisis.

A Appendix

A.1 Proof of Lemma 1

To establish the claim in lemma 1, it is enough to compare equations (6) and (10) for a given common coupon k:

$$\begin{aligned} \frac{\partial V_t^b}{\partial b}\Big|_{b=k} &= \frac{\tau}{r} - \frac{\zeta}{r}\mathcal{D}(x_t, \xi_b), \\ \frac{\partial V_t^c}{\partial c}\Big|_{c=k} &= \frac{\tau}{r} - \frac{\zeta^c}{r}\mathcal{D}(x_t, \chi) \\ \text{with:} \quad \zeta &= \frac{(1-\beta)((r-\mu)\tau + \phi r((1-\tau)-\lambda)}{r-\mu}, \quad \zeta^c = \frac{\tau(\pi - c\beta)}{\pi} \end{aligned}$$

Note that $\xi_b = \phi \cdot (d+k)$ and $\chi = \phi \cdot (d+k)$. Therefore, subtracting $\frac{\partial V_t^b}{\partial b}$ from $\frac{\partial V_t^c}{\partial c}$ yields

$$\frac{\partial V_t^c}{\partial c}\Big|_{c=k} - \frac{\partial V_t^b}{\partial b}\Big|_{b=k} = \frac{\pi\phi r(\beta-1)(\lambda-(1-\tau)) - d(r-\mu)\tau\beta}{\pi r(r-\mu)}\mathcal{D}(x_t, \xi_b),$$

where the numerator determines the sign. Since λ is bounded between 0 and $(1-\tau)$, and $\beta < 0$, it is verified that the numerator is positive and therefore $\frac{\partial V_t^c}{\partial c} - \frac{\partial V_t^b}{\partial b} > 0$ which confirms the claim, that $\frac{\partial V_t^c}{\partial c}\Big|_{c=k} > \frac{\partial V_t^b}{\partial b}\Big|_{b=k}$.

A.2 Expected distress costs

Besides the probability for the occurrence of financial distress, a second measure for the severity financial distress, in which a systemic regulator will be interested, is the expected present value of distress costs. First note that the value $\mathcal{D}(x_t, \xi)$ of one monetary unit paid out in case of default is a monotonically increasing function in ξ.

A.2.1 Complete contracts

Within the complete contract setting, the risk policy is controllable, and thus the same for a bank with straight bonds or CoCo bonds. Therefore, it is immediately obvious that

$$\mathcal{D}(x_t, \xi_c) < \mathcal{D}(x_t, \xi_b).$$

Distress costs, in our model setup, are given by $(1-\lambda)\frac{\xi}{r-\mu}$ so that its present value amounts to $\mathcal{D}(x_t, \xi) \cdot (1-\lambda)\frac{\xi}{r-\mu}$. Since a higher ξ results in both higher distress costs $(1-\lambda)\frac{\xi}{r-\mu}$ given a distress and a higher state price $\mathcal{D}(x_t,\xi)$ for a distress, it is immediately obvious that the following inequality holds:

$$(1-\lambda)\frac{\xi_c}{(r-\mu)}\mathcal{D}(x_t,\xi_c) < (1-\lambda)\frac{\xi_b}{(r-\mu)}\mathcal{D}(x_t,\xi_b).$$

The severity of financial distress in terms of its expected present value of distress costs are strictly lower for CoCo bonds than for straight bonds.

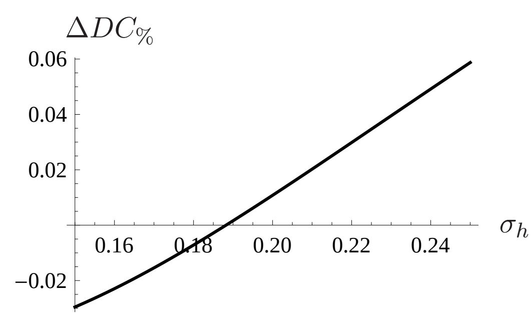
A.2.2 Incomplete contracts

If risk-shifting is possible, we need to keep track of the risk parameter σ, and we extend the notation to $\mathcal{D}(x_t,\xi,\sigma)$.

Denoting the expected present value of default costs as DC, it equals $DC^b = (1-\lambda)\frac{\xi_b}{r-\mu}\mathcal{D}(x_t,\xi_b,\sigma_l)$ for a bank with straight bonds, and $DC^c = (1-\lambda)\frac{\xi_c}{r-\mu}\mathcal{D}(x_t,\xi_c,\sigma_h)$ correspondingly for a bank with CoCo bonds. Note that in contrast to the case of computing the actual probability of distress (for which the physical drift rate is necessary) we use the risk-neutral drift μ to compute DC, since we need the pricing measure.
To gauge the impact, figure 5 reports the difference in expected distress costs as percentage of initial firm value, i.e. $\Delta DC = DC^c/V_t^c - DC^b/V_t^b$.

Figure 5: Expected distress costs.

The figure plots the difference in expected distress costs as percentage of initial bank value between straight bonds and CoCo bonds, i.e. $\Delta DC = DC^c/V_t^c - DC^b/V_t^b$. Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$. $\gamma = 0.4$.



A.3 Proof of Lemma 3

To establish the claim in lemma 3, we analyze the difference between the partial derivatives for a general exogenous threshold ξ. Let $\Delta\partial$ denote the first derivative of the difference in equity values $\Delta S = S_t^c - S_t^b$, i.e. $\Delta\partial = \frac{\partial \Delta S}{\partial \sigma}$. Recall from (4) and (9) that

$$\begin{aligned} S_t^b &= \mathcal{V}(x_t, d+b) - \mathcal{V}(\xi_b, d+b)\mathcal{D}(x_t, \xi_b) \\ S_t^c &= \mathcal{V}(x_t, d+c) - \mathcal{V}(\chi, d+c)\,\mathcal{D}(x_t, \chi) + \\ &\quad (1-\gamma)\,\mathcal{D}(x_t, \chi)\,(\mathcal{V}(\chi, d) - \mathcal{V}(\xi_c, d)\,\mathcal{D}(\chi, \xi_c)) \end{aligned}$$

Since we consider the case where the coupon of CoCo bonds is equal to the initial straight bond coupon, $b = c$, we also have $\chi = \xi_b$ (i.e. conversion takes place when the bank with straight bonds would face financial distress), and ΔS simplifies to

$$\begin{aligned} \Delta S &= (1-\gamma)\,\mathcal{D}(x_t, \chi)\,(\mathcal{V}(\chi, d) - \mathcal{V}(\xi_c, d)\,\mathcal{D}(\chi, \xi_c)) \\ &= (1-\gamma)\;(\mathcal{V}(\chi, d)\,\mathcal{D}(x_t, \chi) - \mathcal{V}(\xi_c, d)\,\mathcal{D}(x_t, \xi_c)) \end{aligned} \tag{A-1}$$

and therefore we get

$$\begin{aligned} \Delta\partial &= \frac{\partial}{\partial \sigma}(1-\gamma)\;(\mathcal{V}(\chi, d)\,\mathcal{D}(x_t, \chi) - \mathcal{V}(\xi_c, d)\,\mathcal{D}(x_t, \xi_c)) \\ &= (1-\gamma)\Big(\mathcal{V}(\chi, d)\mathcal{D}(x_t, \chi) \cdot \log\left(\frac{x_t}{\chi}\right) \cdot \frac{\partial \beta(\sigma)}{\partial \sigma} - \\ &\quad \mathcal{V}(\xi_c, d)\mathcal{D}(x_t, \xi_c) \cdot \log\left(\frac{x_t}{\xi_c}\right) \cdot \frac{\partial \beta(\sigma)}{\partial \sigma}\Big). \end{aligned}$$

Now, for arbitrary $\chi > \xi_c$, it can be verified that $\mathcal{V}(\chi, d) > \mathcal{V}(\xi_c, d)$, $\log\left(\frac{x_t}{\chi}\right) > \log\left(\frac{x_t}{\xi_c}\right)$, and $\mathcal{D}(x_t, \chi) > \mathcal{D}(x_t, \xi_c)$, which implies that $\Delta\partial > 0$. This establishes the claim that

$$\frac{\partial S_t^b}{\partial \sigma}\Big|_{b=b} < \frac{\partial S_t^c}{\partial \sigma}\Big|_{c=b}.$$

A.4 Proof of Proposition 3

Recall from (12), the derivative w.r.t. σ of the equity value with straight bonds, is given as

$$\frac{\partial S_t}{\partial \sigma} = -\mathcal{V}(\xi, \pi)\,\frac{\partial}{\partial \sigma}\,\mathcal{D}(x_t, \xi, \sigma),$$

where $\frac{\partial}{\partial \sigma}\,\mathcal{D}(x_t, \xi, \sigma) = \mathcal{D}(x_t, \xi, \sigma) \cdot \log\left(\frac{x_t}{\xi}\right) \cdot \frac{\partial \beta(\sigma)}{\partial \sigma} > 0$. Thus, the sign of $\frac{\partial S_t}{\partial \sigma}$ is determined by $\mathcal{V}(\xi, \pi)$. For the general constraint $\xi(\pi) = \phi\,\pi$, $\mathcal{V}(\xi, \pi)$ is

$$\mathcal{V}(\xi, \pi) = (1-\tau) \cdot \frac{\phi\, r - (r-\mu)}{r(r-\mu)} \cdot \pi. \tag{A-2}$$

For $\phi > (r-\mu)/r$, $\mathcal{V}(\xi, \pi)$ is positive and increasing in π, while for $\phi < (r-\mu)/r$, $\mathcal{V}(\xi, \pi)$ is negative and decreasing in π.

Furthermore, from lemma 3, we already know that

$$\frac{\partial}{\partial \sigma}(1-\gamma)\;(\mathcal{V}(\chi, d)\,\mathcal{D}(x_t, \chi) - \mathcal{V}(\xi_c, d)\,\mathcal{D}(x_t, \xi_c)) > 0 \tag{A-3}$$

Now, as in Proposition 3 distinguish the case of weak and strong financial constraints, i.e. the cases $\xi < \hat{\xi}$ and $\xi > \hat{\xi}$ respectively.
Consider first weak constraints, i.e. $\xi < \hat{\xi}$, which is equivalent to $\phi < (r-\mu)/r$. From (A-2), we know that $\mathcal{V}(\xi, \pi)$ is negative and decreases in π. Therefore, we find that the derivative of equity value with straight bonds $\frac{\partial S_t^b}{\partial \sigma}$ is positive. From the implication of lemma 1, we know that the optimal coupon of CoCo bonds is larger, i.e. $c^* > b^*$, which implies that $\mathcal{V}(\xi, (d+c^*)) < \mathcal{V}(\xi, (d+b^*)) < 0$. From (A-3), the derivative of the additional term in the equity value function of CoCo bonds is always positive, from which we immediately derive the result, that

$$0 < \frac{\partial S_t^b}{\partial \sigma}\Big|_{b=b^*} < \frac{\partial S_t^c}{\partial \sigma}\Big|_{c=c^*}.$$

Now, consider the case of strong constraints, i.e. $\xi > \hat{\xi}$, which is equivalent to $\phi > (r-\mu)/r$. From (A-2), we know that $\mathcal{V}(\xi, \pi) > 0$ and increases in π. Therefore, we find that the derivative of equity value with straight bonds $\frac{\partial S_t^b}{\partial \sigma}$ is negative. Again, from the implication of lemma 1, $c^* > b^*$, we obtain $\mathcal{V}(\xi, (d+c^*)) > \mathcal{V}(\xi, (d+b^*)) > 0$. However, since from (A-3), the derivative of the additional term in the equity value function of CoCo bonds is always positive, it is not determined which sign $\frac{\partial S_t^c}{\partial \sigma}$ has, while it is unambiguous that $\frac{\partial S_t^b}{\partial \sigma}$ is negative. This shows that

$$\frac{\partial S_t^b}{\partial \sigma}\Big|_{b=b^*} < 0 \quad \text{and} \quad \frac{\partial S_t^c}{\partial \sigma}\Big|_{c=c^*} \gtrless 0$$

as asserted in the proposition.

A.5 Robustness Results

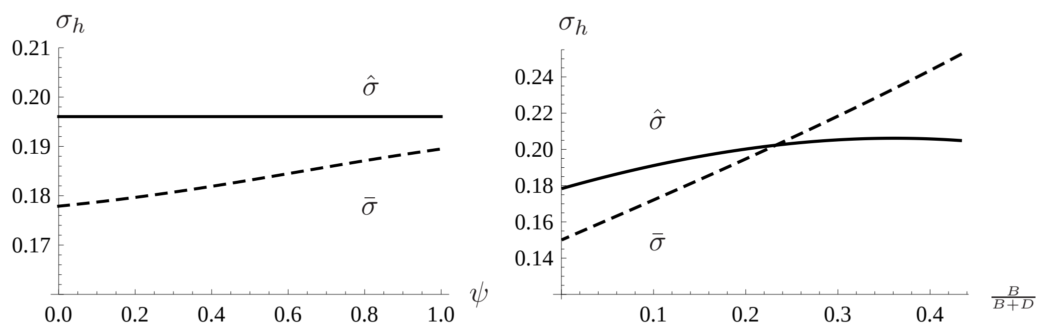
In this appendix, we argue that although the finding in proposition 4 is to be understood as an existence result, it can be shown to hold for a substantial range of parameter values. We focus on two crucial parameters which might have a significant impact on results: The market price of risk and the fraction of total liabilities replaced by CoCo bonds. First, as the market price of risk increases, the physical drift rate must increase as well. Due to a higher μ^P, the physical probability for a default declines. This is, however, true for both CoCo and straight bond financing so that it is not directly clear how this affects the relationship between the default risk under the two types of bond financing. Second, as a higher fraction of total liabilities is replaced by CoCo bonds, the threshold for financial distress decreases. Both effects will impact on results.
The left panel in figure 6 plots graphs for the critical values $\hat{\sigma}$ and $\bar{\sigma}$ when the market

price of risk ψ varies from -0.5 to 2. This range is very wide and obviously covers all reasonable values for the market price of risk.[30] The graph shows that results are largely unaffected by the choice of the risk premium. Since the risk premium results in a higher drift under both forms of bond financing, it is not surprising that the net effect on the critical risk level $\bar{\sigma}$ is marginal. The interval $[\bar{\sigma}, \hat{\sigma}]$ is larger for small risk premia. Thus, our base case scenario seems to be a conservative choice. Hence, we can conclude that our results are not significantly driven by assumptions about the market price of risk.

The right panel in figure 6 plots the critical the critical values $\hat{\sigma}$ and $\bar{\sigma}$ as a function

Figure 6: Robustness results concerning market price of risk and debt structure

The figure shows the critical values $\hat{\sigma}$ and $\bar{\sigma}$ as a function of the market price of risk ψ (left panel) and the fraction of bonds to total liabilities $\frac{B}{B+D}$ (right panel). Parameter values are: $x_t = 1$, $d = 1.5$, $\lambda = 0.55$, $\psi = 0.5$, $r = 0.08$, $\mu = 0.05$, $\tau = 0.35$, $\sigma_l = 0.15$, $\gamma = 0.2$.



of the fraction of optimally-chosen bonds to the total liabilities $\frac{B}{B+D}$. To compute the liability ratio we consider exogenous deposit obligations with d between zero and the maximum level d_{max}, which exhausts the debt capacity of the bank and compute the corresponding optimal straight bond coupon $b^* \geq 0$.[31] Then, these interest rate obligations for deposits and the straight bond are translated into the values of D and B as well as the resulting liability ratio $\frac{B}{B+D}$.

30 Dimson et al. (2006) report that over the period 1900-2005, the ten-year average equity premium never exceeded 20% (and was even slightly negative for some decades). Since the average volatility of equity markets is around 20%, an upper bound for the market price of risk is roughly 1. Dimson et al. (2006) further report that the average equity premium is around 5-8% in the entire period, thus a value for the market price of risk around 0.25 - 0.4 seems reasonable.

31 Technically speaking d_{max} is determined such that the debt capacity of the bank is exhausted, i.e. that $b^* = 0$ in (7).

The fraction $\frac{B}{B+D}$ can be interpreted as the extent to which a bank has exhausted its debt capacity by deposits. For a value of $\frac{B}{B+D}$ close to zero (one), the bank has a small (high) amount of bonds outstanding that can be replaced by a CoCo bond. As mentioned in the introduction, the largest issue of CoCo bonds has been initiated by Lloyds in November 2009 with a volume of £7.5 billion. As stated in its annual report from end of 2008, Lloyds had £245 billion in customer deposits and traded debt securities in issue. Thus, the CoCo bond issue represented roughly 3%.

Figure 6 shows that the critical result $\bar{\sigma} < \hat{\sigma}$ is obtained for liability ratios up to approximately 22%. Thus, our finding in proposition 4 holds for a substantial range of the parameters. In particular, as evidenced by the example of Lloyds, where the CoCo bond issue represents 3% of the sum of deposits and traded debt securities, the largest issue sofar is very well within the range for which our results hold.

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