PAY NOW OR LATER:
FINANCIAL FLEXIBILITY AND SECURITY DESIGN

by

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Abstract

Recent years have witnessed a proliferation of new types of debt securities. Amongst these, PIK-Toggle bonds give a borrower the choice between rolling over coupon payments by issuing additional bonds or paying the coupon in cash. These bonds are characterized as "covenant light". In a setting where there is asymmetric information about the borrowers' cash flows and a possibility of diversion, we show that a PIK-Toggle bond is an optimal contract for financially constrained firms that restructure their operations. A PIK-Toggle contract serves as a screening device so that firms with high cash flows choose regular coupon debt while viable firms with temporary low cash flows choose PIK-Toggle debt to raise funds to restructure. Using data on all PIK-Toggle issues we empirically validate the implications of our model.

JEL classification: G13, G33

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Abstract

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1. Introduction

Investors in corporate debt require the borrowing firm to perform its fiduciary duty towards its debt holders. However, the management of a firm cannot always credibly commit to defend the interests of the debt holders. Therefore, debt contracts often include covenants that restrict certain practices or envisage the possibility of future renegotiation. The covenants underscore that a firm may encounter difficulties in the future. To mitigate negative contingencies, debt contracts can be designed to address conflicts of interest that may arise in such instances.

In recent years there was a proliferation of new types of debt securities. Some include covenants that are not well understood and often regarded as ineffectual or a result of imprudence. Questions arise naturally when there is a significant increase in the use of leverage coupled with rapid innovation, such as what occurred between the years 2003 and 2007. One debt contract that apparently fits the description of imprudence is the PIK-Toggle bond, because of its payout characteristics and use in financing Leveraged Buyouts.[2] A "PIK-Toggle clause" in a corporate bond indenture allows a firm to roll over a coupon payment until the maturity of the debt. Thus, borrowers have a choice between paying a regular coupon in cash or to capitalize the interest by increasing the principal amount of the outstanding notes by issuing a payment-in-kind note (PIK interest). Appendix A provides an example of such a clause. This clause raises the question: why would lenders give a borrower the choice to skip a cash payment on the coupon date? If such a clause is included would not owner/manager have an incentive to continue to operate the firm if continuation offers an opportunity to

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[2] For example see "Payment in kind giving a risky new look to financing", Financial Times, May 25, 2006 and "The PIK of the bunch in financing", Financial Times, June 28th 2006.

divert resources from the firm? Would not lenders prefer to intervene if a firm fails to make a cash coupon payment? This paper addresses two questions:

- Can a PIK-Toggle bond be an optimal debt contract?
- Table I reveals that most of the firms issuing a PIK-Toggle bond are corporations with non-investment grade ratings. What explains the empirical observation that there is no instance of a PIK-Toggle clause in a corporate bond indenture for a company with a good credit rating? In other words, why is a PIK-Toggle issued by lower rated firms?

Firms that restructure operations face two problems. First, cash flows are constrained during the restructuring period. Second, there is concern about the ability and incentives of the management to implement the desired restructuring. PIK-Toggles allow a firm to postpone coupon payments when liquidity is scarce and outside sources of finance are expensive. The automatic stay on coupons implicit in a PIK-Toggle contract mimics an ex-ante rescheduling feature, and provides an opportunity to conserve cash until the firms' prospects improve. In the presence of financial distress costs, this automatic stay benefits both owner/manager and existing bond holders. But a PIK-Toggle contract is also effective in restricting the diversion of cash flows for private benefits by those in control of the assets, as it penalizes missed coupons via a higher rate on skipped payments. Thus, the design of a PIK-Toggle ensures that when cash is available owner/manager pay contracted coupons rather than divert. At the same time any cash on hand is conserved in states when cash is insufficient to service the debt. Such financial flexibility increases debt capacity and firm value. In the case of firms that undergo a period of restructuring to improve their profitability, or firms that are likely to encounter temporary difficulties, this financial flexibility is particularly useful.

We contrast PIK-Toggles with other known debt contracts, such as fixed coupon bonds, zero coupon bonds and income bonds (or revenue bonds). The drawback of regular coupon debt financing is that it requires the firm to pay coupons at a time of a liquidity shortage. As a result, the firm is more likely to go bankrupt before the maturity of the debt. Long term zero coupon bonds avoid intermediate bankruptcy, but are subject to increased diversion and consequently limit the firm's debt capacity. Income bonds work well in a perfect information world, but with asymmetric information and agency problems between owner/manager and debt holders, owner/manager can divert cash and pay based only on the residual reported income. Unless the cash flows generated by the firm are observed by all parties, coupons indexed to cash flow proxies are not optimal in general. This exercise of contrasting different types of debt contracts provides a practical answer on why PIK-Toggles exist.

Given the advantages of PIK-Toggles in ensuring greater financial flexibility and in limiting cash diversion by insiders, why do a majority of firms issue coupon debt but not include a "PIK-Toggle" clause? When a firm decides to issue a PIK-Toggle it gives the market a signal of its quality. A borrower, if it chooses to issue a PIK-Toggle, signals to the market that it might face periods of liquidity constraints in the future (PIK-Toggle issuance reveals "low'' type). Some firms decide to reveal their "low" type because they value the flexibility that PIKs provide (of not paying the coupon during periods of liquidity shortage, thus avoiding distress and saving the cost of rescheduling the debt). For such firms, this flexibility is more valuable than the negative effect on the terms of the debt from disclosing the firm's type. Firms with better prospects (or firms that are more confident about their cash generation ability) do not benefit from the PIK-Toggle clause to the same extent, and prefer to separate themselves from firms with poorer prospects. Similarly, firms for which a debt restructuring process is not very costly may prefer less expensive coupon debt than the embedded cost associated with PIK-Toggles. We construct such a separating equilibrium to demonstrate the manner in which PIK-Toggles can be used as a screening device.

Since PIK-Toggles save the firm cash when the firm is liquidity constrained, it is reasonable to ask why firms do not arrange to have credit lines with banks that can be drawn down when needed? The answer lies in the fact that not all firms are able to obtain credit lines. According to Sufi (2009) and Loukoianova, Neftci and Sharma (2006), banks extend credit lines to firms that meet a certain credit rating level. Granting a credit line is a credible commitment that signals the quality of a firm, and generates a surplus that is partly extracted by the bank in the form of an upfront fee paid on the undrawn portion of the credit line. This fee compensates the bank for having to extend credit to the firm if it faces financial constraints. The likelihood of such a firm facing financial constraints has to be small for the bank to break even on average. Thus, banks use credit lines to compete for good firms and not for firms that are dependent on loans or have low credit. This is consistent with findings in Mosebach (1999) that credit lines are made contingent on firm quality. In short, credit lines are not an option for low rated companies to overcome potential liquidity shortfalls. Interestingly, PIK-Toggles have the features of credit lines that are embedded in and are not separable from the debt.

We test our model's predictions using a Standard and Poors database on PIK-Toggle issues. The database also includes details on all newly high yield issues. Consistent with our theoretical implications, we find that firms that the choice of PIK-Toggle debt relative to regular debt is positively related to a firm's expected financially constraints and negatively related to expected cash flows.

Our paper is related to several strands of the finance literature. Starting with the work of Smith and Warner (1979), the role of bond covenants and their impact on firm value has been extensively examined. Bond covenants help mitigate agency conflicts, reduce the costs related to underinvestment (e.g. Myers (1977)) and lessen the impact of asset substitution. PIKs add a new dimension to the conventional role of bond covenants. PIK Toggles provide financial flexibility when a firm faces liquidity constraints and at the same time reduce the effects of moral hazard.

Our work is also related to the literature on optimal financial contracting. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) derive the optimal contract for a firm operated by a constrained entrepreneur who might divert cash from creditors. These authors present a solution to the set of securities that comprise the firm's optimal capital structure in the presence of agency costs. In these papers the terms of the contract are realigned every period to create a balance between continuation and cancellation. In contrast our paper considers the type of securities issued as exogenous, and PIK-Toggles are a mechanism designed by the borrowers and we provide a rationale for their existence. In addition, we relax the assumption that the terms of contract change every period to capture the features of existing financial contracts. We find that to implement an incentive compatible and truth telling contract, the dividend policy needs to be adjusted every state and subject to the approval of the PIK holders.

We argue that debt heterogeneity in the capital structure, the combining of senior regular debt and junior PIK-Toggle debt, is an optimal outcome for low credit quality firms. Such a multi-tiered structure with different debt contracts is also documented by Rauh and Sufi (2010).

The article is organized as follows: Section 2 presents a two-period discrete time model of a firm where owner/manager have limited resources and need to make an investment to restructure the operations. The firm must choose a debt contract to raise the required funds while at the same time preserving the benefits of owner/manager, who control the firm.

In Section 3 the discrete time model is extended to a continuous time setting. The continuous time setting has the advantage that it allows us to quantify the results and to nest the findings in the literature on capital structure and credit risk that have used this approach in the past. With asymmetric information about the firm's cash flows and the possibility of diversion by the owner/manager, we show that a PIK-Toggle contract is optimal when compared to other debt contracts.

Section 4 discusses how a PIK can be used to screen between different types of firms in a setting with asymmetric information.

Section 5 contains empirical evidence on the choice between PIKs and regular debt, and Section 6 concludes the paper.

2. Motivation- a two-period model

This section outlines a two-period model to illustrate in a rather simple fashion the optimality of a PIK-Toggle bond. The canonical example is extended to a continuous time framework in Section 3. While the continuous time version has the benefit of allowing for a more realistic state space that permits us to do numerical simulations, the two-period example provides the intuition that helps access the relatively more complicated set up.

2.1 The set up

A risk-neutral owner/manager of an existing firm needs to invest an amount $Z > 0$ to restructure the operations of the firm. The owner/manager does not have additional capital to finance the restructuring. Figure 1 provides a depiction of the state space and the associated cash flows that accrue after the investment is made in a two-period setup. The restructuring results in time 1 reported cash flows of $X(s) = 0$, 1 or 2 in states $s = 0$, 1 or 2, respectively. Each state occurs with a probability 1/3. Correspondingly there are six possible states at time 2, shown in Figure 1. The reported cash flows are observable and verifiable at no cost. In addition, the firm has outstanding debt with contractual payments $K(s) = 1$ in all states. A failure to pay these contractual amounts results in default, and subsequent liquidation. For notational simplicity the risk-free rate is set to zero. Accumulated cash flows at time 1 can be used to pay claimants at time 2.

Given this setup, we ask: If creditors operate in a competitive market, what investment amount $Z > 0$ can be raised to finance the restructuring? Assume, for simplicity, that there is no recovery in case of default and that the liquidation values are zero. We want to investigate the nature of the optimal contract in a setting without agency issues when the reported cash flows are the actual cash flows. Later we include the impact of agency costs.

2.2 Optimal contract without agency costs

The cash flows at times 1 and 2 after the restructuring are given in Figure 1. The firm will be liquidated at time 1 in state $s = 0$ because the contractually specified debt payments cannot be made (excess cash flow is negative). However, in state $s = 1$ there is sufficient cash to satisfy the claim of the senior debt-holders, but there is no cash left to service any new claims (excess cash=0). Lastly, for $s = 2$ there is 1 unit of cash flow left to service new claims. Similarly, the cash flows at time 2 are 0, 1 or 2 in the six possible states.

The pledgeable income against which a new contract can be sold is based on the value of the excess cash flows $C(s) = X(s) - K(s)$ that can be apportioned to a newly issued claim to finance the restructuring. If we allow for saving, some of the excess cash flows at time 1 in state 2 can be saved and paid out at time 2. Thus the pledgeable income ($P(t)$) is:

$$\underset{C(2)}{Max}\, P(t = 0) = \frac{1}{3}C(2) + \frac{1}{9}C(1,2) + \frac{1}{9}C(2,2) + \frac{1}{9}C(2,1) \qquad (1)$$

where $C(2) \in [0,1], \quad C(1,2) \in [0,1], \quad C(2,1) \in [0,(1 - C(2)], \quad C(2,2) \in [0,1 + (1 - C(2)]$. Note that saving at time 1 in state 2 results in an excess cash flow at time 2 to that extent. The choice variable is how much to save or pay in state 2. One solution is to set the contractual payments to $C(2) = 1, \quad C(1,2) = 1 \quad C(2,2) = 1$. The resulting contract can be implemented by an income bond and the amount that can be raised is the expected value of the cash flows: $Z = \frac{5}{9}$. The financial claim pays as much as is available after any payments to existing debt-holders have been paid. If there is no cash to satisfy the new investor, default is not triggered. The contractually specified second-period payments do not depend on the first-period payment. The optimal contract gives the maximum amount that can be raised in this setting.

2.3 Optimal contract with agency problems

Suppose now that the owner entrepreneur can hide one unit of first-period cash flow. This yields a benefit to the owner/manager of $\lambda \in [0,1]$ per unit of cash flow hidden or diverted. Such a benefit accrues, for example, by a conversion of cash into perquisites for the owner/manager. Suppose this hiding can occur only at time 1. Time 2 cash flows are observable. Then, such diversion can only occur at time 1 in state *s*=2 when the cash flows are enough to satisfy existing claims.

Denote as $\hat{C}(r,s)$ the excess cash flow reported when *r* is the reported state and *s* is the true state. Thus $\hat{C}(r,s) = \hat{C}((2,2),(2,2))$ denotes the reported excess cash flow when the reported state is *r*=(2,2) and the true state is *s*=(2,2). This would constitute truthful reporting-the true state at time 1 was *s*=(2) and the current state is *s*=(2,2). On the other hand $\hat{C}((1,2),(2,2))$ denotes an instance where the reported state is *r*=(1,2) while the true state is *s*=(2,2). In this case, owner/manager divert one unit of cash flow at time 1. Thus, a reported excess cash flow of 0 at time 1 results in an inference that the state at time 1 is *s*=1 rather than *s*=2. There is no incentive to divert cash flows in state *s*=2 when the following incentive compatibility constraint is satisfied (payoff to owner/manager including diversion is lower than payoffs without diversion at time 1 and 2).

$$\underbrace{\lambda}_{diversion} + \underbrace{\frac{1}{3}(1-\hat{C}((1,2),(2,2))}_{payoff\quad after\quad diversion} \leq \underbrace{\frac{1}{3}(1-\hat{C}((2,2),(2,2))}_{payoff\quad with\quad truthful\quad reporting} \qquad (2)$$

Rearranging, the new incentive constraint gives (when it is binding):

$$\hat{C}((1,2),(2,2)) - \hat{C}((2,2),(2,2)) = 3\lambda \qquad (3)$$

Equation (3) implies that misreporting (no payment at time 1) results in a penalty equal to 3λ at time 2. Thus, the difference in the payoff to claimholders between a reported state *r*=(1,2) and reported state *r*=(2,2) should be 3λ at time 2, when the cash flow at time 2 is 2.

In this setting our objective is to maximize pledgeable income and the choice variable is the first-period payout to the new claim holder. Suppose equation (3) is satisfied and there is truthful reporting. Then, the maximization problem is (suppressing dependence on *r* because reporting is truthful):

$$\underset{\hat{C}(2)}{Max}\, P(t=0) = \left(\frac{1}{3}\hat{C}((2)) + \frac{1}{9}\hat{C}((2,2)) + \frac{1}{9}\hat{C}((2,1)) + \frac{1}{9}\hat{C}((1,2))\right) \qquad (4)$$

Using equation (3) and $\hat{C}((1,2)) = 1$ (time 2 payoffs are observable) yields:

$$\underset{\hat{C}(2)}{Max}\left(\frac{1}{3}\hat{C}((2)) + \frac{1}{9}\left(\underbrace{1-\hat{C}((2)}_{saving} + \underbrace{1-3\lambda}_{\substack{using \\ equation(3)}}\right) + \frac{1}{9}\left(\underbrace{1-\hat{C}((2)}_{saving}\right) + \frac{1}{9}(1)\right) = \frac{4}{9} + \frac{1}{9}\hat{C}((2)) - \frac{1}{3}\lambda$$

(5)

It is optimal to set $\hat{C}((2)) = 1$ and the payoffs to a newly issued financial claim are $\hat{C}((2)) = 1, \hat{C}((1,2)) = 1, \hat{C}((2,2)) = 1 - 3\lambda$. The optimal contract resembles a PIK note. There is no default after a missed first-period payment to the new bondholders, given that the existing bondholders receive their specified payment in state *s*=(1,2). After having missed a payment, the state is assessed to be *s*=1 by the lender, and the second period repayment amount increases from $\hat{C}((2,2)) = 1 - 3\lambda$ to a higher amount equal to $\hat{C}((1,2)) = 1$. The total pledgeable income and the corresponding amount that can be raised in this setting is equal to $Z = \frac{5}{9} - \frac{1}{3}\lambda$. Thus the amount that can be raised is lower than the full information case and depends on the diversion factor.

What would be the pledgeable income if the new incentive constraint were violated? In this case the first-period repayment to new bondholders is zero and the cash is converted to the owner's personal utility. What remains is second period pledgebale income of 1 in each state where the excess cash flow is 1 unit, and whose expected value equals $\frac{2}{9}$. This is smaller than the pledgebale income when the owner is induced to pay out the entire first period cash flow. This pledgebale income corresponds to that of an income bond when diversion is possible. Also, a zero coupon bond pledgebale income is lower than the PIK contract because time 1 cash flows are not pledged and are subject to diversion. Finally, if funds are raised using a coupon bond, the firm would go bankrupt in both states 0 and 1. Thus, a PIK note is preferred over a coupon bond, an income bond or a zero coupon bond. The next few sections provide a continuous time formulation of this model.

3. A continuous time formulation with moral hazard

This section provides a continuous time analogue to the two-period model discussed in the previous section. The continuous time model allows us to quantify the results and to nest our findings in the literature on capital structure that have used this approach before. Again we analyze the contracting problem facing the owner/manager equity of a firm when they need to raise a fixed amount of funds for restructuring via the sale of a financial claim. The owner/manager can conceal and divert a portion of the project's cash flows (moral hazard).

As in the previous section, we start in section 3.1 by defining the state space and the associated cash flows in a continuous time setting. Sections 3.2 and 3.3 show that in certain cases a PIK contract is optimal in the sense that it increases equity value conditional on the amount raised, and increases the potential amount that can be raised from outside creditors (increases pledgeable income and debt capacity) relative to other contracts. The continuous time setting permits numerical simulations and a richer set of implications collected in section 3.3. We retain the notation used in section 2.

3.1 Model of the firm

Consider again a firm that needs to raise a fixed amount Z. The funds are used as a one-time investment made at time 0 to restructure the business, without which the firm will need to suspend operations and declare bankruptcy (recovery is equal to proportion $0 < \alpha_B < 1$ of the assets in place). The time needed to restructure the firm is T. Conditional on continuation the firm will generate operating cash flows, between now and T, that follow a continuous time process:

$$dX(t) = (\mu_0 - K)dt + \sigma dz(t) \tag{6}$$

where μ_0 is the expected cash flow during the restructuring period, σ is the instantaneous volatility of the cash flows and dz is the increment of a standard Brownian motion. K is the continuous coupon flow on existing coupon bearing debt with maturity T and face value F_K. Any excess cash flows in the firm are accumulated in a savings account denoted $S(t)$ earning a rate r. Thus, the accumulated cash balance at time t equals:

$$S(t) = \int_0^t e^{r(t-s)} dX(s) + S(0)e^{rt} \tag{7}$$

The first term in equation (7) is the cash generated at each point from the start to time t. The second term is the initial account balance compounded at the rate of interest until time t. If the restructuring is successful, we assume that the expected cash flows in equation (6) increase to μ_T at the conclusion of the restructuring period (T) where $\mu_T > \mu_0$. The investment in restructuring the firm is efficient: $E\left[\frac{\mu_T - \mu_0}{r}\right] > e^{rT} Z$ where $E[\cdot]$ is the expectations operator regarding success or failure in completing the restructuring.

Investing Z improves the firm's operations in the long run, but might require additional debt service in the immediate future. When the cash flows generated by the firm do not cover coupon payments to new and existing claims, owner/manager can access the saved funds. However when the cash flows plus any accumulated balances are not enough to cover coupon payments, the firm faces financial distress. We assume that the owner/manager is constrained and unable to provide additional capital during the expected restructuring period from time 0 until time T. Also, *the* owner/manager does not receive any cash payments until all the bond holders have been paid off at time T. Later we relax this assumption. If the firm were to go bankrupt, the owner/manager does not receive any payoff. Thus total payoff (benefit) to the owner/manager at time T is given by:

$$B(t) = E\left[e^{-r(T-t)} \Big(\underbrace{V(T)}_{\substack{terminal \\ firm\,value}} + \underbrace{S(T)}_{\substack{accumualted \\ cash}} - \underbrace{F_K}_{\substack{face\,value \\ of\ existing \\ debt}} - \underbrace{C(T)}_{\substack{face\,value \\ of\ new\,debt}} \Big) 1_{\{\tau > T\}} \right] \tag{8}$$

$$\tau = \inf\{t : S(t) < 0\} \text{ and } S(t) = \int_0^t e^{r(t-s)} dX(s) + S(0)e^{rt}.$$

Here $V(T)$ is the unlevered value of the firm at time T, $C(T)$ is the principal repayment associated with the new debt amount Z, and $1_{\{\tau > T\}}$ is an indicator function whose value is 1 if the condition in the brackets is satisfied and zero otherwise. Equation (8) assumes that any outstanding debt is repaid at T and after this date the firm reverts to an all equity firm. The assumption of reverting to an all equity firm at T is not critical to the analysis but helps disentangle the role of the financial contract from the choice of the terminal capital structure and priority considerations. Any cash flows not paid to bond holders accrue to the owner/manager at this point.

The value of the firm at T, $V(T)$ in equation (8), can be characterized as follows: In the absence of any financial constraints, and when there is no debt, the owner/manager can withdraw any excess cash and choose to abandon operations when any incremental cash contributions to the firm do not increase the value of equity, i.e., the value of equity declines to zero. Under our assumptions the value of the equity (and the firm) is state independent and the unlevered value of the unconstrained firm is equal to the present value of cash flows generated by the firm:

$$V(T)=E\left[\int_{t=T}^{\infty}e^{-r(t-T)}dX(t)\right]=E\left[\int_{t=T}^{\infty}e^{-r(t-T)}(\mu_T dt+\sigma dz(t))\right]=\frac{\mu_T}{r}.\text{[3]} \qquad (9)$$

The interim accumulated cash balances create path dependency, and make it impossible to find analytical solutions to the values of the firm's claims at the time of restructuring (t=0). Therefore we resort to numerical techniques. We assume that if the firm survives, it is solvent at the maturity of the existing debt: $\frac{\mu_T}{r}-F_K-C(T)>0$. In the examples we use the following parameter values to illustrate the impact of the various inputs: $\mu_0=80$ and $\mu_T=110$ are the expected cash flows, $\sigma=40$ is the instantaneous volatility of the cash flows, $r=5\%$ is the risk-free rate of interest, $F_K=1000$ is the face value of existing debt and $K=40$ is the required coupon flow on the existing debt. Thus, the unlevered value at maturity of the firm is $\frac{110}{.05}=2200$.

The claim of existing debt holders conditional on the investment Z is given by:

$$D(t=0)=E\left[\int_{t=0}^{\tau}\underbrace{e^{-rt}Kdt1_{\{\tau\le T\}}}_{coupons}+\underbrace{e^{-r\tau}Min(\alpha_B V(\tau),F_K)1_{\{\tau\le T\}}}_{recovery}\right]+E\left[\underbrace{e^{-rT}F_K1_{\{\tau>T\}}}_{\substack{face\\value}}\right] \qquad (10)$$

Note that the existing debt holders promised claim is not diluted from any newly issued security. Existing debt holders are senior, and as in Hart and Moore (1995) do not participate in a claim restructuring. The issuance of new debt satisfies a

[3] See Oksendal (1991), page 22, Theorem 3.7 (iii).

participation constraint for existing debt, such that the value of existing debt with restructuring in equation (10) is strictly more than the value recovered without the additional investment (no dilution).

Our objective is to characterize the most efficient way to raise an amount *Z*. As in the discrete time case, a financial contract specifies a sequence of cash flows $\{C(t)\}$ paid to the lender that is measurable with respect to the cash flow process $X(t)$. The pledgeable income $P(t)$ against which a subordinate claim is sold (the price of the claim) equals:

$$P(t=0) = E\left[\underbrace{\int_{t=0}^{\tau} e^{-rt} dC(t) 1_{\{\tau \le T\}}}_{\text{promised coupons}} + \underbrace{e^{-r\tau} \alpha_P C(T) 1_{\{\tau \le T\}}}_{\text{recovery}}\right] + E\left[\underbrace{e^{-rT} C(T) 1_{\{\tau > T\}}}_{\substack{\text{face} \\ \text{value}}}\right] \qquad (11)$$

where $\tau = \inf\{t : S(t) < 0\}$. In equation (11) $0 < \alpha_P < 1$ is the proportion of the face value of the newly issued debt that is recovered were the firm to go bankrupt, and after existing debt claims have been repaid. Note that α_P for new claim holders depends on the amount of senior debt, face value of the new junior debt, and the proportion of assets recovered α_B when the firm goes bankrupt. Thus $\alpha_P = Max\left(0, \frac{\alpha_B V(\tau) - F_K}{C(T)}\right)$ where *V* is the value of firm assets at bankruptcy. Any distress costs and costs of renegotiation are implicit in this parameter. For most parameter values considered the extent of the recovery is nearly zero after existing debt claims are repaid. When additional financing is raised, the savings account must account for the additional coupon outflow each period: $S(t) = \int_0^t e^{r(t-s)} (dX(s) - dC(s)) + S(0)e^{rt}$.

We compare four different debt claims: an income bond, a zero coupon bond, regular coupon debt and a payment in kind bond (PIK-toggle bond). The cash flows per period to these alternate contracts are specified as:

1. Income bond: $\{C(t)=\theta(t)X(t), t\in(0,T)\}$ and $C(T)=F_{income}$. In general the parameter θ is state dependent.[4]
2. Zero coupon bond: $\{C(t)=0, t\in(0,T)\}$ and $C_{zero}(T)=F_{zero}$.
3. Regular debt: $\{C(t)=C_{rd}, t\in(0,T)\}$ and $C(T)=F_{rd}$.
4. Payment in kind bond $\{C(t)=1_{\{X(t)\}}C_{pik}, t\in(0,T)\}$ where $1_{\{X(t)\}}$ is an indicator function that is equal to 1 if a coupon is paid and 0 otherwise. Also, the face value is given by:

$$\left\{C(T)=F^{*}_{pik}=F_{pik}+\underbrace{\int_{t=0}^{T}\left(1-1_{\{X(t)\}}\right)C_{pik}e^{\beta(T-t)}dt}_{rolled\ over\ coupons}\right\} \qquad (12)$$

where the constant $\beta > r$ is the rate on rolled over interest payments (PIK interest rate). As noted earlier, in a PIK contract the owner/manager defer coupon flows to a later date if they are cash constrained. The payment received by a PIK holder at maturity is equal to the initial amount lent to the firm plus accrued coupons. We assume that both existing debt and PIKs are repaid at *T*.

Our objective is to compare the efficiency of these contracts in terms of their ability to raise the amount *Z* at the least cost to owner/manager. Hence, our problem is to maximize the private benefit of owner/manager subject to their ability to raise the desired investment, and the choice variable is the type of contract to be issued:

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[4] A commodity linked bond, with coupon payments indexed to the output price of a commodity producer is a version of the income bond, but with the advantage that the debt payment is fully observable and set independent of the parties. Thus, it is not subject to possible manipulation by the borrower. However, as long as coupons have to be paid even when output prices are very low, commodity linked bonds still have the problem of regular coupon bonds, which require payments when the firm is cash constrained, and thus force it into default. For an analysis of commodity linked debt and risk management see Morellec and Smith (2007).

$$\underset{\substack{C(t)=income,\\ zero,rd,pik}}{Max} \quad B(t) \text{ s.t. } P(0) = Z = E\left[\int_{t=0}^{\tau} e^{-rt} dC(t) 1_{\{\tau \le T\}} + e^{-r\tau}\alpha_P C(T) 1_{\{\tau \le T\}}\right] + E\left[e^{-rT} C(T) 1_{\{\tau > T\}}\right] \tag{13}$$

In solving equation (13) we also address the issue of whether a particular type of contract is feasible in terms of its ability to raise the amount required. The debt capacity of the firm is different for each type of contract. Thus, if the amount required (Z) is large, it may not be possible to raise the required amount via one particular type of debt contract, while it may be feasible under a different type of debt contract. In our second optimization problem we explore the dual for equation (13), wherein we maximize the debt capacity subject to the equity holder benefit being greater than or equal to zero. Thus, we answer the following question: What is the maximum pledgeable income (maximum amount Z that can be raised) with each type of contract?

$$\underset{\substack{C(t)=income,\\ zero,rd,pik}}{Max} \quad P(0) = E\left[\int_{t=0}^{\tau} e^{-rt} dC(t) 1_{\{\tau \le T\}} + e^{-r\tau}\alpha_P C(T) 1_{\{\tau \le T\}}\right] + E\left[e^{-rT} C(T) 1_{\{\tau > T\}}\right]$$

$$\text{s.t. } B(t) \ge 0 \tag{14}$$

While equation (13) analyzes the problem faced by the owner/manager when the amount required is such that he can choose any of the contracts, equation (14) analyzes the case where Z is the largest amount feasible. We analyze the questions posed in equations (13) and (14) for two cases – first a special case where saving is not permitted in the firm (section 3.2), followed by a more general case where there is a possibility of saving (section 3.3). In each case we analyze the manner in which the possibility of diversion (lower effort) impacts the efficiency of the contract in the sense of maximizing equity value or its feasibility in terms of its ability to raise the desired amount Z.

3.2 The case with no cash savings by the firm

We first consider a simplified setting when saving does not occur, and all excess cash flows are immediately paid out to the owner/manager. The value of the claim sold and the corresponding owner/manager' benefits equal:

$$P(t=0) = E\left[\int_{t=0}^{\tau} e^{-rt} dC(t) 1_{\{\tau \le T\}} + e^{-r\tau} \alpha_P C(T) 1_{\{\tau \le T\}}\right] + E\left[e^{-rT} C(T) 1_{\{\tau > T\}}\right] \qquad (15a)$$

$$B(t=0) = E\left[\int_{t=0}^{\tau} e^{-rt} \underbrace{\left(dX(t) - dC(t)\right)}_{\substack{\text{payout to equityholders} \\ \text{in the absence of saving}}} 1_{\{\tau \le T\}}\right] + E\left[e^{-rT}\left(\underbrace{\frac{\mu_T}{r} - F_K - C(T)}_{\substack{\text{time}\,T\text{ payout to} \\ \text{equityholders}}}\right) 1_{\{\tau > T\}}\right]$$

(15b)

With $\mu_0 = 80$, $\mu_T = 110$, $\sigma = 40$, $r = 0.05$, $F_K = 1000$, $K = 40$, $\alpha_B = 0.2$, $\beta = 0.09$ and $T = 8$, the claim price for the case of an income bond where $\theta(t) = 1$ for all t and $F_{income} = 500$ gives $P_{income} = 46$, and private benefits are $B_{income} = 2$. As we show later, the maximum amount that can be raised for restructuring the company occurs when $\theta(t) = 1$ for all t. In each of the other types of debt contracts (zeroes, regular coupon and PIK) pledgeable income is the value of the cash flows to the creditors conditional on survival, as given in equation (15a). For example, with $F_{zero} = F_{rd} = F_{pik} = 500$ and $C_{rd} = C_{pik} = 40$ the pledgeable income is 1, 10 and 20 for zero coupon bonds, regular coupon bonds and PIKs, respectively, in a full information setting. The corresponding private benefits to the owner/manager are 45, 10 and 26 for zero coupon bonds, regular debt and PIKs, respectively.

In this setting, an income bond pledgeable income is higher because the cash payoff is higher or equal to the flows to each of the other three types of debt contracts, in each state of the world when the firm is solvent. In the case of coupon bonds it is possible that the contracted coupon flows are higher than the cash flows generated by the firm, and consequently the firm becomes insolvent.

PIK bonds allow for the same payoff as the coupon bonds in each state, but increase the survival time because non-payment does not result in bankruptcy.

Finally, zero coupon bonds do not have survival issues from non-payment of coupons, but their pledgeable income is lower to the extent that interim coupon flows are not paid to claimants. With high enough volatility of cash flows, the firm runs out of cash in the interim in many instances, and investors do not receive any cash. Thus, the pledgeable income for zero coupon bonds is low. The higher private benefit to the owner/manager for zero coupon bonds reflects the fact that no interim coupons are paid to bond holders but are paid out to the owner/manager and these are precisely the cash flows that accrue to income bond holders to make their pledgeabale income higher to the same extent.

Lemma 1: *With full information and in the absence of savings,*

(a) *An income bond with maturity T and* $\{\theta(t)=1 \forall t \in (0,T]\}$ *maximizes pledgeable income, where the contract pays any positive cash flows generated until the firm stops operating or the contract matures.*

(b) *The pledgebale income with income bonds when* $\{\theta(t)=1 \forall t \in (0,T]\}$ *is higher than with regular debt, zero coupon bonds and PIKs if recovery is low enough:* $r\alpha_P F_{zero,rd,pik} < (\mu_0 - K)$.

Proof: See Appendix

Consider now the case when the owner/manager can divert cash (moral hazard) for his own private benefit. Here $dX(t)$ is the true cash flow and $d\hat{X}(t)$ is the reported cash flow where $d\hat{X}(t) \le dX(t)$. The owner/manager's private benefit from diversion is a fraction $\lambda \in [0,1]$ of the amount diverted and $1-\lambda$ is the dead-weight costs of concealing or diverting these flows. In the absence of saving, the firm is alive only if $dX(t) > 0$. Thus the immediate private benefit from diversion is given by:

$$\lambda\left(dX(t) - d\hat{X}(t)\right) \qquad (16)$$

In this setting, we ask what type of contract maximizes pledgeable income and is incentive compatible for the owner/manager (solves the optimization problem in equation (13))? The private benefit to the owner/manager is given by:

$$B(0) = E\left[\int_{t=0}^{\tau} e^{-rt}\left(\underbrace{\lambda(dX(t)-d\hat{X}(t))}_{\substack{diversion\\ benefit}}+\underbrace{(d\hat{X}(t)-dC(t))}_{\substack{permitted\\ payout\\ to\ \ equityholders}}\right)1_{\{\tau\le T\}}\right]+E\left[e^{-rT}\left(\underbrace{\frac{\mu_T}{r}-F_K-C(T)}_{\substack{terminal\\ payout}}\right)1_{\{\tau>T\}}\right]$$

(17)

Depending on the particular debt contract, the owner/manager faces tradeoffs. At each point in time, he weighs the immediate benefit of diversion (in equation (17)) against the potential cost of diversion. Note that in the case of an income bond, diversion does not lead to a higher possibility of default or a lower payoff because there is no required coupon whose non-payment is penalized (there is no cost of diversion). Hence, maximum diversion is possible, and the pledgeable income is reduced to that extent, decreasing the amount that can be raised. The pledgeable income for an income bond with $\{\theta(t)=1\forall t\in[0,T)\}$ is:

$$P_{Income}(t=0)=E\left[\int_{t=0}^{\tau} e^{-rt}d\hat{X}(t)1_{\{\tau\le T\}}+e^{-r\tau}\alpha_P(F_{income})1_{\{\tau\le T\}}\right]+E\left[e^{-rT}(F_{income})1_{\{\tau>T\}}\right]$$

(18)

which assumes that the cash flows at time T are perfectly observable. If all the interim cash flows are subject to diversion, the price will be akin to that of a zero coupon bond, because investors expect minimal interim cash flows. Therefore, maximized pledgeabale income is likely to be low in this instance, but at the same time this results in higher private benefits to the owner/manager.

The pledgeable income for a zero coupon bond is lower to the extent that interim coupons are not paid: $P_{Zero}(t=0)=E\left[e^{-r\tau}\alpha_P(F_{zero})1_{\{\tau\le T\}}\right]+E\left[e^{-rT}(F_{zero})1_{\{\tau>T\}}\right]$.

Zero coupon bonds allow the owner/manager to withdraw all interim funds, and there is no benefit of diversion (diversion incurs a fixed deadweight cost in equation (17) and gives no benefit). It is better for the owner/manager to pay himself any excess cash flows and zero coupon bond holders are paid only if the firm survives (this presumes no covenants that restrict dividend payments). Note that the pledgeabale income for a zero coupon bond with diversion is equal to that in the case with full information.

Remark 1: *Zero coupon bonds are feasible only for amounts that are low enough, interim default is not a concern (firms have strong credit) and the borrowing is for short maturity.*

Proof: See Appendix

Next we analyze the case of regular debt. It is in the best interest of the owner/manager not to divert cash flows if this could result in a non-repayment of a coupon, because the firm would then face financial distress and the owner/manager would forego all current and future benefits. Therefore, the net impact of a coupon bond on owner/manager' diversion is that there is less diversion in low cash flow states.

With PIK financing, the owner/manager trades off the cost and benefit of diversion when a coupon can be paid. The benefit to the owner/manager from diversion is the immediate cash inflow associated with the skipped coupon: $\lambda\left(C_{pik}\, dt\right)$. The drawback of a missed payment is the expected penalty incurred because of a higher terminal payoff: $E\left(1_{\{\tau>T\}} e^{-(r-\beta)(T-t)} C_{pik}\, dt\right)$. A PIK holder chooses to pay a coupon rather than divert cash in good states of the world if the expected terminal payoff is higher relative to the immediate benefit from diversion. When the firm cash flows are low, it is in the owner/manager's interest to roll over the coupons and avoid costly bankruptcy. On the other hand, if cash flows are high enough to pay the coupon, there is an incentive to pay the coupon if the present value of rolling over the coupon - $E\left(1_{\{\tau>T\}} e^{-(r-\beta)(T-t)} C_{pik}\, dt\right)$ is larger than the amount diverted- $\lambda\left(C_{pik}\,dt\right)$. This can be achieved with a sufficiently high PIK coupon rate β. Thus, in states with enough cash for coupon payments, owner/manager does not divert more cash because of the large cost of rolling over.

Lemma 2: *With asymmetric information and no internal cash flow savings, a PIK bond with maturity T,*

(a) *Prevents diversion when:* $\lambda < E\left(1_{\{\tau>T\}}e^{-(r-\beta)(T-t)}\right)$.

(b) *Maximizes pledgeable income when* $\lambda < E\left(1_{\{\tau>T\}}e^{-(r-\beta)(T-t)}\right)$, *if recovery is low enough and diversion is high enough.*

Proof: See Appendix

3.3 Optimal contract with cash savings by the firm

This section extends the analysis by allowing any excess cash flows in the firm to be accumulated in an account denoted $S(t)$. The ability to save mitigates the drawbacks of a coupon bond relative to a PIK bond. Consider again the setting where the owner/manager can divert cash flows for his own private benefit:

$$B(t=0) = E\left[\int_{t=0}^{\tau} e^{-rt}\lambda\left(\underbrace{dX(t) - d\hat{X}(t)}_{diversion}\right)1_{\{\tau\leq T\}}\right] + E\left[e^{-rT}\left(\underbrace{\frac{\mu_T}{r} + S(T) - F_K - C(T)}_{\substack{terminal\\payoff}}\right)1_{\{\tau>T\}}\right] \quad (18)$$

where $\tau = \inf\{t : S(t) < 0\}$ and $S(t) = \int_0^t e^{r(t-s)}(d\hat{X}(s) - dC(s)) + S(0)e^{rt}$, where $dX(t)$ is the true cash flow and $d\hat{X}(t)$ is the reported cash flow. This gives the owner/manager the opportunity to report larger losses than actual and divert funds. Note that in the previous section there was no saving within the firm and excess cash was distributed. However now excess cash is accumulated and no payout is permitted prior to maturity of the debt contracts at time *T*.

Clearly, zero coupon bonds and income bonds permit higher diversion because all interim payments are susceptible to diversion.

In the case of regular debt, the owner/manager's best interest is to pay the coupon if he can, because the immediate cost is bankruptcy and loss of future benefits. Any diversion leads to a lower accumulated cash balance, $S(t)$. The future value of total benefits, conditional on truth telling along a sample path is:

$$1_{\{\tau>T\}}e^{-r(T-t)}\left(\frac{\mu_T}{r}+S(T)-F_K-F_{rd}\right) \tag{19}$$

where the stopping time τ depends on whether there is no cash available to pay coupons ($\tau=\inf\{t:S(t)<0\}$). Hence the diversion is traded off by evaluating its incremental impact on the value of continuation in equation (19), primarily as it changes the chances of survival. The marginal impact of these benefits from a change in accumulated cash equals:

$$\frac{\partial}{\partial S(t)}E\left(1_{\{\tau>T\}}e^{-r(T-t)}\left(\frac{\mu_T}{r}+S(T)-F_K-F_{rd}\right)\right) \tag{20}$$

Consider a firm that is doing poorly and is subject to consecutive negative cash flow shocks. In such states additions to the cash balances are helpful for survival and the expected survival time is sensitive to savings- the value of the derivative in equation (20) is high, and it is in the best interest of the owner/manager not to divert if he receives a large payoff at *T*. On the other hand, if the firm has many positive cash flow shocks, the owner/manager would divert more than he would otherwise, because the expected survival chances are high (the value of the derivative in equation (20) is low) and the impact of diversion on firm survival is low. Therefore, the net impact of a coupon bond on equity holder diversion is that there is more diversion when there is more cash accumulated in the firm, rather than in states when there is less cash accumulated.

Such a diversion plan is congruent. The owner/manager diverts when there is more cash sitting in the firm for the reason that any excess cash in the firm has a small shadow value, and it would be paid to him at a later time. At some level of cash balances, it is optimal to pay out cash dividends. Therefore, regular debt inhibits diversion when needed in a better manner as compared to income bonds and zeroes. Their primary drawback is that the firm would face distress more often than it would in the case of a zero coupon bond or a PIK bond.

PIK bonds do not completely reduce diversion but are especially useful in bad states of the world. With PIK financing, an agent chooses to pay a coupon rather than divert cash flows in good states because there are lower deadweight costs associated with such states. When the firm is doing poorly, it is in the best interest of the owner/manager to roll over the coupons and avoid bankruptcy. Hence PIK-Toggles dominate the regular coupon bond in all instances.

The tradeoff facing owner/manager given a PIK contract when there is the possibility of diverting some cash flows at $t \in [0,T)$ for their private benefit takes into account the immediate benefit from not paying a coupon when cash is available:

$$\lambda\left(C_{pik}\, dt\right) \tag{21}$$

The diversion in equation (21) is traded off by evaluating its incremental impact on future benefits primarily as it changes the terminal payoff along each sample path by changing the PIK amount repaid at maturity- $F^{*}_{pik}\left(t, X(t,T)\right)$:

$$1_{\{\tau > T\}} e^{-r(T-t)} \left(\frac{\mu_T}{r} + S(T) - F_K - F^{*}_{pik}\left(t, X(t,T)\right) \right) \tag{22}$$

where the stopping time τ depends on whether there is no cash available to pay from the existing cash balances ($\tau = \inf\{t : S(t) < 0\}$). Again, diversion can be prevented by a higher PIK coupon rate β that increases the terminal face value of debt to be repaid and consequently reduces the benefit.

In states when there is enough cash, the owner/manager does not roll over coupons because of the rollover penalty. In states when there is insufficient cash, PIK coupons are rolled over providing less instances of default. PIKs are beneficial insofar as they help a firm avert bankruptcy in low cash states and induce the firm to pay coupons if there is sufficient cash. However, if the firm accumulates sufficient cash, the owner/manager is able to divert funds as in the case for regular debt and pays the PIK coupons in cash at the same time.

Lemma 3: *With asymmetric information and savings, the owner/manager chooses to pay the cash coupon on a PIK bond and at the same time divert cash when:*

$$\frac{\partial E\left(1_{\{\tau>T\}} e^{-r(T-t)}\right)}{\partial S(t)} < \lambda < E\left[1_{\{\tau>T\}} e^{-(r-\beta)(T-t)}\right]$$

Proof: See Appendix.

Next we compare the various debt contracts in Figure 2 which gives a graphical depiction of the owner/manager benefits as a function of parameters that reflect the nature of the restructuring – the amount required (*Z*) and the time needed to implement the restructuring (*T*).

Figure 2(a) is a solution to equation (12) where the amount raised is the same for each type of contract but the face value of newly issued debt is correspondingly different. The graph shows that when the amount required (*Z*) is small, all debt contracts are feasible and the private benefits to the owner/manager are similar for each of the contracts, but largest for PIK debt. As the amount required *Z* increases, it is not possible to raise the desired amount using income bonds, zero coupon bonds or regular debt. Thus, private benefits are maximized with PIK debt in some instances because this is the only feasible contract (our second optimization problem is the right boundary point for each graph). Thus PIK debt maximizes debt capacity and at the same time allows for some surplus to the owner/manager.

In Figure 2 (b) the face value of the debt is fixed for each contract. It shows that when the restructuring period (*T)* is low, zeroes, income bonds, regular debt and PIK debt are all comparable in terms of value. When the face value is discounted for a short period, the bond prices are closer to the face value in each instance, because coupons contribute less to prices. In contrast, for longer restructuring periods, PIK bonds are beneficial in comparison to the other contracts. With a longer restructuring period, the coupon payments contribute more to the debt value while the face value due at maturity is discounted over a longer period. PIKs capture the value of interim cash flows, but at the same time have the flexibility of allowing the coupons to be deferred in case of adverse changes in cash flows and thus avert bankruptcy (akin to zero coupon bonds). In other words, PIKs increase debt capacity especially when the restructuring period is long.

Figure 3 analyzes pledgeable income as a function of the firm type (parameters that proxy for growth, firm liquidity, costs of diversion and recovery) when the face value of each bond is constant. Figure 3(a), shows that pledgeable income for each contract increases as expected cash flows increase. Note that an increase in expected cash flow is equivalent to a decline in existing interest rate burden (what really matters is the interest rate coverage ratio). Both PIK debt and regular debt converge to the face value of the debt as debt becomes less risky. Thus, higher expected cash flows (or higher interest rate coverage) increase the amount pledegeable when coupon bonds are used, because of an ability to service any interim coupons. In the case of zero coupon bonds, interest rate coverage has an insignificant impact on value. Income bonds have the largest appreciation as more income is available for coupons and diversion does not lead to interim bankruptcy.

Figure 3(b) analyzes the impact of diversion on the pledgeable income. A lower cost of diversion (higher λ) results in a larger decrease in value of income bonds than any of the other contracts. The coupon flow and the value of income bonds are closely related to the reported cash flows at each period in time. If the cost of diversion is low (higher λ), there is a lower incentive to report these cash flows truthfully. However, the impact from such diversion is lower for zero coupons (their value is not derived from the coupon flow) and for regular coupon bonds (the coupons must be paid or the firm defaults). Figures 3(c) graphs the impact of initial cash balances on pledgeable income. Higher starting cash balances decrease the relative benefit of PIKs as there is lesser liquidity shortage risk and the liquidity hedge provided by PIKs is less relevant. Finally, Figure 3(d) shows that a lower cost of distress makes the relative advantage of PIKs lower.

In sum, PIKs increase debt capacity and provide the owner/manager larger the benefits of liquidity while reducing the incentives for diversion. They are beneficial when: (i) restructuring periods are long, (ii) the firm expected cash flows are low relative to the costs of servicing the debt (low interest rate coverage), (iii) the firm's cash flow has temporary low Sharpe ratio $\frac{\mu_0 - r}{\sigma}$, (iv) low cash balances, and (v) the recovery rate from default is low (or high debt restructuring costs).

Time variation in incentives

The parameters of a PIK-Toggle contract (β, C_{pik}, F_{pik}) are determined at the outset when funds are raised, and these parameters remain fixed for the duration of the contract. However, as the firm progresses with its restructuring and nears the end of the restructuring period (T) the contract parameters that make a PIK-Toggle incentive compatible also change. Figure 4 provides a graphical depiction of the manner in which the incentive compatible values of PIK-Toggles vary with time.

In Figure 4 (a) we graph the breakeven rate on PIKed coupons, β, that leaves the owner/manager indifferent at each t between diverting cash immediately and paying a coupon, in order to get higher benefits of continuation at T. The simulation determines how the PIK interest rate (β) evolves as a function of T (time to pay the unpaid coupons). When there is a long time remaining in the restructuring (T is high) there is a higher chance that the firm will run out of cash in the interim, and therefore the probability of survival is low. Also, the terminal value of an unpaid coupon by compounding at a rate β over a longer period makes the cost to the owner/manager of not paying a coupon higher than it would be when T is lower. Because this cost of rolling over a coupon at a rate β for higher T is relatively larger than the corresponding decline in probability of survival, the breakeven rate for β declines as T increases.

In Figure 4 (b) we reverse the question and compute the face value of the PIK debt, F_{pik}, that leaves the owner/manager indifferent between diverting cash and saving the cash, for a given β. Diversion gives an instant private benefit but at the same time decreases the probability of survival at T. If the face value F_{pik} increases because the firm has missed coupons in the past, the payoff to the owner/manager that accrues at maturity of the restructuring – the continuation value - is lower. Therefore, for a given T, the higher is the accumulated debt, F_{pik}, the more likely the owner/manager diverts. Combining this with the finding in the previous paragraph, results in a penalty rate β that should increase with the accumulated unpaid coupons.

The above analysis shows that the incentives for diversion and paying the coupons depend on the benefits of diversion, the continuation value received upon survival at T, the restructuring period, as well as the contract parameters. In some states the owner/manager may find it optimal to divert excess cash rather than wait until the maturity of the PIK to collect the accumulated cash, if the deadweight cost *(1-λ)* is low (even though it is positive). To design a contract where truth telling is optimal in all states, it is necessary to include a state contingent dividend policy in equation (18) that depends on the level of cash balances as well as the reported current period earnings: $\delta(S(t),\hat{X}(t),T)$. The owner/manager prefers dividend payouts relative to diversion because there is no deadweight cost associated with a dividend payment. Hence a contract that prevents diversion in all states and maximizes firm value consists of a dividend policy and a PIK-Toggle coupon rate: $\{\delta(t),\beta\}$.

Remark 2: *There is truthful reporting ($X(t)=\hat{X}(t)$) when there is a dividend policy in conjunction with the PIK-Toggle coupon rate: $\{\delta(t)),\beta\}$ such that the marginal impact on dividends and future benefits from truthful reporting is larger than the diversion payoff:* $\frac{\partial}{\partial \hat{X}(t)}\left(\delta(t)+B(t)\right)\geq\lambda$.

Proof: See Appendix.

There is considerable choice in setting the ex-ante fixed PIK interest rate β. A very high β at the outset ensures that the owner/manager does not avail of the facility to PIK because a decision to roll over the coupon is just too expensive to take. A high β therefore does not provide financial flexibility coupled with incentive compatibility. On the other hand a low β increases financial flexibility but there may be many instances where the initial β is not sufficiently high to be an effective deterrent of diversion in all states of the world. However, a dividend policy in conjunction with the β makes the contract incentive compatible. When there are more accumulated PIKed coupons, there is a higher incentive for diversion because the claim of the owner/manager becomes lower. In such states, the ex-ante β is again not high enough to prevent diversion ex-post, and this implies that the dividend policy must be adjusted to make the contract incentive compatible. The implication is that dividends at each t must be subject to the approval of the creditors.

PIK-Toggles and Control Rights

Our analysis shows that PIKs are optimal for firms that are financially constrained and there is a high possibility of states where the periodic coupon payments are not possible. A high chance of default would normally suggest that debt holders should be allocated the renegotiation and control rights in case of non-payment, rather than automatically permitting a roll-over of coupons. However when renegotiation and deadweight costs are sufficiently high, firms can reduce the deadweight costs if the control rights (of roll over) are given to the owner/manager. The drawback is that the owner/manager would choose more diversion and roll over more frequently than necessary. The higher coupon rate on toggled coupons can be regarded as the contingent cost paid by owner/manager to bond holders for the associated transfer of control rights. The Toggle Note can therefore also be viewed as a contract that requires payment at specified coupon rate but the terms are loosened (actually waived in a rollover), as long as a higher interest rate is paid afterwards on missed coupons (transfer cost of control rights). The ex-ante overall benefit of such a transfer of rights and the associated reduction in deadweight costs is shared by PIK debt and the owner/manager. The debt capacity is enhanced.

Remark 3: *Debt holders optimally transfer control rights to the owner/manager and give an automatic stay (waive intervention ex-ante) when the following two conditions hold:*

(i) There is a high likelihood of default and default is sufficiently costly,

(ii) The cost paid by owner/manager via a higher toggle coupon rate on missed coupon payments is such that truth telling is optimal (satisfies the tradeoffs in equations (21) and (22)).

Proof: See Appendix

4. PIKs versus Debt Rescheduling

The automatic stay in a PIK-Toggle is equivalent to ex-ante rescheduling of the intermediate debt payments, and it is useful for a firm that is likely to face insufficient liquidity to service interim coupons. If the firm were instead financed with regular coupon debt it would have to reschedule its debt when faced with cash shortages. Renegotiation and rescheduling of debt is expensive, especially when lenders are dispersed. Thus, a PIK contract provides an economical ex-ante commitment to reschedule coupon payments to the maturity of the contract. However, a PIK covenant is not a common feature of most debt contracts. A natural question that arises is: Why are PIK provisions included in debt contracts in very few instances?

To answer this question we resort to a setting with asymmetric information about the firm's cash flows. Suppose that there are two types of firms characterized by the level of their expected cash flows: H (high type, with $\mu_0 = \mu_H$) and L (low type, $\mu_H > \mu_L$). The firm type is private information, known only to the owner/manager. The firm needs to raise capital Z at time zero to finance an investment (restructuring). Lenders can offer a menu of contracts to owner/manager. Suppose they are offered a choice between straight debt and PIK-Toggle debt.

If there were no asymmetric information and lenders could identify the firm type, they could offer contracts that depend on the type of firm and the type of contract (PIK or regular debt). Each owner/manager would then pick the type of contract that maximizes his private benefits. A firm that is less likely to fall into distress from not being able to pay its coupon obligations has less need to rely on the automatic stay provided by PIKs (type H firms). A firm that is more likely to have interim cash shortages (an L-type firm) prefers a PIK bond if the expected costs of ex-post debt rescheduling exceed the higher costs associated with the payment in kind coupons.

When the type of firm is not verifiable, the choice of the debt contract provides information about the firm. In general, if the probability that a borrower is of type H is γ, the lender would offer contracts with a coupon rate that is a function of the average firm-type: $\gamma\mu_H + (1-\gamma)\mu_L$. If firm H chooses pure coupon debt and firm L mimics, in equilibrium both firms will raise the same amount and pay the pooled coupon rate: $C_{rd}\left(\gamma\mu_H + (1-\gamma)\mu_L\right)$ where the subscript rd refers to regular debt and the term in brackets indicates that the coupon is set based on the pooled cash flow. This would be the case when μ_H does not differ much from μ_L and the cash flow stream is relatively high and steady (low volatility) compared with the costs of ex-post rescheduling. In such an economy one would not expect PIKs bonds.

In a separating equilibrium where type *H* firms issue regular coupon debt and type *L* firms prefer to issue PIK bonds , an owner/manager of a type *L* firm must be better off than if the firm raises straight coupon debt at the pooling coupon. In choosing to issue PIKs, a type *L* firm signals to the market that it has a high probability of not having enough liquidity to service the debt in the future. Despite the higher spread in the PIK coupon, the company reveals its low type because it values the flexibility that PIK-Toggles offer. It trades off a higher PIK coupon for a lower expected cost of rescheduling and this allows the firm to raise more funds. Not many companies are willing to admit this (an admission that their prospects are risky, instead of being safe), and that may explain why PIKs do not occur frequently. If type *H* firm issues PIK debt it will be perceived as type-*L* and would pay a higher coupon for flexibility that it does not need. Thus the incentive compatibility constraints for equity holder benefits are given by (where the terms in brackets are the firm expected cash flows and the coupon and type of debt to raise *Z*):

$$B\left(\mu_L, C_{pik}(\mu_L); Z\right) > B\left(\mu_L, C_{rd}(\gamma\mu_H + (1-\gamma)\mu_L); Z\right) \tag{23}$$

for the type *L* firm, and

$$B\left(\mu_H, C_{rd}(\mu_H); Z\right) > B\left(\mu_H, C_{pik}(\gamma\mu_H + (1-\gamma)\mu_L); Z\right) \tag{24}$$

for the type *H* firm.

There is no incentive to deviate from this equilibrium because if a type *L* firm mimics a type *H* firm, it will pay a lower coupon on regular coupon debt, but will be unable to service the coupons in many instances. Similarly, type *H* firm has no incentive to be seen as a type *L* firm because the PIK coupon rate is higher while the benefit of flexibility is not needed. These constraints on not deviating from their optimal choice are written as:

$$B\left(\mu_L, C_{pik}(\mu_L); Z\right) > B\left(\mu_L, C_{rd}(\mu_H); Z\right) \qquad (25)$$

for the type L firm, and

$$B\left(\mu_H, C_{rd}(\mu_H); Z\right) > B\left(\mu_H, C_{pik}(\mu_L); Z\right) \qquad (26)$$

for the type H firm.

Figure 5 shows the incremental benefit to owner/manager from issuing PIK debt or regular debt (equations (23) and (24)). Suppose that the pooled expected cash flow during the restructuring period is $\gamma\mu_H + (1-\gamma)\mu_L$ =100 and the proportion of each type of firm is $\gamma = 0.5$. We fix the pooled rate of 110 as well as $\gamma = 0.5$, and change the expected cash flows to type L and H firms so that the expected cash flow remains constant. The solid line shows the incremental benefit to type H firms and the dashed line shows the incremental benefit to type L. The graph shows that there is no benefit to each type of firm in the region between the two vertical lines from revealing their type. However if the type H cash flows are higher than 110, there is a benefit to revealing the type. On the other hand, if the type L firm has expected cash flows lower than 85, the firm has an incentive to reveal its type. Hence the outside region is the separating equilibrium wherein firms will choose to issue PIK (type L firms) or regular debt (type H firms) because the benefit of revealing their type outweighs the cost of mimicking the other firm. Thus, PIKs give type L firms financial flexibility and this is better than the negative effect from paying a higher rate upon revealing the type. For type H firms the value of financial flexibility is reduced, and the cost incurred in terms of their inability to reschedule is more than offset by the lower coupon on the straight coupon debt relative to the cost of issuing PIKs.

PIK-Toggles save type L firms cash when there is a liquidity shortage to make a coupon payment. As an alternative to PIK-Toggles a question that arises is-why don't these firms get a bank line of credit that they could tap to service the coupons? DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007), for example, show that the optimal capital structure can be implemented with a credit line, debt and equity over which the agent controls the payout policy. Credit lines are contingent cash offers by banks to companies, for which the company pays a fee. The fact that a credit line is a far out of the money liquidity option for well rated borrowers, means that banks are not required to allocate capital and reserves against such a line of credit until it is utilized. In contrast, a low rated company is very likely to draw on the credit line and this would require banks to allocate capital and reserves, making the product less profitable. Thus, banks provide credit lines to well rated companies but not to lower rated companies. Huang (2010) shows that credit lines are not a good substitute for cash because the banking system imposes credit constraints disproportionately on the smaller and poorly rated companies. Loukoianova, Neftci and Sharma (2006), and Sufi (2009) find evidence that firms who hold credit lines must have good credit-rating levels. Thus, the credit line is not an alternative for the poorly rated companies. PIK-Toggles do, however, have embedded credit lines in the debt contract.

5. Firm characteristics and the choice of PIKs- Empirical Evidence

We now provide preliminary empirical evidence consistent with the model predictions about the determinants of the choice between PIKs and regular debt. The data provides a context in which to place the theoretical results in the preceding sections and should be regarded as illustrative rather than a complete test of the theory. Our conjecture is that PIKs serve as a screening device between firms with good cash flow prospects (type H firms) vs. lower cash flow prospects (type L) and are issued by firms that are likely to be more financially constrained. As we describe later, we start with a dataset that includes all PIK-Toggle issues. In addition we obtain a listing of all high yield debt issues for a two-year period.

None of these include an income bond or a pure zero coupon bond. Hence we examine the choice between PIKs and regular debt and we anticipate that:

The choice of PIKs relative to Regular Debt is:

(a) Positively related to a firm's financial constraints.

(b) Negatively related to expected cash flows and initial liquidity.

We employ straightforward proxies for a firm's expected financial constraints, expected cash flows and liquidity to investigate the choice between PIKs and regular debt, as explained below.

Data and variable construction

We start with a data set that includes all PIK-Toggle issues (1/2006- 3/2008) and regular debt issues for the period. Also, we obtain PIK data from the listings in the Standard and Poors high-yield database and FISD. The regular debt issue data is obtained from the Citibank database on all high-yield debt issues. The data set contains security specific information such as bid price, coupon, yield-to-maturity, credit ratings from Moody's and S&P, and the issue and maturity dates of PIK notes issued over that period. While our initial toggle sample consists of all toggle issues between 2006 and 2008, data availability on one issuer reduced our toggle sample to effectively the issues in 2006 and 2007. The data set includes 41 PIK issuing firms and 430 regular debt issues after excluding the one issuer in 2008.

We utilize several other databases to collect firm specific information, as well as control variables. These databases include: (i) CapitalIQ from Standard and Poors, (ii) Citibank database on all high yield issues, (iii) Center of Research in Security Prices (CRSP), and (iv) FRED database at the Saint Louis Federal Reserve.

For a PIK note observation to be included in our analysis, the firm must have an associated SEC filing and balance sheet data available on CapitalIQ. This yields a final sample of 41 firms. Table 1 gives the offer date, net proceeds of the offering, years to maturity, credit rating and the treasury spread for the PIK issues. The average net proceeds are $486 million (median $300 million). All the PIK issues are rated below investment grade with a mean spread of 568 basis points (median 547 basis points). The spread is defined as the difference between the yield to maturity on a PIK Note and the yield to maturity on its duration equivalent Treasury security. In those cases where no corresponding Treasury yield is available for a given maturity, the yield spread is calculated using interpolation based on an exponential functional form. Note also that the average maturity of the issues is around eight years. The PIKs are issued by a wide variety of firms in different industries.

Variables

To test our hypothesis on the determinants of the choice between PIKs and regular debt we need firm-specific proxies for financial constraints, expected cash flows and initial liquidity. In addition we include control variables such as firm risk and recovery (these variables are collected in column 1 of Table 3). We proxy for each of the variables using firm level data (these are listed in column 2 of Table 3).

Data for each proxy is collected for the year prior to the issue date (labeled t=-1) as well as the year of the issue date (labeled t=0). Note that period t=0 corresponds to the year in which the bond was issued. Thus this period *straddles the offering date*, and represents in part information that can be construed as private information.

Our primary measure of expected financial constraints is the inverse of the ex-post coverage ratio. We proxy for firm expected cash flows using revenue growth as well as by earnings growth at t=0, information that is revealed after the issue. We also include the revenue growth in the period prior to the issue (t=-1) in one of the models. Finally, our proxy for initial liquidity is cash plus short term investments divided by total assets.

In addition to the variables of interest we include firm level controls for firm risk as proxied by the issuer credit rating and firm size. The credit rating is the average of the S&P and Moody's bond ratings for the firm (a measure that largely depends on the volatility of the firm's assets and the amount of debt). Numerical ratings are computed using a conversion process where AAA rated bonds are assigned a value of 22 and D rated bonds receive a value of one. For example, a firm with an "A1" rating from Moody's and an "A+" from S&P would receive an average score of 18. The conversion numbers for both Moody's and S&P ratings are provided in Appendix B. This is consistent with prior literature (e.g., Molina (2005)). Our proxy for recovery is the ratio of plant and equipment to total assets.

Method

We use a single-stage procedure to analyze the determinants of a firm's choice between PIKs and regular debt. Specifically, we run a Logit regression where the dependent variable is set to 1 if the choice is PIK debt and 0 if the choice is regular debt. Our regression specification is:

$$\begin{aligned} y_i = {} & b_{1-3}(\mathit{Firm\ Expected\ Cashflows})_{i,t} + b_4(\mathit{Initial\ Liquidity}) + b_5(\mathit{Financial\ Constraints}) \\ & + b_{6-7}(\mathit{Firm\_Risk}) + b_8(\mathit{Tangibilty/Recovery}) + \varepsilon_{i,t} \end{aligned} \quad (27)$$

where y_i equals 0 or 1.

Results

Table 2a contains a summary of firm level data categorized by the type of issue – PIK or regular debt. Table 2 shows that the PIK issuer firm size (total assets) has a median of $2,412 million in the year of issuance that is higher than that of regular debt issuers. The firms are on average profitable with a median profitability of about 14% for PIKs (12% for regular debt) in the year prior to the issue date and 8% (10%) in the year of issue. The median revenue growth rate is 14% for regular debt issuers vs. 6% for PIK issuers over the year of issuance. In addition none of the PIK issuers incurred any R&D expenses. Also, PIK issuers are more financially constrained than regular debt issuers (coverage of 2.15 in t=0 relative to 2.6 for regular debt issuers).

Table 2b is a correlation matrix between the indicator function (1 for PIK issuers and 0 for regular debt) and the firm level data. The first column reveals that the PIK indicator is negatively correlated with revenue growth at t=0 and t=-1, consistent with our prediction that PIKs are likely issued by firms with lower expected cash flows. The initial cash ratio is negatively related to PIK choice and the inverse coverage ratio (COV0) is positively related to PIK choice (0.17), as expected.

Table 3 presents the results from the logit regression specified in equation (27). The firm specific proxies listed in column 2 of Table 3 include proxies for expected cash flows, liquidity, financial constraints as well as other controls such for firm risk and recovery, discussed earlier. We report the estimates and t-stats (significance) of each of the parameter estimates for a total of five specifications (Models 1 to 5).

The first variable (revenue growth) is negatively related to the choice of PIK debt in model 1 and 2, and lends support to the screening hypothesis noted in Section 4. For the fully specified model 4 and 5 the sign is again consistent. Revenue growth is a proxy for expected future earnings and cash flows. Thus, when firms expect higher revenue growth and consequently higher cash flows, firms choose regular debt while firms with lower growth chose PIK debt. Recall that the revenue growth for the current period is not entirely known at the time of the issue, and can be construed as private information. In the case of model 3 where earnings growth is the proxy for expected cash flows, the signs and parameter estimates are again consistent, even though the parameter estimate is not statistically significant.

Models 1, 2, 4 and 5 show that the inverse of interest coverage (financial constraints) is positively related to PIK usage- firms that issue PIKs are more constrained while firms that issue regular debt face lower financial constraints. The results are again consistent with the idea that liquidity constrained firms with information about their growth prospects choose PIK debt or regular debt to maximize their values, consistent with our theoretical model. The pseudo R-square for the fully specified models is 0.09 and 0.12 (models 4 and 5).

Finally, the initial liquidity and cash in the firm is negatively related to PIK choice. The parameter estimates are as expected but they are not statistically significant.

In sum, the preliminary evidence is consistent with the idea that many of the firms that issued PIKs were financially constrained (higher inverse coverage ratios) and with lower growth in expected cash flows (as proxied by revenue growth rates).

6. Conclusions

Recent years have witnessed a rapid growth in the amount of outstanding debt. The increase in leverage combined with rapid financial innovation has raised many questions regarding the role of the different types of debt securities as contributing factors in this growth. Many analysts noted the failure of creditors in evaluating the risk of borrowers and the contract structures as a contributing factor in the 2007 financial crisis. One debt contract that has been criticized as "covenant-light" is the PIK-Toggle bond, on the basis that a PIK-Toggle gives a borrower the choice to automatically roll over coupon payments by issuing additional bonds rather than paying cash. In this paper we show that a PIK-Toggle bond by incorporating and ex-ante rescheduling feature of debt payments is indeed an optimal contract for lower rated firms in a setting where there is asymmetric information about the borrower's cash flows and a possibility of diversion. A PIK-Toggle contract serves as a screening device so that firms with good prospects choose regular coupon debt while firms which expect temporary liquidity shortages. This explains the empirical observation that a PIK clause is a not often used in conjunction with regular debt. Using data on all PIK issues over the period 2006 to 2007 we empirically validate our screening model that PIKs are issued by firms with lower expected revenues and firms that expect to be financially constrained in the immediate future. While PIK toggles were viewed with skepticism in prior years, recent reports show that some firms have used the PIK-toggle option to survive the economic downturn of 2009-2010. Analysts have pointed to the positive impact of the toggle option in providing state contingent liquidity and helping these firms survive the recent liquidity events.[5]

[5] See High Yield Weekly 12th May 2009, JP Morgan North America Credit Research.

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Appendix

A. Example of a PIK-Toggle clause

For any interest payment period through October 15, 2010, the Company may, *at its option*, elect to pay interest on the senior notes:

- entirely in cash ("*Cash Interest*") or
- entirely by increasing the principal amount of the outstanding senior notes or by issuing PIK Notes ("*PIK Interest*").

The Company must elect the form of interest payment with respect to each interest period by delivering a notice to the Trustee prior to the beginning of each interest period. The Trustee shall promptly deliver a corresponding notice to the Holders. In the absence of such an election, interest on the senior notes will be payable entirely in cash. Interest for the first interest period commencing on the Issue Date shall be payable entirely in cash. After October 15, 2010, the Company will make all interest payments on the senior notes entirely in cash. Cash Interest on the senior notes will accrue at the rate of 9% per annum and be payable in cash. PIK Interest on the senior notes will accrue at the rate of $9^{3/4}$% per annum and be payable with respect to senior notes represented by one or more global notes registered in the name of, or held by, the Depository Trust Company ("*DTC*") or its nominee on the relevant record date, by increasing the principal amount of the outstanding global senior notes by an amount equal to the amount of PIK Interest for the applicable interest period (rounded up to the nearest $1,000); and with respect to senior notes represented by certificated notes, by issuing PIK Notes in certificated form in an aggregate principal amount equal to the amount of PIK Interest for the applicable interest period (rounded up to the nearest whole dollar).

B. Bond Rating Numerical Conversions

This figure provides bond rating conversion codes for Moody's and S&P ratings used in the analysis. Moody's and S&P ratings of 13 and above are considered investment grade, while those below 13 are non-investment grade. The data covers the period from 1990 to 2000.

Conversion Number	Moody's Ratings	S&P Ratings
22	Aaa	AAA
21	Aa1	AA+
20	Aa2	AA
19	Aa3	AA-
18	A1	A+
17	A2	A
16	A3	A-
15	Baa1	BBB+
14	Baa2	BBB
13	Baa3	BBB-
12	Ba1	BB+
11	Ba2	BB
10	Ba3	BB-
9	B1	B+
8	B2	B
7	B3	B-
6	Caa1	CCC+
5	Caa2	CCC
4	Caa3	CCC-
3	Ca	CC
2	C	C
1	D	D

C. Proofs

Proof of Lemma 1:

(i) The problem is:

$$\underset{\theta(t)\in[0,1]}{Max} P(t=0) = E\left[\int_{t=0}^{\tau} e^{-rt}\theta(t)(dX(t))1_{\{\tau\le T\}}\right] + E\left[e^{-r\tau}\alpha_P\theta(T)\left(\frac{\mu_0}{r}-F_K\right)1_{\{\tau\le T\}}\right]$$
$$+E\left[\theta(T)e^{-rT}\left(\frac{\mu_T}{r}-F_K\right)1_{\{\tau>T\}}\right] \quad \text{(C1)}$$

where $\tau = \inf\{t : dX(t) < 0\}$. Consider two portfolios: $\theta_1(t) = 1\forall t$ and $0 \le \theta_2(t) \le 1\forall t$. Then the difference in the pledgebale income of the two portfolios is given by:

$$P(\theta_1)-P(\theta_2) = E\left[\int_{t=0}^{\tau} e^{-rt}\underbrace{(\theta_1-\theta_2)}_{\ge 0}\left(\underbrace{dX(t)}_{\ge 0\forall t<\tau}\right)1_{\{\tau\le T\}}\right] + E\left[e^{-r\tau}\alpha_P\underbrace{(\theta_1-\theta_2)}_{\ge 0}\underbrace{\left(\frac{\mu_0}{r}-F_K\right)}_{>0}1_{\{\tau\le T\}}\right]$$
$$+E\left[\underbrace{(\theta_1-\theta_2)}_{\ge 0}e^{-rT}\underbrace{\left(\frac{\mu_T}{r}-F_K\right)}_{>0}1_{\{\tau>T\}}\right] \ge 0$$

These components are each greater than zero by construction. Thus, the pledgeable income is maximized when $\theta(t) = 1\forall t$.

(ii) To prove that the pledgebale income is higher for income bonds we need to evaluate the difference in payoffs between income bonds and the alternate bonds (coupon bonds, PIKs and zeroes). First note that the survival time for an income bonds is equal to or larger than the alternate bonds ($\tau_{income} \ge \tau_{rd}$ and $\tau_{income} = \tau_{pik}, \tau_{zero}$) because there are no required coupon payments on income bonds while coupon bonds can default if the cash flows are not sufficient to pay the required coupon. The difference between the pledgeable income for income bonds and other bonds (subscripted by 2) is given by:

$$P_{income} - P_2 = E\left[\int_{t=0}^{\tau_2} e^{-rt} \underbrace{\left(dC_{income}(X(t)) - dC_2(X(t)\right)}_{\geq 0} 1_{\{\tau_2 \leq T\}}\right]$$

$$+ e^{-r\tau_2} E\left[\int_{\tau_2}^{\tau_{income}} e^{-rt} \underbrace{\left(dC_{income}(X(t))\right)}_{\geq 0} 1_{\{\tau_{income} \leq T\}}\right] + \alpha_P F\left(E\left[e^{-r\tau_{income}} 1_{\{\tau_{income} \leq T\}} - e^{-r\tau_2} 1_{\{\tau_2 \leq T\}}\right]\right)$$

$$+ \underbrace{E\left[C_{income}(X(T)) 1_{\{\tau_{income} > T\}} - C_2(X(T) 1_{\{\tau_2 > T\}})\right]}_{\geq 0}$$

(C2)

The first row of the equation (C2) above includes the coupon flow in non-bankruptcy states- flows that accrue to income bonds are higher than those to alternate bonds (from the Lemma 1 (i)). The third row term is non-negative because the income bond has a higher probability of survival, when face values are the same.

We are only left with a consideration of the second row in (C2) for states in which the firm faces bankruptcy. If a firm that issues income bonds is alive, the expected payoff in the next period to the income bond holders is $E[dC_{income}] = (\mu - K)dt$ (instantaneous flow in first part of line 2 of (C2)). On bankruptcy the payoff to any of the other bond holders with face value F is $\alpha_P F$ and the instantaneous return on this payoff is $r\alpha_P F dt$ (incremental instantaneous flow to bond holders of type 2 from early bankruptcy from second part of line 2 of (C2)). Thus, the incremental instantaneous flow in second row terms are together positive if the expected cash flows for the live firm $E[dC_{income}] = (\mu - K)dt$ is higher than the flows to the bankrupt firm from early bankruptcy, i.e., $r\alpha_P F < (\mu - K)$.

Proof of Remark 1:

The pledgeable income on a zero coupon bond is given by $P_{Zero}(0) = E\left[e^{-r\tau}\alpha_P(F_{zero})1_{\{\tau \leq T\}}\right] + E\left[e^{-rT}(F_{zero})1_{\{\tau > T\}}\right]$. Given T, the highest possible pledgeabale income is the default free bond: $e^{-rT}(F_{zero})$. Thus for a zero coupon bond to be feasible given T, the default rate has to be low enough and $Z < e^{-rT}(F_{zero})$. If T can be reduced, the pledgeable income is higher the lower the maturity, such that the absolute bound on the amount raised is $Z < (F_{zero})$. Hence, zero coupon bonds could be feasible only if the amount required is low enough, default rate is low enough and/or the maturity is low enough.

Proof of Lemma 2:

(i) The benefit of diversion of a coupon *Cdt* is the immediate payoff: $\lambda(Cdt)$. The present value of the expected terminal payoff by owner/manager from not paying the bondholders now is: $E\left(1_{\{\tau>T\}}e^{-(r-\beta)(T-t)}Cdt\right)$. The present value of the terminal payoff must exceed the current benefit to prevent diversion at any point in time.

(ii) The survival time for a PIK bond is equal to or larger than the alternate bonds ($\tau_{pik} \geq \tau_{rd}$ and $\tau_{pik} = \tau_{icnome}, \tau_{zero}$). PIK coupon payments can be rolled over while regular debt cash flows may not be sufficient to pay the required coupon. Both income bonds and zero coupon, as is the case for PIKs, will not default unless the cash flow shock is negative. The coupon flows and pledgeable income of PIKs (denoted with a subscript of *pik*) relative to zeroes, coupon bonds and income bonds (denoted with a subscript of 2) is given by:

$$\begin{aligned} P_{pik} - P_2 = {} & E\left[\int_{t=0}^{\tau_2} e^{-rt}\underbrace{\left(dC_{pik}(X(t)) - dC_2(X(t)\right)}_{\geq 0}1_{\{\tau_2 \leq T\}}\right] \\ & + e^{-r\tau_2}E\left[\int_{\tau_2}^{\tau_{pik}} e^{-rt}\left(dC_{pik}(X(t))\right)1_{\{\tau_{pik}\leq T\}}\right] + \alpha_P\left(E\left[e^{-r\tau_{pik}}F_{pik}^* - e^{-r\tau_2}F_2\right]1_{\{\tau_{pik}\leq T\}}\right) \\ & + \underbrace{E\left[e^{-r\tau_{pik}}F_{pik}^*1_{\{\tau_{pik}>T\}} - e^{-r\tau_2}F_2 1_{\{\tau_{pik}>T\}})\right]}_{\geq 0} \end{aligned} \quad \text{(C3)}$$

The proof follows Lemma 1. The first term of (C3) is positive when the alternate bond is a zero coupon or regular debt. However, diversion must be large enough for the average coupon flow for income bonds to be lower than that for PIKs on average so that: [$E[dC_{pik}(X(t)) - dC_{income}(\hat{X}(t))] > 0$.

The third row of (C2) is non-negative because the PIK bond has an equal or higher probability of survival compared to other debt, and assuming initial face values are the same.

Again, the second row of (C2) represents states in which the firm faces bankruptcy. On bankruptcy the payoff to any of the bond holders with face value F is $\alpha_p F$ and the instantaneous return on this payoff is $r\alpha_p Fdt$. However if a firm that issues PIK bonds is alive, the expected payoff in the next period to the *pik* bond holders is $E[dC_{pik}]$, the first component of the second row. Thus, the second row terms are together positive if the expected cash flows for the live firm $E[dC_{pik}]$ are higher than $r\alpha_P F_2$ (or recovery is low enough).

Proof of Lemma 3:

The right side of the inequality follows from Lemma 2- the current marginal payoff per unit to owner/manager from diversion is λ. The cost is the expected final payoff from increasing the PIK coupon amount based on the PIK coupon rate β. To prevent diversion the current marginal payoff from diversion should be lower than the present value of the payoff from paying a higher coupon amount at the maturity of the contract.

If *S(t)* is high, the probability of firm survival is large: $E\left(1_{\{\tau>T\}}\right)$ is high and the cost of diversion in terms the impact of a change in accumulated cash on survival probability is low: $\frac{\partial E\left(1_{\{\tau>T\}}\right)}{\partial S(t)}$ is low. Diversion today gives λ per unit diverted and the expected current value of the payoff from waiting is $E\left(e^{-r(T-t)}1_{\{\tau>T\}}\right)$ per unit. If λ is high enough (or (1- λ) is low), the payoff from diversion is higher than the present value of the cost when:

$$\lambda > \frac{\partial E\left(e^{-r(T-t)}1_{\{\tau>T\}}\right)}{\partial S(t)}.$$

Proof of Remark 2:

Consider the private benefits to owner/manager with dividend payments.

$$B(t=0) = E\left[\int_{t=0}^{\tau} e^{-rt}\left(\underbrace{\lambda(dX(t)-d\hat{X}(t))}_{\text{diversion}} + \underbrace{\delta(\hat{X}(t),S(t),T)}_{\substack{\text{permitted}\\ \text{dividend}\\ \text{payout}}}\right)1_{\tau\le T}\right] + E\left[e^{-rT}\left(\underbrace{\frac{\mu_T}{r} - F_K - C(T)}_{\substack{\text{terminal}\\ \text{payout}}}\right)1_{\tau>T}\right]$$

The immediate benefit of diversion is λ per unit of diversion. A change in reported income results in a change in current dividend and the value of future benefits: $\frac{\partial}{\partial\hat{X}(t)}\left(\delta(\hat{X},S,T)+B(t)\right)$. There is no diversion at time *t* if the sensitivity of dividend payouts and the value of future benefits to reported income are greater than or equal to the benefit from diversion (λ).

Proof of Remark 3:

This remark follows from the previous analysis. For Pareto Optimality, owner/manager must be no worse off and at the same time he ex-ante debt value (pledgeable income) is higher when (i) and (ii) hold. First note that when (i) holds and in a setting with full information the pledgebale income for regular debt is lower than PIKs because of a lower survival time: $P_{rd}(C,t,T) < P_{pik}(C,t,T)$. When (ii) holds, truth telling is optimal and therefore $P_{rd}(C,t,T) < P_{pik}(C,t,T)$. Also, equity holder payoffs are the same for both contracts (regular debt and PIKs) in non-bankruptcy states. Also, equity holder payoffs are strictly larger for PIKs than regular debts in some states because of a longer survival time: $B_{PIK} > B_{rd}$.

Table 1. Recent issues of Payment-In-Kind bonds

This table provides a list of firms that have issued payment-in-kind bonds over the years 2006 and 2007. PIK data is obtained from Standard and Poors. The table reports the proceeds as well as the long term credit rating assigned to the company at the time of the issue.

Name	Offer Date	Net Proceeds	Years to Mat	Moodys Rating	Treasury Spread
Affinion Group Inc	04/21/06	350	9	Caa1	673
Ainsworth Lumber	04/11/06	75	7	B2	400
Aleris International Inc.	12/13/06	600	8	B3	453
Allison Transmission, Inc.	10/12/07	550	8	Caa1	657
Alltel Communications, Inc.	11/16/07	915	10	Caa1	768
BevMo Intermediate Holdings, Inc.	03/15/07	29	5	NR	905
Biomet, Inc.	10/11/07	88	10	B3	547
Blaze Recycling & Metals, LLC	06/22/07	15	6	NR	1154
Brickman Group	01/23/07	198	10	NR	671
Ceridian Corp.	10/26/07	475	8	Caa2	797
CHR Intermediate HoldCo (Compucom)	05/24/07	148	6	Caa1	725
Claire's Stores, Inc.	05/22/07	350	8	Caa1	480
Digicel Group Limited	07/18/06	154	6	B3	352
Dollar General Corporation	06/28/07	725	10	Caa2	668
Dresser Inc	01/23/06	350	10	B2	227
Energy Future Holdings Corp.	10/24/07	2445	10	B3	730
Freescale Semiconductor, Inc.	11/16/06	1500	8	B1	449
General Nutrition Centers, Inc.	03/07/07	297	7	Caa1	450
Hawker Beechcraft Acquisition Company LLC	03/16/07	400	8	B3	434
HCA, Inc.	11/09/06	1500	11	B2	499
Intelsat (Bermuda), Ltd.	06/19/06	1590	17	Caa1	612
iPayment Investors L.P. (Ipayment Inc.)	06/30/06	74	8	NR	669
IPCS Inc	04/11/07	175	7	Caa1	325
Kronos Inc	04/05/06	481	7	B2	287
Local TV Finance, LLC	05/02/07	190	8	Caa1	463
Metals USA Holdings Corp.	12/19/06	145	6	Caa1	600
Momentive Performance Materials Inc.	11/29/06	300	8	B3	562
NMH Holdings, Inc.	06/29/07	172	7	Caa2	638
Noranda Aluminum Acquisition Corp. (Holding	05/10/07	510	8	B3	400
Penhall International Corp	07/18/06	175	8	B3	682
Pharmaceutical Tech & Services (Cardinal Health	04/04/07	866	18	Caa1	1052
PNA Intermediate HoldCo	02/06/07	167	6	Caa1	700
Realogy Corporation	04/05/07	543	7	Caa1	665
Rexnord Corp	07/14/06	485	8	B3	445
Surgical Care Affiliates	06/21/07	150	8	B3	375
Texas Competitive Electric Holdings Company Ll	11/29/07	1632	9	B3	790
United Surgical Partner International	04/11/07	200	10	Caa1	153
Universal Hospital Services, Inc.	05/22/07	230	8	B3	500
Univision Communications Inc.	03/01/07	1500	8	B3	520
US Oncology Holdings, Inc.	11/30/06	345	5	Caa1	450
Verso Paper Holdings LLC	07/26/06	250	9	B1	375
Average		486	8.1		568
Median		300	8.0		547

Table 2a. Firm Characteristics and PIK choice

This table describes provides medians of financial ratios for the sample of firms that issued PIKs as well as non-investment rated companies that issued regular debt during the same period. The time period -1 refers to the year prior to the issue and 0 refers to the issue period. The data includes a list of firms that have issued payment-in-kind bonds over the years 2006 and 2007 and all high yield issues over the comparable period. Data is obtained from Standard and Poors and Citibank.

	Prior Year (t=-1)		**Issue Period (t=0)**	
	PIK	**Regular**	**PIK**	**Regular**
Total Assets (TA)	1616	1502	2412	2235
Total Liabilities (TL)	751	945	2743	1686
Long Term Debt (LTD)	368	478	1260	1082
Cash and Short Term Investments (CASH)	43	44	100	68
Capital Expenditures (CAPEX)	53	66	78	87
Plant and Equipment (PE)	357	345	437	474
R&D Expenses (RD)	0	32	0	431
EBITDA	224	191	247	246
Leverage (LTD/Book Equity)	0.60	1.05	2.81	1.68
Liquidity (Cash +ST Inv/TA)	0.03	0.04	0.03	0.03
Interest Coverage	5.85	3.40	2.15	2.60
1/Interest Coverage	0.17	0.29	0.47	0.38
RD/TA	0.00	0.02	0.00	0.01
PE/TA	0.22	0.29	0.14	0.26
EBITDA/TA (PROF)	0.14	0.12	0.08	0.10
Revenue Growth (1 year)	0.09	0.13	0.06	0.14
Treasury Spread			554	450
Average Rating			6.0	7.0
Net Proceeds (millions)			300	296

Table 2b. Correlation Matrix: Firm Characteristics and PIK choice

This table describes provides a correlation matrix of financial ratios for the sample of firms that issued PIKs as well as non-investment rated companies that issued regular debt during the same period. The time period -1 refers to the year prior to the issue and 0 refers to the issue period. The data includes a list of firms that have issued payment-in-kind bonds over the years 2006 and 2007 and all high yield issues over the comparable period. Data is obtained from Standard and Poors and Citibank.

		PIK	RG1	RG0	EB1	EB0	CR	COV1	COV0	RAT	PE/TA	TA1	TA0
PIK indicator	PIK	1.00											
Revenue Growth (t=-1)	RG1	-0.12	1.00										
Revenue Growth (t=0)	RG0	-0.17	0.26	1.00									
EBITDA Growth (t=-1)	EB1	-0.10	0.62	0.26	1.00								
EBITDA Growth (t=0)	EB0	-0.15	0.11	0.68	0.12	1.00							
Cash Ratio (t=-1)	CR	-0.04	-0.01	0.04	0.01	0.07	1.00						
Inverse Coverage (t=-1)	COV1	-0.14	-0.06	-0.02	-0.11	0.10	-0.01	1.00					
Inverse Coverage (t=0)	COV0	0.17	-0.05	-0.14	-0.07	-0.09	-0.03	0.07	1.00				
Rating	RAT	-0.14	0.20	0.16	0.10	0.09	0.02	0.03	-0.13	1.00			
PE / TA at t=-1	PE/TA	-0.07	0.07	0.02	0.03	0.05	-0.09	-0.04	-0.15	0.09	1.00		
Total Assets (t=-1)	TA1	0.13	-0.14	-0.21	-0.07	-0.14	-0.04	0.01	0.00	0.16	-0.05	1.00	
Total Assets (t=0)	TA0	0.13	-0.14	-0.01	-0.02	-0.04	-0.02	-0.02	0.00	0.15	-0.01	0.92	1.00

Table 3. Firm Characteristics and PIK choice- Logit Analysis

This table describes provides the results of a Logit regression. The dependent variable is set to 1 if the firm chooses PIK debt and 0 otherwise. The data includes a list of firms that have issued payment-in-kind bonds over the years 2006 and 2007 and all high yield issues over the comparable period. Data is obtained from Standard and Poors and Citibank. Time period -1 refers to the year prior to the issue and 0 refers to the issue period.

		Model				
Varaible	**Proxy**	**1**	**2**	**3**	**4**	**5**
Firm Expected Cash Flows						
	Revenue Growth(t=-1)	-0.98*			-0.98*	
	Revenue Growth(t=0)		-0.96****			-0.96****
	EBITDA Growth (t=-1)			-0.99		
Initial Cash/Liquidity						
	(Cash + ST Investments)/TA (t=-1)	-0.07	-0.08	-0.09	-0.15	-0.15
Expected Financial Constraints						
	1/Coverage (t=0)	0.03***	1.11*	1.29***	1.25**	1.08
Controls						
	Credit Rating (t=-1)				-0.69***	-0.67**
	Size: log(TA) (t=-1)				1.39***	1.27*
	Tangibility Ratio: PE/TA (t=-1)				-0.65	-0.62
Pseudo R-squared		0.07	0.09	0.04	0.09	0.12

*, **,***,**** indicate significance at 25%, 10%, 5% and 1% levels respectively

Figure 1
A Two-Period Model

This figure illustrates the cash flows at time 1 and time 2 from an initial investment *Z* at time 0. Each state (denoted s) is associated with a cash flow (reported cash flow *X*(s)) and corresponding payments to existing debt (*K*(s)). The excess cash flow is denoted C. The firm defaults if excess cash flows are negative.

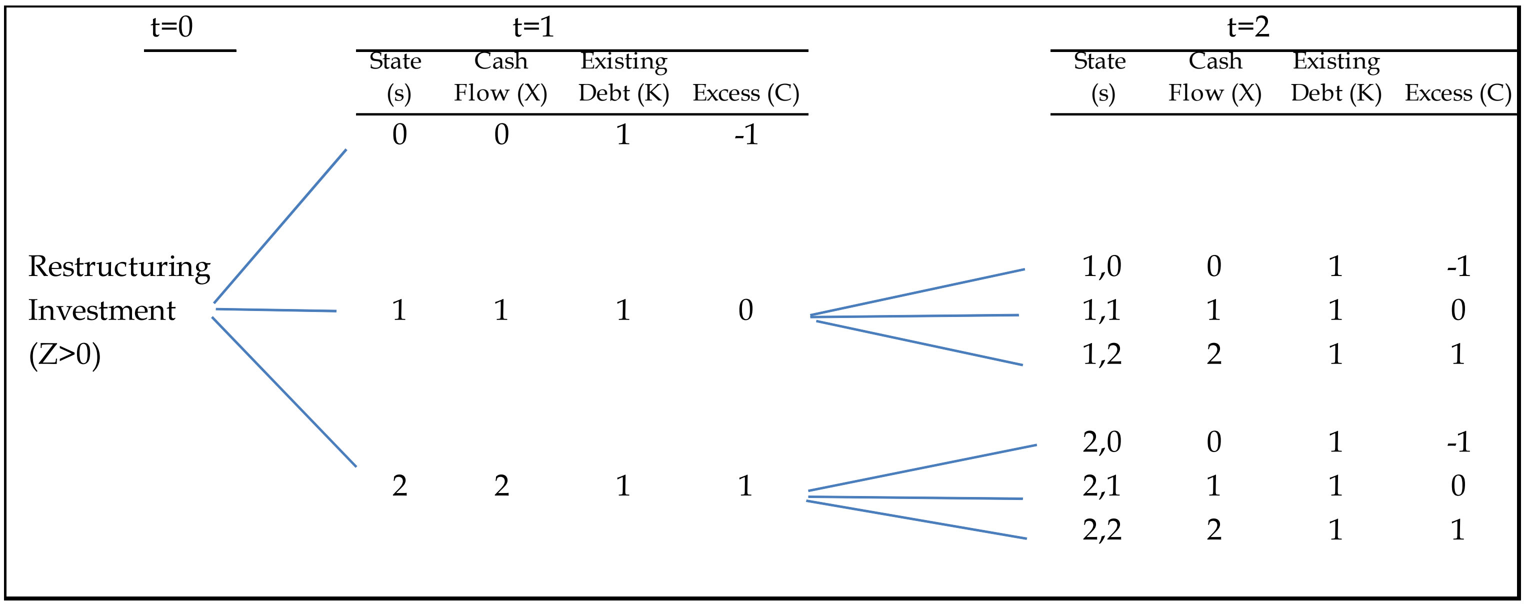
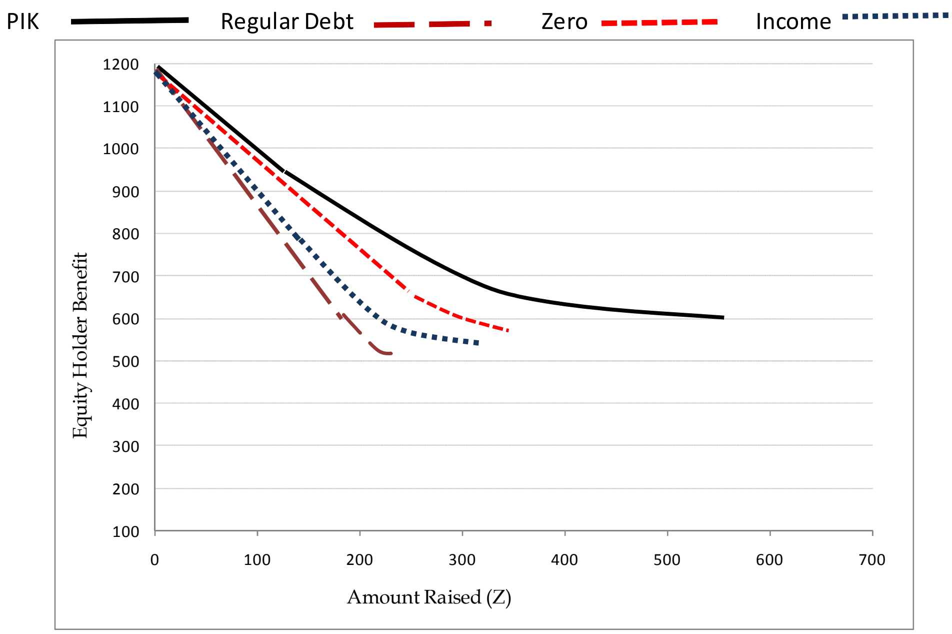


Figure 2

Impact of Restructuring Parameters: Amount (Z) and Restructuring Time (T)

This figure illustrates equity holder benefits as a function of the amount required for restructuring (Z) and the restructuring period (*T*) when Income Bonds, Zeroes, PIKs and Coupon Bonds are used to raise funds for restructuring. In all the computations we fix the parameter values where required as: $\mu_0 = 80$, $\mu_T = 110$, $\sigma = 40$, $r = 0.05$, $K = 40$, $T = 8$, $\lambda = 0.8$, $\beta = 0.09$, $F_{income} = F_{zero} = F_{rd} = F_{pik} = 500$, $C_{rd} = C_{pik} = 24$, $\alpha_P = 0$ and $S(0) = 5$.

(a)



(b)

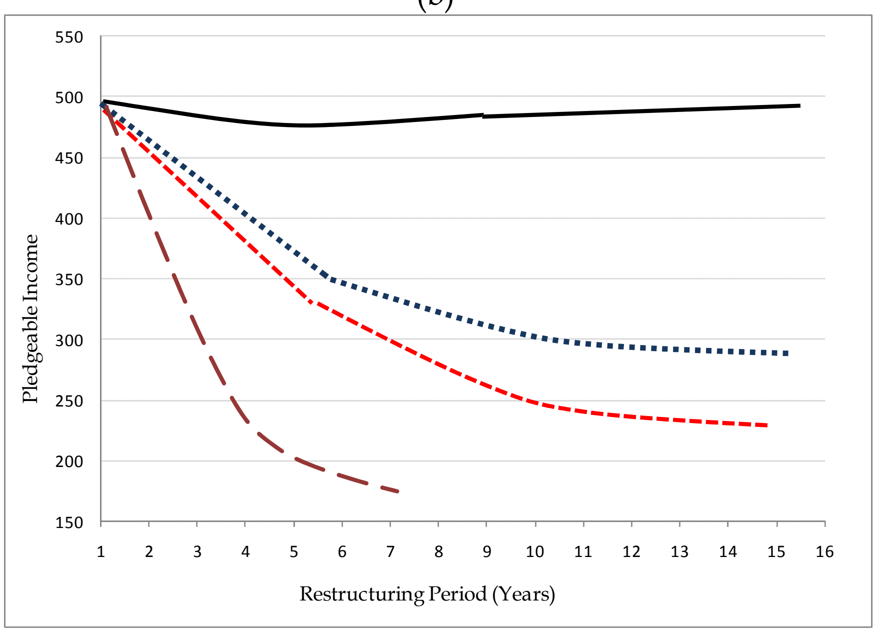
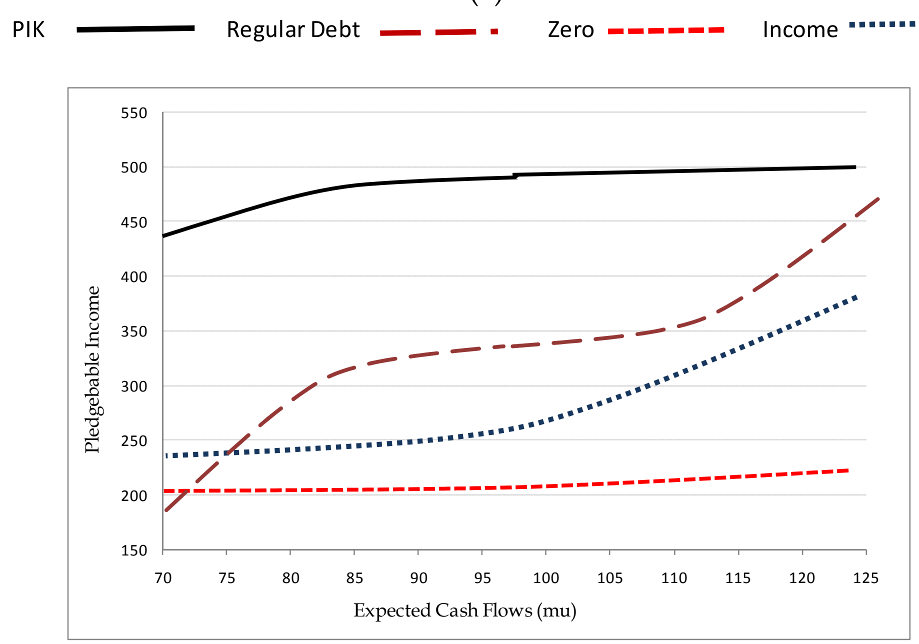


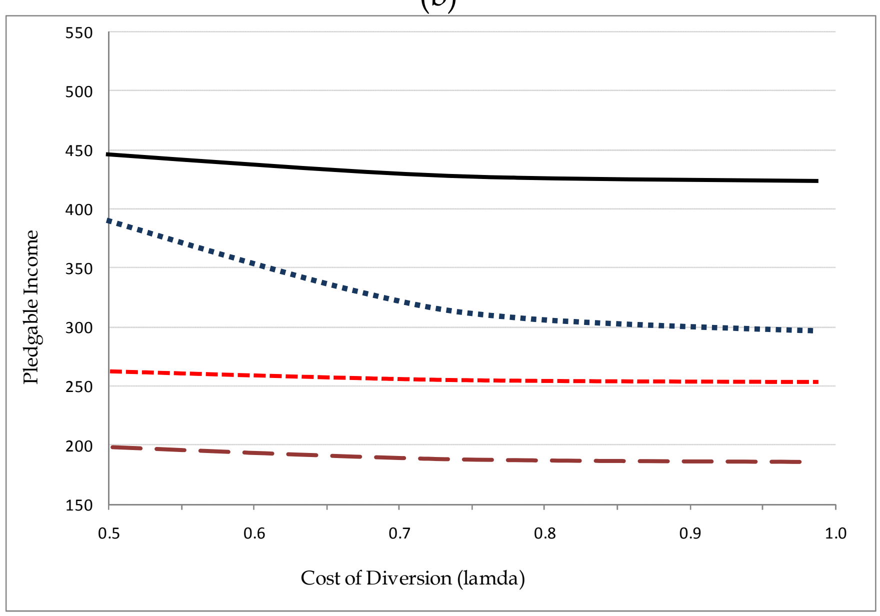
Figure 3
Impact of Firm Characteristics

This figure illustrates pledgeable income as a function of firm characteristics (expected cash flows, cost of diversion, starting cash balance and recovery) when Income Bonds, Zeroes, PIKs and Coupon Bonds are used to raise funds for restructuring. In all the computations we fix the parameter values where required as:, $\mu_0 = 80$, $\mu_T = 110$, $\sigma = 40$, $r = 0.05$, $K = 40$, $T = 8$, $\lambda = 0.8$, $\beta = 0.09$, $F_{income} = F_{zero} = F_{rd} = F_{pik} = 500$, $C_{rd} = C_{pik} = 24$, $\alpha_P = 0$ and $S(0) = 5$.

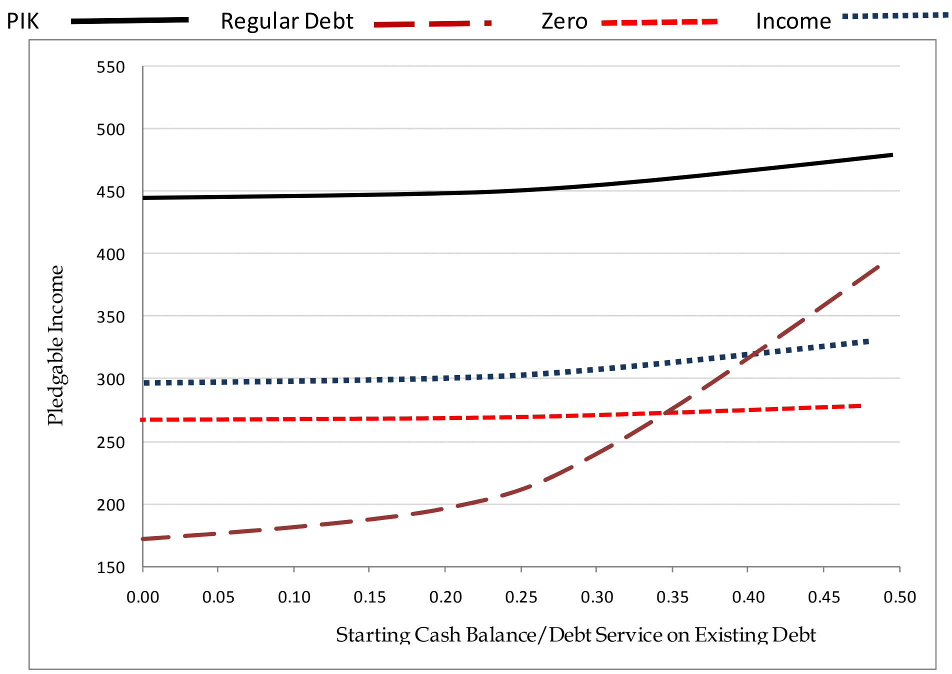
(a)



(b)



(c)


PIK
Regular Debt
Zero
Income
550
500
450
400
350
300
250
200
150
Pledgable Income
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
Starting Cash Balance/Debt Service on Existing Debt

(d)

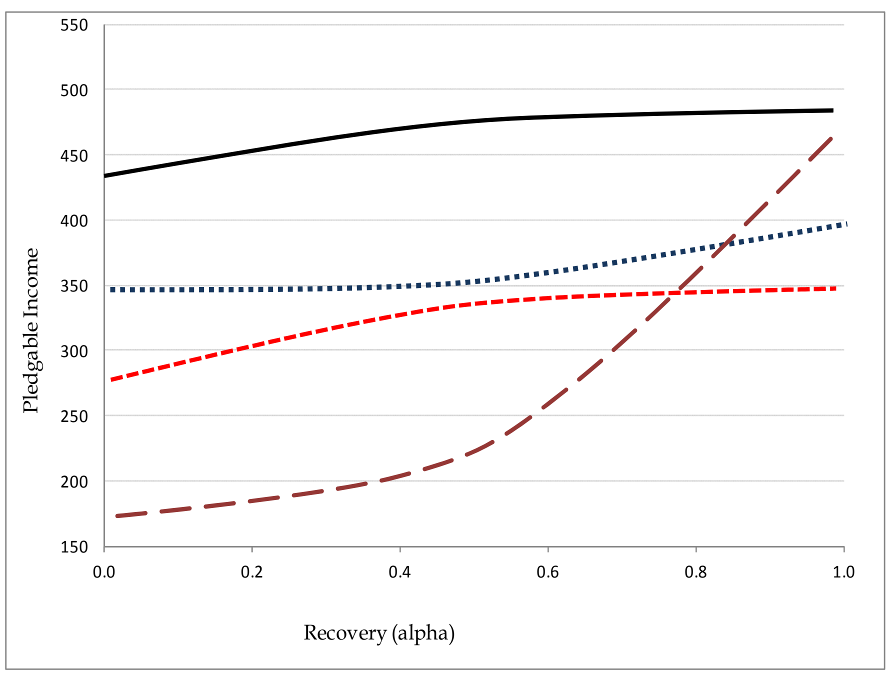
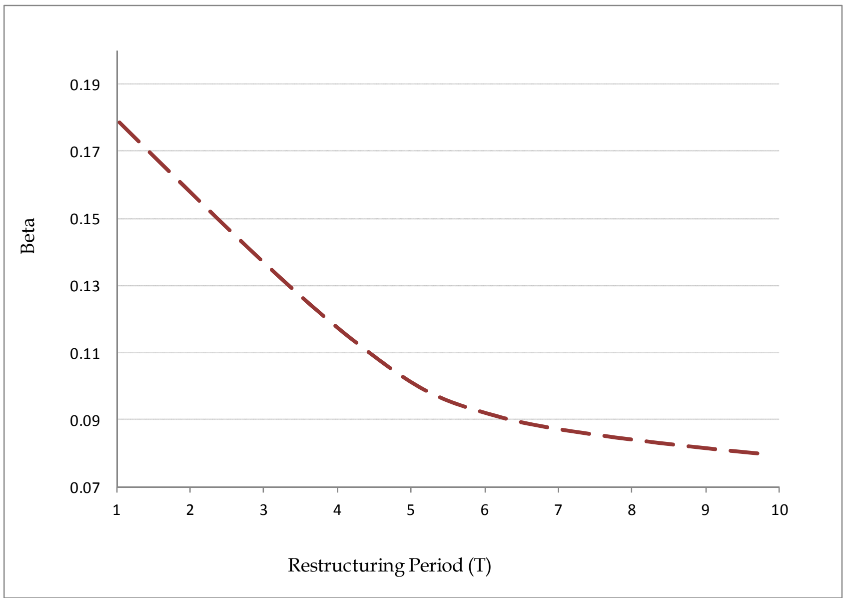

550
500
450
400
350
300
250
200
150
Pledgable Income
0.0
0.2
0.4
0.6
0.8
1.0
Recovery (alpha)

Figure 4
Incentives for Diversion

This figure illustrates the values of the PIKed interest rate (β) and the face value of the PIK-Toggles (F_{pik}) that make the owner/manager indifferent between diversion and paying coupons (in Figure (a)) and saving (in Figure (b)). We assume that. $\mu_T = 110$, $\sigma = 40$, $r = 0.05$, $K = 40$, $\lambda = 0.8$, $\alpha_P = 0$ and $S(0) = 5$. The cash flow is set at $\mu_0 = 80$ and $F_{pik} = 500$ in (a), $\mu_0 = 70$ in (b).

(a)



(b)

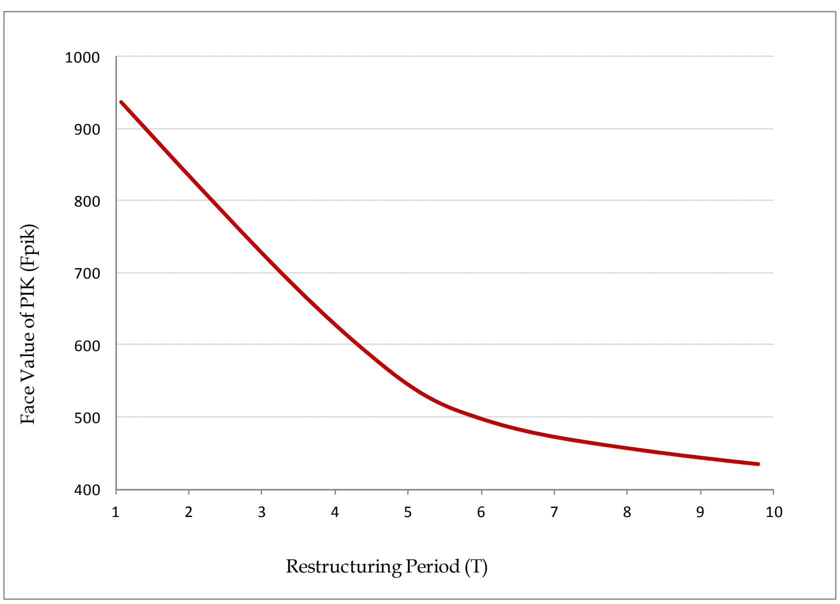


Figure 5
Separating Equilibrium- PIKs vs Regular Debt

This figure illustrates the incremental Equity Holder Benefits from choosing PIKs or regular debt as a function of the pooled expected cash flows during the restructuring period. In all the computations we fix the parameter values where required as: $\mu_T = 110$, $\sigma = 40$, $r = 0.05$, $F_K = 1000$, $K = 40$, $\lambda = 0.8$, $\alpha_P = 0$ and the amount raised as $Z = 500$.