

Strategic Complementarity, Fragility, and Regulation

Xavier Vives*

IESE Business School

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The paper analyzes a very stylized model of crises and demonstrates how the degree of strategic complementarity in the actions of investors is a critical determinant of fragility. It is shown how the balance sheet composition of a financial intermediary, parameters of the information structure (precisions of public and private information), and the level of stress indicators in the market impinge on the degree of strategic complementarity. The model distinguishes between solvency and liquidity risk and characterizes them. Both a solvency (leverage) and a liquidity ratio are required to control the probabilities of insolvency and illiquidity. It is found that in a more competitive environment (with higher return on short-term debt) the solvency requirement has to be strengthened, and in an environment where the fire sales penalty is higher and fund managers are more conservative the liquidity requirement has to be strengthened while the solvency one relaxed. Higher disclosure or introducing a derivatives market may backfire, aggravating fragility (in particular when the asset side of a financial intermediary is opaque) and, correspondingly, liquidity requirements should be tightened. The model is applied to interpret the 2007 run on SIV and ABCP conduits.

Keywords: stress, crises, illiquidity risk, insolvency risk, leverage ratio, liquidity ratio, disclosure, transparency, opaqueness, panic, run, derivatives market. JEL Codes G21, G28.

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1. Introduction

In a crisis situation, and the present financial crisis is a good example, things seem to go wrong at the same time and an adverse shock is magnified by the actions and reactions of the investors.¹ In particular, liquidity evaporates while short-term investors rush for the exit, and a solvency problem may arise. The increased reliance on market funding of financial intermediaries, investment banks in particular but also commercial banks, has been blamed for the increased fragility. The demise of Northern Rock in 2007, and Bear Stearns and Lehman Brothers in 2008, or of the German IKB or Hypo Real State, are cases in point where the short-term leverage of the institutions was revealed as a crucial weakness of their balance sheet.² In this context it has proved difficult to disentangle liquidity from solvency risk. The influence of the opaqueness of financial products, the impact of public news (such as those provided by the ABX index on residential-based mortgage securities, public statements about the health of banks³, or stigma associated to borrowing from the discount window which becomes known⁴), as well as the stabilizing or de-stabilizing influence of derivative markets are also debated. The crisis has put regulatory reform in the agenda. Policy makers and regulators are struggling with how to reform capital requirements, introduce liquidity requirements, and control markets for derivatives.⁵

¹ See, for example, Brunnermeier (2009).

² For example, in June 2007 wholesale funds represented about 26% of liabilities in Northern Rock (Shin (2009)) and short term financing represented an extremely high percentage of total liabilities in Lehman Brothers before the crisis (Adrian and Shin (2010)). Washington Mutual faced a withdrawal of \$16.5 billion of large deposits just in the two weeks before its collapse (according to the Office of Thrift Supervision). See also the evidence in Ivashina and Scharfstein (2010).

³ Such as the case of the run on IndyMac Bancorp in June 2008 which followed shortly after the public release of letters by Senator Schumer of the Banking Committee.

⁴ See Armantier et al. (2010).

⁵ See, for example, FSA (2009) and BIS (2009). The Dodd-Frank Act (2010) introduces a leverage limitation for financial holding companies above a certain size. BIS (2009) proposed two new liquidity ratios: a liquidity coverage ratio to cover short term cash outflows with highly liquid assets and a net stable funding ratio to cover required stable funding with available stable funds. It is worth noting that Bear Stearns was regulated by the SEC and was in fact subject to a liquidity requirement which proved ineffective in the crisis.

The recent financial crisis has reopened the debate on whether crises are fundamentals or panic-driven. Friedman and Schwartz (1963) in their analysis of the Great Depression argued forcefully that many bank failures arose out of panics, that is, because of liquidity rather than solvency problems. This would be consistent with the self-fulfilling view of crises of Bryant (1980) and Diamond and Dybvig (1983). Gorton (1988) disputed the view that crises are panic driven with a study of crises in the National Banking Era concluding that crises were predictable (see also Gorton (1985) and Schotter and Yorulmazer (2009)). The information view of crises has been developed, among others by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998).⁶ Calomiris and Mason (2003) dispute also the analysis of Friedman and Schwartz (1963) and conclude that some episodes of banking crises in the 1930s in the US can be explained by deteriorating fundamentals while others could be explained by a panic component dominating (as the crises in January and February of 1933). Starr and Yilmaz (2007) claim that both fundamentals and panic elements coexist in the explanation of the dynamics of bank runs in Turkey.

In this paper I present a model that bridges the panics and fundamentals view of crises and show how a worsening of some stress indicators may trigger a downward spiral by increasing strategic complementarity in the actions of investors and magnify the impact of bad news. The model disentangles liquidity from solvency risk and shows how their control needs both a solvency (leverage) and a liquidity requirement. It is found also that more transparency or adding a derivatives market may be counterproductive in terms of stability. This is particularly so when the asset side of a financial intermediary is very opaque.

The paper presents a very stylized model of a crisis where investors have to decide whether to keep the investment or run. The investment may be in a currency, bank, or short-term debt. A financial intermediary will be our main focus. The degree of strategic

⁶ Postlewaite and Vives (1987) present a model with incomplete information about the liquidity shocks suffered by depositors featuring a unique Bayesian equilibrium with a positive probability of bank runs. In their model there is no uncertainty about the fundamental value of the banks' assets and no solvency problems.

complementarity in the actions of investors, that is, how the actions of investors move together, is shown to be a critical determinant of fragility. It is characterized how the degree of strategic complementarity depends on the balance sheet composition of a financial intermediary, parameters of the information structure of investors (precisions of public and private information), and the level of stress indicators in the market. In the case of a financial intermediary, strategic complementarity increases with a weaker balance sheet (higher leverage) and with stress indicators such as the required return of short-term debt of the bank (competitive pressure) or the level of the fire-sale penalty of early asset liquidation. A weaker balance sheet or an increase in stress indicators raises fragility, the probability of a crisis, and the range of fundamentals for which there is coordination failure from the point of view of the institution attacked (that is, when the institution is solvent but illiquid). Moreover, the impact of bad news, say a public signal about weak fundamentals, is magnified when strategic complementarity is high. At the same time public signals, coming for example, from a derivatives market, may be destabilizing (over and above the strict content of the news), and the more so the more precise they are.

The model is based on the theory of games with strategic complementarities with incomplete information (an example of which are the “global games” of Carlsson and van Damme (1993) and Morris and Shin (1998); see Vives (2005)).⁷ The model provides a general framework, which nests the contributions of Morris and Shin (1998, 2004) and Rochet and Vives (2004), and the following incremental contributions over the received literature: (1) Show the link between strategic complementarity and fragility (by showing how the degree of strategic complementarity is the crucial parameter to characterize equilibrium and how strategic complementarity relates to the deep parameters in the model); (2) deliver predictions both in the case where there is a unique equilibrium and where there are multiple equilibria; (3) characterize illiquidity and insolvency risk and show how do they depend on the composition of the balance sheet of a financial intermediary; (4) show how a regulator to control the probabilities of insolvency and

⁷ The importance of strategic complementarities in macro models has been highlighted by Cooper and John (1988).

illiquidity has to set solvency and liquidity requirements; and (5) apply the model to interpret the 2007 run on SIVs and the role of a derivatives market.

The policy message that follows from the analysis is that a regulator, with the tools available, needs to pay attention to the composition of the balance sheet of financial intermediaries, in particular to the ratio of cash to unsecured short-term debt and to the short-term leverage ratio (ratio of unsecured short-term debt to equity or, more in general, stable funds). Those two ratios, together with the required return of short-term debt and parameters of the information structure, are crucial determinants of the probabilities of insolvency and illiquidity. Often minimum ratios on solvency (inverse of short-term leverage) and liquidity will be sufficient to control the probabilities of insolvency and illiquidity. It is found that in a more competitive environment (with higher return on short-term debt) the solvency requirement has to be strengthened, and in an environment where the fire sales penalty increases and fund managers become more conservative the liquidity requirement has to be strengthened while the solvency one relaxed. The introduction of a derivatives market should go together with tightened regulation.

The plan of the paper is as follows. Section 2 presents the basic model and examples. Section 3 characterizes equilibria and its links with strategic complementarity. Section 4 analyzes illiquidity and insolvency risk and develops the comparative statics properties of equilibria. Section 5 deals with the literature connections and evidence, and Section 6 with solvency and liquidity regulation. Section 7 applies and extends the model to the 2007 run on SIV. Concluding remarks close the paper. The Appendix collects the proofs of the propositions.

2. The investors' game and a stylized crisis model

Consider the following binary action game among a continuum of investors of mass one. The action set of player i is $\{0,1\}$, with $y_i = 1$ interpreted as “acting” and $y_i = 0$ “not acting”. To act may be to attack a currency (Morris and Shin (1998)), refuse to roll over

debt (Morris and Shin (2004)), run on a bank or SIV or not renew a certificate of deposit in the interbank market (Rochet and Vives (2004)).

Let $\pi^1 = \pi(y_i = 1, y; \theta)$ and $\pi^0 = \pi(y_i = 0, y; \theta)$ where y is the fraction of investors acting and θ is the state of the world. The differential payoff to acting is $\pi^1 - \pi^0 = B > 0$ if $y \geq h(\theta; \alpha)$, and $\pi^1 - \pi^0 = -C < 0$ if $y < h(\theta; \alpha)$, where $h(\theta; \alpha)$ is the critical fraction of investors above which it pays to act and α will be an index of vulnerability or stress. We have that

$\pi^1 - \pi^0$	$y > h(\theta; \alpha)$ $B > 0$	$y \leq h(\theta; \alpha)$ $-C < 0$
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Let $\gamma \equiv C/(B + C)$ be the critical success probability of the collective action such that it makes an agent indifferent between acting and not acting. This is the ratio of the cost of acting to the differential incremental benefit of acting in case of success in relation to failure. An investor will “act” if his assessed probability of successful mass action is larger than γ . It is assumed that $h(\cdot; \alpha)$ is strictly increasing, crossing 0 at $\theta = \underline{\theta}$ ($\lim_{\theta \uparrow \underline{\theta}} h(\theta; \alpha) = 0$) and 1 at $\theta = \tilde{\theta} > \underline{\theta}$. Note that this allows the function $h(\cdot; \alpha)$ to be discontinuous at $\theta = \underline{\theta}$ with $h(\underline{\theta}; \alpha) > 0$ and $h(\theta; \alpha) < 0$ for $\theta < \underline{\theta}$. More specifically, let h be linear in θ :

$$h(\theta; \alpha) = h_0(\alpha) + h_1(\alpha)(\theta - \underline{\theta}) \text{ for } \theta \geq \underline{\theta}$$

with $h_0(\alpha) \geq 0$, $h_1(\alpha) > 0$, $\partial h_0 / \partial \alpha \leq 0$ and $\partial h_1 / \partial \alpha < 0$, and $h(\theta; \alpha) < 0$ for $\theta < \underline{\theta}$.

A larger α means more vulnerability or a more stressful environment for the institution attacked since the threshold for the attack to be successful is lower (see Figure 1).

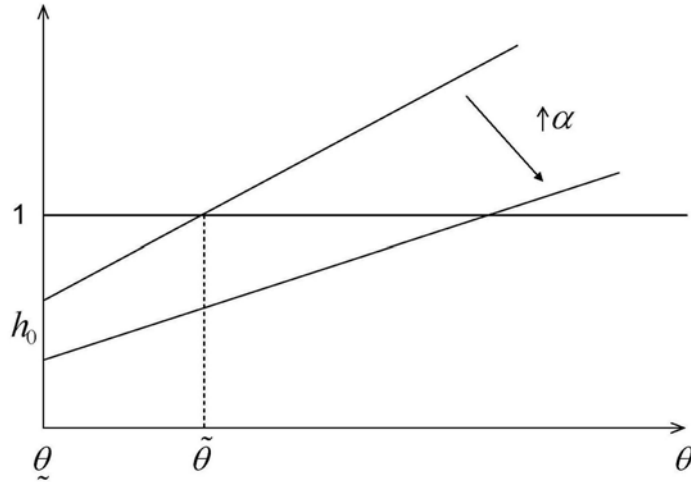


Figure 1. The function $h(\theta; \alpha)$ is the critical fraction of investors above which it pays to act; α is the stress indicator.

The game is of strategic complementarities since $\pi^1 - \pi^0$ is increasing in y .⁸ In fact, $\pi(y_i, (y, \theta))$ has increasing differences in $(y_i, (y, -\theta))$. It follows from these payoffs, if the state of the world is known, that if $\theta < \underline{\theta}$ then it is a dominant strategy to act; if $\theta > \tilde{\theta}$ then it is a dominant strategy not to act; and for $\theta \in (\underline{\theta}, \tilde{\theta})$ there are multiple equilibria. Both everyone acting and no one acting are equilibria. Since the game is a game of strategic complementarities there is a largest and a smallest equilibrium. That is, there are extremal equilibria. The largest equilibrium is $y_i = 1$ for all i if $\theta \leq \tilde{\theta}$, and $y_i = 0$ for all i if $\theta > \tilde{\theta}$, and it is (weakly) decreasing in θ . This is a consequence of $\pi^1 - \pi^0$ being decreasing in θ .

From now on I consider an incomplete information version of the game where investors have a Gaussian prior on the state of the world $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$ and investor i observes a

⁸ In a game of strategic complementarities the marginal return of the action of a player is increasing in the level of the actions of rivals. This leads to best replies being monotone increasing. See Vives (2005).

private signal $s_i = \theta + \varepsilon_i$ with Gaussian i.i.d. distributed noise $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$.⁹ It is worth noting that the prior mean μ_θ of θ can be understood as a public signal of precision τ_θ .

The model encompasses several crisis situations studied in the literature: currency attacks, loan foreclosure and bank runs. Our leading example will be the third.

Currency attacks. A first example of the model is a streamlined version of the currency attacks model of Morris and Shin (1998) where θ represents the reserves of the central bank (with $\theta \leq 0$ meaning that reserves are depleted). Each speculator has one unit of resources to attack the currency ($y_i = 1$) at a cost C . Letting $h(\theta; \alpha) = \alpha^{-1}\theta$, where $\alpha > 0$ is the mass of attackers or α^{-1} the proportion of uncommitted reserves of the central bank, the attack succeeds if $y \geq \alpha^{-1}\theta$. Still α could be interpreted as the wealth available to a fixed mass of speculators. The capital gain if there is depreciation is fixed and equal to $\hat{B} = B + C$. We have that $\gamma \equiv C/(B + C)$ is likely to be small.

Loan foreclosure. A second example, formally equivalent to the first, is foreclosing a loan to a firm (Morris and Shin (2004)).¹⁰ Here θ is the ability of the firm to meet short-term claims (where $\theta \leq 0$ means no ability). There are many creditors and creditor i forecloses if $y_i = 1$. In this case $h(\theta; \alpha) = \alpha^{-1}\theta$ where $\alpha > 0$ is the mass of creditors (or α^{-1} proportion of uncommitted liquid resources of the firm) and the project fails if $y \geq \alpha^{-1}\theta$. The face value of the loan is L , the value of collateral (at interim liquidation) is $K < L$ and let $B = K$ and $C = L - K$. We have that $\gamma \equiv 1 - K/L$.

9 This is referred to in the literature as a “global game”. Those games were introduced by Carlsson and van Damme (1993) as games of incomplete information with types determined by each player observing a noisy signal of the underlying state. The goal is to select an equilibrium with a perturbation in a complete information game with multiple equilibria. The basic idea is that players entertain the “global picture” of slightly different possible games being played. Each player has then a noisy signal of the game being played.

10 Bebchuk and Goldstein (2009) consider a variation of the model to study self-fulfilling credit market freezes and use it to assess the responses of the government in the present crisis.

A bank run. The third example models a run in a bank (or, more generally, a leveraged financial institution) after Rochet and Vives (2004). Traditional bank runs were the outcome of massive deposit withdrawal by individual depositors. Modern bank runs are the outcome of non-renewal of short-term credit in the interbank market, like in the case of Northern Rock, the 2007 run on SIV, or the 2008 run by short-term creditors in the case of Bear Stearns or Lehman Brothers.¹¹

Consider a market with three dates: $t = 0, 1, 2$. At date $t = 0$, the bank has own funds E (taken to include stable resources: equity, long-term debt and even insured deposits) and uninsured short term debt (e.g. uninsured wholesale deposits, certificates of deposit (CDs)) in amount $D_0 \equiv 1$.¹² These funds are used to finance risky investment I and cash reserves M . The balance sheet constraint at $t = 0$ is thus $E + D_0 = I + M$. The returns θI on these assets are collected at date $t = 2$ and if the bank can meet its obligations, the short-term debt is repaid at face value D , and the equityholders of the bank obtain the residual (if any). Investors are also entitled to the face value D if they withdraw in the interim period $t = 1$. Let $m \equiv M/D$ be the liquidity ratio, $\ell = D/E$ be the short-term leverage ratio, and $d = D/D_0$ the return of the short-term debt.

A continuum of fund managers makes investment decisions in the short-term debt market. At $t = 1$ each fund manager, after the observation of a private signal about the future realization of θ , decides whether to cancel ($y_i = 1$) or renew his position ($y_i = 0$). If $y > M$ then the bank has to sell some of its assets¹³ in a secondary market to meet payments. The early liquidation value of the assets of the bank involves a fire sales penalty $\lambda > 0$ (retrieving only $\theta/(1 + \lambda)$ for each unit invested).¹⁴

¹¹ This example can be reinterpreted also replacing bank by country and short-term debt for foreign-denominated short-term debt.

¹² The distinction of stable funds within liabilities is made also in the BIS (2009) document dealing with liquidity risk.

¹³ Or borrow against collateral in the repo market.

¹⁴ In case of secured collateral λ relates to the haircut required. Haircuts on asset backed securities rose dramatically after the collapse of Lehman Brothers (see, e.g., Gorton and Metrick (2010)). It is

A fund manager is rewarded for taking the right decision (that is, withdrawing if and only if the bank fails). The cost of canceling the investment is C and the benefit for getting the money back or canceling when the bank fails is $\hat{B} = B + C$. Again, $\gamma \equiv C/(B + C)$ is likely to be small since $B + C$ is the benefit to make the right decision. What is crucial is that investors, whatever the reason, adopt a behavioral rule of the type: cancel the investment if and only if the probability that the bank fails is above threshold γ . This rule will arise also if investors expect a fixed return when withdrawing, nothing if they withdraw and the bank fails, and there is a (small) cost of withdrawing. Note that a larger γ is associated to a less conservative investor. Risk management rules may therefore influence γ .

Let $\underline{\theta} \equiv (D - M)/I = (1 - m)/(\ell^{-1} + d^{-1} - m)$ (using the balance sheet constraint) be the solvency threshold of the bank¹⁵, such that if $\theta < \underline{\theta}$ the bank fails even if all fund managers renew credit to the bank; and $\tilde{\theta} \equiv (1 + \lambda)\underline{\theta}$ the “supersolvency” threshold, such that a bank does not fail even if no fund manager renews his CDs. Under these conditions the bank fails if

$$y \geq h(\theta; \alpha) \equiv m + \frac{\ell^{-1} + d^{-1} - m}{\lambda}(\theta - \underline{\theta})$$

for $\theta \geq \underline{\theta}$, and $h(\theta; \alpha) < 0$ otherwise. Here we have that $h_0 = m > 0$. Note that we can let $\alpha \equiv \lambda$, $\alpha \equiv d$ (in the latter case $\underline{\theta}$ is decreasing in d and therefore increasing d lowers the threshold $h(\theta; \alpha)$), and $\alpha \equiv \ell$ (we have that $\partial h/\partial \ell < 0$ since $\partial \underline{\theta}/\partial \ell > 0$).

important to remark that the liquidity requirements on broker-dealers in the US were related to unsecured funding while, for example, the demise of Bear Stearns in the end happened because of its failure to renew its secured funding. (See SEC’s Oversight of Bear Stearns and Related Entities (SEC, 2008).)

¹⁵ It is worth noting that the solvency threshold is related to what the FDIC calls the “Net Non-core Funding Dependence Ratio”, as part of the CAMELS assessment, computed as non-core liabilities, less short-term investments divided by long-term assets.

Note also that $\text{sign}\{\partial\theta/\partial m\} = \text{sign}\{1 - \ell^{-1} - d^{-1}\}$ and if $1 - \ell^{-1} - d^{-1} < 0$ then $\partial\theta/\partial m < 0$ and $\partial h/\partial m > 0$ (despite the fact that $\partial h_1/\partial m < 0$).¹⁶ (See Figure 1).

In the balance sheet of a financial intermediary typically we have $1 - \ell^{-1} - d^{-1} < 0$. Indeed, the ratio of (uninsured) short-term debt over stable funds (equity, long term debt and insured deposits) $\ell = D/E$ is below 1 for commercial banks, and although it is above 1 for investment and wholesale banks typically $d^{-1} \geq .9$ (with an interest rate of at most 10%), and therefore we would need $\ell^{-1} < .1$ or $\ell > 10$ to have $1 - \ell^{-1} - d^{-1} > 0$.¹⁷ (See Figure 2.) For a typical SIV we have also that $\ell < 1$.¹⁸ We assume henceforth, unless otherwise stated, that $1 - \ell^{-1} - d^{-1} < 0$.

¹⁶ We have that $\partial h/\partial m = \lambda^{-1}(1 + \lambda - \theta) > 0$ since $\theta < (1 + \lambda)\theta < 1 + \lambda$ as $1 - \ell^{-1} - d^{-1} < 0$ implies that $\theta < 1$.

¹⁷ For example, at September 20, 2008, among commercial banks Wells Fargo had $\ell = .64$, Wachovia $\ell = .54$ and Bank of America $\ell = .89$ where we take $D = \text{Deposits (uninsured) + Short term debt + Other liabilities}$ and $E = \text{Equity + Long term debt + deposits (insured)}$. Citigroup had $\ell = 1.30$, JP Morgan Chase $\ell = 1.95$, Bank of NY Mellon $\ell = 4.53$, Goldman Sachs $\ell = 3.78$, Morgan Stanley: $\ell = 2.70$, and Merrill Lynch $\ell = 1.52$. Only State Street Corp. would go above $\ell = 10$ with $\ell = 14.77$ (which would yield $1 - \ell^{-1} - d^{-1} = 0$ with an interest rate of 7.26%). (Own computation from Veronesi and Zingales (2010), see Figure 2). Lehman Brothers had $\ell = 3.76$ at the end of 2007 (derived from Adrian and Shin (2010)).

¹⁸ According to the Global Financial Stability Report of the IMF (April 2008), the typical funding profile of SIV in October 2007 was 27% in asset-backed commercial paper (ABCP) and the rest in medium-term notes and capital notes. This would mean that $\ell = .34$.

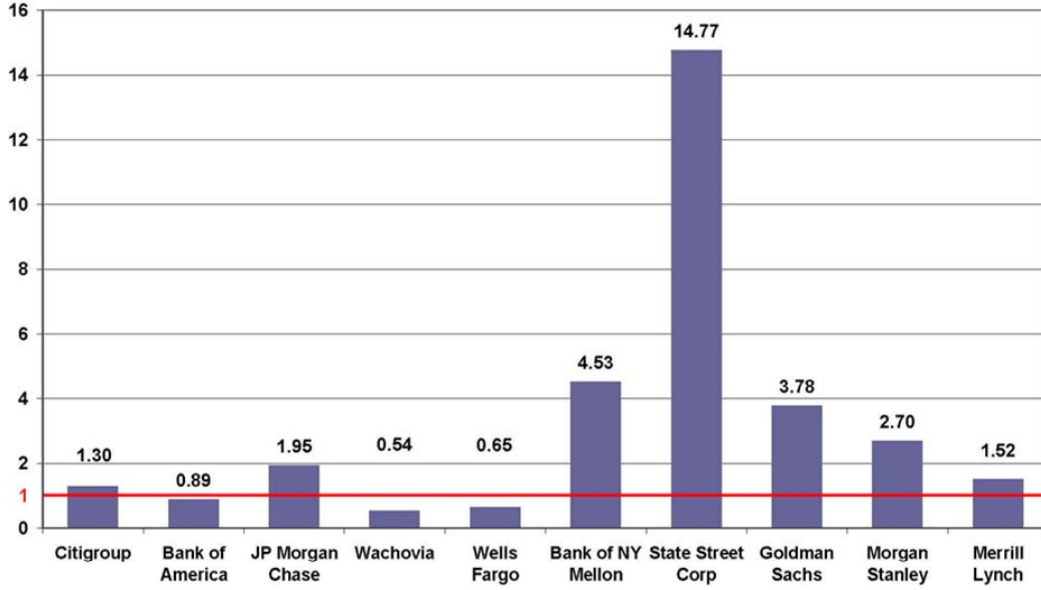


Figure 2: Short-term leverage ratio of US banks at September 30, 2008: $\ell = D/E$, D = Deposits (Uninsured) + Short Term Debt + Other Liabilities, E = Equity + Long Term Debt + Deposits (Insured). Source: Veronesi and Zingales (2010), (data for Goldman Sachs and Morgan Stanley as of 08/31/2008).

3. Equilibrium and strategic complementarity

In this section we study equilibria of the investors' game, the factors influencing the degree of strategic complementarity, and characterize precisely insolvency and illiquidity risk.

The following proposition characterizes the equilibria of the investors' game. The proof is standard and provided in the Appendix for completeness.

Proposition 1. Let $\gamma \equiv C/(B + C)$.

- (i) An equilibrium is characterized by two thresholds (s^*, θ^*) with s^* yielding the signal threshold below which an investor acts and $\theta^* \in [\underline{\theta}, \tilde{\theta}]$ the state-of-the-world critical threshold, below which the acting mass is successful. The probability of a crisis conditional on $s = s^*$ is γ .

(ii) There are at most three equilibria. There is a critical $\bar{h}_0 \in (0,1)$ such that $\theta^* = \underline{\theta}$ for $h_0 \geq \bar{h}_0$, and for $h_0 < \bar{h}_0$ we have that $\theta^* > \underline{\theta}$ and then the equilibrium is unique if $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$. When $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$ there is a range of γ for which there are multiple equilibria.

In order to gain some intuition into the structure of the game and the result let us think in terms of the best reply of a player to the (common) signal threshold used by the other players. Let $P(s, \hat{s})$ be the conditional probability that the acting players succeed if they use a (common) threshold \hat{s} when the player considered receives a signal s (and denote by Φ the standard Normal cumulative distribution). We have that

$$P(s, \hat{s}) \equiv \Pr(\theta < \hat{\theta}(\hat{s})/s) = \Phi\left(\frac{\sqrt{\tau_\theta + \tau_\varepsilon}\left(\hat{\theta}(\hat{s}) - \frac{\tau_\theta\mu_\theta + \tau_\varepsilon s}{\tau_\theta + \tau_\varepsilon}\right)}{\sqrt{\tau_\theta + \tau_\varepsilon}}\right),$$

where $\hat{\theta}(\hat{s})$ is the critical θ below which there is success when players use a strategy with threshold \hat{s} ($\hat{\theta}(\hat{s})$ is the solution in θ of $\Phi(\sqrt{\tau_\varepsilon}(\hat{s} - \theta)) - h(\theta) = 0$, which is increasing in \hat{s}). It is immediate then that $\partial P/\partial s < 0$ and $\partial P/\partial \hat{s} \geq 0$. Given that other players use a strategy with threshold \hat{s} , the best response of a player is to use a strategy with threshold s^* where $P(s^*, \hat{s}) = \gamma$: act if and only if $P(s, \hat{s}) > \gamma$ or, equivalently, if and only if $s < s^*$. This defines a best-response function in terms of thresholds

$$r(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \hat{\theta}(\hat{s}) - \frac{\tau_\theta}{\tau_\varepsilon} \mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon} \Phi^{-1}(\gamma).$$

We have that $r' = -(\partial P/\partial \hat{s})/(\partial P/\partial s) \geq 0$ and the game is, indeed, of strategic complementarities: a higher threshold \hat{s} by others induces a player to use also a higher threshold. It can be shown that if $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$ then $r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \hat{\theta}'(\hat{s}) \leq 1$.

This ensures that $r(\cdot)$ crosses the 45° line only once and that the equilibrium is unique. Note that r' is increasing in τ_θ since $\hat{\theta}'(\hat{s})$ is independent of τ_θ . A sufficient condition to have multiple equilibria (necessary also for regular equilibria for which $r'(\hat{s}) \neq 1$) is that $r'(s) > 1$ for $r(s) = s$. (See Figure 3a).

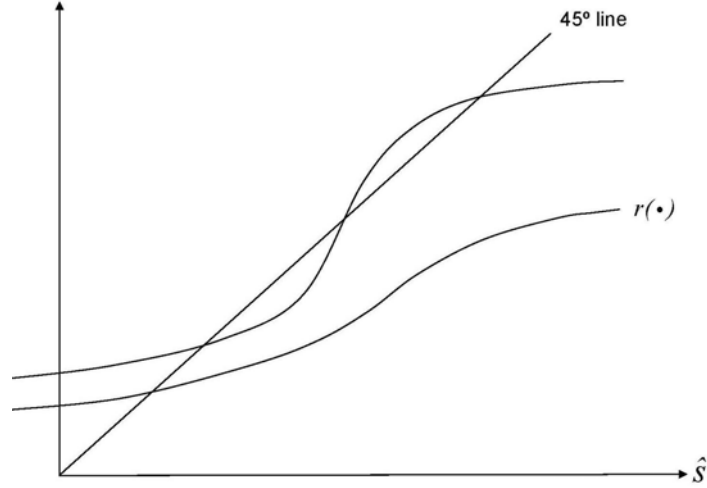


Figure 3a: Best response of a player to the threshold strategy \hat{s} used by rivals (the flatter best response corresponds to the case $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$ while the steeper one to the case $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$ when there is multiplicity).

An example of Figure 3a would be the following. When $h_0 < \bar{h}_0$ and τ_θ is low enough we know that there is a unique equilibrium. As τ_θ increases the degree of strategic complementarity increases and we have typically (and generically) three equilibria.

In Figure 3b it is illustrated how the equilibrium set changes as we increase α when γ is an intermediate range: from a unique equilibrium to multiple and back to uniqueness (for example, $h(\theta) = \alpha^{-1}\theta$ with $\alpha \in \{1, 4, 8\}$, $\tau_\theta = \tau_\varepsilon = 1$, $\mu_\theta = 0.5$, and $\gamma = .85$).¹⁹ In both cases as α increases, strategic complementarity increases in the relevant range inducing multiple equilibria. If strategic complementarity increases further we go back to a unique equilibrium since then strategic complementarity is strong in an irrelevant range.

¹⁹ When $h_0 < \bar{h}_0$ and h_1 is large enough (α low) we know that there is a unique equilibrium. It is possible to check also that, for an intermediate range of γ , as h_1 decreases (α increases) we have multiple equilibria (generically three), and that if h_1 decreases further (α increases more) we go back to a unique equilibrium. A similar situation arises with movements in γ , but then the best reply moves vertically (with $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$, for example, when $h(\theta) = \theta/10$, $\tau_\theta = 1$, $\tau_\varepsilon = 5$, $\bar{\theta} = 5$ and $\gamma \in \{.1, .5, .9\}$).

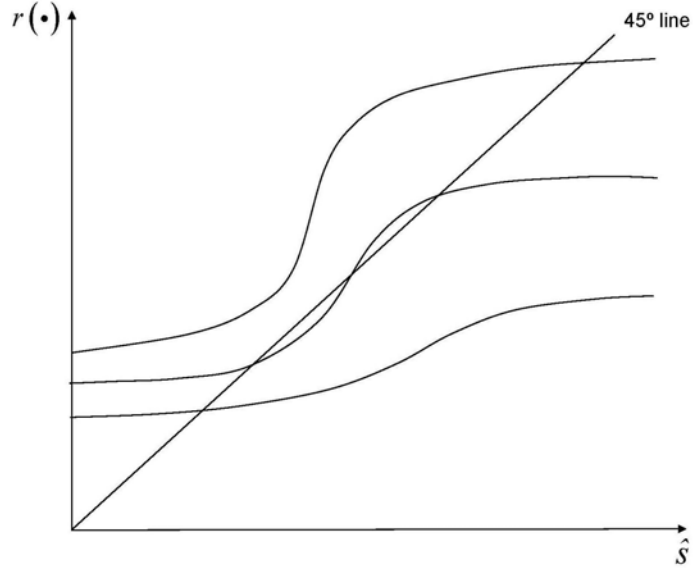


Figure 3b: Possible best responses of a player to the threshold strategy \hat{s} used by rivals for an intermediate range of γ . The lower (upper) branch corresponds to α low (high), the intermediate to α intermediate.

A necessary condition for multiple equilibria is that strategic complementarity be strong enough, a sufficient condition is that strategic complementarity be strong enough at relevant points (candidate equilibria). Indeed, when strategic complementarity is moderate always then there is a unique equilibrium, when it is not there may be multiple equilibria (see Figure 3a,b).

The strength of the strategic complementarity among the actions of the players depends on the slope of the best response. The maximal value of the slope is

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1 \sqrt{2\pi\tau_\varepsilon}},$$

which is increasing in h_1^{-1} and in τ_θ , and decreasing and then increasing in τ_ε (with $\bar{r}' \rightarrow \infty$ as $\tau_\varepsilon \rightarrow 0$ and $\bar{r}' \uparrow 1$ as $\tau_\varepsilon \rightarrow \infty$).²⁰ Note that $\bar{r}' \leq 1$ if and only if $(\tau_\theta / \sqrt{\tau_\varepsilon}) \leq h_1 \sqrt{2\pi}$. The degree of strategic complementarity will be higher in a more

²⁰ It is easily checked that $\text{sign}\{\partial \bar{r}' / \partial \tau_\varepsilon\} = \text{sign}\{(1/2)h_1 \sqrt{2\pi\tau_\varepsilon} (1 - \tau_\theta \tau_\varepsilon^{-1}) - \tau_\theta\}$, in particular $\partial \bar{r}' / \partial \tau_\varepsilon < 0$ for $\tau_\varepsilon < \tau_\theta$.

stressful situation (α larger which increases h_1^{-1}) and/or with a precise prior (τ_θ large) and/or imprecise signals (τ_ε low). The equilibrium is unique with α and/or $\tau_\theta/\sqrt{\tau_\varepsilon}$ small because when those parameters are small the strength of the strategic complementarity among the actions of the players is low and the slope of the best response is less than 1 ($r' \leq 1$, Figure 3).

When α is smaller a change in fundamentals θ changes more the critical threshold $h(\theta)$. This implies that a change in the strategy threshold \hat{s} used by other investors leads to a smaller optimal reaction since the induced change in the conditional probability that the acting players succeed is smaller. When noise in the signals is smaller a player faces greater uncertainty about the behavior of others and the strategic complementarity is lessened. Consider the limit cases $\tau_\varepsilon \rightarrow +\infty$ (or, equivalently, a diffuse prior $\tau_\theta \rightarrow 0$). Then the distribution of the proportion of acting players \tilde{y} is uniformly distributed over $[0,1]$ conditional on $s_i = s^*$. This means that players face maximal strategic uncertainty. In contrast, at any of the multiple equilibria with complete information when $\theta \in (\underline{\theta}, \tilde{\theta})$, players face no strategic uncertainty (e.g. in the equilibrium in which everyone acts, a player has a point belief that all other players will act).

In the bank example case maximal strategic complementarity among investors is increasing in $h_1^{-1} = \lambda/(\ell^{-1} + d^{-1} - m)$. That is, strategic complementarity is more likely to be higher with higher short-term leverage ℓ , face value of short-term debt d , or fire sales penalty λ . Interestingly, strategic complementarity will tend to be higher when the liquidity ratio is higher but recall that if $1 - \ell^{-1} - d^{-1} < 0$ then $\partial h/\partial m > 0$ and a run is less likely to be successful.

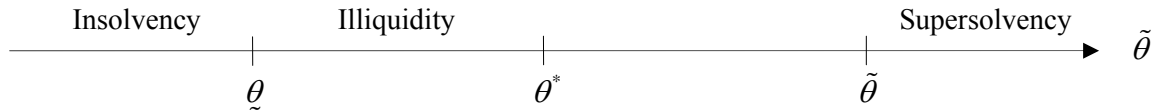
4. Illiquidity risk and insolvency risk

This section characterizes coordination failure, liquidity and solvency risk, and how they depend on the stress indicators and parameters of the information structure. To do so it

develops the comparative statics properties of equilibria, both when the equilibrium is unique or multiple, and works out the implications in the examples considered.

4.1. Coordination failure, illiquidity risk, and insolvency risk

At equilibrium with threshold θ^* , there is a crisis when $\theta < \theta^*$. In the range $[\theta^*, \tilde{\theta})$ there is coordination failure from the point of view of investors, because if all of them were to act then they would succeed. For example, in the range $[\theta^*, \tilde{\theta})$ if currency speculators were to coordinate their attack then they would succeed, but in fact the currency holds. In the range $[\underline{\theta}, \theta^*)$ there is coordination failure from the point of view of the institution attacked. In the bank case in the range $[\underline{\theta}, \theta^*)$ the bank is solvent but illiquid, that is, the bank would have no problem if only investors would renew their short-term debt, but in the range they do not and the bank is illiquid.²¹



The risk of illiquidity is therefore given by $Pr(\underline{\theta} \leq \theta < \theta^*)$ and the risk of insolvency by $Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$. The latter is the probability that the bank is insolvent when there is no coordination failure from the point of view of the bank. The overall probability of a “crisis” is $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$. Note that $Pr(\underline{\theta} \leq \theta < \theta^*) = Pr(\theta < \theta^*) - Pr(\theta < \underline{\theta})$. A crisis occurs for low values of the fundamentals. In contrast, in the complete information model there are multiple self-fulfilling equilibria in the range $(\underline{\theta}, \tilde{\theta})$.

²¹ As argued by Rochet and Vives (2004) this provides a rationale for a Lender of Last Resort intervention with the discount window.

4.2. The comparative statics of risk

The comparative statics results hold when the equilibrium is unique, and when there are multiple equilibria then the results hold for the extremal equilibria (the largest and the smallest). Furthermore, the results hold for any equilibrium, even the middle unstable one, if out-of-equilibrium adjustment is adaptive. With best-reply dynamics at any stage after the parameter perturbation from equilibrium, a new state of the world θ is drawn independently and a player responds to the strategy threshold used by other players at the previous stage. Then a parameter change that moves monotonically the best reply will induce a monotone adjustment process with an unambiguous prediction. For instance, if we are at the higher equilibrium of the middle branch in Figure 3b, a decrease in α may induce a movement to the lower branch and best reply dynamics would settle at the unique (and lower) equilibrium.²² The following proposition states the results (see the proof in the Appendix).

Proposition 2. Comparative statics. Let $h_0 < \bar{h}_0$. At extremal equilibria (or under adaptive dynamics):

- (i) Both θ^* , s^* and the probability of crisis $P(\theta < \theta^*)$ are decreasing in γ (i.e., with less conservative investors) and in the expected value of the state of the world μ_θ , and increasing in stress indicator α .
- (ii) The range of fundamentals $[\underline{\theta}, \theta^*)$ for which there is coordination failure and illiquidity (from the point of view of the institution attacked) is decreasing in γ and increasing in α .
- (iii) A release of a public signal μ_θ has a multiplier effect on equilibrium thresholds (i.e. over and above the impact on the best response of an investor), which is enhanced if τ_θ is higher.
- (iv) If $\gamma < 1/2$ and μ_θ is low then a more precise public signal increases θ^* , $P(\theta < \theta^*)$ and the range $[\underline{\theta}, \theta^*)$ while a more precise private signal reduces it.

²² See Echenique (2002) and Vives (2005).

Remark: The region of potential multiplicity $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1(\alpha)\sqrt{2\pi}$ is enlarged with an increase in stress indicator α and/or an increase in the precision of the public signal in relation to the private one $\tau_\theta/\sqrt{\tau_\varepsilon}$.

With potential multiplicity a marginal change may have a large effect. Suppose that we start in the uniqueness region and that by increasing α we move to the multiplicity region. Then since increasing α moves the best reply r up, best reply dynamics starting at the initial equilibrium would settle at the low threshold equilibrium with the new α . This would represent a marginal change. However, a further increase in α may make disappear all equilibria except the high threshold one. This will imply a discrete jump to the high threshold equilibrium. Note that if the initial position is at the middle unstable equilibrium an increase in α will imply a discrete jump to the high threshold equilibrium. (See Figure 3.b.) Similarly, if we start at the high threshold equilibrium and we *decrease* α and move to the uniqueness region then best reply dynamics will lead us to the low threshold equilibrium.

Similar effects apply to increased public information τ_θ . This means that releasing more public information (e.g., by the Central Bank), is not necessarily good. To start with it will not be good if fundamentals are weak (μ_θ low). Furthermore, a public signal, which becomes common knowledge, has the capacity to move to a higher threshold equilibrium with a higher probability of a crisis. The analysis may therefore rationalize oblique statements (and “constructive ambiguity”) by central bankers and other regulatory authorities which seem to add noise to a basic message. And the more so in situations where vulnerability is high.

We can specialize the comparative static results in Proposition 2 to the examples. For instance, from result (i) we have immediately that the probability of a currency crisis is decreasing in the relative cost of the attack γ and in the expected value of the reserves of the central bank μ_θ . The following Corollary collects further results.

Corollary 1:

- (i) In the currency crisis or credit foreclosure cases ($h(\theta) = \alpha^{-1}\theta$), the probability of illiquidity $Pr(\underline{\theta} \leq \theta < \theta^*)$ increases with α while the probability of insolvency $Pr(\theta < \underline{\theta})$ is unaffected by α .
- (ii) In the bank case ($h(\theta) = m + \lambda^{-1}(\ell^{-1} + d^{-1} - m)(\theta - \underline{\theta})$ with $\underline{\theta} \equiv (1 - m)/(\ell^{-1} + d^{-1} - m)$) there is a critical liquidity ratio $\bar{m} \in (0,1)$ such that for $m < \bar{m}$, $\theta^* > \underline{\theta}$ and $\theta^* = \underline{\theta}$ for $m \geq \bar{m}$. Assume $1 - \ell^{-1} - d^{-1} < 0$ and $m < \bar{m}$. Then:
- The probability of insolvency $Pr(\theta < \underline{\theta})$ is decreasing in the liquidity ratio m and in the expected return on the bank's assets μ_θ , increasing in the short-term leverage ratio ℓ and the face value of debt d , and independent of the fire-sales penalty λ , and the critical withdrawal probability γ .
 - The probability of failure $Pr(\theta < \theta^*)$ (and the critical θ^*) are decreasing in m , γ , and μ_θ , and increasing in λ , ℓ , and d .
 - The range of illiquidity $[\underline{\theta}, \theta^*)$ is decreasing in m and increasing in d . Both the range and the probability of illiquidity $Pr(\underline{\theta} \leq \theta < \theta^*)$ are decreasing in γ and increasing in λ and ℓ .

Remark: A tax δ on short-term liabilities D_0 (like the one proposed by the Obama administration or the UK government) changes the balance constraint at $t = 0$ to $E + (1 - \delta)D_0 = I + M$ and therefore increases d to $d = D/((1 - \delta)D_0)$. Therefore, for a given D_0 the tax will increase the probability of failure. This will be counteracted by a lower propensity to take short-term debt.

In all situations an increase in α (vulnerability or stress), be it the mass of attackers/creditors, the inverse of the proportion of uncommitted reserves of the central bank or of liquid resources of the firm, face value of deposits of bank, or level of fire-sales penalty for early liquidation, increases fragility by increasing the degree of strategic

complementarity. The increase in the face value of debt can be interpreted as an increase in competitive pressure. An increase in the liquidity ratio makes an attack less likely (whenever $1 - \ell^{-1} - d^{-1} < 0$ since then $\partial h / \partial m > 0$) but need not decrease the degree of strategic complementarity. Indeed, with a high enough liquidity ratio the risk of illiquidity can be eliminated, however this will come at the cost of less investment in the risky asset.

5. Literature connections and evidence

Previous literature has obtained results related to the ones in the presented and a growing body of evidence is consistent with them. Let us deal with both issues in turn.

5.1. Received results

Goldstein and Pauzner (2005) also show how increasing the deposit rate increases the probability of a run of depositors in a model of the global games type. Chang and Velasco (2001) in a model of financial crisis in emerging markets in the Diamond and Dybvig (1983) tradition find that financial liberalization increases the expected welfare of depositors but may increase also fragility. In Matutes and Vives (1996), in a model which combines the banking model of Diamond (1984) with a differentiated duopolistic structure à la Hotelling, an increase in rivalry does increase the probability of failure in an interior equilibrium of the depositor's game where banks have positive market shares. Cordella and Yeyati (1998) find that disclosure of a risk exposure of a bank (which is not controlled by the bank manager) may increase fragility by increasing the deposit rates demanded by investors.

According to result (iii) in Proposition 2 public information has a coordinating potential beyond its strict information content as in Morris and Shin (2002). Every player knows that an increase in μ_θ will shift downward the best replies of the rest of the players and everyone will be more cautious in acting. This happens because public information becomes common knowledge and affects the equilibrium outcome. This phenomenon may be behind the apparent overreaction of financial markets to Fed announcements.

Morris and Shin (2009) also study how insolvency risk and illiquidity risk vary with the balance sheet composition of a financial institution in a model where future fundamental uncertainty interacts with the strategic uncertainty of the present. In their model there would be no illiquidity risk if there was no future insolvency risk (ex post uncertainty) since it is assumed that partial liquidation of assets has no long run effect. The authors show that illiquidity risk is (i) decreasing in the ratio of cash plus interim realizable assets to short term liabilities; (ii) increasing in the "outside option ratio" (opportunity cost of the funds of short run debt holders); and (iii) increasing in the ex post variance of the asset portfolio ("fundamental risk ratio"). The results and the broad message of the paper are consistent with ours: regulation needs to pay attention to the balance sheet composition of a financial intermediary.

Several negative feedback loops may aggravate a crisis. For example, in the bank crisis model, the fire-sales penalty will be related to adverse selection. Vives (2010) shows how the asset fire-sale penalty is increasing in the noise in the signals of the bidders and in the amount auctioned, and decreasing in the number of bidders. In a crisis scenario it is plausible to expect noisier signals, an increased amount auctioned, and fewer bidders. All this will mean that when the bank tries to sell more assets because it is in distress it will face a larger discount, and this, in turn, will induce more sales to face the commitments, and further discounts. In the extreme the market may collapse because adverse selection is very severe in relation to the number of bidders. A similar thing may happen with the face value of debt since when a bank in distress needs refinancing it will be offered worse terms, and this aggravates in turn the fragility of the bank.²³

The presence of large players and market power may introduce further issues. In the interbank market example market power may either facilitate liquidity provision (because liquidity is a public good and then sound banks may have an incentive to provide liquidity to a bank in trouble to avoid contagion (Allen and Gale (2004), Sáez and Shi

²³ See Brunnermeier and Pedersen (2009) for a model of a liquidity spiral combining market and funding liquidity, and Bernardo and Welch (2004) for a model where the fear of having to liquidate assets after a crisis may contribute to the frenzy by incentivating sales in the middle of the run. See also Eisenbach (2010) for a feedback model of short-term debt and rollover risk.

(2004)) or may impede it (as banks with surplus funds underprovide lending strategically to induce fire-sales of bank-specific assets of needy intermediaries (Acharya et al. (2010)).²⁴

The literature has also found strategic complementarities in the decisions of individual banks that correlate their decisions to force the central bank to bail them out collectively. Farhi and Tirole (2011) show how private leverage choices of financial intermediaries display strategic complementarities through the response of monetary policy. This makes optimal for banks to adopt a risky balance sheet. Acharya and Yorulmazer (2007) show how the regulator has ex post incentives to bail out failed banks when many fail. This provides incentives to banks, particularly small ones, to herd in their investment policies and increase the risk of collective failure.

5.2 Empirical evidence

There is evidence that in banking crises are driven by both solvency and liquidity issues. Gorton (1988) in his study of crises in the US National Banking Era (1865-1914) concluded that panics were triggered when the "fundamentals" (a leading indicator of recession) reached a certain level. Calomiris and Mason (2003) find that some episodes of banking crises in the 1930s in the US can be explained by deteriorating fundamentals while others are open to being interpreted as the panic component dominating (as the crises in January and February of 1933). Starr and Yilmaz (2007) study bank runs in Turkey and conclude that both fundamentals and panic elements coexist in the explanation of the dynamics of the crises. Schotter and Yorulmazer (2009) find in an experimental study that the severity of bank runs depends on the fundamentals (the state of the economy).

Identifying and estimating strategic complementarities is not an easy task since the strategic aspect must be disentangled from responses of investors to common shocks. Chen et al. (2010) identify strategic complementarities in mutual fund investment by relying precisely on the fact that illiquid funds (those with a higher fire sales penalty λ in

²⁴ See also Corsetti et al. (2004, 2006) for other effects of the presence of large players.

our model) generate larger strategic complementarities for investors than liquid ones since redemptions impose larger costs in the former funds. Hertzberg et al. (2011)) use a natural experiment with a credit registry expansion in Argentina to identify complementarities.

Consistently with result (ii) in Proposition 2, there is experimental evidence that bank runs occur less frequently when banks face less stress (a lower α in our model) in the sense of a larger number of withdrawals being necessary to induce insolvency (Madies (2006) and Garratt and Keister (2009)). There is also evidence of the multiplier effect of public information (Proposition 2 (iii)) in the credit registry expansion in Argentina (Hertzberg et al. (2011)) and with discount window stigma (Armantier et al. (2010)). The latter refers to the reluctance of banks to borrow from the discount window because of fear that the bad news will become publicly known. Armantier et al. (2010) find that during the crisis banks borrowed from the Term Auction Facility, in which the borrowing bank is one of many, at higher rates than those available at the discount window, and that this spread was increasing with more stressed conditions in the interbank market. The spread indicates how much a bank is willing to pay to avoid the release of a public bad signal. This is consistent with the result of having a higher publicity multiplier associated to a higher stress indicator α (corresponding, for example, to a higher fire sales penalty λ). Consistently with the results in the Corollary 1 (ii) among the 72 largest commercial banks in OECD countries, those which relied less on wholesale funding, and had higher capital cushions and liquidity ratios, fared better during the crisis (in the sense of having smaller equity value declines and being subject to less government intervention, see Ratnovski and Huang (2009)).

6. Liquidity and solvency regulation of financial intermediaries

This section examines regulation in the banking model and how the likelihood of crises can be controlled.

Using the liquidity and the leverage ratios it is possible for a regulator to control the maximum probabilities of insolvency q and of a crisis p , and therefore the probability of illiquidity. If the regulator wants that $p = q$ then from Corollary 1 (ii) this can be done setting $m \geq \bar{m}$ and inducing $\theta^* = \underline{\theta}$. In general this will be too costly, because of the reduction in the level of investment in the risk asset, and the regulator will set $p > q$.

We have that $Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$ and therefore $Pr(\theta < \underline{\theta}) \leq q$ if and only if $\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}$ where $\underline{\theta} \equiv (1 - m)/(\ell^{-1} + d^{-1} - m)$. The probability of insolvency is thus decreasing in m , ℓ^{-1} and d^{-1} since $\underline{\theta}$ is. Furthermore, $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ (where θ^* is the largest equilibrium) and therefore $Pr(\theta < \theta^*) \leq p$ if and only if $\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta}$. The cutoff θ^* and the probability of a crisis are decreasing in m , ℓ^{-1} (and also in γ , d^{-1} and λ^{-1}).

Now, choosing m and ℓ^{-1} large enough so that

$$\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}$$

and

$$\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta},$$

(noting that both $\underline{\theta}, \theta^*$ are decreasing in m and ℓ^{-1}) we ensure that $Pr(\theta < \underline{\theta}) \leq q$ and $Pr(\theta < \theta^*) \leq p$. With $p > q$ we have that $(1 + \lambda)\underline{\theta}_q > \theta_p^* > \underline{\theta}_q$ since $(1 + \lambda)\underline{\theta}_q$ is the supersolvency threshold. The constraints are tighter when there is bad news (μ_θ low) and/or the precision of public information τ_θ is larger (but note that, in general, both μ_θ and τ_θ influence also θ^*).

We have, therefore, that the regulator can control the maximum probabilities of insolvency q and of a crisis p , and therefore the probability of illiquidity, by an appropriate induced choice of liquidity m and leverage ℓ ratios. It follows that when $p > q$ both the solvency and the liquidity constraints needed to control the probabilities

of insolvency and illiquidity will have, in principle, to become tighter in a more competitive environment where d is higher. Furthermore, the liquidity constraint (which helps to control the probability of a crisis over and above the strict probability of insolvency when there is no coordination failure) will have to become tighter, in principle, in a situation where λ is higher. Note, however, that there is a partial substitutability between m and ℓ^{-1} since they both decrease $\underline{\theta}$ and θ^* .

The limit case where $\tau_\varepsilon \rightarrow \infty$ allows for a closed-form solution and an explicit expression for the regulatory constraints. Then it is easy to see that

$$s^* = \theta^* = \underline{\theta} \left(1 + \frac{\lambda}{1-m} (\max\{1-\gamma-m, 0\}) \right)$$

and $\bar{m} = 1 - \gamma$.²⁵ When $m < 1 - \gamma$, both θ^* and $\theta^* - \underline{\theta}$ are decreasing in γ and in m (provided that $1 - \ell^{-1} - d^{-1} < 0$), and increasing in λ , ℓ , and d .

It is easy to see that to control the maximum probabilities of insolvency q and of a crisis p the regulator has to choose:

$\underline{\theta} \leq \underline{\theta}_q$ or

$$\ell^{-1} \geq (\underline{\theta}_q)^{-1} - d^{-1} - \left((\underline{\theta}_q)^{-1} - 1 \right) m, \quad (S)$$

and $\theta^* \leq \theta_p^*$, or

$$\ell^{-1} \geq \frac{1 + (1 - \gamma)\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1 + \lambda}{\theta_p^*} - 1 \right) m. \quad (L)$$

The ratios m and ℓ^{-1} interact and are partially substitutable. The solvency constraint (S) becomes tighter when d increases; the liquidity constraint (L) becomes tighter when d increases or γ decreases (and it becomes steeper when λ increases). Note that γ and λ only affect the liquidity constraint (see Figure 4).

²⁵ Note that in this case θ^* is independent of μ_θ and τ_θ .

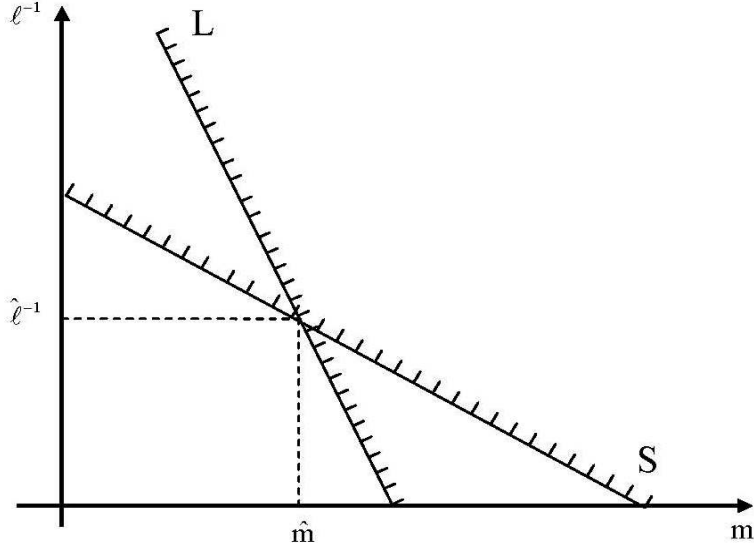


Figure 4: Solvency (S) and liquidity (L) constraints to control probabilities of insolvency and crisis with a short-term leverage ratio ($\ell = D/E$) and a liquidity ratio ($m = M/D$).

We see that liquidity m and leverage ℓ ratios are both needed in general to control the risk of insolvency and illiquidity. The regulator has to propose therefore a region in (m, ℓ^{-1}) space where the ratios of a bank have to lie. This region is limited by a kinked downward sloping schedule reflecting the (partial) *substitutability* between m and ℓ^{-1} (Figure 4). The bank will choose then the least cost combination (m, ℓ^{-1}) which will necessarily lie on the frontier of one of the constraints. It is worth noting that often there will be no loss of efficiency if the regulator sets minimum levels for (m, ℓ^{-1}) given the kink in the constraint set. For example, when $\tau_e \rightarrow \infty$ we can eliminate the risk of illiquidity by setting $\bar{m} = 1 - \gamma$ and then control q with a requirement on ℓ^{-1} .

The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios will be given by the intersection of the boundaries of the solvency and the liquidity constraints. When $\lambda = 0$ the two constraints collapse into one and in equilibrium $\theta^* = \theta$.

The following proposition summarizes the results.

Proposition 3. Let $1 - \ell^{-1} - d^{-1} < 0$. Then:

(i) To control for the probabilities of insolvency (q) and of a crisis (p) both a leverage and a liquidity constraint have to be fulfilled. They are partially substitutable and both become tighter when return prospects μ_θ are lower or the precision of public information τ_θ larger.

(ii) When $\tau_\varepsilon \rightarrow \infty$:

- If $p = q$ then $\hat{m} = 1 - \gamma$.

- If $p > q$ and $(1 - \gamma)\lambda$ is not too small then $1 - \gamma > \hat{m} > 0$, the solvency constraint becomes tighter when d increases, and the liquidity constraint becomes tighter when d increases or γ decreases. The minimal regulatory ratios for liquidity and solvency are

$$\hat{m} = \max \left\{ 1 - \gamma \left(1 - \lambda^{-1} \left(\frac{\theta_p^*}{\theta_q} - 1 \right) \right)^{-1}, 0 \right\} \text{ and } \hat{\ell}^{-1} = (\theta_q)^{-1} - d^{-1} - \left((\theta_q)^{-1} - 1 \right) \hat{m}.$$

We have then $\partial \hat{m} / \partial \lambda > 0$, $\partial \hat{m} / \partial \gamma < 0$, $\partial \hat{m} / \partial d = 0$, $\partial \hat{m} / \partial \mu_\theta > 0$, $\partial \hat{m} / \partial \tau_\theta > 0$, $\partial \hat{\ell}^{-1} / \partial \lambda < 0$, $\partial \hat{\ell}^{-1} / \partial \gamma > 0$, $\partial \hat{\ell}^{-1} / \partial d > 0$, $\partial \hat{\ell}^{-1} / \partial \mu_\theta < 0$ and $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$.

A higher return on short-term debt d will increase the solvency requirement and leave unaffected the liquidity requirement; a higher fire sales penalty λ and more conservative investment managers (lower γ) will increase the liquidity requirement and decrease the solvency one. This means that in a more competitive environment (higher d) the solvency requirement has to be strengthened, and in an environment where λ is high and γ is low the liquidity requirement has to be strengthened while the solvency one relaxed. An increase in return prospects or the precision of public information calls for a higher liquidity requirement and a lower solvency one.²⁶ The result implies that the regulator should set together disclosure and prudential policy.

²⁶ The result that $\partial \hat{m} / \partial \mu_\theta > 0$ and $\partial \hat{m} / \partial \tau_\theta > 0$ if $p > q$ is driven by the fact that $\partial(\theta_p^* / \theta_q) / \partial \mu_\theta < 0$ and $\partial(\theta_p^* / \theta_q) / \partial \tau_\theta < 0$.

Remark: It is worth exploring what happens in the extreme case when $1 - \ell^{-1} - d^{-1} > 0$. Banks with intense investment banking or wholesale activity have $1 - \ell^{-1} > 0$, since $\ell = D/E$ is generally above 1, and therefore $1 - \ell^{-1} - d^{-1} > 0$ becomes possible. If leverage is high enough, and $1 - \ell^{-1} - d^{-1} > 0$, then $\partial \underline{\theta} / \partial m > 0$, $\underline{\theta} > 1$, and $\partial \theta^* / \partial m > 0$ if γ or λ are small. In this case increasing the liquidity ratio leads to a higher likelihood of insolvency and crisis. In the interesting case the regulator requires that $\underline{\theta}_q > 1$ (assume also that $(\underline{\theta}_q)^{-1} - d^{-1} > 0$ and $(1 + (1 - \gamma)\lambda)(\theta_p^*)^{-1} - d^{-1} > 0$). We have then that the solvency constraint is upward sloping, and the liquidity constraint is also upward sloping if $\theta_p^* > 1 + \lambda$. If the liquidity constraint is downward sloping then the potential choices of the bank when faced with the constraints are $(m, \ell^{-1}) = (0, (1 + (1 - \gamma)\lambda)(\theta_p^*)^{-1} - d^{-1})$ and $(m, \ell^{-1}) = (\hat{m}, \hat{\ell}^{-1})$. If both constraints are downward sloping then only the first choice survives. This means that when $1 - \ell^{-1} - d^{-1} > 0$ it may be optimal (to keep the probabilities of insolvency and illiquidity under control) to induce the intermediary to choose to keep no liquid reserves and just impose a leverage limit. In short, for a highly leveraged institution it is better not to keep liquid reserves.

7. An interpretation of the 2007 run on SIV

A slowdown in house prices and tightening of monetary policy led to increasing doubts about subprime mortgages that were reflected in a sharp decline in 2007 in the asset-based securities index ABX. This index had been launched in January 2006 to track the evolution of residential mortgage-based securities (RMBS). The index is a credit derivative based on an equally weighted index of 20 RMBS tranches, and there are also subindexes of tranches with different rating, for different vintages of mortgages. The ABX index has provided two important functions: information about the aggregate market valuation of subprime risk, and an instrument to cover positions in asset-based

securities, for example by shortening the index itself (Gorton (2008, 2009)).²⁷ The decline in the ABX index during 2007 seems to have played a major role in the unfolding of the crisis and the run on SIV and ABCP conduits in particular. Indeed, at the end of 2006 sub-indexes for triple-B securities moved somewhat downward after trading at par, and dropped dramatically in 2007 (see Figure 5).²⁸ Something similar happened to the CMBX, a synthetic index corresponding to the ABX including 25 credit default swaps on commercial mortgages.

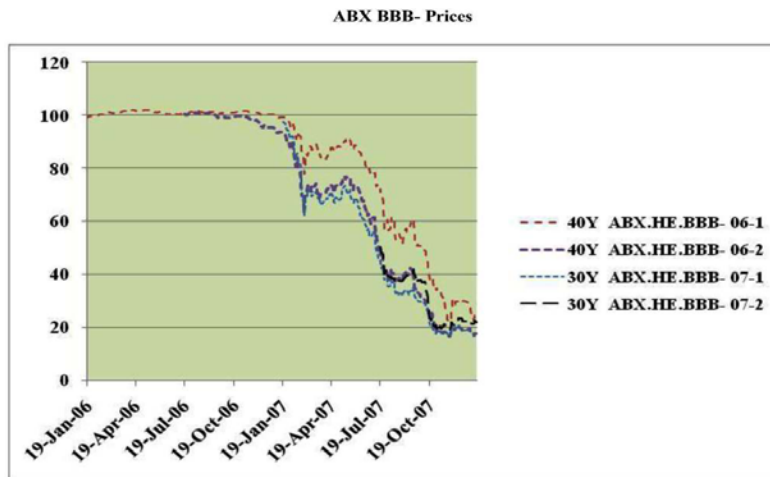


Figure 5: Prices of the 2006-1, 2006-2, 2007-1 and 2007-2 vintages of the ABX index for the BBB- tranche. (Source: Gorton (2008).)

These indexes were highly visible and had a strong influence on markets and the evolution of the indexes went together with a sequence of bad news on subprime mortgages (bankruptcies and earning warnings for originators, downgrading of ratings for RMBS bonds and CDO –collateralized debt obligations, and large losses for hedge funds) from January to August 2007. Indeed, the run seems to have been triggered by an unexpected decline in the ABX index.

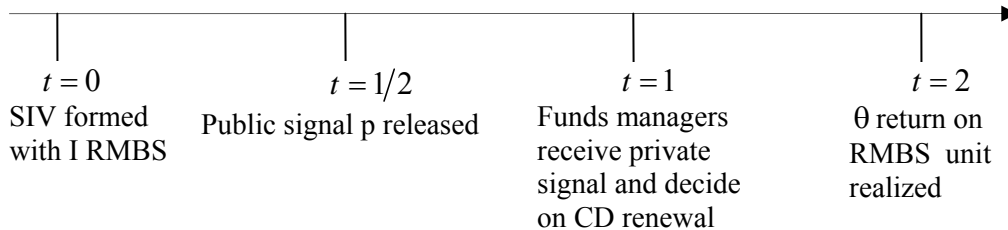
The runs began on ABCP conduits and SIV which had some percentage of securities backed by subprime mortgages. These vehicles were funded with short maturity paper

²⁷ In fact, trading in the ABX indices (by Paulson & Co. and by Goldman Sachs) has delivered two of the largest payouts in the history of financial markets. See Stanton and Wallace (2009) who argue also that ABX prices are imperfect measures of subprime security values.

²⁸ The index starts trading at par except in the case of the 2007-2 index which opened significantly below par.

and the run amounted to investors not rolling over the paper. While for a typical SIV, ABCP liabilities were 27% of the total at the end of 2007, for a typical conduit they were at 100%.²⁹ Those vehicles need not have a high proportion of assets directly contaminated by the subprime mortgages but they had a large indirect exposure through a large share of assets issued by the financial sector. As short-term financing dried up, bank sponsors intervened and absorbed many of these vehicles onto their balance sheets.³⁰

Consider the following time line in the basic banking model. At time $t = 0$ mortgage loans are awarded and securitized. At $t = 0$ a SIV is formed and holds I loans and M reserves financed by equity E and short-term debt (CDs) D_0 . At $t = 1/2$ a public signal p about θ is released. At $t = 1$ each fund manager, after receiving a private signal about θ , decides whether to cancel ($y_i = 1$) or renew his CD ($y_i = 0$). At $t = 2$ the returns θI on the RMBS assets are collected, if the bank can meet its obligations, the CDs are repaid at their face value D , and the equityholders of the SIV obtain the residual (if any).



The public signal p may be the value of the ABX index or the price of a derivatives' market with RMBS as underlying asset. Denote by τ the precision of the public signal. In the simplest scenario neither the SIV nor the fund managers in the short-term debt market participate in the derivatives market.

²⁹ See the Global Financial Stability Report of the IMF (April 2008).

³⁰ See Acharya and Schnabl (2010) and Covitz et al. (2009) for evidence on the runs in the ABCP market.

In this case the introduction of the ABX index implies a discrete increase in the public precision together with (public) bad news. This will lead to a higher probability of a crisis, a higher θ^* , both because of the direct effect of bad news ($E(\theta|p)$ low) and of increased public precision (with $E(\theta|p)$ low).

A high level of noise in the signals will push in the same direction also (Proposition 2 (iv)). Recall that a lower τ_ε increases strategic complementarity when τ_ε is already low (the maximal slope of $r(\cdot)$ tends to infinity as $\tau_\varepsilon \rightarrow 0$). Imprecise signals of SIV investors are likely given the opaqueness of the structured subprime products and distance from loan origination.³¹ In this case we may expect $\tau/\sqrt{\tau_\varepsilon}$ to be large also. The reason is that the precision of the signal of the fund managers (investors in the SIV) τ_ε may be low (think of the German Landesbank investing in structured subprime products) and much lower than the precision of the private signals of the sophisticated traders in the derivatives market (think of investment banks such as Goldman Sachs or hedge funds such as Paulson&Co) which influence positively the public precision τ (together with their risk tolerance).³² In case the derivatives market breaks down then the public signal that the derivatives price represents disappears and this, somewhat paradoxically, may have a stabilizing influence by lessening strategic complementarity.

The impact of the bad news is magnified when short-term leverage ℓ , the cost of funds d , and fire sales penalty λ for early asset sales are high (all those factors make h_1^{-1} and strategic complementarity high). This was the situation in the crisis. In fact, the fire-sale penalties increased dramatically with the market becoming practically illiquid. Those SIV then had to be absorbed back by the parent banking institution.

³¹ See Pagano and Volpin (2009) for a model where issuers of structured bonds choose opaque ratings to enhance the liquidity of their primary market at the cost of diminishing (perhaps drastically) the liquidity of the secondary market. Wagner (2007) argues that financial development may incentivate banks to move into more opaque assets.

³² See Angeletos and Werning (2006), Tarashev (2007), and Vives (Section 4.4, 2008) for related models.

The upshot of the discussion is that an increase in public precision should tighten in principle solvency and liquidity constraints. For example, in the case $\tau_\varepsilon \rightarrow \infty$ we know that \hat{m} should be increased and $\hat{\ell}^{-1}$ decreased due to their partial substitutability. Therefore, the introduction of a derivatives market should go together with revised regulatory ratios.

The presence of a derivative market may have other consequences. First of all, it may allow hedging the risk associated to the subprime products. This can be done by the SIV itself, by the fund managers with exposure to SIV by providing short-term financing, and by other investors with subprime exposure.³³ The SIV by shorting the index may reduce exposure to subprime risk and increase reserves for potential non-renewal of CDs at $t = 1$ at the cost of not profiting from the full appreciation potential of the subprime investment I . If the SIV hedges completely its position then it is completely safe but the expected return is low, a partial hedge will increase the reserves to diminish the failure probability. Interestingly, the private information of the SIV hurts its hedging possibilities creating adverse selection in the derivatives market.³⁴

8. Concluding remarks

This paper presents a stylized model of a financial crisis which characterizes solvency and liquidity risk and highlights how the degree of strategic complementarity among actions of investors is a key parameter to understand fragility. The results characterize how market outcomes depend on the balance sheet structure (leverage and liquidity), market stress parameters (degree of competition, the fire sales penalty of early liquidation of investments), and the informativeness of public and private signals. Fragility increases

³³ Public precision of the price in the derivatives market will decrease in the degree of risk aversion of informed investors in the market and in the sensitivity of the hedger demand to their endowment shock (Vives (Section 4.4, 2008)).

³⁴ When the SIV has market power and precise information in relation to the prior, and the hedgers have a much correlated endowment shock then the derivatives market dries up (Medrano and Vives (2004)). This is likely to be the case in a crisis situation.

with balance sheet stress (short-term leverage, low liquidity, high return on short-term debt); with market stress (fire sales penalty, more conservative investors), and with the precision of public information when fundamentals are weak. A high degree of opaqueness on the asset side of a financial intermediary together with a strong public signal (say from a derivatives market) will increase the degree of strategic complementarity and potential fragility.

The main general policy conclusion on regulatory reform is that a piecemeal approach will not work. The regulator will need to pay attention to the composition of the balance sheet of a financial intermediary and to the level of disclosure to control the probabilities of insolvency and illiquidity. In order to do so a leverage limitation and a liquidity requirement are needed, they are partially substitutable, and have to be set together taking into account the level of transparency. Indeed, in an environment with high market illiquidity and conservative investors the liquidity requirement has to be tightened while the solvency one relaxed; prudential constraints have to be tightened overall with higher disclosure or in the presence of a derivatives market (with stricter liquidity requirement and relaxed solvency). Competition policy and prudential regulation are not independent: In a more competitive situation leverage limits have to be strengthened.³⁵

The analysis has several important limitations. First of all, the balance sheet of the intermediary is exogenous and, correspondingly, the objectives of the regulator are also exogenous. Both could be endogenized introducing, for example, a moral hazard or commitment problem on the part of the intermediary which would rationalize the short-term debt structure and indicate an optimal closure policy for the regulator.³⁶ Second, the analysis is basically static while we are trying to capture dynamic phenomena.³⁷ Third, the investors are symmetric³⁸; and finally, the analysis focuses on a single institution and

³⁵ This theme is developed in Vives (2011).

³⁶ See Calomiris and Kahn (1991), Diamond and Rajan (2000), Gale and Vives (2002) and Rochet and Vives (2004) for related approaches to the issue.

³⁷ See, for example, the dynamic analysis of panic debt runs in He and Xiong (2009).

³⁸ Steiner and Sákovic (2009) study a global game example where players are ex ante asymmetric.

takes market parameters as given (e.g., the fire sales penalty); consequently, it does not take into account systemic effects.³⁹ These issues are left for further research.

³⁹ See Acharya and Yorulmazer (2007) and Farhi and Tirole (2009).

Appendix

Proof of Proposition 1: The game is “monotone supermodular” (i.e., of strategic complementarities with a monotone information structure) since $\pi(y_i, y; \theta)$ has increasing differences in $(y_i, (y, -\theta))$, that is, the differential payoff to act $\pi^1 - \pi^0$ is increasing in the aggregate action and the negative of the state of the world $(y, -\theta)$, and signals are affiliated. This means that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies in type (Van Zandt and Vives (2007)). Since there are only two possible actions, the strategies must then be of the threshold form: $y_i = 1$ if and only if $s_i < \hat{s}$ where \hat{s} is the threshold. It follows also that the extremal equilibrium thresholds, denoted \bar{s} and \underline{s} , bound the set of strategies which are the outcome of iterated elimination of strictly dominated strategies. If $\bar{s} = \underline{s}$ the game is dominance solvable and the equilibrium is unique. An equilibrium will be characterized by two thresholds (s^*, θ^*) with s^* yielding the signal threshold to act and θ^* the state-of-the-world critical threshold, below which the acting mass is successful and an acting player obtains the payoff $B - C > 0$. In equilibrium the fraction of acting players $y(\theta^*, s^*) = Pr(s \leq s^* / \theta^*) \equiv \Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*))$ must be no larger than the critical fraction above which it pays to act $h_0 + h_1(\theta^* - \underline{\theta})$. This yields $\theta^* = \underline{\theta} + h_1^{-1}[\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) - h_0]$. Note that this implies that $\theta^* \in [\underline{\theta}, \tilde{\theta}]$ since $h(\tilde{\theta}) = 1$. Furthermore, at the critical signal threshold the expected payoff of acting and not acting should be the same:

$$\begin{aligned} & E[\pi(1, y(\theta, s); \theta) - \pi(0, y(\theta, s); \theta) / s = s^*] \\ &= Pr(\theta \leq \theta^* / s^*)B + Pr(\theta > \theta^* / s^*)(-C) = 0; \end{aligned}$$

$$\text{or } Pr(\theta \leq \theta^* / s^*) \equiv \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma, \text{ where } \gamma \equiv C / (B + C) < 1.$$

If $h_0 > 0$ and $\Phi(\sqrt{\tau_\varepsilon}(s^* - \underline{\theta})) \leq h_0$ then $\theta^* = \underline{\theta}$. Otherwise, $\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*)$ and $\theta^* > \underline{\theta}$. There is a critical $\bar{h}_0 \in (0, 1)$ such that for

$h_0 < \bar{h}_0$ we have that $\theta^* > \underline{\theta}$ and $\theta^* = \underline{\theta}$ for $h_0 \geq \bar{h}_0$. Since $\theta^* = \underline{\theta}$ when $s^* \leq \underline{\theta} + \Phi^{-1}(h_0)/\sqrt{\tau_\varepsilon}$ and by replacing in $\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma$, it

follows that $\bar{h}_0 = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(\gamma)\right)$. Let $h_0 < \bar{h}_0$, then equations (1)

$\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*)$ and (2) $\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma$ combine into

equation

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0,$$

by substituting the value of s^* from (1) into (2). This equation may have multiple solutions in θ^* . As $\theta \rightarrow \tilde{\theta}$ we have that $\Phi^{-1}(h(\theta))$ tends to $+\infty$ and $\varphi \rightarrow -\infty$; as $\theta \rightarrow \underline{\theta}$ we have that $h(\theta) \rightarrow h_0$ and $\varphi \rightarrow \varphi(\underline{\theta}) > 0$ whenever $h_0 < \bar{h}_0$. There is at least a solution $\theta^* \in [\underline{\theta}, \tilde{\theta}]$. The solution will be unique if $\varphi' < 0$; there will be multiple

solutions (three in fact) if $\varphi'(\theta) > 0$ for a potential solution $\varphi(\theta) = 0$. When there is a θ such that $\varphi(\theta) = 0$ and $\varphi'(\theta) = 0$ there will be two equilibria. As γ tends to 0 (1)

we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ ($+\infty$), s^* tends to (minus) infinity and θ^* tends to $\tilde{\theta}$ ($\underline{\theta}$). This follows from equation (2) and the fact that $\theta^* \in [\underline{\theta}, \tilde{\theta}]$. There is a unique

solution if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$. Indeed, $\varphi' = 1 - (\sqrt{\tau_\varepsilon}/\tau_\theta)h_1[\phi(\Phi^{-1}(h(\theta)))]^{-1}$, where

ϕ is the density of the standard normal. Since ϕ is bounded above by $1/\sqrt{2\pi}$, it follows

that φ' is bounded above: $\varphi' \leq 1 - h_1\sqrt{2\pi\tau_\varepsilon}/\tau_\theta$ (with strict inequality, except when $h(\theta) = 1/2$ because then $\Phi^{-1}(1/2) = 0$ and ϕ attains its maximum: $\phi(0) = 1/\sqrt{2\pi}$).

Therefore, if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$ then $\varphi' \leq 0$. In this case the equilibrium is unique and the game is dominance solvable because then $\bar{s} = \underline{s}$. Furthermore, it should be clear that

the critical thresholds θ^* and s^* move together. If $\tau_\theta/\sqrt{\tau_\varepsilon} > h_1\sqrt{2\pi}$ then there is a range of γ for which there are multiple equilibria. Indeed, choose γ such that $h(\theta^*) = 1/2$,

then $\varphi'(\theta^*) > 0$ and there must be three equilibria, and by continuity there is a

neighborhood of such γ with multiple equilibria. Note that for γ small we will have a unique equilibrium (high) and for γ high a unique equilibrium (low) since as γ tends to 0 (1) we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ ($+\infty$) and φ to $+\infty$ ($-\infty$) for any given $\theta^* \in (\underline{\theta}, \tilde{\theta})$. Given the shape of φ (decreasing-increasing-decreasing) for γ small we obtain an equilibrium in the third decreasing portion and for γ high an equilibrium in the first decreasing portion. ■

Claim 1: If $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$ then $r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \hat{\theta}'(\hat{s}) \leq 1$.

Proof: From the equation $\Phi(\sqrt{\tau_\varepsilon}(\hat{s} - \theta)) = h(\theta)$ we can solve for the inverse function and obtain $\hat{s}(\theta) = \theta + (1/\sqrt{\tau_\varepsilon})\Phi^{-1}(h(\theta))$ with derivative $\hat{s}' = 1 + (1/\sqrt{\tau_\varepsilon})h'(\phi(\Phi^{-1}(h(\theta))))^{-1}$, where ϕ is the density of the standard normal. Since ϕ is bounded above by $1/\sqrt{2\pi}$, it follows that \hat{s}' is bounded below: $\hat{s}' \geq 1 + \sqrt{2\pi/\tau_\varepsilon}h_1$. Hence, $\hat{\theta}'(\hat{s}) \leq (1 + \sqrt{2\pi/\tau_\varepsilon}h_1)^{-1}$, with strict inequality, except when $h(\theta) = 1/2$ because then $\Phi^{-1}(1/2) = 0$ and ϕ attains its maximum: $\phi(0) = 1/\sqrt{2\pi}$. ■

Proof of Proposition 2: When $h_0 < \bar{h}_0$, the equation

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0$$

determines θ^* . We obtain the results by looking at how parameter changes impinge on $\varphi(\cdot)$. When $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$, we have that $\varphi' < 0$ and there is a unique equilibrium. The usual comparative static analysis applies. When $\tau_\theta/\sqrt{\tau_\varepsilon} > h_1\sqrt{2\pi}$ there may be three equilibria and the results will apply to the two extremal ones. With adaptive dynamics the results apply in general.

(i) The result for θ^* follows since φ is decreasing in γ and μ_θ , and increasing in α . The threshold s^* moves with θ^* .⁴⁰ The result for $P(\theta < \theta^*)$ is immediate for γ and α , and for μ_θ also since increases in μ_θ move the distribution of θ to the right. (ii) Note that $[\underline{\theta}, \theta^*)$ must increase with α and decrease with γ from $\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\epsilon} \Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\epsilon} \Phi^{-1}(\gamma) = 0$ since θ^* is increasing in α and decreasing in γ , $\partial h_0 / \partial \alpha \leq 0$ and $\partial h_1 / \partial \alpha < 0$ and therefore $h(\theta; \alpha) = h_0(\alpha) + h_1(\alpha)(\theta - \underline{\theta})$ is decreasing in α for given $(\theta - \underline{\theta}) > 0$, and $\Phi^{-1}(\cdot)$ is increasing. (iii) The equilibrium signal threshold is determined by $r(s^*; \mu_\theta) - s^* = 0$. From which it follows that for a marginal change in μ_θ

$$\left| \frac{ds^*}{d\mu_\theta} \right| = \frac{|\partial r / \partial \mu_\theta|}{1 - r'} > \left| \frac{\partial r}{\partial \mu_\theta} \right|$$

whenever $r' < 1$ is met and $r' > 0$. In consequence, an increase in μ_θ will have a larger effect on the equilibrium threshold s^* than the direct impact on the best response of a player $\partial r / \partial \mu_\theta = -\tau_\theta / \tau_\epsilon$. The same is true for discrete changes even with multiple equilibria if we restrict attention to extremal equilibria or in general with adaptive dynamics. This multiplier effect is largest when r' is close to 1 at equilibrium, that is, when strategic complementarities are strong, and we approach the region of multiplicity of equilibria. This is so when τ_θ is large since r' is decreasing in h_1 which is in turn strictly decreasing in τ_θ . (iv) Let θ^* be the smallest equilibrium. The first part follows since $\partial \varphi / \partial \tau_\theta = \theta^* - \mu_\theta - (\tau_\theta + \tau_\epsilon)^{-1/2} \Phi^{-1}(\gamma) / 2$ and therefore $\partial \varphi / \partial \tau_\theta > 0$ if $\theta^* > \mu_\theta$ and $\gamma < 1/2$ (since $\gamma < 1/2$ implies that $\Phi^{-1}(\gamma) < 0$). Note that θ^* is decreasing in μ_θ and therefore for low enough μ_θ we will have that $\theta^* > \mu_\theta$. Note that since $\theta^* > \mu_\theta$ when τ_θ increases $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ also increases. For the second part,

⁴⁰ The result for μ_θ follows also from a general argument in monotone supermodular games. We know that extremal equilibria of monotone supermodular games are increasing in the posteriors of the players (Van Zandt and Vives (2007)). A sufficient statistic for the posterior of a player under normality is the conditional expectation $E[\theta | s] = (\tau_\theta \mu_\theta + \tau_\epsilon s) / (\tau_\theta + \tau_\epsilon)$, which is increasing in μ_θ . It follows then that extremal equilibrium thresholds $(-\theta^*, -s^*)$ increase with μ_θ .

using the equation $\varphi(\theta^*) = 0$ in $\partial\varphi/\partial\tau_\varepsilon$ we obtain $\partial\varphi/\partial\tau_\varepsilon = -(\theta^* - \mu_\theta - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma))\tau_\theta/2\tau_\varepsilon$. It follows that $\partial\varphi/\partial\tau_\varepsilon < 0$ if $\theta^* > \mu_\theta$ since $\gamma < 1/2$. ■

Proof of Corollary 1:

(i) When $h(\theta) = \alpha^{-1}\theta$ we have always that $\underline{\theta} < \theta^*$ and that $Pr(\underline{\theta} \leq \theta < \theta^*)$ increases with α (the latter since in this case $\underline{\theta} = 0$ and we know that θ^* increases with α).

(ii) From the proof of Proposition 1 it follows that the critical ratio \bar{m} fulfils $m = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(\gamma)\right)$ where $\underline{\theta} \equiv (1 - m)/(\ell^{-1} + d^{-1} - m)$. The right hand side is decreasing in m if $\partial\underline{\theta}/\partial m < 0$ (which obtains according to our maintained assumption $1 - \ell^{-1} - d^{-1} < 0$). We have also that $\partial\underline{\theta}/\partial d > 0$, $\partial\underline{\theta}/\partial\ell > 0$ and $\underline{\theta}$ is independent of λ and γ . We know also that $\partial h/\partial m > 0$ (under the maintained assumption), $\partial h/\partial\ell^{-1} > 0$ and $\partial h/\partial d^{-1} > 0$. The results follow from Proposition 2 or by direct inspection of the equilibrium condition $\varphi(\theta) = 0$. ■

Proof of Proposition 3:

(i) Argument in the text.

(ii) When $\tau_\varepsilon \rightarrow \infty$ and $m < 1 - \gamma$ we have that

$$\begin{aligned}\theta^* &= \underline{\theta}\left(1 + \frac{\lambda}{1 - m}(1 - \gamma - m)\right) = (1 - m + \lambda(1 - \gamma - m))/(\ell^{-1} + d^{-1} - m) > \underline{\theta} \\ \theta^* - \underline{\theta} &= \underline{\theta}\left(\frac{\lambda}{1 - m}(1 - \gamma - m)\right) = (\lambda(1 - \gamma - m))/(\ell^{-1} + d^{-1} - m) > 0\end{aligned}$$

Both θ^* and $\theta^* - \underline{\theta}$ are decreasing in γ and in m (we have that $sign\{\partial\theta^*/\partial m\} = sign\{(1 + \lambda)(1 - \ell^{-1} - d^{-1}) - \lambda\gamma\}$ and the result for m holds in particular when $1 - \ell^{-1} - d^{-1} < 0$), and increasing in λ , ℓ , and d . Now, $\underline{\theta} \leq \underline{\theta}_q$ or

$$\ell^{-1} \geq m - d^{-1} + (1 - m)(\underline{\theta}_q)^{-1} = (\underline{\theta}_q)^{-1} - d^{-1} - ((\underline{\theta}_q)^{-1} - 1)m \quad (\text{S})$$

and

$$\begin{aligned} \varrho \left(1 + \frac{\lambda(1-\gamma-m)}{1-m} \right) &\leq \theta_p^*, \text{ where } \varrho \equiv (1-m)/(\ell^{-1} + d^{-1} - m), \text{ or} \\ m &\geq \frac{1+(1-\gamma)\lambda - (\ell^{-1} + d^{-1})\theta_p^*}{1+\lambda - \theta_p^*} \quad \text{or, equivalently,} \\ \ell^{-1} &\geq \frac{1+(1-\gamma)\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m, \end{aligned} \quad (\text{L})$$

Note that when $1 - \ell^{-1} - d^{-1} < 0$ $(\varrho_q)^{-1} - d^{-1} > (\varrho_q)^{-1} - 1 > 0$ and $(1+\lambda)(\theta_p^*)^{-1} > (\varrho_q)^{-1} > 1$, and $(1+(1-\gamma)\lambda)(\theta_p^*)^{-1} > (\varrho_q)^{-1}$ (for gamma small).

The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios will be given by the intersection of the boundaries of the solvency

$$\ell^{-1} = (\varrho_q)^{-1} - d^{-1} - \left((\varrho_q)^{-1} - 1 \right) m$$

and the liquidity

$$m = \frac{1+(1-\gamma)\lambda - (\ell^{-1} + d^{-1})\theta_p^*}{1+\lambda - \theta_p^*} \quad \text{or} \quad \ell^{-1} = \frac{1+(1-\gamma)\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m$$

constraints (note that $(\varrho_q)^{-1} - d^{-1} > (\varrho_q)^{-1} - 1 > 0$). We obtain for positive solutions

$$\begin{aligned} \hat{m} &= 1 - \frac{\gamma}{1 - \lambda^{-1} \left(\frac{\theta_p^*}{\varrho_q} - 1 \right)} \quad \text{and} \\ \hat{\ell}^{-1} &= (\varrho_q)^{-1} - d^{-1} - \left((\varrho_q)^{-1} - 1 \right) \hat{m}. \end{aligned}$$

Note that, indeed, $\hat{m} = 1 - \gamma$ if it is required that $\theta_p^* = \varrho_q$, and $\hat{m} = 0$ if $\gamma \geq 1 - \lambda^{-1} \left(\frac{\theta_p^*}{\varrho_q} - 1 \right)$. If $\theta_p^* > \varrho_q$ and for $(1-\gamma)\lambda$ is not too small it follows that $\partial \hat{m} / \partial \lambda > 0$, $\partial \hat{m} / \partial \gamma < 0$, $\partial \hat{m} / \partial d = 0$ and $\partial \hat{\ell}^{-1} / \partial \lambda < 0$, $\partial \hat{\ell}^{-1} / \partial \gamma > 0$, and $\partial \hat{\ell}^{-1} / \partial d > 0$.

We have also that $\partial \hat{m} / \partial \mu_\theta > 0$ and $\partial \hat{m} / \partial \tau_\theta > 0$ if $p > q$ since then $\partial(\theta_p^* / \theta_q) / \partial \mu_\theta < 0$ and $\partial(\theta_p^* / \theta_q) / \partial \tau_\theta < 0$. Correspondingly we have that $\partial \hat{\ell}^{-1} / \partial \mu_\theta < 0$ and $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$. ■

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