CDOs and the Financial Crisis: Credit Ratings and Fair Premia

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Abstract

We use the market standard Gaussian copula model to show that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. Yield enhancement on tranches is attributed to concentration of risk premia. Our findings imply that credit ratings are not sufficient for pricing, which is surprising given their central role in structured finance markets. This illustrates limitations of the rating methodologies being solely based on estimates of real-world payoff prospects. We further demonstrate that prices and ratings of CDO tranches have low stability and therefore they are likely to decline significantly more than prices and ratings of corporate bonds if credit conditions deteriorate. The pace and severity of re-pricing and downgrading of CDO tranches is further exacerbated by default contagion.

Keywords: Collateralized debt obligations, Credit ratings, Fair premia, Structured finance, Rating agencies
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1. Introduction

There is no doubt that structured credit products only developed so rapidly prior to the 2007-2009 financial crisis because they offered higher coupons relative to equally-rated corporate bonds. Yield enhancement on structured securities was particularly appealing to investors who assumed that credit ratings represent

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a universal and robust indication of payoff prospects across different asset classes. For example, the premise that the highest rating grade is a guarantee of very low default risks encouraged many institutional investors to add triple-A securitized tranches to their portfolios. The frailty of such a rating-based approach became evident only in 2007-2009 when the mounting losses associated with subprime mortgages eventually led to the collapse of the structured finance markets.

The main result of this article is that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds due to concentration of risk premia in spreads of non-equity tranches. Finance theory indicates that credit ratings, which measure only pure default risk, cannot fully account for fair premia due to risk aversion of investors. Strictly speaking, credit ratings represent expected losses or default probabilities calculated under the physical measure. In contrast, fair premia are closely related to expected losses calculated under the risk-neutral measure that is derived from (significantly) higher market-implied default probabilities. We show that CDO structuring leads to high sensitivity of expected tranche losses to default probabilities of the underlying bonds. CDO tranches can therefore be tailored to combine low real-world expected losses with much higher risk-neutral expected losses. For this reason, CDO tranches can qualify for high credit ratings, while offering significant yield enhancement relative to similarly-rated bonds.

In modeling credit ratings and fair premia of CDO tranches, we rely on the market standard one-factor Gaussian copula model. As a stylized example, we consider a portfolio of hundred 'BBB-' bonds[1] with a market spread of 111.95 bps that decomposes into 53.06 bps of pure default risk compensation and 58.89 bps of risk premium. By securitizing this bond portfolio, we create a mezzanine CDO tranche of the same 'BBB-' credit quality and hence roughly the same spread to compensate for pure default risk. Yet this tranche has a much higher fair spread of 320.69 bps, which is an almost three-fold increase in the total spread due to a five-fold multiplication of the risk premia of the underlying bonds. This illustrates that tranche spreads cannot be derived on the basis of similarly-rated bonds. Yield enhancement is even stronger when we re-securitize the 'BBB-' tranches into a CDO-squared.[2] For example, we construct a CDO-squared tranche with a 'BBB-' rating and a fair spread of 795.71 bps.

[1]Whenever we discuss a credit rating without indicating the rating agency, e.g. a 'BBB-' bond, we always refer to the S&P rating.

[2]A CDO-squared is a CDO-type security backed by a collateral pool consisting of tranches from other CDO deals.

Our results on the tranche yield enhancement demonstrate that the current rating system can be gamed if it is used for pricing purposes. Producing CDOs allows for boosting premia on highly rated securities. This creates vast possibilities for rating arbitrage, which made the structured finance industry so profitable. The excess tranche spreads can be distributed between CDO investors and issuers. The investors are able to increase their returns on highly rated portfolios, while the issuers are compensated for their efforts and risks associated with originating and structuring CDOs. These results complement the study by Brennan et al. (2009) who propose an analytical model based on the CAPM and the Merton model to analyze the gains of an investment banker selling CDO tranches at the spreads of equally-rated corporate bonds.

Finally, we explain why structured securities are likely to perform poorly during unfavorable market conditions. The key to this analysis lies in high sensitivity of expected tranche payoffs to default probabilities of the underlying bonds. Consequently, we show that an increase in default probability estimates of the underlying bonds, which is typical for a deterioration in credit conditions, has a much stronger effect on ratings and prices of CDO tranches than of corporate bonds. Moreover, the pace and severity of downgrading of CDO tranches is exacerbated by default contagion. We also discuss how model risk and possible inaccuracies in rating assumptions can have a dramatic effect on CDO tranches.

The main contribution of this article is that it provides a comprehensive analysis of CDO tranches focusing on credit ratings and fair spreads. Our approach combines perspectives of CDO originators, rating agencies and investors. Since we use the market-standard models, we capture the most important aspects of structured finance in a simple yet meaningful way. We show that CDO tranches have much higher fair spreads than equivalently-rated bonds. The key novelty of this paper is the clear cut explanation of yield enhancement on tranches, which is directly related to their risk properties (i.e. high sensitivity of expected tranche payoffs to default probabilities of the underlying bonds). This result clearly illustrates that CDO tranches are subject to a trade-off between higher fair spreads and lower risks of downgrading and re-pricing. The key limitation of using credit ratings for investment decisions is due to the fact that ratings capture solely pure default risk, while pricing is done under the risk-neutral measure.

The rest of the paper is organized as follows. Section 2 discusses the background of the structured finance markets. Section 3 explains the modeling approach and assumptions. In Section 4 we present our findings on the CDO yield enhancement and in Section 5 we analyze the sensitivity of tranche payoffs. In Section 6 we discuss the stability of ratings and prices, while Section 7 concludes.

2. Background

Structured finance transforms lower quality assets into securitized tranches that are better suited for investors' risk appetite. Producing structured securities begins with pooling (collateral) assets into large well-diversified portfolios, which leads to a substantial reduction of idiosyncratic risks. Subsequent prioritization of cash flows associated with each underlying portfolio creates several securities (tranches) of varying credit quality. Tranche investors bear the credit losses incurred by the underlying portfolio within pre-agreed limits and in return they receive premium payments. Most of the credit risk is concentrated in the first-loss equity tranche. More senior tranches are characterized by higher credit quality compared to the average quality of the collateral pool. In practice, highly rated 'AA' and 'AAA' tranches constitute about 60% of the volume of securitized portfolios rated by Fitch (2007). Prior to the 2007-2009 financial crisis, the ability of structured finance to produce such large volumes of highly rated tranches turned out particularly successful to meet the large market demand for very safe securities originating from institutional investors such as pension funds or money-market funds.

Benmelech and Dlugosz (2009a) analyze the practice of rating CDOs[3] and they use the term 'alchemy' to describe the apparent disparity between the credit quality of CDO tranches and the credit quality of their underlying collaterals. Equally intriguing is that highly rated tranches offer a significant yield enhancement relative to similarly-rated bonds. For example, in the run-up to the 2007-2009 financial crisis, triple-A tranches provided as much as 50 bps in the case of CDO-squareds, while such attractive coupons were not common for triple-A assets within the corporate bond universe.[4] The originators of CDOs attributed the tranche yield enhancement to the 'leveraging' and 'correlation risk' created by prioritizing tranche payoffs (ABC of CDOs, 2004). However, most likely the implications of 'leveraging' and 'correlation risk' were not fully understood by investors. Crouhy et al. (2008) point out that "the argument could be made that as the yields on structured instruments exceeded those on equivalently rated corporations, the market knew they were not of the same credit and/or liquidity risk. But investors still misjudged the risk".

[3]Benmelech and Dlugosz (2009a) focus on collateralized loan obligations (CLOs), which are CDOs backed by portfolios of loans.

[4]We compared the spreads on a few dozen CDO tranches rated by S&P in 2006 (from S&P Ratings Direct database).

Our focus on credit ratings is motivated by their predominant importance in the structured finance markets. Credit ratings are essential because the complexity of securitized products limits the ability of unsophisticated investors to conduct independent risk assessment (Crouhy et al., 2008). There is also a growing consensus in academic literature that investors rely heavily on credit ratings not only for risk management, but also to infer fair premia; for a discussion we refer to Brennan et al. (2009), Coval et al. (2009), Crouhy et al. (2008), Krahnen and Wilde (2008) and Firla-Cuchra (2005). For example, Krahnen and Wilde (2008) point out that "Ratings are used almost universally by investors, bankers, supervisors, and regulators as the relevant risk metric. The familiarity of markets with these letter ratings has probably encouraged investors to add these instruments to their portfolios, and has helped to establish the market for various ABS [CDO] products in the first place".

The rating agencies are ambiguous about the meaning of credit ratings. On the one hand, they advertise credit ratings as "a uniform measure of credit quality globally and across all types of debt instruments" (S&P, 2007).[5] The same document further reads, "In other words, an 'AAA' rated corporate bond should exhibit the same degree of credit quality as an 'AAA' rated securitized debt issue". On the other hand, the agencies assert that credit ratings are merely "opinions about a relative creditworthiness of a security" (S&P, 2009). Similarly, the rating agencies indicate that credit ratings are not sufficient for pricing, but they do not explain fundamental differences in risks between like-rated bonds and securitized assets. For illustration, an S&P document explaining the "meaning behind structured finance ratings" states: "We recognize that the global capital markets may not always price similarly rated debt types the same, all things being equal. This is also true when comparing different securitized issues. Such differences may be based on both credit and non-credit or market considerations, including perceived prepayment risks based on asset or structural characteristics; seller/servicer characteristics; the asset class' historical track record; the availability of historical performance data; and market liquidity considerations, including the depth of secondary markets in certain sectors or markets." (S&P, 2007).

Credit ratings are an assessment of a security's credit quality. In case of corporate bonds, the rating process depends heavily on qualitative as well as quantitative components. Bonds are categorized into a number of grades according

[5]This quote and a broader discussion about the meaning of credit ratings is given by Ashcraft and Schuermann (2008).

to their relative payoff prospects. These rating grades are not meant to represent precise estimates of default probabilities. Actual default performance of bonds typically varies between years. For example, 'BBB' bonds rated by S&P have an average annual default rate of 0.26% with a standard deviation of 0.27% (based on 1985-2009 period), see S&P (2010). The rating agencies also publish cumulative default probabilities of bonds, which are more stable than annual default rates. For example, triple-A bonds rated by S&P have a 10-year historical default probability of 0.36% (S&P, 2005). Such statistics give investors an intuitive meaning to the 'relative ranking of payoff prospects' implied by credit ratings.

The rating methodologies for structured securities are based on the principle that their credit ratings should be comparable to bond ratings. The rating agencies use quantitative models to estimate default probabilities or expected losses of CDO tranches. The values of these risk measures are then mapped into letter-grade ratings according to pre-specified bounds corresponding to different rating categories. The S&P's methodology aims to ensure that CDO tranches have the same cumulative (real-world) default probabilities as equally-rated bonds, while the Moody's methodology aims to match (real-world) expected losses. For example, a tranche qualifies for the triple-A rating by Standard and Poor's if its 10-year default probability is equal or less than 0.36%, which is the historical default probability of triple-A corporate bonds over the same time horizon. Similarly, Moody's assigns the triple-A rating to a tranche if its 10-year expected loss is equal or less than 0.0055%, which corresponds to the historical loss on equally-rated bonds.[6]

3. Model and assumptions

3.1. Modeling approach

In this part we introduce the market standard method for modeling CDOs. We start with discussing how a CDO structure allocates losses incurred on the underlying assets to the tranches. We then explain the approach to modeling defaults of the collateral bonds. We also define credit ratings and fair spreads of tranches. Last, we discuss how we conduct the study by means of Monte Carlo simulations.

We construct a CDO backed by a collateral pool consisting of $i = 1, ..., n$ bonds with each bond i having a notional N_i. The total notional of the portfolio is thus equal to $N_{total} = \sum_{i=1}^{n} N_i$. The CDO's maturity time is T. Default times of the

[6] We refer to S&P (2002) and S&P (2005) for details on the S&P rating methodology. See Moody's (2005) and Moody's (2007) for details on the Moody's methodology.

obligors are denoted $\tau_1, \tau_2, ..., \tau_n$ and the corresponding recovery rates are denoted by R_i. The cumulative loss on the collateral pool up to time t is given by:

$$L(t) = \sum_{i=1}^{n} N_i (1 - R_i) \mathbf{1}_{\tau_i < t}, \tag{1}$$

where $\mathbf{1}$ is the indicator function defined as usual.

The CDO structure splits the total portfolio risk into several tranches with each tranche being defined by its attachment point K_L and its detachment point K_U. Tranche investors cover the portfolio losses exceeding K_L up to the limit of the tranche notional $K_U - K_L$. The lower attachment point is also referred to as the tranche subordination level. For example, if the total portfolio notional is \$100 million and the tranche attachment and detachment points are, respectively, \$3 and \$7 million, then the cumulative portfolio losses between \$3 and \$7 million are passed on as losses to the tranche investors. Formally, the CDO tranche losses up to time t are given by:

$$L(K_L, K_U, t) = \min\left[\max\left(L(t) - K_L, 0\right), K_U - K_L\right]. \tag{2}$$

The key challenge in rating and pricing structured securities lies in modeling defaults of the collateral assets. The appropriate modeling framework must capture not only the univariate risk properties of the underlying assets, but also dependence between their defaults. That is because tranche payoffs are linked to the portfolio loss rate.

The univariate risk properties of the underlying assets are summarized by the cumulative distribution functions of their default times τ_i:

$$F_i(t) = Pr(\tau_i < t) = 1 - S_i(t), \tag{3}$$

where $S_i(t)$ is the survival function to time t. The specification of $F_i(t)$ or $S_i(t)$ depends on the purpose of modeling as it can reflect probabilities either under the physical measure or under the risk-neutral measure. The survival functions together with the recovery rates give all security-specific information needed for analyzing expected cash flows on single-name securities.

We model default dependence using the Gaussian copula approach. Let us introduce a series of random variables:

$$V_i = \Phi^{-1}\left(F_i(\tau_i)\right), \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. We further assume that $V_1, ..., V_n$ jointly follow the multivariate standard normal distribution with specified pair-wise correlations between any V_i and V_j. For modeling CDOs, the market typically assumes a one-factor model such that correlations between collateral assets are due to their exposure to a single common factor. It this case, each V_i can be expressed as:

$$V_i = \sqrt{\rho_i} Y + \sqrt{1-\rho_i} X_i, \tag{5}$$

where $Y \sim N(0,1)$ is the common (systemic) factor, $X_i \sim N(0,1)$ is the idiosyncratic (obligor-specific) component and $\rho_i \in [0,1]$ is the parameter determining correlations. It is standard to interpret $V_i \sim N(0,1)$ as the scaled asset value of obligor i, which is in line with the Merton approach of default modeling. The market typically assumes that all ρ_i are equal to a common ρ, which simplifies the correlation structure. We can then interpret ρ in Eq. 5 as the asset value correlation between any two obligors in the collateral portfolio.

For modeling CDO-squareds, we assume a more complex correlation structure, which is captured by the two-factor model. We consider a CDO-squared with a collateral pool composed of $j = 1, ..., K$ underlying CDO tranches. In turn, each of the underlying CDO tranches is backed by a portfolio of bonds indexed $i = 1, ..., n$. The scaled value of obligor i belonging to the reference portfolio of the (underlying) jth CDO is denoted as $V_{i,j}$ and can be expressed as:

$$V_{i,j} = \sqrt{\alpha\rho_i} Y + \sqrt{(1-\alpha)\rho_i} Z_j + \sqrt{1-\rho_i} X_{i,j}, \tag{6}$$

where Y and ρ_i are as defined below Eq. 5, $X_{i,j} \sim N(0,1)$ is the idiosyncratic (obligor-specific) component, $Z_j \sim N(0,1)$ is the factor specific to the reference portfolio of the jth CDO, and finally, parameter $\alpha \in [0,1]$ determines the relative exposure to the common factor Y and to the CDO-specific factor Z_j. In this setting, the credit risk of underlying tranches is partly driven by tranche-specific factors, which provides additional diversification. If all ρ_i are equal to a common ρ, then ρ gives the asset value correlation between any two obligors within the same CDO collateral pool, while $\alpha\rho$ is the asset value correlation between any two obligors belonging to collateral pools of different underlying CDOs.

3.1.1. Rating Measures

Standard and Poor's ratings are based on tranche default probability, while Moody's ratings are based on expected tranche loss. Tranche default probability is the likelihood that the cumulative portfolio loss exceeds the subordination level of the tranche before maturity time T:

$$PD_{tranche} = \mathbb{E}^{P}\left(\mathbf{1}_{L(T)>K_L}\right), \tag{7}$$

where P denotes the physical default probability measure. Expected tranche loss is defined as the loss on the tranche notional until maturity:

$$EL_{tranche} = \frac{\mathbb{E}^{P} L\left(K_L, K_U, T\right)}{K_U - K_L}. \tag{8}$$

We emphasize that credit ratings are determined under the physical measure. The physical measure captures the actual (real-world) default probabilities and it is typically estimated from historical data on default frequencies. The physical measure is the appropriate choice for modeling credit ratings because they are meant to reflect real-world payoff prospects of a security (i.e. real-world default probability or expected loss).

3.1.2. Fair Premia

Holders of a tranche incur losses if the portfolio loss rate exceeds the subordination level of the tranche. The series of cash flows equal to the tranche losses associated with credit events is called the default leg. The present value of the default leg is given by:

$$V_{default} = \mathbb{E}^{Q} \int_0^T B\left(0, t\right) dL\left(K_L, K_U, t\right), \tag{9}$$

where Q denotes the risk-neutral measure and $B\left(0, t\right)$ is the discount factor for the time interval $(0, t)$.

In return for taking on default risk, tranche investors receive premium payments based on the running spread s. The present value of the premium leg is given by:

$$V_{premium}\left(s\right) = \mathbb{E}^{Q}\left[\sum_{i=1}^{qT} B\left(0, \frac{i}{q}\right) \frac{s}{q} \left(\left(K_U - K_L\right) - L\left(K_L, K_U, \frac{i}{q}\right)\right)\right], \tag{10}$$

where q is the frequency of coupon payments, e.g. $q = 4$ for quarterly payments.

Determining the fair spread is equivalent to finding the level of tranche spread,

s^*, equating the default and the premium legs. Since the premium leg is linear as a function of s, the fair tranche spread equals:

$$s^* = \frac{V_{default}}{V_{premium}\,(s = 1bp)}. \tag{11}$$

For the purpose of calculating fair spreads, we use the risk-neutral measure implied by market information. Risk-neutral default probabilities are typically much higher than their physical counterparts because they incorporate risk premia. The risk-neutral measure can be derived from the term structure of CDS spreads of the collateral bonds given the recovery rate assumptions. The recovery rate estimates are assumed to be exogenous and can be based on the rating agencies' studies of historical data.

3.1.3. Implementation

The aforementioned tranche statistics are most easily calculated using Monte Carlo simulations. In each simulation run, we draw realizations of the random variables Y and X_i independently from the standard normal distribution. Next, we compute default times τ_i of the underlying assets using formulas (5) or (6) and the inverse of formula (4):

$$\tau_i = F_i^{-1}\left(\Phi\left(v_i\right)\right). \tag{12}$$

Once the default times and the corresponding recoveries are determined for all simulation runs, the calculation of tranche default probabilities, expected losses and fair spreads using formulas (7), (8) and (11) is straightforward.

In addition to the fair tranche spreads calculated under the risk-neutral measure Q, we also calculate tranche spreads under the physical measure P. This gives the spreads compensating for default risk in the real-world (pure default risk). Similarly, we calculate tranche default probabilities and expected losses under the risk-neutral measure Q instead of the physical measure P.

3.2. Manufacturing structured assets

Manufacturing structured assets consists of two steps. The first step is to select the collateral portfolio. The second step is to choose the capital structure (tranching). We first discuss how we produce a stylized CDO and then we turn to the CDO-squared case.

3.2.1. CDO collateral portfolio

We choose a homogeneous collateral portfolio of one hundred bonds with a maturity of 10 years. Each bond has a default probability of 10% until maturity, which results in a 'BBB-' rating by Standard and Poor's.[7] We make a simplifying assumption that the survival functions of the underlying bonds have an exponential form, $S_i(t) = e^{-t\lambda_i}$, with a constant default intensity parameter λ_i.[8] The intensity parameter is calibrated by equating the assumed default probability until maturity (e.g. $p_i = 10\%$) to the default probability implied by the exponential survival function: $p_i = 1 - e^{-T\lambda_i}$.[9]

We assume that the collateral bonds have random recovery rates drawn from a Beta distribution with mean of 50% and standard deviation of 20%, which implies that the expected loss of each bond is 5%. Hence, according to the Moody's criteria the bonds qualify for the 'Ba1' rating.[10]

The ability to produce highly rated tranches is critically dependent on the joint default behavior of their collateral assets. According to the Standard and Poor's rating assumptions, the asset value correlation between corporate obligors belonging to different industry sectors is 5%, while the asset value correlation within an industry sector is 15% (for U.S. bonds), see S&P (2005). We set the asset value correlation to 12.5%, which is a realistic average correlation if the bonds belong to several industries.[11]

We also assume that each collateral bond has a market-implied default probability of 20% until maturity of 10 years, which is double the physical default probability. It is equivalent to a market spread of 111.95 bps on the collateral bonds.[12] Moreover, this assumption is cautious since it corresponds to the lower bound of estimates given in the literature suggesting that risk-neutral default probabilities for 'BBB' rated bonds are 2 to 5 times higher than their physical default

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[7] A 'BBB-' bond rated by S&P has a 10-year default probability between 5.88% and 10.64% according to the S&P benchmark tables (S&P, 2005).

[8] The exponential (real-world) survival functions are a good approximation when compared to exact survival functions provided by S&P (2005).

[9] The intensity parameter for the risk-neutral default probability is calibrated similarly.

[10] A 'Ba1' bond rated by Moody's has a 10-year expected loss between 3.25% and 5.17% (Moody's, 2007).

[11] In practice, the average correlation between collateral assets is typically lower than 12.5% because collateral portfolios include not only U.S. bonds, but also European or Asian bonds as well as RMBS or ABS tranches. For asset value correlation lower than 12.5%, the yield enhancement on tranches is even higher as demonstrated in the sensitivity analysis in Section 5.

[12] We also assume a fixed discount factor of 2% per annum.

probabilities, see Berndt et al. (2005), Driessen (2005), Delianedis and Geske (2003) or Hull et al. (2005)). For higher risk-neutral default probabilities, the yield enhancement on tranches is even stronger as explained in Section 5.

3.2.2. CDO structuring

The capital structure of the CDO is chosen for the purpose of minimizing the cost of financing of the underlying debt. Given investors' reliance on credit ratings, the cost of financing of a tranche is decreasing in its credit quality. This leads to clear incentives to maximize the volume of tranches with as good ratings as possible. The market practice is to look at the criteria of the rating agencies and to produce tranches that just qualify for their credit ratings. The structuring of CDO tranches is therefore strongly interrelated with the rating process.

We now describe the junior mezzanine tranche which is tailored to have identical credit quality as the underlying corporate bonds. It means that we not only ensure that this tranche has the same credit ratings as the underlying bonds, but we impose a stronger condition that the tranche and the bonds have the same (real-world) default probabilities and (real-world) expected losses. This is very convenient for our further analysis of the rating-premia relationship, but it slightly departs from the typical structuring process. The lower attachment point of the junior mezzanine tranche is chosen as a 90% quantile of the real-world portfolio loss distribution such that the tranche meets the assumed 10-year default probability of 10%. Next, we fix the upper attachment point such that 10-year expected tranche loss is 5%. For our portfolio, this implies that the lower and upper attachment points of the tranche are 9.90% and 14.75%, respectively. It follows from the obtained default probability and expected loss of the tranche that it receives a 'BBB-' rating from S&P and 'Ba1' from Moody's.

The more senior tranches are tailored in line with the market practice of maximizing the size of tranches with as high ratings as possible, which means that these tranches just meet the criteria for their credit ratings. While we report ratings of both S&P and Moody's, we structure the more senior tranches based solely on the S&P methodology measuring default probability. Within this methodology, tranche default probability is determined only by its lower attachment point. Therefore, we first choose the subordination level of the super-senior 'AAA' tranche as an appropriate quantile of the real-world portfolio loss distribution such that this tranche meets the benchmark 10-year default probability of 0.36%.[13]

[13]Starting from 2005, S&P uses different default probability benchmarks for CDO tranches, which are no longer based on historical bond performance. After the change, senior CDO tranches

Similarly, we construct the senior 'AA' tranche by choosing its lower attachment point such that the tranche meets the 10-year default probability target of 0.87%. The upper attachment point of the 'AA' tranche is the lower attachment of the super-senior tranche. For our portfolio, these two tranches have subordinations of 17.08% and 19.45%, respectively.

We also obtain two other tranches, which have both attachment and detachment points implied by the tranches defined so far. The first one is the unrated equity tranche, which is at the bottom of the capital structure. Its lower attachment is 0% and its upper attachment is given by the subordination level of the junior mezzanine tranche, i.e. 9.90%. Another tranche is in between the junior mezzanine tranche and the senior tranche, so it goes from 14.75% to 17.08% of the CDO notional. It is rated 'A-' by S&P and 'Baa1' by Moody's.

Table 1 presents the results of structuring and rating in columns (1)-(5). These results are obtained under the physical measure appropriate for modeling credit ratings. We can see how CDO prioritization of cash flows adjusts the risks of tranches. The default probabilities and expected losses of the tranches are decreasing with tranche seniority. Most of the credit risk is contained in the equity tranche absorbing all portfolio losses up to the limit of 9.90% of the CDO notional. The lowest default risk is associated with the super-senior 'AAA' tranche, which has almost 80% of the CDO notional. The remaining columns (6)-(8) of Table 1 present the risk-neutral results relevant for tranche pricing as discussed in the next section.

A similar CDO structuring exercise was done by Krahnen and Wilde (2008) who use (Moody's) historical default rates to determine tranche subordination levels as quantiles of the portfolio loss distribution. This corresponds to the practice of choosing tranche subordinations to just meet the rating criteria, which is common in the market for CDOs backed by non-synthetic assets. In contrast, tranche subordination levels are fixed and pre-defined in the markets for CDS index tranches. For example, the iTraxx Europe index is divided into six tranches with subordinations of 0%, 3%, 6%, 9%, 12%, and 22%.

have higher target default probabilities corresponding to historical tranche performance; however, in this paper we still use the corporate bond benchmarks to preserve direct comparability of ratings.

Table 1: CDO tranche risk statistics, ratings and premia.

This table reports the results of CDO structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO. The remaining columns report (10-year horizon) tranche default probabilities and expected losses as well as annualized spreads. These results are obtained under the physical and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The results obtained under the physical measure are related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk, while column (8) gives the fair (market) spreads. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	98.33% 'NR'	47.50% 'NR'	636.54	99.90%	78.53%	1475.40
tranche 2 junior mezz.	9.90%	10.00% 'BBB-'	5.00% 'Ba1'	48.25	44.64%	30.24%	320.69
tranche 3 senior mezz.	14.75%	1.97% 'A-'	1.35% 'Baa1'	12.76	18.43%	14.55%	143.83
tranche 4 senior	17.08%	0.87% 'AA'	0.58% 'A2'	5.43	11.09%	8.46%	81.81
tranche 5 super-senior	19.45%	0.36% 'AAA'	0.01% 'Aa1'	0.10	6.21%	0.27%	2.52
corporate bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

3.2.3. Creating a CDO-squared

We also analyze CDO-squared securities which have incurred particularly large losses during the financial crisis. CDO-squared tranches are rated according to the same principles as CDOs. The market standard is to use the 'the bottom up' approach, which derives the cash-flows on the underlying tranches directly from the cash flows on their collateral bonds. This approach accounts for specific characteristics (e.g. credit quality) and overlap among the collaterals of the underlying CDO tranches.

We construct a CDO-squared collateral pool as follows. We create thirty CDOs that are all identical to the stylized CDO deal described earlier in this section; however, we assume that these CDOs reference portfolios of different bonds implying no overlap among their collateral portfolios. Consequently, each of these CDOs contains a junior mezzanine tranche with the attachment of 9.90% and detachment of 14.75%, which is rated 'BBB-' by S&P and 'Ba1' by Moody's. These thirty mezzanine tranches constitute the CDO-squared collateral portfolio. We further assume that the asset value correlation between any two obligors within the same CDO collateral pool is 12.5%, while the asset value correlation between any two obligors belonging to collaterals of different underlying CDOs is 3.5%. This is equivalent to assuming that $\rho = 12.5\%$ and $\alpha\rho = 3.5\%$ in Eq. 6. We thus obtain additional diversification at the level of the underlying tranches, which is critical for the ability to produce highly rated CDO-squared tranches. A similar approach was used by Hull and White (2010) in the analysis of ABS CDOs. In practice, such diversification can be achieved by selecting tranches backed by collateral bonds belonging to different industry sectors and geographic locations.[14] In addition, collateral pools of CDO-squareds very often include some tranches of asset backed securities (e.g. RMBS or ABS).

We choose the capital structure of the CDO-squared to ensure that its tranches have similar credit quality to the corresponding tranches of the stylized CDO. For this purpose, we apply the same structuring scheme as in the CDO case. The results of structuring for the CDO-squared are reported in columns (1) – (5) of Table 2. Note that the corresponding CDO and CDO-squared tranches have the same credit ratings from both S&P and Moody's, which is simply due to the assumed structuring process. Furthermore, the junior mezzanine tranche of the CDO-squared is tailored to have a default probability of 10% and an expected loss of 5%, which results in a 'BBB-' rating by S&P and a 'Ba1' rating by Moody's. We also produce two CDO-squared tranches with 'AAA' and 'AA' ratings by S&P, which just meet the default probability benchmarks of 0.87% and 0.36% required for these rating categories. Finally, we obtain the senior mezzanine tranche and the equity tranche with both attachment and detachment points implied by other tranches of the CDO-squared.

4. Credit ratings and fair premia

In this section we analyze the relation between credit ratings and fair premia. We start with calculating tranche default probabilities, expected losses and

[14]For example, S&P assumes a correlation of 0% between two corporate bonds belonging to different industry sectors and different regions (regions are defined as Asia, Europe etc.), see S&P (2005).

Table 2: CDO-squared tranche risk statistics, ratings and premia.

This table reports the results of CDO-squared structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO. The remaining columns report (10-year horizon) tranche default probabilities and expected losses as well as annualized spreads. These results are obtained under the physical and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The results obtained under the physical measure are related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk, while column (8) gives the fair (market) spreads. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordi-nation	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	77.88% 'CCC-'	32.46% 'Caa2'	338.84	99.62%	91.87%	1498.01
tranche 2 junior mezz.	13.27%	10.00% 'BBB-'	5.00% 'Ba1'	46.89	80.60%	68.16%	795.71
tranche 3 senior mezz.	24.92%	2.07% 'A-'	1.38% 'Baa1'	12.86	55.67%	49.27%	520.66
tranche 4 senior	31.25%	0.87% 'AA'	0.58% 'A2'	5.36	43.09%	37.50%	379.86
tranche 5 super-senior	37.50%	0.36% 'AAA'	0.04% 'Aa1'	0.40	32.21%	7.61%	71.83
corporate bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

spreads under the assumption that each underlying bond has a market-implied default probability of 20% until maturity, which implies a market spread of 111.95 bps. The obtained results are reported in columns (6)-(8) of Tables 1 and 2 for the CDO and CDO-squared, respectively.

A number of observations can be made. Firstly, it is seen that the transition from the physical to the risk-neutral measure corresponds to a huge increase in the default probabilities, expected losses and spreads for all tranches. Secondly, the magnitude of the changes, regardless of the tranches considered, is much higher for the CDO-squared than for the CDO. Thirdly, from a market pricing perspective, the most important observations follow from the analysis of the junior mezzanine tranches, which have the same (real-world) default probabilities and (real-world) expected losses as the collateral bonds. In Table 1 we see that while the underlying portfolio of 'BBB-' bonds has a fair spread of 111.95 bps, the similarly-rated CDO tranche has a fair spread of 320.69 bps. Hence the fair spread is almost 3 times as high. In Table 2 we observe that the corresponding 'BBB-' CDO-squared tranche has a fair spread of 749.52 bps, which is almost 7 times higher than the spread on the similarly-rated underlying bonds. These results clearly demonstrate that fair spreads on tranches are much higher than fair spreads on corporate bonds even when there are absolutely no differences in credit quality.

A closer examination of Tables 1 and 2 reveals that the yield enhancement is attributable to concentration of risk premia in spreads of non-equity tranches. In general, fair spreads on credit-sensitive instruments can be decomposed into compensation for pure default risk and additional risk premia (compensating for the uncertainty of securities' payoffs). The compensation for pure default risk can be read in column (5) of Table 1 or Table 2. The risk premia are simply calculated by subtracting the compensation for pure default risk from the total fair spread reported in column (8). For the corporate bonds, the assumed spread of 111.95 bps can be decomposed into 53.06 bps of pure default risk compensation and 58.89 bps of risk premia. For the junior mezzanine tranches, the compensation for pure default risk is 48.25 bps and 46.89 bps, respectively, for the CDO and CDO-squared. These values are slightly lower compared to 53.06 bps of pure default risk compensation for the corporate bonds. In contrast, the risk premia on the aforementioned tranches are respectively equal to 272.44 bps and 748.82 bps, so they are much higher than 58.89 bps of corporate risk premia. Relative to the similarly-rated corporate bonds, the risk premia are thus multiplied by a factor of almost 5 for the CDO tranche and by a factor of 13 for the CDO-squared tranche.

The results for the junior-mezzanine tranches are striking. They demonstrate that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds, which means that credit ratings are by far insufficient for pricing. The yield enhancement is possible because risk premia are concentrated in non-equity tranches, while the rating methodologies capture solely pure default risk. In other words, structured finance can produce securities that have low pure default risk and thus obtain high credit ratings, but have inherently high risk premia. On the one hand, it allows investors to earn higher spreads on highly rated portfolios. On the other hand, investors who overly rely on credit ratings for inferring fair spreads are likely to accept insufficient risk compensation on structured products.

The foregoing analysis can be generalized to the case of the more senior tranches; however, it requires consideration of similarly-rated corporate bonds.

Table 3: Comparison of fair spreads on tranches and risk-equivalent bonds.

In columns (1)-(5) we summarize tranche ratings and fair spreads reported previously in Tables 1 and 2. Note that the corresponding tranches and their risk-equivalent bonds have the same credit ratings by S&P and Moody's. In column (6) we report fair spreads on risk-equivalent corporate bonds (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the respective tranches. To calculate fair spreads on these bonds, we assume that their risk-neutral default probabilities are double the physical probabilities.

Tranche	S&P rating	Moody's rating	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	'BBB-'	'Ba1'	320.69	795.71	111.95 / 111.95
tranche 3 senior mezz.	'A-'	'Baa1'	143.83	520.66	28.03 / 28.23
tranche 4 senior	'AA'	'A2'	81.81	379.86	11.62 / 11.83
tranche 5 super-senior	'AAA'	'Aa1'	2.52	71.83	0.22 / 0.83

For this purpose, we create corporate bonds that have identical (real-world) default probabilities and (real-world) expected losses as the corresponding tranches, which implies the same credit quality. We call these bonds *risk-equivalent* to the respective tranches.[15] To determine fair spreads on the risk-equivalent bonds, we assume that their risk-neutral default probabilities are double the historical probabilities regardless of credit quality (robustness to this assumption is explained further down). The obtained results are summarized in Table 3. In column (6) we report fair spreads on the risk-equivalent bonds, while columns (1)-(5) summarize tranche ratings and fair premia previously shown in Tables 1 and 2.

The main message from Table 3 is that fair premia on the non-equity tranches are much higher than fair premia on their risk-equivalent bonds. The magnitude of the yield enhancement critically depends on tranche seniority and on whether a particular tranche belongs to the CDO or CDO-squared. For example, the spread on the super-senior CDO tranche is equal to 2.52 bps, while the spread on the corresponding risk-equivalent 'AAA' bond is equal to 0.22 bps. For the CDO-

[15]Since the expected losses and default probabilities of the corresponding CDO and CDO-squared tranches are different, we construct risk-equivalent bonds separately for the CDO and CDO-squared. In most cases the differences are small.

squared, the spread on the super-senior tranche is as high as 71.83 bps, while the corresponding risk-equivalent 'AAA' bond yields 0.83 bps.

We check robustness of the above results with respect to the ratio of risk-neutral to real-world default probabilities of the risk-equivalent bonds. The market evidence suggests that for highly rated bonds this ratio can be much higher than the assumed value of 2. For example, Hull et al. (2005) report a ratio of 16.8 for 'AAA' bonds. Under such assumption, the 'AAA' bonds that are risk-equivalent to the super-senior CDO and CDO-squared tranches would have fair spreads of 1.88 bps and 7.01 bps, respectively. Since these spreads are respectively lower than fair spreads of 2.52 bps and 71.83 bps on the corresponding super-senior tranches, we find that the yield enhancement on tranches is still achieved, which demonstrates robustness of our results.

The tranche yield enhancement is also critically dependent on the assumed risk premia on the underlying bonds. In the appendix in Table A.I we replicate the results of Table 3 under the assumption of 180.72 bps market spread on the collateral 'BBB-' bonds, i.e. higher bond risk premia as suggested by Hull et al. (2005).[16] It is seen, for example, that the fair spreads on the super-senior CDO and CDO-squared tranches are equal to 13.46 bps and 428.03 bps, respectively, compared to 2.52 bps and 71.83 bps in the baseline case. This indicates that the yield enhancement on tranches is increasing with higher risk premia on the underlying bonds relative to the baseline case. Since the baseline level of risk premia on the underlying 'BBB-' bonds is fairly low compared to the estimates given in the literature (see Section 3), the results of this paper likely underestimate the true magnitude of the tranche yield enhancement.

The documented large differences in fair spreads between similarly-rated tranches and bonds create opportunities for rating arbitrage, which means that excess spreads can be distributed between tranche investors and CDO issuers. This explains why structured finance securities are so appealing to originators as well as investors. Since investors are typically risk averse and CDO tranches are tailored to their risk appetites, the total risk compensation paid on the tranches of a CDO can be lower than the total spread received on the collateral portfolio. In other words, the risk-return profiles of tranches can be attractive to investors at the spread levels, which are below the model-implied fair spreads. In this sense the ability of

[16]The market spread of 180.72 bps is equivalent to assuming that risk-neutral default probability of each of the underlying 'BBB-' bonds is triple the 10-year physical default probability of 10%. That is motivated by Hull et al. (2005) who find that the ratio of risk-neutral to physical default probabilities is equal to 5.1 for 'BBB' bonds and 2.1 for 'BB' bonds.

financial engineering to tailor the risks of tranches creates value. The remaining share of the yield can then be allocated to CDO issuers compensating them for the risks associated with their part of structured finance activities. These risks arise because the originators are often unable to sell the total notional of all CDO tranches and have to retain and hedge remaining risks. For example, in the market for synthetic CDOs, single tranche issues were very popular, so the originating banks were only partly securitizing the underlying CDS portfolios. There are also reputational risks, which can lead financial institutions to possibly bail-out their CDOs as was in the case of Bear Stearns.

Fair tranche spreads reported in this paper are computed using the Gaussian copula model with the same correlation parameter for all tranches. This is a simplification compared to the market practice of pricing different tranches using different correlation parameters, which are summarized by the base correlation curve. This market practice is often interpreted as a way to correct for limitations of the Gaussian copula dependence structure, market appetite for risk or exposure of tranches to systemic risk. Nevertheless, the use of the base correlation curve merely reallocates some yields between different tranches, so our simplified approach still captures the economics of the tranche yield enhancement and of the rating arbitrage.

A question can be raised on whether our theoretical approach gives results that can be directly generalized to the true world. We argue that credit ratings and fair spreads reported in this paper can be considered to be accurate and unbiased as they are obtained using the market standard models on the basis of consistent and realistic assumptions. Specifically, we use the same default dependence to generate data for both rating and pricing securities. This simplification allows us to analyze the meaning of credit ratings without concern about any market imperfections, changing circumstances or model limitations. Therefore, our results demonstrate that even if the rating agencies could accurately estimate (real-world) default probabilities and (real-world) expected losses of CDO tranches, then credit ratings would still imply little about the level of fair spreads. That is because fair spreads are mainly driven by risk premia, while the rating methodologies capture solely pure default risk. While the standard rating approach has proven to be adequate for corporate bonds, structured securities might require a different rating approach as they are specifically tailored by originators to maximize the yield enhancement on tranches.

5. Sensitivity analysis

In this part we examine the sensitivity of tranche payoffs to default probabilities of the underlying bonds with the aim of providing a clear-cut explanation of the yield enhancement on tranches and illustrating their associated risk properties. We depict sensitivity of tranche default probabilities and expected tranche losses because these risk measures determine credit ratings. Moreover, expected losses are also closely related to spreads, so we can make inference about tranche spreads.[17]

Figure 1 presents the sensitivity results for CDO tranches. To benchmark tranche sensitivities, we plot curves corresponding to the underlying portfolio of bonds. To facilitate inference, we add vertical lines representing the real-world measure, i.e. at the collateral default probability equal 10%, and the risk-neutral measure, i.e. at the collateral default probability equal 20%. Note that these lines cross the CDO tranche curves at the values corresponding to the results of Table 1.

Panel A of Figure 1 explores the sensitivity of tranche default probabilities to changes in default probabilities of the collateral bonds. It is seen that the sensitivity of the tranche default probabilities is generally higher than the corresponding sensitivity of the collateral bonds. In Panel B we present the sensitivity of expected tranche losses and we observe qualitatively similar results. Only the expected loss of the super-senior tranche appears to be fairly insensitive to a modest increase in the collateral default probability. However, the relative (percentage) increase in the expected loss of the super-senior tranche is very large in the 10-20% interval of the collateral default probability. Table 1 shows that the expected tranche loss increases 27 times (from 0.01% to 0.27%) when the collateral default probability doubles (from 10% to 20%).

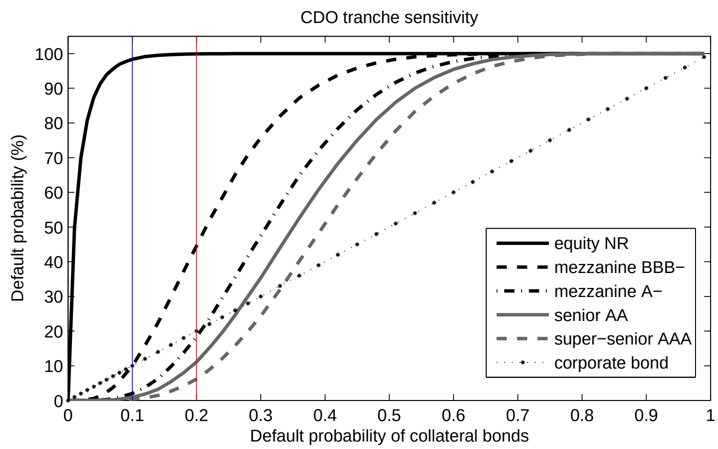
Figure 1 is the key to understanding the mechanics of the yield enhancement on tranches. On the one hand, CDO tranches must be structured to have sufficiently low (real-world) default probabilities and (real-world) expected losses to qualify for investment grade credit ratings. On the other hand, CDO tranches must have sufficiently large risk-neutral expected losses to provide higher spreads than similarly-rated corporate bonds. These two properties can only be combined due to high sensitivity of expected tranche payoffs to collateral default probabilities seen in Figure 1.

To illustrate the argument, let us consider the junior mezzanine tranche and

[17] It follows from Eq. 8 and Eq. 9 that the expected tranche loss is equal to the tranche default leg rescaled by the tranche notional and corrected for discounting.

the (underlying) corporate bond. Clearly, these two securities have equal default probabilities and expected losses at the real-world level of the collateral default probability (‘0.1’ line). Consequently, the tranche and the bonds have the same

Panel A: Sensitivity of CDO tranche default probabilities



Panel B: Sensitivity of CDO expected tranche losses

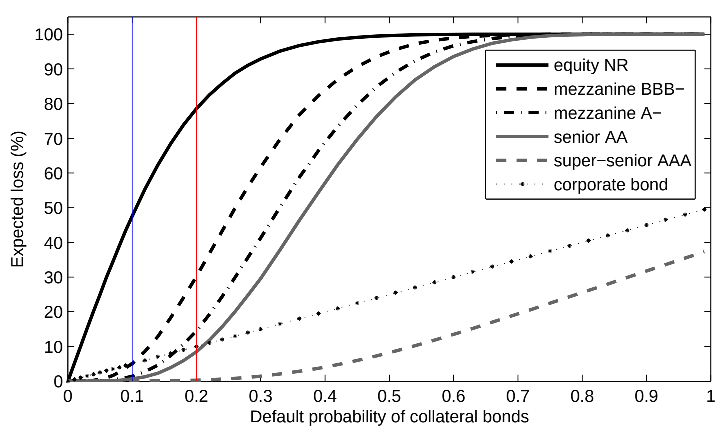


Figure 1: This figure illustrates sensitivity of payoff prospects of CDO tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value ‘0.1’ correspond to the real-world level of collateral default probabilities, whereas the ‘0.2’ lines correspond to the risk-neutral level. For ease of comparison, we add curves representing the collateral bonds.

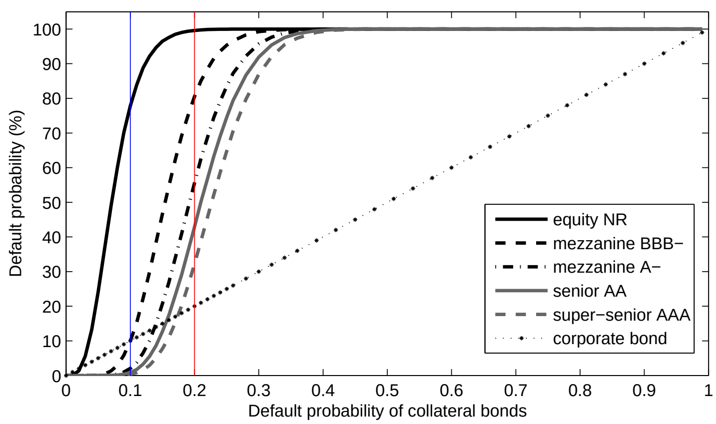
credit ratings. In contrast, the expected tranche loss at the risk-neutral level of the collateral default probability ('0.2' line) is considerably higher than the expected bond loss. This is only possible because the curve of the expected tranche loss is steeper than the curve of the expected bond loss in the 10-20% interval of the collateral default probability. Consequently, the fair spread on the junior-mezzanine tranche is much higher than the fair spread on the corporate bonds.

The yield enhancement mechanics for CDO-squared tranches is analogous to that for CDO tranches. It is evident from Figure 2 that the sensitivity of CDO-squared tranches is much higher compared to the CDO case. It is seen that non-equity CDO-squared tranches are structured to meet very low default probability and expected loss benchmarks that are required for their rating categories. However, as soon as the default probability of the collateral bonds increases beyond the real-world level of 10%, we observe a huge increase in both risk measures. Particularly, the largest increments in the tranche default probabilities and expected tranche losses are visible in the 10-20% interval of the collateral default probability, which explains the magnitude of the yield enhancement on CDO-squared tranches.

Structured securities are commonly referred to in the literature as leveraged instruments. In general, securities are highly leveraged when their payoffs are highly volatile with changes in credit conditions. Figures 1 and 2 illustrate that CDO-squareds are more leveraged than CDOs and also that tranches have different risk properties compared to corporate bonds. In particular, CDO-squareds are structured at the 'critical' points such that expected tranche losses are still low, but they rise dramatically if the collateral default probability increases beyond the real-world level. The implication of this feature is that CDO-squared tranches have little upside potential relative to their real-world expected payoffs, while adverse market changes are likely to generate huge losses. Similar asymmetry of payoffs is seen for CDO tranches although it is of lesser magnitude. In contrast, a pool of bonds is characterized by symmetry in payoff prospects.

From the analysis of Figures 1 and 2, it is clear that tranche yield enhancement increases with higher sensitivity of expected tranche payoffs. Let us therefore analyze how to construct CDOs maximizing the yield enhancement on tranches. A critical feature determining the sensitivity of tranche payoffs is diversification of the collateral portfolio. Note that portfolio diversification can be increased by selecting collateral bonds with lower pair-wise asset value correlations or by increasing the number of bonds for a given portfolio notional. By replicating Figures 1 and 2 (unreported results), we find that tranche sensitivity curves become steeper for more diversified portfolios. To illustrate the impact of portfolio diversification on the tranche yield enhancement, in Table A.II in the appendix we report tranche spreads under the assumption of 5% asset value correlation (in this analysis we re-calculate tranche subordination levels according to the new corre-

Panel A: Sensitivity of CDO-squared tranche default probabilities



Panel B: Sensitivity of CDO-squared expected tranche losses

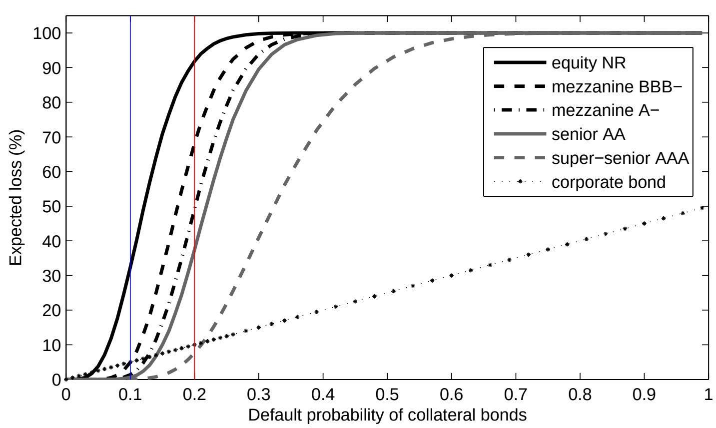


Figure 2: This figure illustrates sensitivity of payoff prospects of CDO-squared tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value '0.1' correspond to the real-world level of collateral default probabilities, whereas the '0.2' lines correspond to the risk-neutral level. For ease of comparison, we add curves representing the collateral bonds.

lation assumption). In such case, the junior mezzanine CDO tranche has a fair spread 522.85 bps and the corresponding CDO-squared tranche has a fair spread of 1271.46 bps compared to, respectively, 320.69 bps and 749.52 bps under the baseline assumption of 12.5% correlation. This shows that tranches backed by more diversified collateral portfolios have much higher fair spreads than tranches backed by less diversified portfolios, ceteris paribus. This finding sharply contrasts with the widespread view that high diversification of a collateral pool is an advantage for CDO investors. This view is also shared by the rating agencies; for example, when S&P discusses strengths and weaknesses of a newly issued CDO[18], then high diversification of the collateral portfolio is always classified as a 'strength'.[19]

Finally, the tranche yield enhancement is also dependent on the size of risk premia of the collateral bonds. In Table A.I in the appendix we showed that higher risk premia on the underlying 'BBB-' bonds can result in substantially higher yield enhancement on tranches. This result is confirmed by Figures 1 and 2 where it is seen that expected tranche losses are monotonically increasing as a function of collateral default probabilities. Note that the rating agencies derive physical default probabilities on the basis of credit ratings, so by construction these probabilities are the same for similarly-rated bonds. For 'BBB-' bonds, the real-world default probability is equal to 10%. However, CDS spreads of 'BBB-' bonds (or equivalently their market-implied default probabilities) can vary substantially from one bond to another. A CDO issuer aiming to maximize tranche spreads should therefore select collateral bonds with relatively high CDS spreads for their credit ratings. This leads to the adverse selection problems as high CDS spreads indicate higher risks that are not captured by the rating agencies. There is anecdotal evidence that such adverse selection was one of the reasons for the poor performance of CDOs during the financial crisis (Fitch, 2008).

[18]Upon announcing credit ratings of newly issued CDO tranches, S&P releases so-called 'New Issue' reports that provide details of S&P's analysis and justification for the assigned ratings.

[19]Of course, if we had considered tranches with fixed subordination levels, then lower asset value correlation would be to the benefit of investors in senior CDO tranches. In practice, however, CDO originators select more diversified collateral portfolios specifically to lower tranche subordinations such that to maximize the volume of tranches with as high ratings as possible.

6. Rating and price stability of CDO tranches

The results of the previous section show that even a highly rated tranche, which is structured to have a minute expected loss under the physical measure, can incur heavy losses if the realized default rate of the collateral pool exceeds the assumed threshold. In a dynamic setting, tranche prices might become depressed even prior to the realization of collateral losses. When credit conditions deteriorate, then CDS spreads widen as a consequence of a rise in actual default probabilities as well as in corresponding risk premia. In this case, investors should re-price CDO tranches using the revised market-implied default probabilities of the underlying bonds. The changes in prices of CDO tranches are typically much higher than the changes in prices of corporate bonds due to high sensitivity of tranche payoffs documented in Section 5. If a deterioration in credit conditions is prolonged relative to time until CDO maturity, then the prices of CDO tranches are unlikely to recover. This might go a long way in explaining the dramatic decline in prices of structured securities during the 2007-2009 financial crisis.

In case of unfavorable market conditions, tranche ratings can be expected to come under severe stress as well. Hereby, we enumerate key factors that are likely to cause a deterioration in tranche ratings. Firstly, credit ratings are highly sensitive to credit enhancement levels, which are reduced once defaults hit the underlying portfolios. Secondly, possible downgrades within a collateral pool lead to an increase of the rating agencies' estimates of (real-world) default probabilities of the collateral bonds. This has a stronger effect on tranche ratings than on bond ratings again due to higher sensitivity of tranche payoffs. Thirdly, due to reputational concerns, the re-pricing of CDO tranches is likely to trigger further downgrades of CDO tranches as explained further in this section.

To illustrate the divergence between the stability of CDO tranches and corporate bonds, we analyze a scenario corresponding to a fairly severe deterioration in credit conditions, i.e. a one notch downgrade of the entire collateral portfolio from 'BBB-' to 'BB+'. The CDO and CDO-squared are structured and rated under the baseline assumptions given in Section 3. We assume that soon after the issuance, the estimates of 10-year default probabilities of the underlying bonds increase from 10% to 13%. We suppose that the tranches are next re-rated using the revised default probabilities of the collateral bonds, but keeping the tranche subordination levels fixed. Table 4 presents the results for the CDO and CDO-squared.[20]

Table 4 documents a large deterioration in the credit quality of the tranches, particularly for the senior and super-senior tranches.[21] The super-senior CDO tranche is downgraded from the initial rating of 'AAA' to 'AA-' and the corresponding CDO-squared tranche is downgraded as far as to the 'BBB+' grade. In other words, a one notch downgrade of the collateral pool triggers downgrades of the super-senior tranches by as many as 3 and 7 notches.

An argument can be made that the scenario analyzed in Table 4 is not very realistic as the rating agencies are unlikely to downgrade the entire collateral portfolio and subsequently re-run their rating models using the revised default probabilities. However, we note that a similar deterioration in credit quality of the tranches can occur if a large portion of the collateral bonds is downgraded by more than one notch. Furthermore, default contagion can lead to a situation when a default of a single collateral bond can by itself explain a substantial increase in default probabilities of the surviving bonds.

To study the impact of default contagion, we consider a scenario when a single default within the collateral portfolio occurs soon after CDO origination. Let us first assume that this early default does not change market expectations about default probabilities of the surviving bonds. In this setting, the possible impact on tranche ratings is limited because the portfolio loss rate increases by about 0.5% (given a 50% recovery rate). It might trigger, for example, a one notch downgrade of the tranches if they were tailored to just satisfy the rating criteria.[22]

The impact of a single default is likely to be very different in the presence of credit contagion when an early default signals a low realization of the common economic factor Y in Eq. 5 or Eq. 6.[23] The credit-worthiness of the tranches is then reduced not only due to a partial loss of tranche credit enhancements, but also due to the increased likelihood of a market-wide deterioration in credit conditions. For illustration, we calculate the conditional default probabilities of the

[20]The values reported in Table 4 correspond to those at which tranche curves in Figures 1 and 2 would cross vertical lines at 10% and at 13% of the collateral default probability.

[21]For ease of exposition, in this section we consider only S&P ratings.

[22]Nevertheless, tranches are highly sensitive to several defaults within a collateral pool, which follows from the sensitivity analysis in Section 5.

[23]We consider information-driven default contagion as discussed by Schonbucher (2004). This means that default contagion arises because an obligor's default reveals some information about the common economic factor Y driving the riskiness of all obligors. This definition is different from a more standard one according to which default contagion is associated with "a direct causal relationship between two obligor's defaults" (Schonbucher, 2004).

Table 4: Impact of a deterioration in collateral credit quality on tranche ratings.

This table analyzes the impact of a deterioration in credit conditions corresponding to an increase in 10-year default probabilities of the collateral bonds from 10% to 13%. This is equivalent to a one-notch downgrade from BBB- to BB+. The first two columns summarize the capital structure of the CDO and CDO-squared. Columns (4) and (5) report tranche default probabilities under the standard market conditions (i.e. collateral default probability of 10%) and credit ratings by S&P. In columns (7) and (8) we report tranche default probabilities after the market conditions have deteriorated (i.e. collateral default probability of 13%). In these columns we also report the revised tranche ratings and we calculate the severity of downgrading (in number of notches).

		Standard market conditions (PD = 10%)			Deteriorated market conditions (PD=13%)		
Tranche	Tranche subordination CDO/CDO-sq.	Default probability			Default probability		
		Bond	CDO	CDO-squared	Bond	CDO	CDO-squared
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0% / 0%		98.33% 'NR'	77.88% 'CCC-'		99.34% 'NR'	92.23% 'NR'
tranche 2 junior mezz.	9.90% / 13.27%	10.00% 'BBB-'	10.00% 'BBB-'	10.00% 'BBB-'	13.00% 'BB+' 1 notch	19.14% 'BB-' 3 notches	29.83% 'B+' 4 notches
tranche 3 senior mezz.	14.75% / 24.92%		1.97% 'A-'	2.07% 'A-'		4.96% 'BBB' 2 notches	10.15% 'BBB-' 3 notches
tranche 4 senior	17.08% / 31.25%		0.87% 'AA'	0.87% 'AA'		2.40% 'A-' 4 notches	5.46% 'BBB' 6 notches
tranche 5 super-senior	19.45% / 37.50%		0.36% 'AAA'	0.36% 'AAA'		1.06% 'AA-' 3 notches	2.87% 'BBB+' 7 notches

surviving obligors given that the first default in the collateral portfolio occurs at time t within one year after CDO issuance. This analysis is based on the CDO collateral portfolio of 100 bonds whose detailed description along with assumptions is presented in Section 3.[24]

The results plotted in Figure 3 show that a sudden and early default event can lead to a large increase in default probabilities of the surviving names. If the first default occurs after 1, 3 and 6 months, then the conditional default probabilities of the surviving bonds jump to, respectively, 16.19%, 13.31%, 11.14%. This means that a deterioration of credit conditions considered in Table 4 can be easily caused

[24] Appendix B explains how the conditional default probabilities are calculated.

by a single default within the collateral pool after roughly 3 months, which is not an unlikely scenario under stressful market conditions.[25]

Our analysis of default contagion shows that abrupt and severe downgrades of CDO tranches can be explained within the market-standard models. Let us now generalize this finding to the actual market setting. In practice, investors derive default probability estimates based on the pace and severity of defaults within the universe of assets correlated with a particular collateral portfolio; or equivalently, they look at overall (corporate) credit conditions. Note that collateral portfolios are typically comprised of bonds with low investment-grade ratings (such as 'BBB'), which are known to perform pro-cyclically with much higher default rates in economic recessions.[26] Since the rating agencies derive default probability estimates for collateral bonds solely based on their credit ratings, these are

[25]In this analysis, it is critical that the first default occurs relatively early, so that it sends a strong negative signal about credit conditions. If the first default occurs after 1 year, then the conditional default probability of the surviving bonds is equal to 8.86%, which roughly corresponds to the unconditional default probability of 1% per annum over the remaining 9 years until maturity. That is because such default event is neutral information for market participants as it is line with ex-ante expectations about credit quality.

[26]An S&P definition of the 'BBB' rating states: "An obligor rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitments" (S&P, 2009).

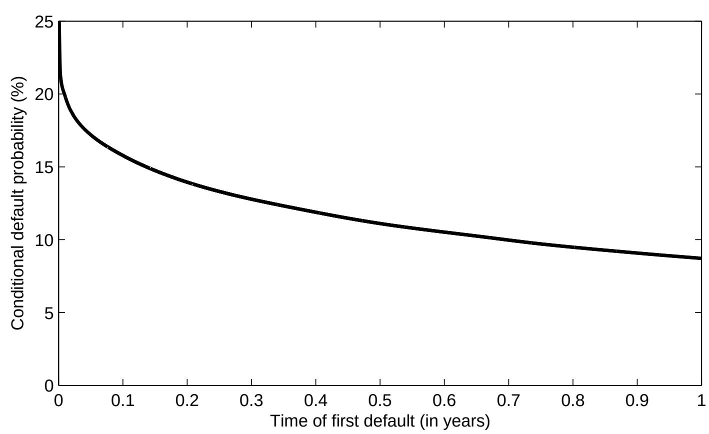


Figure 3: This figure plots default probability of the surviving bonds conditional on the information that the first default in the collateral pool occurs at time t.

'through-the-cycle' estimates reflecting historical default frequencies of similarly-rated bonds. During economic recessions, therefore, the actual default probabilities of collateral bonds can be expected to significantly exceed the rating agencies' estimates. This means that a significant deterioration in collateral credit quality considered in Table 4 is most likely to occur when the economy enters a recession.

When the economy enters a recession, then investors will re-price CDO tranches as soon as CDS spreads of their collateral bonds widen. Nevertheless, in such case, the rating agencies might be reluctant to downgrade tranches for the sake of rating stability. In particular, CDO tranches might still be far from defaulting even when their actual default probabilities increase significantly such that they no longer meet the criteria for their credit ratings. However, the rating agencies will feel the pressure to timely downgrade CDO tranches if their market prices are declining. Otherwise, the resultant disparity between high credit ratings of CDO tranches and (at some point) low market value would undermine the reliability of the rating agencies. The rating agencies' decisions to downgrade CDO tranches are rather arbitrary and they can be made before realization of significant collateral losses. To support downgrading, the rating agencies can quote changing market circumstances or revise rating assumptions.[27] For example, during the 2007-2009 financial crisis, S&P increased its estimate of the baseline correlation between RMBS tranches from 0.3 to 0.35-0.75, which resulted in massive downgrades of ABS CDOs (S&P, 2008). Similarly, credit ratings of other structured securities have failed as an indication of payoff prospects. Benmelech and Dlugosz (2009b) find that CDO tranches experienced more frequent and severe downgrades than like-rated corporate bonds in 2007-2009, in particular for triple-A securities.

Coval et al. (2009) show that default risks of CDO tranches are concentrated in systematically adverse economic states. This result is in line with intuition because tranches that reference portfolios of assets have little exposure to idiosyncratic risks. Consequently, if we condition on the realization of an economic recession, then (non-equity) tranches can be expected to incur substantially more downgrades and losses than similarly-rated corporate bonds. Conversely, tranches should outperform similarly-rated corporate bonds during favorable market conditions. This means that even triple-A tranches inherit procyclicality of performance from their lower-rated collaterals.

[27] According to Fitch (2008) "The [rating] committees may make adjustment to standard assumption, or call for bespoke analysis. In addition, general economic outlook for certain sectors or industries may be taken into account".

In practice, model error and possible bias in rating assumptions are additional risk factors for CDO tranches. From the sensitivity analysis of CDO tranches and also from Table 4, it follows that if collateral default probabilities are underestimated, then CDO tranches might turn out to be dramatically more risky than initially assumed. Moreover, rating methodologies for CDO tranches are dependent on default correlation, which of course plays no role for rating corporate bonds.

In light of the foregoing discussion, the comparability of rating stability between corporate bonds and CDO tranches is doubtful, particularly for highly rated tranches. In the corporate bond markets, the highest credit quality can be considered a guarantee of very low default risks and good rating stability. Notably, very few corporates qualify for the 'AAA' rating, which in case of Fitch is about 1% of its total corporate coverage (Fitch, 2007). In contrast, almost 60% of the volume of CDO tranches rated by Fitch is assigned 'AAA' ratings (Fitch, 2007). It seems that lower rating stability is the downside of the rating methodologies allowing for issuance of such large volumes of highly rated tranches, which in addition provide higher spreads relative to similarly-rated bonds.

7. Conclusions

This paper shows that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds implying that credit ratings are not sufficient for pricing. This creates rating arbitrage possibilities, which explains why structured securities can be so appealing to investors as well as issuers. Credit ratings reflect real-world payoff prospects, while pricing is done under the risk-neutral measure. On the one hand, CDO tranches are tailored to have sufficiently low pure default risks to meet the criteria for the highest credit ratings. On the other hand, CDO tranches must have sufficiently large risk-neutral expected losses to provide higher spreads relative to similarly-rated corporate bonds. The recipe for maximizing yield enhancement is to produce tranches backed by highly diversified portfolios of bonds with high CDS spreads relative to their credit ratings.

We have further examined risk properties of CDO tranches and we have shown that expected tranche payoffs are highly sensitive to default probabilities of the collateral bonds. This risk property is necessary for achieving the yield enhancement on tranches. However, the downside of such risk profile is that CDO tranches are inherently prone to incur large losses and massive downgrades when market conditions deteriorate. Tranche downgrading is likely to be exacerbated by default contagion and by corresponding re-pricing of CDO tranches. It follows that CDO tranches have very different risk properties relative to similarly-rated corporate bonds. For CDO investors, there is a clear trade-off between yield enhancement and severe downgrades and losses if credit conditions deteriorate.

Our findings show limitations of credit ratings understood as a universal measure of credit quality. Even if the rating agencies provide accurate and unbiased estimates of real-world default risks, then credit ratings are still not sufficiently informative about the level of fair spreads. This suggests that improving the rating methodologies should go beyond minor changes aiming to better account for parameter uncertainty or introducing more conservative assumptions. The key issue to be considered is whether structured securities require a different rating approach accounting for their distinguishing risk characteristics.

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Appendix A.

Table A.I: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 30% risk-neutral default probability of the underlying bonds.

In columns (1)-(3) we summarize tranche subordination levels and credit ratings. The structuring and rating is done under the baseline assumptions. In columns (4) and (5) we report fair tranche spreads, while column (6) reports fair spreads on the corresponding risk-equivalent bonds (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the respective tranches. To calculate fair spreads, we assume that risk-neutral default probabilities of bonds are triple the physical probabilities.

Tranche	Tranche subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	9.90% / 13.27%	'BBB-'/'Ba1'	784.92	1717.63	180.72 / 180.72
tranche 3 senior mezz.	14.75% / 24.92%	'A-'/'Baa1'	453.97	1407.68	42.49 / 42.75
tranche 4 senior	17.08% / 31.25%	'AA'/'A2'	306.75	1228.89	17.83 / 17.34
tranche 5 super-senior	19.45% / 37.50%	'AAA'/'AA1'	13.46	428.03	0.33 / 1.21

Table A.II: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 5% asset value correlation.

In columns (1)-(5) we summarize tranche subordination levels, ratings and fair spreads. The structuring and rating is done under the assumption that asset value correlation is 5% instead of 12.5% used in the baseline case. In column (6) we report fair spreads on corporate bonds that are risk-equivalent to the corresponding tranches (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. To calculate fair spreads on these bonds, we assume that their risk-neutral default probabilities are double the physical probabilities.

Tranche	Tranche subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	8.45% / 12.52%	'BBB-'/'BA1'	522.85	1271.46	111.95 / 111.95
tranche 3 senior mezz.	11.40% / 22.07%	'A-'/'Baa1'	282.38	1057.71	27.34 / 28.16
tranche 4 senior	12.64% / 27.35%	'AA'/'A2'	186.13	906.41	11.46 / 11.09
tranche 5 super-senior	13.96% / 33.41%	'AAA'/'Aa1'	4.15	237.00	0.12 / 0.62

Appendix B.

Consider a random vector $(V_1, V_2, ..., V_{100})$ which has a multivariate standard normal distribution with a common pair-wise correlation parameter ρ. Each V_i represents the scaled value of obligor i. We want to calculate default probability of the surviving obligors conditional on the information that the first obligor in the collateral pool defaults at a given time t. Without loss of generality (due to symmetry), we calculate the default probability of the first obligor conditional on the default of the 100th obligor at time t. In mathematical terms, this probability is given by:

$$Pr\left(V_1 \leq K | V_1 > k^*, V_2 > k^*, ..., V_{99} > k^*, V_{100} = k^*\right), \tag{B.1}$$

where K is the threshold corresponding to a default after 10 years and k^* is the threshold corresponding to a default at time t (e.g. 3 months).

A feature of the multivariate normal distribution is that the conditional distribution of $(V_1, V_2, ..., V_{99})$ given a known value of V_{100} is again normally distributed with adjusted conditional mean and variance matrices. The conditional mean of each V_i (corresponding to the surviving bonds) is ρk^*, the variance is $1 - \rho^2$ and the pair-wise correlations between V_i and V_j are equal to $\rho - \rho^2$. Using this conditional distribution, we can easily calculate the probability given by formula B.1 using simulations.

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