Risk-sharing or risk-taking? Counterparty risk, incentives and margins*

Bruno Biais† Florian Heider‡ Marie Hoerova§

First version: March 2010
This version: July 2011

Abstract

We analyze hedging contracts under asymmetric information. We model hedging as the design of a contract between a protection buyer seeking to reduce his risk exposure and a protection seller. If the seller learns that the hedge is likely to be loss-making for her in the future, her incentives to take risk on her other positions (balance sheet risk) increase. The seller's risk-taking incentives limit hedging and can generate endogenous counterparty risk. We show that variation margins enhance the scope for risk-sharing but may, in fact, lead to more risk-taking. Initial margins, in turn, address the market failure caused by unregulated trading of hedging contracts.

JEL classification: G21, G22, D82

Keywords: Insurance; Complete contracts; Moral hazard; Counterparty risk; Margin requirements

*We would like to thank our discussants Ulf Axelson, Sugato Bhattacharyya, Bruce Carlin, Simon Gervais, Artashes Karapetyan and seminar participants at the NBER Summer Institute, the American Finance Association meeting (Denver), the 3rd Conference of the Paul Woolley Centre for the Study of Capital Market Dysfunctionality (LSE), the Carefin conference "Matching Stability and Performance" (Bocconi), the 6th MTS Conference on Financial Markets (London), the CEPR ESSFM meeting (Gerzensee), the Conference on Government Intervention and Moral Hazard in the Financial Sector (Norges Bank), the Conference on Organization of Markets (UCSB), the Conference on Risk Management after the Crisis (Toulouse School of Economics), the UCLA, Queen's University, University Paris X, the University of Minho, the Bank of Canada and the ECB. The views expressed do not necessarily reflect those of the European Central Bank or the Eurosystem.

†Toulouse School of Economics, email: biais@cict.fr

‡European Central Bank, Financial Research Division, email: florian.heider@ecb.int

§European Central Bank, Financial Research Division, email: marie.hoerova@ecb.int

1 Introduction

We analyze the benefits and costs of hedging contracts under asymmetric information. The benefits arise since hedging enhances risk-sharing opportunities between agents with different risk-bearing capacities. The costs arise when hedging creates *hidden liability* that increases risk-taking incentives. We build on this trade-off to develop an incentive-based rationale for margin requirements.

We model hedging as the design of an optimal contract between a risk-averse buyer of insurance who seeks protection against a risk exposure and a risk-neutral seller of insurance who provides the protection. Examples of hedging contracts are forwards, futures or credit default swaps (CDS).

Financial institutions selling protection are exposed to risk from their own assets and liabilities (balance sheet risk). Managing balance sheet risk is costly. For example, financial institutions must devote resources to scrutinize the default risk of their borrowers or to manage their maturity mismatch.[1] Not managing balance sheet risk (risk-taking) can lead to the failure of the protection seller and to the default on his contractual obligations. Protection buyers are therefore exposed to counterparty risk. For example, Lehman Brothers and Bear Stearns defaulted on their CDS derivative obligations because of losses incurred on their other investments, in particular sub-prime mortgages.

Our main assumption is that the care with which financial institutions manage their balance sheet risk is unobservable to outsiders and that financial institutions are protected by limited liability. This creates a moral hazard problem between the buyer and seller of protection. In this context, we show that hedging creates hidden liability: Ex-ante, when entering the position, the hedge is neither an asset nor a liability for the protection seller. For example, the seller of a credit default swap pays the buyer in case of credit events (default, restructuring) but collects an insurance premium otherwise. On average, the seller must at

[1] For example, in the wake of the 2007-09 crisis many financial institutions financed themselves through short-term debt. While such financing was relatively easy to establish, it left these institutions exposed to the risk of not being able to roll-over their liabilities (Brunnermeier and Oehmke, 2009).

least break even for her to be willing to enter the position. But, if negative information about the underlying position arrives after the contract is signed, the hedge is more likely to be a liability rather than an asset for the protection seller. For instance, after bad news about the future solvency of firms, the seller of a CDS is more likely to pay out the insurance than after good news. The implicit liability embedded in the hedging position undermines the protection seller's risk-management incentives. This is because she bears the full cost of risk-management while the benefit accrues in part to the protection buyer.

Given the incentives of the protection seller, the protection buyer faces a trade-off. If he wants to curb seller's risk-taking incentives, he must reduce the hidden liability and opt for a contract with limited risk-sharing. Such under-insurance is costly and he may instead choose a contract providing full insurance, recognizing that it will encourage seller's risk-taking and lead to counterparty risk. We show that the latter type of contract is optimal when the seller's moral hazard is severe and, at the same time, the probability of seller's default is perceived to be small.

Our analysis identifies a channel through which hedging under asymmetric information can lead to increased aggregate risk. In the absence of a moral hazard problem, the risk the protection buyer is hedging and the balance sheet risk of the protection seller are independent. Under asymmetric information, however, the hidden liability can lead to risk-taking by the seller after bad news about the buyer's risk. Hence, hedging contracts create interconnectedness: bad news about buyer's risk lead to an increased default risk of the seller.

We use the model to develop an incentive-based theory of margin requirements. The buyer and seller of protection can agree as part of their risk-sharing transaction that the seller deposits cash on a separate account when the buyer makes a margin call. This is costly since deposited cash earns a lower rate of return than assets kept on the balance sheet. The benefit is that the cash in the margin account is ring-fenced from the seller's risk-taking. The margin thus reduces the size of the moral hazard problem between the protection buyer and seller.

We show that margins are called after bad news about the underlying position since only then are the seller's risk-management incentives jeopardized. That is, *variation* margins are optimal. In the contract with limited insurance but with the seller's risk-management incentives intact, variation margins enhance the scope for risk-sharing. In the contract with risk-taking by the seller but full insurance in no-default states, variation margins insure the protection buyer against the risk of the seller's default. The overall effect of margins on aggregate risk is ambiguous: while they enhance risk-sharing opportunities, they also make the protection buyer more willing to tolerate risk-taking by the protection seller since they protect him against counterparty risk.

We show that our benchmark model with one protection buyer and one protection seller is isomorphic to a model in which a protection buyer acquires insurance from multiple protection sellers taking correlated risks. However, if protection sellers can re-trade hedging contracts and transfer contractual obligations, the constrained efficient contract is no longer incentive-compatible. Efficiency can be restored using *initial* margins that are imposed as soon as a seller's position exceeds a threshold.

Rajan (2006) provides an informal account of how hedging can lead to more risk-taking in the economy. The paper closest to ours is Thompson (2010). He also analyzes financial insurance contracts and the effects of counterparty risk. In his model, there is an adverse selection problem on the protection buyer's side as he privately observes the hedged risk. The protection seller is subject to moral hazard as she makes an unobservable choice between a liquid or an illiquid portfolio. Choosing an illiquid portfolio is more profitable but if the protection buyer turns out to be a high risk, illiquid assets need to be liquidated prematurely to pay out insurance. Such liquidation is costly and the protection seller may default, giving rise to counterparty risk. This provides ex ante incentives to high risk protection buyers to reveal their type truthfully so that the protection seller chooses a liquid portfolio. Stephens and Thompson (2011) analyze the effects of competition among multiple protection sellers. Acharya and Bisin (2010) focus on the contractual externality between protection buyers.

Once a protection seller sold insurance to one buyer, selling additional insurance to another buyer increases the seller's incentives to default strategically. Bolton and Oehmke (2011) show that derivatives can be useful risk-sharing tools but argue that they should not be senior in bankruptcy relative to other creditors. Otherwise, derivatives markets can becomes inefficiently large from a social perspective.

We show how information shocks can create hidden liabilities and weaken incentives to exert risk-management effort. Holmström and Tirole (1998), by contrast, examine how liquidity shocks weaken incentives. Our hidden liability operates like a debt-overhang (Myers, 1977). But instead of exogenous debt, we have endogenous liabilities as a result of optimal contracting.

Our approach differs from other models of systemic risk, e.g., Freixas, Parigi and Rochet (2000), Cifuentes, Shin and Ferrucci (2005), and Allen and Carletti (2006), since in our analysis contagion arises because of incentive problems in the market for financial insurance.

The remainder of the paper is organized as follows. In Section 2, we describe the model setup. In Section 3, we analyze the benchmark case in which risk-management effort is observable and there is no moral hazard. In Section 4, we analyze the optimal contract when risk-management effort is unobservable. To highlight the basic trade-off between risk-sharing and risk-taking, we abstract from margins in this section. We also show that our formulation with risk-management effort (or risk-taking) by the protection seller is similar to a formulation with the (lack of) risk-shifting by the protection seller. In Section 5, we analyze the optimal contract with margins. In Section 6, we analyze the case when there are multiple protection sellers who can reinsure or trade with each other. Section 7 concludes. All proofs are in the Appendix.

2 The model

There are three dates, $t = 0, 1,2$, and two agents, the protection buyer and the protection seller, who can enter a risk-sharing contract at $t = 0$.

Protection buyer. The protection buyer is risk-averse with twice differentiable concave utility function, denoted by u. At $t = 0$ he is endowed with a risky exposure of size 1 and return $\tilde{\theta}$. The return is realized at $t = 2$. It can take on two values: $\bar{\theta}$ with probability π and $\underline{\theta}$ with probability $1 - \pi$. The protection buyer seeks insurance against the risk $\tilde{\theta}$. For example, the protection buyer could be a commercial bank seeking to hedge credit risk of its loan portfolio in order to reduce its capital requirement.

Protection seller. The protection seller is risk-neutral. At time $t = 0$ she has an amount $A > 0$ of assets in place which have an uncertain per unit return $\tilde{R}$ at $t = 2$ (balance sheet risk). The protection seller could be an investment bank or an insurance company.[2]

At $t = 1$ the protection seller has to exert costly unobservable effort e to manage the risk of her assets. To capture the moral hazard problem in the simplest possible way, we assume that the protection seller can choose between effort, $e = 1$, and no effort, $e = 0$. If she exerts risk-management effort, we assume that $A\tilde{R}(e = 1) = AR > A$. If she does not exert effort, then $A\tilde{R}(e = 0) = AR$ with probability p and $A\tilde{R}(e = 0) = 0$ with probability $1 - p$. The seller is protected by limited liability. Hence, she may default on her obligations if she does not manage the risk of her balance sheet. In this case, the protection buyer is exposed to counterparty risk.

If the seller does not exert effort, she obtains a private benefit B per unit of assets on her balance sheet. That is, the cost of managing balance sheet risk is proportional to the size of the balance sheet. Note that the impact of the seller's effort on $\tilde{R}$ does not depend on the return of the buyer's asset $\tilde{\theta}$. We assume that the opportunity cost of not managing risk is higher than the private benefit: $(1 - p)\, R > B$. Hence, the protection seller prefers effort to

[2] According to public financial statements of AIG, 72% of notional amounts of CDS sold by AIG Financial Products as of December 2007 were used by banks for capital relief (European Central Bank, 2009).

no effort if she is solely concerned with managing the risk of her assets.

Advance information. Information about the risk $\tilde{\theta}$ is publicly revealed at $t = 0.5$, before the seller makes her effort decision at $t = 1$. Specifically, a signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed. Let λ be the probability of a correct signal:

$$\lambda = \text{prob}[\bar{s}|\bar{\theta}] = \text{prob}[\underline{s}|\underline{\theta}]$$

The probability π is updated to $\bar{\pi}$ upon observing $\bar{s}$ and to $\underline{\pi}$ upon observing $\underline{s}$, where

$$\begin{aligned}
\bar{\pi} &= \text{prob}[\bar{\theta}|\bar{s}] = \frac{\text{prob}[\bar{s}|\bar{\theta}]\text{prob}[\bar{\theta}]}{\text{prob}[\bar{s}]} = \frac{\lambda\pi}{\lambda\pi + (1-\lambda)(1-\pi)} \\
\underline{\pi} &= \text{prob}[\bar{\theta}|\underline{s}] = \frac{\text{prob}[\underline{s}|\bar{\theta}]\text{prob}[\bar{\theta}]}{\text{prob}[\underline{s}]} = \frac{(1-\lambda)\pi}{(1-\lambda)\pi + \lambda(1-\pi)}
\end{aligned}$$

according to Bayes' Law.

We assume that $\lambda \geq \frac{1}{2}$. If $\lambda = \frac{1}{2}$, $\bar{\pi} = \pi = \underline{\pi}$ and the signal is completely uninformative. It is as if there was no advance information about the risk θ. For $\lambda > \frac{1}{2}$, $\bar{\pi} > \pi > \underline{\pi}$, observing $\bar{s}$ increases the probability of $\tilde{\theta} = \bar{\theta}$ (good signal) whereas observing $\underline{s}$ decreases the probability of $\tilde{\theta} = \bar{\theta}$ (bad signal). If $\lambda = 1$, the signal is perfectly informative and it is as if the realization of $\tilde{\theta}$ was already observed at $t = 0.5$.

Margins. A margin is a technology that allows the protection seller to liquidate a fraction α of her assets for cash and to deposit the cash on a separate account when requested to do so by the protection buyer. Thus, αA earns the risk-free rate (which we normalize to one), while $(1-\alpha)A$ continues to earn the return $\tilde{R}$ whose distribution depends on the effort choice of the protection seller. The liquidation of assets into cash and the transfer of cash into a separate account removes the fraction α of the seller's assets from her balance sheet. Those assets are ring-fenced from the seller's moral hazard problem. The cash is unaffected by the default of the seller and can be used to service the buyer's claim. At the same time, the seller no longer obtains private benefits on those assets since he no longer controls them.

We will consider two types of margins. An initial margin is a requirement to deposit cash at $t = 0$, when the protection buyer and seller enter a risk-sharing contract. A variation margin is a requirement to deposit cash at $t = 0.5$, after advance information about the risk θ is observed.

Contract. The contract specifies a transfer τ from the protection seller to the protection buyer, conditional on all contractible information (in case $\tau < 0$, the buyer pays the seller). The contract can also specify a margin α to be deposited by the protection seller as cash with a third party. We assume that the realization of $\tilde{\theta}$, the return on the seller's assets $\tilde{R}$ and the advance signal $\tilde{s}$ are all publicly observable and contractible. Hence, transfers are given by $\tau = \tau(\tilde{\theta}, \tilde{s}, \tilde{R})$. Transfers must be consistent with the limited liability of the protection seller, so that $A\tilde{R} > \tau(\tilde{\theta}, \tilde{s}, \tilde{R})$. We assume $A > \pi\Delta\theta$, which, as we will show below, implies that the limited liability constraint binds only if the protection seller has no assets available to meet her obligations.

The sequence of events is summarized in Figure **??** below.

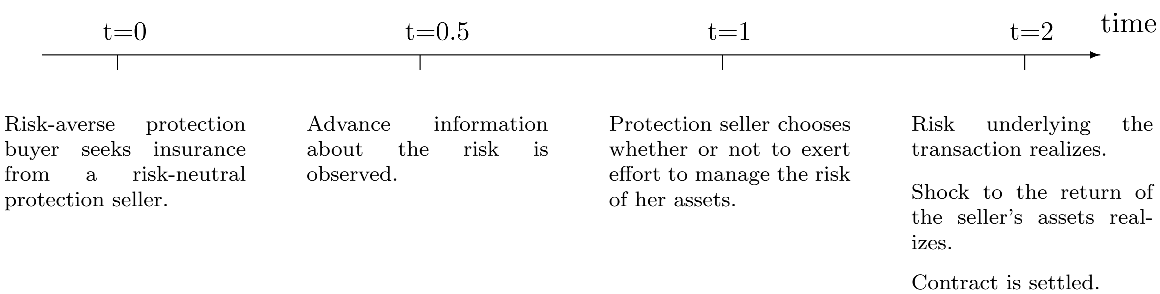

Figure 1: The timing of events

3 First-best: observable effort

In this section we consider the case where the protection buyer can observe the effort level of the protection seller so that there is no moral hazard problem. While implausible, this case will enable us to identify the inefficiencies generated by moral hazard.

Consider the case where the protection buyer instructs the protection seller to exert risk-management effort after both a good and a bad signal. In that case the seller's assets always return $\tilde{R}(1) = R$. Hence we don't need explicitly write $\tilde{R}$ when writing the variables upon which τ is contingent. Since the seller never defaults when exerting effort, the margin is never transferred to the protection buyer. Also, as will be clear below, under our assumption that $A > \pi\Delta\theta$, the limited liability constraint of the seller does not bind. Hence, for simplicity we neglect this constraint.

The protection buyer solves

$$\max_{\alpha,\tau(\tilde{\theta},\tilde{s})} \pi\lambda u(\bar{\theta} + \tau(\bar{\theta}, \bar{s})) + (1-\pi)(1-\lambda)u(\underline{\theta} + \tau(\underline{\theta}, \bar{s})) \tag{1}$$
$$+\pi(1-\lambda)u(\bar{\theta} + \tau(\bar{\theta}, \underline{s})) + (1-\pi)\lambda u(\underline{\theta} + \tau(\underline{\theta}, \underline{s}))$$

subject to the seller's participation constraint

$$\pi\lambda[\alpha A + (1-\alpha)\, AR - \tau(\bar{\theta}, \bar{s})] + \pi(1-\lambda)[\alpha A + (1-\alpha)\, AR - \tau(\bar{\theta}, \underline{s})]$$
$$+ (1-\pi)\lambda[\alpha A + (1-\alpha)\, AR - \tau(\underline{\theta}, \underline{s})] + (1-\pi)(1-\lambda)[\alpha A + (1-\alpha)\, AR - \tau(\underline{\theta}, \bar{s})] \geq AR$$

where $0 \leq \alpha \leq 1$ is a fraction of assets to be deposited as an initial margin.[3] The expression on the right-hand side is seller's payoff if she does not enter the transaction. It is given by the return on her capital, AR.

The participation constraint can be written as

$$\alpha A\,(1-R) \geq \pi\left[\lambda\tau(\bar{\theta}, \bar{s}) + (1-\lambda)\tau(\bar{\theta}, \underline{s})\right] + (1-\pi)\left[\lambda\tau(\underline{\theta}, \underline{s}) + (1-\lambda)\,\tau(\underline{\theta}, \bar{s})\right] \equiv E[\tau] \tag{2}$$

where the expectation is over $\tilde{\theta}$ and $\tilde{s}$. If margins are not used ($\alpha = 0$), the protection seller agrees to the contract as long as the average payment to the buyer is non-positive. If

[3] The case of a variation margin can be written analogously. Unlike an initial margin, which affects the seller's return on assets in all four states of the world, a variation margin is called after a particular signal, thus affecting only two states.

margins are used ($\alpha > 0$), the protection seller needs to be compensated for the opportunity cost of putting cash aside, $R - 1$. Proposition 1 states the solution to this maximization problem. It is easy to show that the corresponding value function is greater than what would be obtained if effort was not always requested. Thus, we can state our first result.

Proposition 1 (First-best contract) *When effort is observable, the optimal contract entails effort after both signals, provides full insurance, and is actuarially fair. Margins are not used. The transfers are given by:*

$$\begin{aligned}\tau^{FB}(\bar{\theta}, \bar{s}) &= \tau^{FB}(\bar{\theta}, \underline{s}) = -(1-\pi)\Delta\theta = E[\tilde{\theta}] - \bar{\theta} < 0 \\ \tau^{FB}(\underline{\theta}, \bar{s}) &= \tau^{FB}(\underline{\theta}, \underline{s}) = \pi\Delta\theta = E[\tilde{\theta}] - \underline{\theta} > 0\end{aligned}$$

In the first-best contract, the consumption of the protection buyer is equalized across states (full insurance). The contract does not react to the signal. Since the only effect of margins here is to tighten the seller's participation constraint, they are not used. Expected transfers are zero and there are no rents to the protection seller. The seller pays the buyer if $\tilde{\theta} = \underline{\theta}$ and vice versa if $\tilde{\theta} = \bar{\theta}$. The payments are proportional to the riskiness of the position, measured by $\Delta\theta$.

It is optimal for the protection buyer to demand effort after both signals. He is fully insured and the seller's assets are safe so there is no counterparty risk. If there was no effort, the buyer would be exposed to counterparty risk and full insurance would no longer be possible.

Finally, note that the values of the transfers given in the proposition confirms our initial claim that, under our assumption that $A > \pi\Delta\theta$, the limited liability condition does not bind.

4 Unobservable effort, no margins

4.1 Effort after both signals

We now turn to the case in which the effort level of the protection seller is not observable. In this section, we characterize the optimal contract assuming that margins are not used, $\alpha = 0$. This provides a useful benchmark to assess the effect of margins on incentives in Section **??**.

We first consider a contract that induces effort of the seller after both a good and a bad signal.

As the protection buyer expects the seller to always exert risk-management effort, $\tilde{R}(1) = R$. Hence, the contract does not need to be contingent on $\tilde{R}$. Thus, the protection buyer solves (**??**) subject to (**??**) and the seller's incentive compatibility constraints. Since the signal about the risk θ is observed before the effort decision is made, the incentive constraints are conditional on the realization of the signal.

Suppose a good signal, $\tilde{s} = \bar{s}$, is observed. Then, the incentive-compatibility constraint is given by

$$\bar{\pi}[AR - \tau(\bar{\theta}, \bar{s})] + (1 - \bar{\pi})[AR - \tau(\underline{\theta}, \bar{s})] \geq$$

$$\bar{\pi}[p(AR - \tau(\bar{\theta}, \bar{s}))] + (1 - \bar{\pi})[p(AR - \tau(\underline{\theta}, \bar{s}))] + AB$$

The expression on the right-hand side is seller's (out-of-equilibrium) expected payoff if she does not exert effort. With probability $1 - p$, the seller's assets return zero and she cannot make any positive payment to the protection buyer. The buyer, in turn, has no interest in making a payment to the seller since it would make it more difficult to satisfy the incentive constraint. Hence $\tau(\tilde{\theta}, \tilde{s} \mid \text{default}) = 0$. The incentive-compatibility constraint after a bad signal, $\tilde{s} = \underline{s}$, is derived analogously.

Simplifying the incentive constraint for each realization of the signal, we get:

$$\begin{aligned} A\mathcal{P} &\geq \bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s}) \\ A\mathcal{P} &\geq \underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}) \end{aligned}$$

where

$$\mathcal{P} \equiv R - \frac{B}{1-p} \tag{3}$$

denotes "pledgeable income" per unit of assets (as in Tirole, 2005). Total pledgeable income $A\mathcal{P}$ puts an upper bound on the expected transfer to the protection buyer, conditional on the observed signal. Note that $\mathcal{P} > 0$ since we assumed $(1-p)\,R > B$.

It will be useful to introduce the following notation:

$$\bar{\tau} \equiv \bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s}) \tag{4}$$

$$\underline{\tau} \equiv \underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}). \tag{5}$$

The incentive constraints become

$$A\mathcal{P} \geq \bar{\tau} \tag{6}$$

$$A\mathcal{P} \geq \underline{\tau} \tag{7}$$

and the participation constraint (**??**) becomes

$$0 \geq \text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]\underline{\tau} \tag{8}$$

Lemma 1 (First-best attainable) *When effort is not observable and the signal is informative, the first-best can be achieved if and only if the protection seller has enough pledgeable income, i.e., for* $A\mathcal{P} > (\pi - \underline{\pi})\Delta\theta = E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]$.

For sufficiently high pledgeable income levels, incentive-compatibility constraints are not binding and the first-best allocation can be reached even when effort is not observable. The threshold level of pledgeable income beyond which the first-best is attainable, $(\pi - \underline{\pi})\Delta\theta$, is proportional to the riskiness of the position $\Delta\theta$ and to the informativeness of the signal λ (which induces a higher wedge between the prior and the updated probability). We can state the following corollary.

Corollary 1 *When the signal is uninformative, $\lambda = \frac{1}{2}$, the first-best is always reached: $A\mathcal{P} > (\pi - \underline{\pi})\Delta\theta = 0$.*

In what follows, we focus on the case when the signal is sufficiently informative. In particular, we assume that:

$$\lambda \geq \lambda^* \equiv \frac{1 - \sqrt{p}}{1 - p} > \frac{1}{2} \tag{9}$$

where the last inequality is satisfied for all p.[4]

To ensure that the protection seller always exerts effort when the first-best is not attainable, the optimal contract must satisfy two incentive-compatibility constraints. The next lemma states that only one of them will be binding.

Lemma 2 (Incentives given the signal) *When effort is not observable and the first-best is not attainable, $A\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$, the incentive constraint after a good signal is slack whereas the incentive constraint after a bad signal is binding.*

Ex ante, before the signal is observed, the derivative position is neither an asset nor a liability for the protection seller. After observing a good signal about the underlying risk, the position is more likely to be an asset for the seller. He is more likely to be paid by the buyer than the other way around. Thus, good news do not generate incentive problems. Negative news, on the other hand, make it more likely that the position moves against the seller. Now it is the seller who is more likely to pay the buyer. For $A\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$, this

[4]This assumption is not very restrictive: note that $\lambda^*(p)$ is decreasing in p and $\lambda^* \to \frac{1}{2}$ as $p \to 1$. As an example, $\lambda^* = 0.59$ for $p = \frac{1}{2}$.

undermines her incentives to exert effort. She has to bear the full cost of effort while the benefit accrues in part to the protection buyer. This is reminiscent of the debt-overhang effect (Myers, 1977). The derivative position contains *hidden liability* that affects seller's risk-management incentives when she has limited pledgeable income.

The following proposition characterizes the optimal contract with effort after both signals.

Proposition 2 (Optimal contract with risk-management) *When effort is not observable and the first-best is not attainable, $A\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$, the optimal contract that induces effort after both signals provides full insurance conditional on the signal. The contract is actuarially fair. The transfers are given by:*

$$
\begin{aligned}
\tau^{e=1,e=1}(\bar{\theta}, \bar{s}) &= -(1-\bar{\pi})\Delta\theta - \frac{prob[\underline{s}]}{prob[\bar{s}]}A\mathcal{P} = E([\tilde{\theta}|\bar{s}] - \bar{\theta}) - \frac{prob[\underline{s}]}{prob[\bar{s}]}A\mathcal{P} < 0 \\
\tau^{e=1,e=1}(\underline{\theta}, \bar{s}) &= \bar{\pi}\Delta\theta - \frac{prob[\underline{s}]}{prob[\bar{s}]}A\mathcal{P} = (E[\tilde{\theta}|\bar{s}] - \underline{\theta}) - \frac{prob[\underline{s}]}{prob[\bar{s}]}A\mathcal{P} > 0 \\
\tau^{e=1,e=1}(\bar{\theta}, \underline{s}) &= -(1-\underline{\pi})\Delta\theta + A\mathcal{P} = (E[\tilde{\theta}|\underline{s}] - \bar{\theta}) + A\mathcal{P} < 0 \\
\tau^{e=1,e=1}(\underline{\theta}, \underline{s}) &= \underline{\pi}\Delta\theta + A\mathcal{P} = (E[\tilde{\theta}|\underline{s}] - \underline{\theta}) + A\mathcal{P} > 0
\end{aligned}
$$

As in the first-best contract, the participation constraint binds and there are no rents to the protection seller. Expected transfers are zero so that the contract is actuarially fair.

The key difference to the first-best contract is that the transfers now depend on the signal:

$$
\tau^{e=1,e=1}(\tilde{\theta}, \underline{s}) < \tau^{FB}(\tilde{\theta}, \underline{s}) = \tau^{FB}(\tilde{\theta}, \bar{s}) < \tau^{e=1,e=1}(\tilde{\theta}, \bar{s})
$$

To preserve the seller's incentives to exert effort, the buyer must reduce the hidden liability by accepting incomplete risk-sharing. In particular, the incentive-compatible amount of insurance is smaller following a bad signal. Hence, the protection buyer must bear signal risk. Correspondingly, the protection seller must be left with some rent after a bad signal in order to induce effort. The protection buyer "reclaims" this rent after a good signal so that the expected rent to the seller is zero.[5]

Conditional on the signal, the optimal contract provides full insurance against the underlying risk $\tilde{\theta}$:

$$\tau(\underline{\theta}, \bar{s}) - \tau(\bar{\theta}, \bar{s}) = \tau(\underline{\theta}, \underline{s}) - \tau(\bar{\theta}, \underline{s}) = \Delta\theta > 0 \tag{10}$$

Since there is full insurance conditional on the signal, we can rewrite the objective function of the risk-averse protection buyer (**??**) as

$$\text{prob}[\bar{s}]u(E[\theta|\bar{s}] + \bar{\tau}) + \text{prob}[\underline{s}]u(E[\theta|\underline{s}] + \underline{\tau}) \tag{11}$$

where we have used (**??**) and (**??**).

Figure 2 illustrates our results so far in the contract space $(\underline{\tau}, \bar{\tau})$. The relevant part of the contract space is when $\underline{\tau} \geq 0$ (x-axis) and when $\underline{\tau} \leq 0$ (y-axis). After a bad signal the protection seller is more likely to pay the protection buyer than vice versa. The opposite holds after a good signal.

Insert Figure 2 here

The participation constraint of the protection seller (**??**) is a line through the origin with slope $-\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}$. The protection seller agrees to any contract that lies on or below this line. Contracts that lie on the line are actuarially fair since expected transfers are zero. The slope gives the "relative price" at which the risk-neutral protection seller is willing to exchange expected transfers after a good and a bad signal.

The indifference curves corresponding to (**??**) are decreasing, convex curves in the contract space $(\underline{\tau}, \bar{\tau})$.[6] The utility of the protection buyer increases as he moves to the north-east in the figure.

The first-best allocation is given by point A where the indifference curve of the protection

[5]Interpreting signals as types, there is then cross-subsidization between types.

[6]The slope of the indifference curve is given by $\frac{d\bar{\tau}}{d\underline{\tau}} = -\frac{\text{prob}[\underline{s}]\underline{u}'}{\text{prob}[\bar{s}]\bar{u}'} < 0$, where $\underline{u}' \equiv u'(C + IE[\theta|\underline{s}] + \underline{\tau})$ and $\bar{u}' \equiv u'(C + IE[\theta|\bar{s}] + \bar{\tau})$. The change in the slope is $\frac{d^2\bar{\tau}}{d\underline{\tau}^2} = -\frac{\text{prob}[\underline{s}]\underline{u}''\bar{u}'}{(\text{prob}[\bar{s}]\bar{u}')^2} > 0$.

buyer is tangent to the participation constraint of the protection seller. The first-best allocation achieves full insurance. In the first-best, transfers are independent from the realization of the signal for a given realization of θ, i.e., $\tau(\tilde{\theta},\bar{s}) = \tau(\tilde{\theta},\underline{s})$. Using (**??**), (**??**) and (**??**), such transfers yield

$$\bar{\tau} = -(E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}|\underline{s}]) + \underline{\tau} \tag{12}$$

Thus, in the $(\underline{\tau},\bar{\tau})$ plane, these transfers lie on the 45^o line that intersects the x-axis at $\underline{\tau} = (E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}|\underline{s}])$ and that intersects the participation constraint at point A.

Point B illustrates the optimal contract with effort. The vertical line that intersects the x-axis at $\underline{\tau} = A\mathcal{P}$ represents the incentive constraint. The protection seller only exerts effort after a bad signal if the contract lies on or to the left of the line. The figure is drawn for $A\mathcal{P} < (E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}])$ so that the first-best allocation is not attainable when effort is not observable (Lemma **??**). The contract achieving the highest utility for the protection buyer lies at the intersection of the incentive and the participation constraint. He is worse off than with the first-best allocation. The indifference curve passing through B lies strictly below the one passing through A.

4.2 No effort after a bad signal

The protection buyer may find the reduced risk-sharing in the contract with effort after both signals too costly. He may instead choose to accept risk-taking by the protection seller in exchange for more sharing of the risk θ. In the previous subsection, we showed that bad news generate incentive problems and make it harder to induce effort by the protection seller. In this subsection, we characterize the optimal contract in which effort is induced only after good news.

After good news, the contract entails risk-management by the protection seller so that $\tilde{R}(1) = R$. After bad news, the contract entails risk-taking so that $\tilde{R}(0) = R$ with probability p while $\tilde{R}(0) = 0$ with probability $1-p$. Hence, the contract is contingent on $\tilde{R}$. The objective function of the protection buyer is given by:

$$\max_{\tau(\tilde{\theta},\tilde{s},\tilde{R})} \pi\lambda u(\bar{\theta}+\tau(\bar{\theta},\bar{s},R)) + (1-\pi)(1-\lambda)u(\underline{\theta}+\tau(\underline{\theta},\bar{s},R)) \tag{13}$$
$$+\pi(1-\lambda)[pu(\bar{\theta}+\tau(\bar{\theta},\underline{s},R)) + (1-p)u(\bar{\theta}+\tau(\bar{\theta},\underline{s},0))]$$
$$+(1-\pi)\lambda[pu(\underline{\theta}+\tau(\underline{\theta},\underline{s},R)) + (1-p)u(\underline{\theta})]$$

With probability $1-p$ the seller's balance sheet returns zero and she cannot make any transfers to the protection buyer. It may be optimal, however, for the protection buyer to make transfers to the protection seller when the seller is in default but $\bar{\theta}$ is realized, i.e. $\tau(\bar{\theta},\underline{s},0)\leq 0$. On the other hand, the transfer when $\underline{\theta}$ is realized and the seller is in default is optimally set to zero, $\tau(\underline{\theta},\underline{s},0)=0$, since the protection buyer is due a transfer in the $\underline{\theta}$ state but the protection seller is unable to make it.

The constraint inducing risk-management after good news is, as before, given by

$$A\mathcal{P} \geq \bar{\pi}\tau(\bar{\theta},\bar{s},R) + (1-\bar{\pi})\tau(\underline{\theta},\bar{s},R) \tag{14}$$

The constraint inducing risk-taking after bad news is given by

$$\underline{\pi}[AR-\tau(\bar{\theta},\underline{s})] + (1-\underline{\pi})[AR-\tau(\underline{\theta},\underline{s})] \leq$$
$$\underline{\pi}[p(AR-\tau(\bar{\theta},\bar{s})) - (1-p)\,\tau(\bar{\theta},\underline{s},0)] + (1-\underline{\pi})[p(AR-\tau(\underline{\theta},\bar{s}))] + AB$$

or, equivalently,

$$A\mathcal{P} \leq \underline{\pi}\tau(\bar{\theta},\underline{s},R) + (1-\underline{\pi})\tau(\underline{\theta},\underline{s},R) - \underline{\pi}\tau(\bar{\theta},\underline{s},0) \tag{15}$$

Following bad news, the seller prefers to run the risk that her assets return zero when the expected transfers to the buyer are sufficiently high.

The seller's participation constraint is

$$-\operatorname{prob}[\underline{s}](1-p)A\mathcal{P} \geq \operatorname{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)\right]+ \tag{16}$$
$$\operatorname{prob}[\underline{s}]p\left[\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)\right]+\operatorname{prob}[\underline{s}]\left(1-p\right)\underline{\pi}\tau(\bar{\theta},\underline{s},0)$$

The expected transfer from the seller to the buyer (right-hand side) is negative. If seller enters the position, she must be compensated for the potential loss of pledgeable income due to the lack of effort after bad news (left-hand side). Note that higher pledgeable income makes it more difficult for a protection seller to accept a contract with no effort. Higher returns on seller's assets AR increase the outside opportunity of the seller, and they may not materialize after entering the contract. Similarly, a smaller private benefit B reduces the value of the contract by reducing the benefit of not exerting effort after a bad signal. An implication is that apparently expensive derivative contracts sold by well established names (high $\mathcal{P}$) are an early warning of future risk-taking.[7]

The following proposition characterizes the optimal contract with effort after a good signal and no effort after a bad signal.

Proposition 3 (Optimal contract with risk-taking) *If risk-taking after bad news is optimal, then the optimal contract with risk-taking provides full insurance except when the seller defaults in $\underline{\theta}$ state. The contract is actuarially unfair. The transfers are given by:*

$$\begin{aligned}
\tau^{e=1,e=0}(\bar{\theta},\bar{s},R) &= \tau^{e=1,e=0}(\bar{\theta},\underline{s},R)=\tau(\bar{\theta},\underline{s},0)=\frac{\pi\Delta\theta-prob[\underline{s}]\left(1-p\right)A\mathcal{P}}{1-prob[\underline{s}]\left(1-\underline{\pi}\right)\left(1-p\right)}-\Delta\theta<0\\
\tau^{e=1,e=0}(\underline{\theta},\bar{s},R) &= \tau^{e=1,e=0}(\underline{\theta},\underline{s},R)=\frac{\pi\Delta\theta-prob[\underline{s}]\left(1-p\right)A\mathcal{P}}{1-prob[\underline{s}]\left(1-\underline{\pi}\right)\left(1-p\right)}>0
\end{aligned}$$

Again, there are no rents to the protection seller (the participation constraint is binding).

[7]This is reminiscent of Biais, Rochet and Woolley (2009) although the mechanism is different. In that paper, large rents are a precursor of risk-taking. When rents become too large, investors prefer to give up on incentives and accept risk-taking. In our analysis, however, the protection seller never earns rents.

The seller pays the buyer if $\tilde{\theta} = \underline{\theta}$ and $\tilde{R} = R$ and vice versa if $\tilde{\theta} = \bar{\theta}$:

$$\tau^{e=1,e=0}(\underline{\theta}, \tilde{s}, R) > 0 > \tau^{e=1,e=0}(\bar{\theta}, \tilde{s}, \tilde{R})$$

There are three differences between the optimal contract with and without effort after bad news. First, the contract with risk-taking does not react to the signal:

$$\tau^{e=1,e=0}(\tilde{\theta}, \bar{s}) = \tau^{e=1,e=0}(\tilde{\theta}, \underline{s})$$

Except for a default in $\underline{\theta}$ state, the consumption of the buyer is equalized across states (as in the first-best contract). Second, unlike in the contract with effort, the buyer is now exposed to counterparty risk. He is uninsured when the seller is in default but the insurance payment is due ($\underline{\theta}$ is realized). Third, the contract with no effort after a bad signal is not actuarially fair since expected transfers are not equal to zero.

4.3 Risk-sharing and risk-taking

The contract with effort after both signals entails limited risk-sharing but no risk-taking, while the contract with no effort after a bad signal entails full risk-sharing but allows risk-taking after bad news. In this section, we examine under what conditions it is privately optimal to allow risk-taking.

Proposition 4 (Endogenous counterparty risk) *Suppose effort is not observable and the first-best is not attainable, $A\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$. There exists a threshold level of pledgeable income $A\hat{\mathcal{P}}$ such that the contract with risk-management is optimal if and only if $A\mathcal{P} \geq A\hat{\mathcal{P}}$. If the probability of default is sufficiently small, $A\hat{\mathcal{P}} > 0$.*

The key factor in the choice between the optimal contract with risk-management and with risk-taking is whether counterparty or signal risk is more costly for the protection buyer. For low levels of pledgeable income, the moral hazard problem is severe. Providing incentives to avoid risk-taking after a bad signal requires a considerable reduction in hidden liability. The buyer then has to bear a lot of signal risk. If, at the same time, default is unlikely (p is high), the counterparty risk under the risk-taking contract is small. It is then optimal for the protection buyer to allow risk-taking by the protection seller.

Counterparty risk thus arises endogenously due to moral hazard. In particular, counterparty risk occurs when the probability of default $1 - p$ is *small*. Note that lower return on seller's assets, R, lowers pledgeable income $\mathcal{P}$. Hence, privately optimal risk-sharing contracts are more likely to allow risk-taking in an environment, in which both the return on riskless investments is low and riskiness of risky investments is perceived to be low.

4.4 Risk-taking or risk-shifting?

So far, we considered the moral hazard problem due to risk-taking by the protection seller. In this subsection, we show the equivalence between this problem and the moral hazard problem due to risk-shifting by the protection seller.

Consider the following modification of the actions of the protection seller, which are unobservable by the protection buyer. The per-unit return on the protection seller's balance sheet, $\tilde{R}$, can be high (H), medium (M), or low (L). We assume $H > M > 1 > L = 0$. Instead of choosing between "effort" (risk-management) and "no effort" (risk-taking), the protection seller chooses between "no risk-shifting" and "risk-shifting". If she chooses no risk-shifting, the per-unit return on her balance sheet is high, $\tilde{R} = H$, with probability $1 - \mu$ and medium, $\tilde{R} = M$, with probability μ. We denote the expected per-unit return in this case by $E[R]$. If she chooses risk-shifting, the return is high with probability $1 - \mu + \alpha$, medium with probability $\mu - (\alpha + \beta)$, and low, $\tilde{R} = 0$, with probability β. We denote the expected per-unit return in this case by $\hat{E}[R]$. We assume that $E[R] > \hat{E}[R]$ so that the expected return under risk-shifting is lower than under no risk-shifting and the protection seller exposes herself to the possibility of default. Note that there is no private benefit B to the protection seller in this set-up.

As before, consider a hedging contract between the protection seller and the protection buyer. If the contract entails no risk-shifting after both good and bad news, the participation constraint of the protection seller is given by

$$AE\left[R\right]-\left[\text{prob}[\bar{s}]\bar{\tau}+\text{prob}[\underline{s}]\underline{\tau}\right]\geq AE\left[R\right]$$

or, equivalently,

$$\text{prob}[\bar{s}]\bar{\tau}+\text{prob}[\underline{s}]\underline{\tau}\leq 0$$

Note that it is the same as the participation constraint (**??**) in the problem with risk-taking. Turning to the incentive constraints of the protection seller, which ensure that she prefers no risk-shifting to risk-shifting after both signals, we have the following two constraints:

$$\begin{aligned}(1-\mu)\left(AH-\bar{\tau}\right)+\mu\left(AM-\bar{\tau}\right) &\geq (1-\mu+\alpha)\left(AH-\bar{\tau}\right)+\left(\mu-(\alpha+\beta)\right)\left(AM-\bar{\tau}\right)\\ (1-\mu)\left(AH-\underline{\tau}\right)+\mu\left(AM-\underline{\tau}\right) &\geq (1-\mu+\alpha)\left(AH-\underline{\tau}\right)+\left(\mu-(\alpha+\beta)\right)\left(AM-\underline{\tau}\right)\end{aligned}$$

or, equivalently,

$$A\bar{\mathcal{P}} \geq \bar{\tau} \tag{17}$$

$$A\bar{\mathcal{P}} \geq \underline{\tau} \tag{18}$$

where

$$\bar{\mathcal{P}}\equiv-\left[\frac{\alpha}{\beta}\left(H-M\right)-M\right] \tag{19}$$

denotes the per-unit "pledgeable income" in this case.[8]

[8]Note that in our benchmark set-up, risk-taking by the protection seller reduced the total per-unit return on the protection seller's assets from R to $pR+B$. The difference in per-unit returns was given by $(1-p)\left[R-\frac{B}{1-p}\right]=(1-p)\,\mathcal{P}$. In the set-up with risk-shifting, the difference in per-unit returns under no risk-shifting and under risk-shifting is $E\left[R\right]-\hat{E}\left[R\right]=\beta\bar{\mathcal{P}}$. Hence, $\bar{\mathcal{P}}$, like $\mathcal{P}$ in our benchmark set-up, represents the loss due to taking a "wrong", return-reducing action. Similarly, β and $(1-p)$, respectively, are the probabilities of default of the protection seller if the return-reducing action is taken.

These constraints are similar to the incentive constraints (**??**) and (**??**) in the problem with risk-risk-management effort. The objective function of the protection buyer is unchanged and is given by (**??**), since the limited liability constraint of the protection seller never binds when she chooses no risk-shifting.

We conclude that the optimal contract with no risk-shifting after both good and bad news is the same as the one characterized in Section **??**, up to a re-definition of the pledgeable income from $\mathcal{P}$ to $\bar{\mathcal{P}}$.

Consider now the contract with no risk-shifting after good news and risk-shifting after bad news. The objective function of the protection buyer is given by:

$$\begin{aligned} \max_{\tau(\tilde{\theta},\tilde{s},\tilde{R})} \; & \pi\lambda u(\bar{\theta}+\tau(\bar{\theta},\bar{s},\tilde{R})) + (1-\pi)(1-\lambda)u(\underline{\theta}+\tau(\underline{\theta},\bar{s},\tilde{R})) \\ & + \pi(1-\lambda)[(1-\beta)\,u(\bar{\theta}+\tau(\bar{\theta},\underline{s},\tilde{R}\geq M)) + \beta u(\bar{\theta}+\tau(\bar{\theta},\underline{s},0))] \\ & + (1-\pi)\lambda[(1-\beta)\,u(\underline{\theta}+\tau(\underline{\theta},\underline{s},\tilde{R}\geq M)) + \beta u(\underline{\theta})] \end{aligned} \tag{20}$$

where conditioning on $\tilde{R} \geq M$ indicates that the protection seller is able to honor her obligations under the contract only if $\tilde{R} = \{H, M\}$. The objective function is similar to the objective function (**??**) in the problem with risk-taking, with p being replaced by $(1-\beta)$. Turning to the participation constraint of the protection seller, it is given by:

$$\begin{aligned} & -\operatorname{prob}[\underline{s}]\beta A\bar{\mathcal{P}} \geq \operatorname{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta},\bar{s},\tilde{R}) + (1-\bar{\pi})\tau(\underline{\theta},\bar{s},\tilde{R})\right] + \\ & \operatorname{prob}[\underline{s}]\,(1-\beta)\left[\underline{\pi}\tau(\bar{\theta},\underline{s},\tilde{R}\geq M) + (1-\underline{\pi})\tau(\underline{\theta},\underline{s},\tilde{R}\geq M)\right] + \operatorname{prob}[\underline{s}]\beta\underline{\pi}\tau(\bar{\theta},\underline{s},0) \end{aligned} \tag{21}$$

Once again, it is similar to the participation constraint (**??**) in the problem with risk-taking. As for the incentive constraints, we know from Section **??** that if the contract with risk-taking after bad news is optimal, the incentive constraints do not bind. Hence, there is no need to consider them explicitly.

We conclude that the problem with risk-shifting after bad news is isomorphic to the problem with risk-taking after bad news characterized in Section **??**.

5 Margins

In this section, we analyze the incentive effects of margins. In the case of the contract with effort, when risk-sharing is limited by the incentive constraint of the protection seller, we show how margins can help increase the amount of insurance for the protection buyer. In the case of the contract with risk-taking, when the buyer is exposed to counterparty risk, we examine the role of margins in providing insurance against the seller's default.

5.1 Margins and risk-management effort

When the protection seller exerts effort, she does not default on her contractual obligations and the margin need not be transferred to the protection buyer. Hence, the objective function of the protection buyer is unchanged and is given by (**??**) or, equivalently, by (**??**). We also know that the incentive constraint of the seller does not bind after a good signal. Hence, the protection buyer will not make a margin call in this case. However, the buyer may want to call a margin after a bad signal when seller's incentives to exert effort may be jeopardized. The seller's participation constraint is thus given by:

$$\text{prob}[\bar{s}]AR + \text{prob}[\underline{s}]\left[\alpha A + (1-\alpha)AR\right] - E[\tau] \geq AR$$

or, equivalently,

$$E[\tau] \leq -\alpha A\,(R-1)\,\text{prob}[\underline{s}] \tag{22}$$

The expression on the right-hand side is negative and represents the opportunity cost of depositing cash rather than keeping it on the balance sheet after a bad signal. The seller forgoes the net return of assets over cash, $R-1$. Placing a higher margin α makes it more difficult for the protection seller to accept the contract. The opportunity cost of the margin makes the contract actuarially unfair (expected transfers are no longer equal to zero).

The incentive-compatibility constraint after a bad signal is given by:

$$\alpha A + (1-\alpha) AR - \underline{\tau} \geq p \left[\alpha A + (1-\alpha) AR - \underline{\tau}\right] + (1-\alpha) AB$$

The expression on the right-hand side is seller's (out-of-equilibrium) expected payoff if she does not exert effort. She earns the private benefit B only on the assets on her balance sheet. There is no private benefit associated with the deposited cash. Higher margins thus reduce the private benefit of risk-taking: the cash is ring-fenced from moral hazard. When seller's assets return zero, she also loses the cash deposited as it is seized by the buyer.[9] We can re-write the incentive constraint as

$$\alpha A + (1-\alpha) A\mathcal{P} \geq \underline{\tau} \tag{23}$$

where $\mathcal{P}$ denotes, as before, the pledgeable income per unit of assets. For $\mathcal{P} < 1$, the margin relaxes the incentive constraint. We can now state the following result.

Lemma 3 *If the per-unit pledgeable income is at least* 1*,* $\mathcal{P} \geq 1$*, margins are not used,* $\alpha^* = 0$*.*

The following proposition characterizes the optimal contract with margins and risk-management effort.

Proposition 5 (Optimal margins with risk-management) *Consider the case when* $\mathcal{P} < 1$*. Let* $\varphi(\alpha) \equiv \frac{u'(E[\theta|\underline{s}]+\underline{\tau}(\alpha))}{u'(E[\theta|\bar{s}]+\bar{\tau}(\alpha))}$*. If* $\varphi(0) < 1 + \frac{R-1}{1-\mathcal{P}}$*, then* $\alpha^* = 0$*. If* $\varphi(1) > 1 + \frac{R-1}{1-\mathcal{P}}$*, then* $\alpha^* = 1$*. Otherwise,* $\alpha^* \in (0,1)$ *is given by*

$$\varphi(\alpha^*) = 1 + \frac{R-1}{1-\mathcal{P}} \tag{24}$$

[9] It is optimal for the buyer to seize the margin whenever the seller is in default, i.e. both in $\underline{\theta}$ state when an insurance payment is due and in $\bar{\theta}$ state when it is not. This is because returning the margin to the seller would only make it more difficult to satisfy the incentive constraint.

In the contract with margins and risk-management effort, the expected transfers are given by:

$$\begin{aligned}
\underline{\tau}(\alpha^*) &= \alpha^* A + (1-\alpha^*)\mathcal{P} \\
\bar{\tau}(\alpha^*) &= -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}\left[\alpha^* AR + (1-\alpha^*)\mathcal{P}\right]
\end{aligned}$$

The benefit of margins is improved risk-sharing via the transfers $\bar{\tau}(\alpha^*)$ and $\underline{\tau}(\alpha^*)$. The margin itself is never paid to the protection buyer since the protection seller does not default when she exerts effort. The first-best is obtained when the buyer's marginal utilities conditional on the bad and the good signal are equalized, $\varphi(\alpha) = 1$, so that there is full insurance against signal risk. But to preserve the seller's incentives to exert effort when the first-best is not attainable, the protection buyer must bear signal risk so that the ratio of the marginal utilities is strictly higher than 1. Since $\frac{\partial \bar{\tau}}{\partial \alpha^*} < 0$ and $\frac{\partial \underline{\tau}}{\partial \alpha^*} > 0$, higher margins reduce the left-hand side of (**??**), moving the expected transfers closer to full insurance.

The cost of margins is that they tighten the participation constraint (**??**) and make the contract with effort actuarially unfair. The optimal margin balances enhanced insurance against signal risk and actuarial fairness. The right-hand side of (**??**) gives the rate at which the trade-off occurs. The numerator of the fraction, $R-1$, is the opportunity cost of foregone asset return causing the actuarial unfairness of the contract. The denominator measures the severity of the incentive problem that the margin helps relax. Margins can relax the incentive constraint (**??**) only for $\mathcal{P} < 1$, and they are more beneficial the lower the pledgeable income per unit of assets, $\mathcal{P}$.

Figure 3 illustrates the case with margins and risk-risk-management.

Insert Figure 3 here

The margin affects the participation and incentive constraint but leaves the objective function of the protection buyer unchanged. The straight line from point B to point D illustrates how the margin changes the set of feasible contracts. The line is the parametric plot of the binding participation constraint (**??**) and incentive constraint after bad news (**??**) as α varies from 0 to 1. The point B represents the optimal contract with effort and no margin (see Section **??**). The optimal margin α^* is given by the point of tangency of the protection buyer's indifference curve to the line BD (point E).[10]

The slope of the line BD gives the relative price at which the protection seller is willing to exchange the transfers $\underline{\tau}$ and $\bar{\tau}$ when margins are used, $\alpha > 0$. The slope is steeper than in the case without margins (the line through points B and A) as long as $R > 1$. The protection seller requires more compensation in terms of $\bar{\tau}$ for a higher expected transfer $\underline{\tau}$ since depositing cash carries an opportunity cost.

5.2 Margins and risk-taking

If the protection seller engages in risk-taking after bad news, her assets return zero with probability $1 - p$. As in the case without margins, we allow for transfers in $\tilde{R}(0) = 0$ state. With margins, the deposited cash, which is default-free, can be used to make transfers to the protection buyer. Hence, the protection buyer may receive payments even when seller's assets return zero.

The objective function of the protection buyer is now given by:

$$\begin{aligned}\max_{\alpha,\tau(\tilde{\theta},\tilde{s},\tilde{R})} \; & \pi\lambda u(\bar{\theta} + \tau(\bar{\theta}, \bar{s}, R)) + (1-\pi)(1-\lambda)u(\underline{\theta} + \tau(\underline{\theta}, \bar{s}, R)) \\ & + \pi(1-\lambda)[pu(\bar{\theta} + \tau(\bar{\theta}, \underline{s}, R)) + (1-p)u(\bar{\theta} + \tau(\bar{\theta}, \underline{s}, 0))] \\ & + (1-\pi)\lambda[pu(\underline{\theta} + \tau(\underline{\theta}, \underline{s}, R)) + (1-p)u(\underline{\theta} + \tau(\underline{\theta}, \underline{s}, 0))]\end{aligned} \tag{25}$$

where $\tau(\tilde{\theta}, \underline{s}, 0) \leq \alpha A$.

[10] At the point of tangency, we have $-\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}\varphi(\alpha^*) = -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]} - \frac{(R-1)\text{prob}[\underline{s}]}{(1-\mathcal{P})\text{prob}[\bar{s}]}$. Multiplying both sides of the equality with $-\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}$ recovers condition (**??**) for $\alpha^* > 0$.

The participation constraint of the protection seller is given by

$$-\text{prob}[\underline{s}]\left[\alpha A\left(R-1\right)+(1-p)(1-\alpha)A\mathcal{P}\right] \geq \text{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)\right] \tag{26}$$
$$+\text{prob}[\underline{s}]p\left[\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)\right]+\text{prob}[\underline{s}]\left(1-p\right)\left(\underline{\pi}\tau(\bar{\theta},\underline{s},0)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},0)\right)$$

The left-hand side is the sum of the opportunity cost of a margin due to the foregone asset return $R-1$ and the loss of pledgeable income due to the lack of effort after bad news. The right-hand side is the expected transfer from the protection seller to the protection buyer.

The constraint inducing risk-taking after bad news is given by

$$\left(1-\alpha\right)A\mathcal{P} \leq \underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)-\underline{\pi}\tau(\bar{\theta},\underline{s},0)-(1-\underline{\pi})\tau(\underline{\theta},\underline{s},0) \tag{27}$$

The incentive constraint after good news is unchanged:

$$A\mathcal{P} \geq \bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)$$

We can now state the following result.

Lemma 4 *If the expected per-unit return on the seller's assets under risk-taking is smaller than* 1*,* $pR+B<1$*, then* $\alpha^*=1$ *in the contract with risk-taking. Such contract is weakly dominated by the contract with risk-management and margins.*

Under risk-taking, the expected per-unit return on the seller's balance sheet including the private benefit is $pR+B$. If $pR+B<1$, this return is lower than the return on cash. Hence, it is more profitable to put seller's assets in the margin. In this case, the seller's balance sheet is fully ring-fenced from her moral hazard. The protection buyer can do at least as well by offering the seller the contract with risk-management and margins.

It follows that the contract with margins and risk-taking after bad news can be optimal only if $pR+B\geq 1$. The following proposition characterizes the optimal margin in this case.

Proposition 6 (Optimal margins with risk-taking) *Suppose risk-taking after bad news is optimal. Let* $\phi(\alpha) \equiv \frac{u'(\underline{\theta}+\tau(\underline{\theta},\underline{s},0))}{u'(\underline{\theta}+\tau(\underline{\theta},\underline{s},R))}$. *If* $\phi(0) < 1 + \frac{pR+B-1}{(1-p)(1-\underline{\pi})}$, *then* $\alpha^* = 0$. *If* $\phi(1) > 1 + \frac{pR+B-1}{(1-p)(1-\underline{\pi})}$, *then risk-taking after bad news is not optimal. Otherwise,* $\alpha^* \in (0,1)$ *is given by*

$$\phi(\alpha^*) = 1 + \frac{pR+B-1}{(1-p)(1-\underline{\pi})} \tag{28}$$

In the contract with margins and risk-taking, the protection buyer gets full insurance when the seller does not default on the contract (marginal utilities are equalized across the no default states). Specifically, the transfer $\tau(\underline{\theta},\underline{s},R)$ is given by

$$\tau(\underline{\theta},\underline{s},R) = \frac{\pi\Delta\theta - \text{prob}[\underline{s}]\,(1-p)\,A\mathcal{P}}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)} - \alpha^* A\frac{\text{prob}[\underline{s}]\,[pR+B-1+(1-\underline{\pi})\,(1-p)]}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}$$

The transfer when the seller defaults on the contract is given by $\tau(\underline{\theta},\underline{s},0) = \alpha^* A$ so that the protection buyer seizes the margin in this case.

The benefit of margins under risk-taking is the insurance they provide against counterparty risk (left-hand side of (**??**)). The wedge between the marginal utilities under default and under no default is reduced. Margins increase the buyer's consumption if the seller defaults since $\frac{\partial\tau(\underline{\theta},\underline{s},0)}{\partial\alpha} > 0$. At the same time, margins reduce his consumption when there is no default, $\frac{\partial\tau(\underline{\theta},\underline{s},R)}{\partial\alpha} < 0$.

The cost of margins is given by their opportunity cost under risk-taking, $pR+B-1$. The optimal margin balances this cost with the benefit of protecting the buyer from counterparty risk. The right-hand side of (**??**) gives the rate at which the trade-off occurs. The numerator of the fraction is the opportunity cost of margins, while the denominator is the probability of protection seller's default on her contractual obligations. Margins are more beneficial the higher the counterparty risk faced by the buyer.

5.3 Margins, risk-sharing and risk-taking

Margins can be implemented as escrow accounts set up by a protection buyer or via a market infrastructure such as a central counterparty (CCP). It is privately optimal to use margins whenever $\alpha^* > 0$. When the contract with margins entails risk-management, the margin acts as a commitment device for the protection seller not to take risks once she observes negative news about her hedging position. When the contract entails risk-taking, the margin protects the buyer against the default of the seller.

The choice between the contract with margins and risk-management and the contract with margins and risk-taking depends again on whether counterparty or signal risk is more costly for the protection buyer. As in Section **??**, the contract with risk-taking may be chosen when pledgeable income is low and the moral hazard problem is severe.

The overall effect of margins on risk-taking, and hence counterparty risk, is ambiguous. On the one hand, margins reduce the signal risk faced by the buyer and make risk-management by the seller more attractive. On the other hand, margins protect the buyer from counterparty risk and make risk-taking by the seller more attractive. If the latter effect is small, then margins reduce the risk-taking effect of hedging. If the buyer benefits a lot from the insurance against counterparty risk, then margins lead to more risk-taking.

6 Extensions

In this section, we analyze the hedging contract with multiple protection sellers. We first characterize the optimal contract between the protection buyer and several protection sellers. We then analyze reinsurance, i.e., the possibility that sellers write additional hedging contracts among themselves. We show that reinsurance is not feasible. We finally consider the possibility of retrading whereby sellers are able to transfer all contractual obligations vis-a-vis the buyer among themselves. We show that such retrading fails to implement the second-best contract. We argue that *initial* margins can restore optimality.

6.1 Multiple sellers

Suppose a protection buyer splits the hedging contract among several protection sellers. For now, we assume that protection sellers cannot reinsure or sell off the original contract (we will relax these assumptions below).

We extend the benchmark model as follows. There are N identical, risk-neutral protection sellers. At time $t = 0$, each protection seller has an amount $\frac{A}{N}$ of assets in place, which have an uncertain per unit return $\tilde{R}$ at $t = 2$. If a seller does not manage her balance sheet risk, she defaults with probability $1-p$. We assume that the default risk is a common or "macro" shock that is non-diversifiable across sellers. If the risk materializes (with probability $1-p$), all sellers fail at the same time when not managing balance sheet risk. Hence, risk-taking among sellers amounts to taking perfectly correlated risks.[11]

As before, the protection buyer solves

$$\max_{\bar{\tau}_i, \underline{\tau}_i} \text{prob}[\bar{s}] u(E[\tilde{\theta}|\bar{s}] + \sum_{i=1}^{N} \bar{\tau}_i) + \text{prob}[\underline{s}] u(E[\tilde{\theta}|\underline{s}] + \sum_{i=1}^{N} \underline{\tau}_i),$$

where subscript i stands for protection seller i, $i = 1, \ldots, N$. Seller i's incentive constraints are given by

$$A\mathcal{P}_i \geq \bar{\tau}_i \text{ and } A\mathcal{P}_i \geq \underline{\tau}_i$$

where

$$A\mathcal{P}_i = \frac{A}{N}\left(R - \frac{B}{1-p}\right)$$

is seller i's total pledgeable income. Seller i's participation constraint is given by

$$E[\tau_i] \leq 0$$

We can now state the following proposition.

[11] For example, Rajan (2006) argues that when the performance of managers in financial institutions is evaluated vis-a-vis their peers, they have incentives to engage in correlated investments. This ensures that they do not underperform.

Proposition 7 (Multiple sellers) *The optimal contract with multiple sellers, $N > 1$, that maintains their risk-management incentives is given by $\bar{\tau}_i = \frac{\bar{\tau}}{N}$ and $\underline{\tau}_i = \frac{\underline{\tau}}{N}$, $i = 1, \ldots, N$, where $\bar{\tau}$ and $\underline{\tau}$ are the optimal expected transfers after a good and a bad signal, respectively, for $N = 1$.*

Protection sellers are risk-neutral and competitive. Summing up their (linear) participation and incentive constraints, the optimization problem with N protection sellers of size $\frac{A}{N}$ is equivalent to the problem with one seller of size A. In this sense, our model with a single protection seller is representative of an entire insurance sector.

6.2 Reinsurance

Suppose a protection buyer splits the contract with risk-management effort among two identical, risk-neutral protection sellers as described in the previous section, i.e., $N = 2$. Each protection seller holds a contract $\left(\frac{\bar{\tau}}{2}, \frac{\underline{\tau}}{2}\right)$ (see Proposition **??**). We now allow for sellers to reinsure each other after the contract is signed but before the signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed.[12] Denote the complete reinsurance contract between the two protection sellers as $\rho(\theta, s, R_1, R_2)$, where R_i is the return on the assets of seller i. We make the convention that $\rho > 0$ means that seller 2 is paying seller 1.

For the sellers to agree on reinsurance, there must be gains from trade. Suppose without loss of generality that seller 2 is reinsuring seller 1. After a bad signal, seller 2 therefore expects having to pay seller 1, $\underline{\rho} > 0$. Since seller 2 will not provide reinsurance if he expects to lose money, it must be that $E[\rho] \leq 0$, and hence $\bar{\rho} < 0$. According to Proposition **??**, each seller's incentive constraint after a bad signal is binding, $\frac{\underline{\tau}}{2} = A\mathcal{P}_2$. An additional expected payout after a bad signal, $\underline{\rho} > 0$, induces risk-taking by seller 2. Assuming for the moment that seller 1 does risk-management effort, the expected gain to seller 2 from

[12] After the signal is observed, there is no scope for reinsurance since the position is no longer neutral. After good news, the hedge is more likely to be an asset and a protection seller does not require reinsurance, while after bad news, the hedge is more likely to be a liability and another protection seller is not willing to provide reinsurance.

providing reinsurance, $E[\rho]$, is

$$\text{prob}[\bar{s}]\left(-\bar{\rho}\right)+\text{prob}[\underline{s}]\left[\frac{AB}{2}-(1-p)\frac{AR}{2}+(1-p)\left(\frac{\tau}{2}\right)+p\left(-\underline{\rho}\right)\right]$$

The first term is the payment from seller 1 after a good signal. The term in square brackets is the gain after a bad signal. The gain has four components. First, seller 2 obtains the private benefit of no longer managing her balance sheet risk. Second, she defaults with probability $(1-p)$ and loses her assets. However, she also gains by defaulting since she no longer has to honor the original hedging contract with the buyer (this is the third term inside the square brackets). Finally, seller 2 does not default with probability p and makes the payment to seller 1.

Using the binding incentive constraint, $\frac{\tau}{2}=\frac{A}{2}\left(R-\frac{B}{1-p}\right)$, seller 2's expected gain simplifies to

$$\text{prob}[\bar{s}]\left(-\bar{\rho}\right)+\text{prob}[\underline{s}]p\left(-\underline{\rho}\right). \tag{29}$$

What is the expected gain from reinsurance to seller 1? Since we assumed that she does not shirk, her expected gain from reinsurance is

$$\text{prob}[\bar{s}]\left(\bar{\rho}\right)+\text{prob}[\underline{s}]p\left(\underline{\rho}\right). \tag{30}$$

Seller 1 receives the payment after a bad signal only if seller 2 has not defaulted, which happens with probability p.

Comparing (**??**) and (**??**), we conclude that there are no gains from reinsurance. Whenever the expected gain for seller 2 is positive, it is negative for seller 1. The conclusion extends to the case when seller 1 too shirks under reinsurance. The expressions (**??**) and (**??**) for the gains to seller 2 and seller 1 remain unchanged since we assume that protection sellers are exposed to a common macro shock if they fail to manage their balance sheet risk. All sellers default with probability $1-p$ if they don't manage risk after observing a bad signal. The following proposition summarizes our result.

Proposition 8 (Reinsurance) *The optimal hedging contract between a buyer and multiple sellers that maintains their risk-management incentives leaves no room for sellers to reinsure each other.*

6.3 Retrading and initial margins

Suppose, as in the previous section, that a protection buyer splits the contract with risk-management effort among two protection sellers. We now consider a possibility of retrading: seller 1 can acquire the contract held by seller 2. This transaction frees seller 2 from all obligations stemming from the contract.

Before the signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed, seller 2 is indifferent between selling the contract for a value of zero and keeping it since a seller's participation constraint binds (see Proposition **??**). What is seller 1's expected gain from acquiring seller 2's contract at a price of zero? Prior to acquiring seller 2's contract, seller 1's incentive constraint after observing a bad signal was binding, $\frac{\underline{\tau}}{2} = A\mathcal{P}_1$ (see Proposition **??**). Hence, increasing her position from $\frac{\underline{\tau}}{2}$ to $\underline{\tau}$ induces risk-taking by seller 1. Her expected gain from acquiring seller 2's contract is:

$$\text{prob}[\bar{s}]\left(-\frac{\bar{\tau}}{2}\right) + \text{prob}[\underline{s}]\left[\frac{AB}{2} - (1-p)\frac{AR}{2} + (1-p)\left(\frac{\underline{\tau}}{2}\right) + p\left(-\frac{\underline{\tau}}{2}\right)\right].$$

The first term is the extra payment from the protection buyer to seller 1 after a good signal. The term in square brackets is the gain after a bad signal. The gain has four components. First, seller 1 obtains the private benefit of no longer managing her balance sheet risk. Second, she defaults with probability $(1-p)$ and loses her assets. However, she also gains by defaulting since she no longer has to pay the protection buyer (this is the third term inside the square brackets). Finally, seller 1 does not default with probability p and has to make the payment to the protection buyer that he would have obtained from seller 2 in the absence of retrading.

Using $\frac{\tau}{2} = \frac{A\mathcal{P}}{2} = \frac{A}{2}\left(R - \frac{B}{1-p}\right)$ and $E\left[\frac{\tau}{2}\right] = 0$ from the binding incentive and participation constraints, seller 1's gain simplifies to

$$\text{prob}[\underline{s}]\,(1-p)\,\frac{A\mathcal{P}}{2} > 0. \tag{31}$$

The expected gain from acquiring the hedging contract arises from exploiting limited liability, i.e., from taking risk after a bad signal and not having to pay the protection buyer with probability $1-p$. The following Proposition summarizes our result:

Proposition 9 (Retrading) *Retrading the hedging contract among the sellers undermines their risk-management incentives.*

If protection sellers can retrade contracts, they have incentives to accumulate contracts and build up hedging positions beyond their pledgeable income. Sellers take on concentrated risks and benefit from the protection offered by limited liability. Anticipating this, the protection buyer does not enter such contracts. Hence, unregulated trading whereby those selling the contracts free themselves from any contractual obligation towards the protection buyer leads to a market failure.

To restore optimality, such unregulated trading has to be banned. Alternatively, requesting an *initial* margin restores protection sellers' risk-management incentives when retrading is possible. By preventing sellers from accumulating excessive hedging positions, initial margins counter sellers' desire to take on concentrated risks. The margin must be deposited *before* the signal realizes since the scope for retrading exists only then. This is in contrast to the variation margin, analyzed earlier, which is deposited after the signal realizes. An initial margin makes it costly for a seller to acquire another seller's contract since she has to liquidate some of her assets.

Consider again the case of two protection sellers. If both sellers retain their half of the optimal contract, $\left(\frac{\tau}{2}, \frac{\tau}{2}\right)$, an initial margin is not required. It is only when a seller wants to accumulate a position that is larger than her pledgeable income, $A\mathcal{P}_i = \frac{A\mathcal{P}}{2}$, that she must put up an initial margin. We can compute the optimal amount at which a position in excess of $\left(\frac{\bar{\tau}}{2}, \frac{\tau}{2}\right)$ should be margined. The cost of liquidating assets to comply with the initial margin, denoted by α_0, must be large enough to outweigh the gain from retrading, which is given by equation (**??**):

$$\text{prob}[\underline{s}]\,(1-p)\,\frac{A\mathcal{P}}{2} \leq \frac{A}{2}\,(R-1)\,\alpha_0,$$

In equilibrium, the condition holds as an equality to minimize the opportunity cost of the initial margin. We can therefore state the following result:

Proposition 10 (Initial margin) *Initial margins maintain sellers' risk-management incentives when retrading is possible. The optimal size of the initial margin is*

$$\alpha_0 = \frac{prob[\underline{s}]\,(1-p)\,\mathcal{P}}{R-1},$$

where, as before, $\mathcal{P}$ denotes the per-unit pledgeable income.

The size of the optimal initial margin depends not only on the (negative) signal risk of the underlying position, but also on the characteristics of traders' balance sheets, i.e., default risk $(1-p)$, net asset return $(R-1)$ and pledgeability $\mathcal{P}$.

Anticipating that initial margins will be requested whenever the seller undertakes a position in excess of her pledgeable income, the seller does not acquire any additional contracts. Hence, initial margins act as an out-of-equilibrium threat.

7 Conclusion

We analyze hedging contracts between protection sellers and a protection buyer. We show how this contract, designed to facilitate risk-sharing, can generate incentives for risk-taking. When the position of the protection seller is more likely to become loss-making in the future, then the position becomes a liability and undermines a seller's incentive to exert risk management effort. Shirking on risk management may lead to a seller's default and, hence, expose the protection buyer to counterparty risk. Hedging can thus propagate risk from derivatives positions to other businesses of financial institutions.

When the seller's moral hazard problem is moderate, margins enhance the scope for risk-sharing. Initial margins discourage retrading and the accumulation of excessive derivatives positions, while variation margins discourage risk-taking for a given position. However, when the moral hazard problem is severe, margins can actually undermine risk-management incentives.

References

Acharya, V. and A. Bisin, 2010, “Counterparty Risk Externality: Centralized versus Over-the-counter Markets,” NBER working paper No. 17000.

Allen, F. and E. Carletti, 2006, “Credit risk transfer and contagion,” *Journal of Monetary Economics*, 53, 89–111.

Allen, F. and D. Gale, 1994, *Financial Innovation and Risk Sharing*, MIT Press, Cambridge, Massachusetts.

Biais, B. and C. Casamatta (1999), “Optimal Leverage and Aggregate Investment,” *Journal of Finance*, 54(4), 1291-1323.

Biais, B., D. Martimort, and J.-C. Rochet, 2000, “Competing Mechanisms in a Common Value Environment,” *Econometrica*, 68, 799-837.

Biais, B., J.-C. Rochet, and P. Woolley, 2009, “The Life-Cycle of the Financial Sector and Other Speculative Industries,” IDEI Working Paper No. 549.

Bolton, P. and M. Oehmke, 2011, “Should Derivatives be Senior?,” working paper, Columbia University.

Brunnermeier, M., 2009, “Deciphering the 2007-08 Liquidity and Credit Crunch,” *Journal of Economic Perspectives* 23, 77-100.

Brunnermeier, M. and M. Oehmke, 2009, “The Maturity Rat Race,” Working Paper, Princeton University.

Brunnermeier, M. and L. Pedersen, 2009, “Market Liquidity and Funding Liquidity,” *Review of Financial Studies*, 22(6), 2201-2238.

Cifuentes, R., H. Shin and G. Ferrucci, 2005, “Liquidity Risk and Contagion,” *Journal of the European Economic Association*, 3 (2-3), 556-566.

European Central Bank, 2009, “Credit Default Swaps and Counterparty Risk,” August.

Freixas, X., Parigi, B.M. and J.-C. Rochet, 2000, “Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank,” *Journal of Money, Credit and Banking*, 32, 611-638.

Holmström, B. and J. Tirole, 1998, "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1-40.

Myers, S., 1977, "Determinants of Corporate Borrowing," *Journal of Financial Economics*, 5, 147-75.

Parlour, C. and G. Plantin, 2008, "Loan Sales and Relationship Banking," *Journal of Finance*, 63(3), 1291-1314.

Parlour, C. and A. Winton, 2008, "Laying Off Credit Risk: Loan Sales and Credit Default Swaps," Working Paper, UCLA.

Rajan, R., 2006, "Has Financial Development Made the World Riskier?", *European Financial Management*, 12, 499-533.

Stephens, E. and J.R. Thompson, 2011, "CDS as Insurance: Leaky Lifeboats in Stormy Seas," working paper, University of Alberta and University of Waterloo.

Thompson, J.R., 2010, "Counterparty Risk in Financial Contracts: Should the Insured Worry about the Insurer?," *Quarterly Journal of Economics*, 125(3), 1195-1252.

Tirole, J., 2005, *The Theory of Corporate Finance*, Princeton University Press, Princeton, USA.

Appendix

Proof of Proposition ??

Let μ denote the Lagrange multiplier on the participation constraint (**??**). Let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha \geq 0$ and $\alpha \leq 1$. The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s})$, $\tau(\underline{\theta}, \bar{s})$, $\tau(\bar{\theta}, \underline{s})$, $\tau(\underline{\theta}, \underline{s})$ and margin α are given by:

$$
\begin{aligned}
\pi\lambda u'(\bar{\theta} + \tau(\bar{\theta}, \bar{s})) - \mu\pi\lambda &= 0 \\
(1-\pi)(1-\lambda)u'(\underline{\theta} + \tau(\underline{\theta}, \bar{s})) - \mu(1-\pi)(1-\lambda) &= 0 \\
\pi(1-\lambda)u'(\bar{\theta} + \tau(\bar{\theta}, \underline{s})) - \mu\pi(1-\lambda) &= 0 \\
(1-\pi)\lambda u'(\underline{\theta} + \tau(\underline{\theta}, \underline{s})) - \mu(1-\pi)\lambda &= 0 \\
\mu A\,(1-R) + \mu_0 - \mu_1 &= 0
\end{aligned}
$$

It follows that the marginal utility of the buyer of insurance is equalized across $(\tilde{\theta}, \tilde{s})$ states (full insurance) and that the participation constraint is binding:

$$\bar{u}'(\tau(\bar{\theta}, \bar{s})) = \bar{u}'(\tau(\bar{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \bar{s})) = \mu > 0 \tag{A.1}$$

where we use a shorthand $\bar{u}'(\tau(\bar{\theta}, \tilde{s}))$ to denote marginal utility in state $\bar{\theta}$ conditional on the signal $\tilde{s}$ and, similarly, $\underline{u}'(\tau(\underline{\theta}, \tilde{s}))$ to denote marginal utility in state $\underline{\theta}$ conditional on the signal $\tilde{s}$. Since $\mu > 0$ and $1 - R < 0$, it must be that $\mu_0 > 0$ and $\mu_1 = 0$. Hence, $\alpha = 0$ must hold in the optimum and margins are not used.

The optimal transfers are obtained by using the fact that the participation constraint is binding and that consumption is the same across $(\tilde{\theta}, \tilde{s})$ states.

Proof of Lemma ??

Let $\mu_{\bar{s}}$ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the incentive compatibility constraints (**??**) and (**??**), respectively (μ again denotes the multiplier on the participation constraint (**??**)). The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s})$, $\tau(\underline{\theta}, \bar{s})$, $\tau(\bar{\theta}, \underline{s})$ and $\tau(\underline{\theta}, \underline{s})$ are given by:

$$
\begin{aligned}
\pi\lambda u'(\bar{\theta} + \tau(\bar{\theta}, \bar{s})) - \mu_{\bar{s}}\bar{\pi} - \mu\pi\lambda &= 0 \\
(1-\pi)(1-\lambda)u'(\underline{\theta} + \tau(\underline{\theta}, \bar{s})) - \mu_{\bar{s}}(1-\bar{\pi}) - \mu(1-\pi)(1-\lambda) &= 0 \\
\pi(1-\lambda)u'(\bar{\theta} + \tau(\bar{\theta}, \underline{s})) - \mu_{\underline{s}}\underline{\pi} - \mu\pi(1-\lambda) &= 0 \\
(1-\pi)\lambda u'(\underline{\theta} + \tau(\underline{\theta}, \underline{s})) - \mu_{\underline{s}}(1-\underline{\pi}) - \mu(1-\pi)\lambda &= 0
\end{aligned}
$$

We re-write the first-order conditions as

$$\begin{aligned}
\bar{u}'(\tau(\bar{\theta},\bar{s})) &= \mu + \mu_{\bar{s}}\frac{\bar{\pi}}{\pi\lambda} && \text{(A.2)}\\
\underline{u}'(\tau(\underline{\theta},\bar{s})) &= \mu + \mu_{\bar{s}}\frac{1-\bar{\pi}}{(1-\pi)(1-\lambda)} && \text{(A.3)}\\
\bar{u}'(\tau(\bar{\theta},\underline{s})) &= \mu + \mu_{\underline{s}}\frac{\underline{\pi}}{\pi(1-\lambda)} && \text{(A.4)}\\
\underline{u}'(\tau(\underline{\theta},\underline{s})) &= \mu + \mu_{\underline{s}}\frac{1-\underline{\pi}}{(1-\pi)\lambda} && \text{(A.5)}
\end{aligned}$$

where we use a shorthand $\bar{u}'(\tau(\bar{\theta},\tilde{s}))$ to denote marginal utility in state $\bar{\theta}$ conditional on the signal $\tilde{s}$ and, similarly, $\underline{u}'(\tau(\underline{\theta},\tilde{s}))$ to denote marginal utility in state $\underline{\theta}$ conditional on the signal $\tilde{s}$.

Since

$$\begin{aligned}
\frac{\bar{\pi}}{\pi\lambda} &= \frac{\text{prob}[\bar{\theta}|\bar{s}]}{\text{prob}[\bar{\theta}\cap\bar{s}]} = \frac{1}{\text{prob}[\bar{s}]}\\
\frac{1-\bar{\pi}}{(1-\pi)(1-\lambda)} &= \frac{\text{prob}[\underline{\theta}|\bar{s}]}{\text{prob}[\underline{\theta}\cap\bar{s}]} = \frac{1}{\text{prob}[\bar{s}]}\\
\frac{\underline{\pi}}{\pi(1-\lambda)} &= \frac{\text{prob}[\bar{\theta}|\underline{s}]}{\text{prob}[\bar{\theta}\cap\underline{s}]} = \frac{1}{\text{prob}[\underline{s}]}\\
\frac{1-\underline{\pi}}{(1-\pi)\lambda} &= \frac{\text{prob}[\underline{\theta}|\underline{s}]}{\text{prob}[\underline{\theta}\cap\underline{s}]} = \frac{1}{\text{prob}[\underline{s}]}
\end{aligned}$$

holds, it follows that there is full risk-sharing conditional on the signal:

$$\begin{aligned}
\bar{u}'(\tau(\bar{\theta},\bar{s})) &= \underline{u}'(\tau(\underline{\theta},\bar{s}))\\
\bar{u}'(\tau(\bar{\theta},\underline{s})) &= \underline{u}'(\tau(\underline{\theta},\underline{s}))
\end{aligned}$$

As in the first-best case, we therefore have

$$\tau(\underline{\theta},\bar{s}) - \tau(\bar{\theta},\bar{s}) = \tau(\underline{\theta},\underline{s}) - \tau(\bar{\theta},\underline{s}) = \Delta\theta > 0 \qquad \text{(A.6)}$$

It follows that, conditional on the signal, the transfer to the buyer when the asset return is low is higher than when the asset return is high, $\tau(\underline{\theta},\tilde{s}) > \tau(\bar{\theta},\tilde{s})$.

Next, we show that the participation constraint must bind. Suppose not, i.e. $\mu = 0$. Then, equations (**??**) and (**??**) imply that $\mu_{\bar{s}} > 0$. Similarly, (**??**) and (**??**) imply that $\mu_{\underline{s}} > 0$. Both incentive constraints bind so that $A\mathcal{P} = \bar{\tau} = \underline{\tau}$. Since the participation constraint is slack, it must be that

$$\begin{aligned}
0 &> E[\tau] \equiv \text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]\underline{\tau}\\
&= A\mathcal{P}\left(\text{prob}[\bar{s}] + \text{prob}[\underline{s}]\right)\\
&= A\mathcal{P}
\end{aligned}$$

which contradicts $A\mathcal{P} > 0$. Hence, the participation constraint binds, $E[\tau] = 0$.

It follows that at least one incentive constraint must be slack. If not, then $\bar{\tau} = \underline{\tau} = A\mathcal{P} > 0$, which contradicts $E[\tau] = 0$.

Suppose both incentive constraints are slack, $\mu_{\bar{s}} = \mu_{\underline{s}} = 0$. Then, we obtain full insurance as in (**??**) and the contract is given by proposition **??** (first-best). The conditions under which the incentive constraints are indeed slack are given by:

$$\begin{aligned} A\mathcal{P} &> \bar{\pi}\tau^{FB}(\bar{\theta}, \bar{s}) + (1 - \bar{\pi})\tau^{FB}(\underline{\theta}, \bar{s}) = (\pi - \bar{\pi})\Delta\theta \\ A\mathcal{P} &> \underline{\pi}\tau^{FB}(\bar{\theta}, \underline{s}) + (1 - \underline{\pi})\tau^{FB}(\underline{\theta}, \underline{s}) = (\pi - \underline{\pi})\Delta\theta \end{aligned}$$

When the signal is informative, $\lambda > \frac{1}{2}$, we have $\bar{\pi} > \pi > \underline{\pi}$. The result in the lemma follows.

Proof of Lemma ??

We have shown above that at least one incentive constraint must be slack. They cannot both be slack since we assume that $A\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$. We now show that it is the incentive constraint following a *bad* signal that is binding. Suppose not, so that $A\mathcal{P} = \bar{\tau} > 0 > \underline{\tau}$ where the last inequality follows from $E[\tau] = 0$. Then, $\mu_{\underline{s}} = 0$ and $\mu_{\bar{s}} \geq 0$ and equations (**??**) through (**??**) yield

$$\bar{u}'(\tau(\bar{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \underline{s})) = \mu \leq \bar{u}'(\tau(\bar{\theta}, \bar{s})) = \underline{u}'(\tau(\underline{\theta}, \bar{s}))$$

Comparing the first with the third term and the second with the fourth term yields

$$\begin{aligned} \tau(\bar{\theta}, \underline{s}) &\geq \tau(\bar{\theta}, \bar{s}) \\ \tau(\underline{\theta}, \underline{s}) &\geq \tau(\underline{\theta}, \bar{s}) \end{aligned}$$

Using $\tau(\underline{\theta}, \tilde{s}) > \tau(\bar{\theta}, \tilde{s})$ (equation (**??**)) and $\bar{\pi} > \underline{\pi}$, we can write

$$\begin{aligned} 0 &< \bar{\tau} \equiv \bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \bar{\pi})\tau(\underline{\theta}, \bar{s}) \\ &< \underline{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \bar{s}) \\ &\leq \underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \underline{s}) \equiv \underline{\tau} \end{aligned}$$

But $\underline{\tau} < 0$, a contradiction. Hence, only the incentive constraint after a bad signal binds.

Proof of Proposition ??

The optimal contract is given by the binding incentive constraint following a bad signal:

$$A\mathcal{P} = \underline{\tau},$$

the binding participation constraint

$$\text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]\underline{\tau} = 0,$$

and full risk-sharing conditional on the signal (**??**).

Proof of Proposition ??

Let $\mu_{\bar{s}}$ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the incentive compatibility constraints (**??**) and (**??**), respectively, and let μ denote the multiplier on the participation constraint (**??**). The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s}, R)$, $\tau(\underline{\theta}, \bar{s}, R)$, $\tau(\bar{\theta}, \underline{s}, R)$, $\tau(\underline{\theta}, \underline{s}, R)$ and $\tau(\bar{\theta}, \underline{s}, 0)$ are:

$$\bar{u}'(\tau(\bar{\theta}, \bar{s}, R)) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.7}$$

$$\underline{u}'(\tau(\underline{\theta}, \bar{s}, R)) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.8}$$

$$\bar{u}'(\tau(\bar{\theta}, \underline{s}, R)) = \mu - \frac{\mu_{\underline{s}}}{p\,\text{prob}[\underline{s}]} \tag{A.9}$$

$$\underline{u}'(\tau(\underline{\theta}, \underline{s}, R)) = \mu - \frac{\mu_{\underline{s}}}{p\,\text{prob}[\underline{s}]} \tag{A.10}$$

$$\bar{u}'(\tau(\bar{\theta}, \underline{s}, 0)) = \mu + \frac{\mu_{\underline{s}}}{(1-p)\,\text{prob}[\underline{s}]} \tag{A.11}$$

where we use a shorthand $\bar{u}'(\tau(\bar{\theta}, \tilde{s}, \tilde{R}))$ to denote marginal utility in state $\bar{\theta}$ conditional on the signal $\tilde{s}$ and return $\tilde{R}$ and, similarly, $\underline{u}'(\tau(\underline{\theta}, \tilde{s}, \tilde{R}))$ to denote marginal utility in state $\underline{\theta}$ conditional on the signal $\tilde{s}$ and return $\tilde{R}$.

We first show that the participation constraint binds. Suppose instead that the constraint is slack, implying $\mu = 0$. Since $\mu_{\underline{s}} \geq 0$, equations (**??**) and (**??**) cannot hold. A contradiction.

Next, we show that the incentive constraint after a bad signal (**??**) is slack, implying $\mu_{\underline{s}} = 0$. Suppose that the constraint binds and

$$A\mathcal{P} + \underline{\pi}\tau(\bar{\theta}, \underline{s}, 0) = \underline{\pi}\tau(\bar{\theta}, \underline{s}, R) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}, R)$$

implying that

$$\underline{\pi}\tau(\bar{\theta}, \underline{s}, R) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}, R) < A\mathcal{P} \tag{A.12}$$

since $\tau(\bar{\theta}, \underline{s}, 0) < 0$. Since the participation constraint also binds, we have that

$$-\,\text{prob}[\underline{s}](1-p)\left[A\mathcal{P} + \underline{\pi}\tau(\bar{\theta}, \underline{s}, 0)\right] = \text{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta}, \bar{s}, R) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s}, R)\right] + \text{prob}[\underline{s}]p\left[\underline{\pi}\tau(\bar{\theta}, \underline{s}, R) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}, R)\right]$$

Using the binding incentive constraint (**??**) in the equation above and simplifying yields

$$\text{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta}, \bar{s}, R) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s}, R)\right] + \text{prob}[\underline{s}]\left[\underline{\pi}\tau(\bar{\theta}, \underline{s}, R) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}, R)\right] = 0 \tag{A.13}$$

Equations (**??**) and (**??**) imply that the optimal transfers $\tau(\bar{\theta}, \bar{s}, R)$, $\tau(\underline{\theta}, \bar{s}, R)$, $\tau(\bar{\theta}, \underline{s}, R)$ and $\tau(\underline{\theta}, \underline{s}, R)$ satisfy the incentive-compatibility condition inducing effort after bad news (**??**) and the participation constraint (**??**) in the contract with effort after both signals. Hence, inducing effort after both signals is feasible with these transfers. We now show that, given these transfers, the expected utility of the contract with effort after both signals, $EU^{e=1,e=1}$, is at least as high as the expected utility of the contract with risk-taking after bad news, $EU^{e=1,e=0}$, i.e.:

$$\begin{aligned}&\pi\lambda u(\bar{\theta}+\tau(\bar{\theta},\bar{s},R))+(1-\pi)(1-\lambda)u(\underline{\theta}+\tau(\underline{\theta},\bar{s},R))+\pi(1-\lambda)u(\bar{\theta}+\tau(\bar{\theta},\underline{s},R))\\&+(1-\pi)\lambda u(\underline{\theta}+\tau(\underline{\theta},\underline{s},R))-\pi\lambda u(\bar{\theta}+\tau(\bar{\theta},\bar{s},R))-(1-\pi)(1-\lambda)u(\underline{\theta}+\tau(\underline{\theta},\bar{s},R))\\&-\pi(1-\lambda)[pu(\bar{\theta}+\tau(\bar{\theta},\underline{s},R))+(1-p)u(\bar{\theta}+\tau(\bar{\theta},\underline{s},0))]\\&-(1-\pi)\lambda[pu(\underline{\theta}+\tau(\underline{\theta},\underline{s},R))+(1-p)u(\underline{\theta})]\geq 0\end{aligned}$$

The left-hand side is equal to:

$$\pi(1-\lambda)(1-p)\left[u(\bar{\theta}+\tau(\bar{\theta},\underline{s},R))-u(\bar{\theta}+\tau(\bar{\theta},\underline{s},0))\right]+(1-\pi)\lambda(1-p)\left[u(\underline{\theta}+\tau(\underline{\theta},\underline{s},R))-u(\underline{\theta})\right]$$

It follows from equations (**??**) and (**??**) that

$$\bar{u}'(\tau(\bar{\theta},\underline{s},R))\leq\bar{u}'(\tau(\bar{\theta},\underline{s},0))$$

and thus $\tau(\bar{\theta},\underline{s},R)\geq\tau(\bar{\theta},\underline{s},0)$. Hence, the expression in the first square bracket is non-negative. The expression in the second square bracket is positive since $\tau(\underline{\theta},\underline{s},R)>0$. Hence, the protection buyer prefers to induce effort after bad news, contradicting the optimality of the contract with risk-taking after bad news. We conclude that if risk-taking after bad news is optimal, the incentive constraint after a bad signal (**??**) must be slack and $\mu_{\underline{s}}=0$.

Hence, we have full sharing of the $\tilde{\theta}$ risk conditional on the signal, except for a default in $\underline{\theta}$ state:

$$\begin{aligned}\bar{u}'(\tau(\bar{\theta},\bar{s},R))&=\underline{u}'(\tau(\underline{\theta},\bar{s},R))\\\bar{u}'(\tau(\bar{\theta},\underline{s},R))&=\underline{u}'(\tau(\underline{\theta},\underline{s},R))=\bar{u}'(\tau(\bar{\theta},\underline{s},0))\end{aligned}$$

and hence

$$\tau(\underline{\theta},\tilde{s})-\tau(\bar{\theta},\tilde{s})=\Delta\theta>0 \tag{A.14}$$

We now show that the incentive constraint after a good signal (**??**) is also slack, implying $\mu_{\bar{s}}=0$. When the constraint is slack, there is full insurance except when the seller defaults in $\underline{\theta}$ state, i.e. we have:

$$\tau(\tilde{\theta},\bar{s},R)=\tau(\tilde{\theta},\underline{s},R)\text{ and }\tau(\bar{\theta},\underline{s},R)=\tau(\bar{\theta},\underline{s},0) \tag{A.15}$$

The buyer is, however, exposed to counterparty risk.

The optimal contract in this case is given by equations (**??**), (**??**) and the binding participation constraint. We now check under what conditions the incentive constraint following a good signal is indeed slack. Starting with the binding participation constraint and using

(**??**) and (**??**), we get

$$
\begin{aligned}
-\operatorname{prob}[\underline{s}](1-p)A\mathcal{P} = &\operatorname{prob}[\bar{s}][\tau(\underline{\theta},\underline{s},R)-\bar{\pi}\Delta\theta] \\
&+\operatorname{prob}[\underline{s}]p[\tau(\underline{\theta},\underline{s},R)-\underline{\pi}\Delta\theta]+(1-p)\operatorname{prob}[\underline{s}]\underline{\pi}\left[\tau(\underline{\theta},\underline{s},R)-\Delta\theta\right]
\end{aligned}
$$

Hence,

$$
\tau(\underline{\theta},\underline{s},R)=\frac{\pi\Delta\theta-\operatorname{prob}[\underline{s}]\,(1-p)\,A\mathcal{P}}{1-\operatorname{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)} \tag{A.16}
$$

For the incentive constraint following a good signal (**??**) to be slack, it must be that

$$
A\mathcal{P}>\bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)=\tau(\underline{\theta},\underline{s},R)-\bar{\pi}\Delta\theta
$$

or, after substituting for $\tau(\underline{\theta},\underline{s})$ and simplifying,

$$
A\mathcal{P}>\Delta\theta\frac{\pi-\bar{\pi}\left[1-\operatorname{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)\right]}{1+\operatorname{prob}[\underline{s}]\underline{\pi}\,(1-p)} \tag{A.17}
$$

Condition (**??**) is always satisfied if

$$
\pi-\bar{\pi}\left[1-\operatorname{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)\right]<0 \tag{A.18}
$$

since $A\mathcal{P}>0$. Condition (**??**) is equivalent to

$$
\lambda^2(1-p)-2\lambda+1<0
$$

This inequality holds under our assumption (**??**), i.e. for all $\lambda\geq\lambda^*\equiv\frac{1-\sqrt{p}}{1-p}>\frac{1}{2}$. This is because the left-hand side of the inequality above is decreasing in λ and it is equal to zero for λ^*.

Proof of Proposition ??

The proof proceeds in three steps. First, we show that the expected utility of the contract with effort after both signals is increasing in $\mathcal{P}$:

$$
\begin{aligned}
\frac{\partial EU^{e=1,e=1}}{\partial\mathcal{P}} &= -\frac{\operatorname{prob}[\underline{s}]A}{\operatorname{prob}[\bar{s}]}\left[\pi\lambda\bar{u}'(\tau(\bar{\theta},\bar{s}))+(1-\pi)\,(1-\lambda)\,\underline{u}'(\tau(\underline{\theta},\bar{s}))\right] \\
&\quad+\pi\,(1-\lambda)\,\bar{u}'(\tau(\bar{\theta},\underline{s}))+(1-\pi)\,\lambda\underline{u}'(\tau(\underline{\theta},\underline{s})) \\
&= \operatorname{prob}[\underline{s}]A\left[\bar{u}'(\tau(\bar{\theta},\underline{s}))-\bar{u}'(\tau(\bar{\theta},\bar{s}))\right]>0
\end{aligned}
$$

since $\tau(\bar{\theta},\underline{s})<\tau(\bar{\theta},\bar{s})$ due to the signal risk.

Second, we show that the expected utility of the contract with no effort following a bad signal is decreasing in $\mathcal{P}$:

$$\begin{aligned}\frac{\partial EU^{e=1,e=0}}{\partial \mathcal{P}} &= -\frac{\text{prob}[\underline{s}]\,(1-p)\,A}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}\left[\pi\lambda\bar{u}'(\tau(\bar{\theta},\bar{s}))+(1-\pi)\,(1-\lambda)\,\underline{u}'(\tau(\underline{\theta},\bar{s}))\right.\\ &\quad \left.+\,\pi\,(1-\lambda)\,\bar{u}'(\tau(\bar{\theta},\underline{s}))+(1-\pi)\,\lambda p\underline{u}'(\tau(\underline{\theta},\underline{s}))\right]\\ &= -\frac{\text{prob}[\underline{s}]\,(1-p)\,A}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}\left[\pi\bar{u}'(\tau(\bar{\theta},\bar{s}))\right.\\ &\quad \left.+(1-\pi)((1-\lambda)+p\lambda)\underline{u}'(\tau(\underline{\theta},\bar{s}))\right]<0\end{aligned}$$

Third, we provide sufficient condition for $EU^{e=1,e=1}(\mathcal{P}=0)<EU^{e=1,e=0}(\mathcal{P}=0)$ so that no effort after a bad signal is optimal for low $\mathcal{P}$.

We have

$$\begin{aligned}EU^{e=1,e=1}(\mathcal{P}=0) &= [\pi\lambda+(1-\pi)\,(1-\lambda)]\,u\,(\underline{\theta}+\bar{\pi}\Delta\theta)+[\pi\,(1-\lambda)+(1-\pi)\,\lambda]\,u\,(\underline{\theta}+\underline{\pi}\Delta\theta)\\ &= \text{prob}[\bar{s}]u\,(\underline{\theta}+\bar{\pi}\Delta\theta)+\text{prob}[\underline{s}]u\,(\underline{\theta}+\underline{\pi}\Delta\theta)\\ &= \text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}]) \qquad \text{(A.19)}\end{aligned}$$

and

$$\begin{aligned}EU^{e=1,e=0}(\mathcal{P}=0) &= \left(\text{prob}[\bar{s}]+\text{prob}[\underline{s}]\,(p+(1-p)\,\underline{\pi})\right)u\left(\underline{\theta}+\frac{\pi\Delta\theta}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}\right)\\ &\quad +(1-p)\,\text{prob}[\underline{s}]\,(1-\underline{\pi})\,u(\underline{\theta})\\ &= \left(\text{prob}[\bar{s}]+p\text{prob}[\underline{s}]\right)u\left(\hat{E}[\tilde{\theta}]\right)\\ &\quad +(1-p)\,\text{prob}[\underline{s}]\left(\underline{\pi}u\left(\hat{E}[\tilde{\theta}]\right)+(1-\underline{\pi})\,u(\underline{\theta})\right) \qquad \text{(A.20)}\end{aligned}$$

where

$$\hat{E}[\tilde{\theta}]\equiv\hat{\pi}\bar{\theta}+(1-\hat{\pi})\underline{\theta}$$

and

$$\hat{\pi}\equiv\frac{\pi}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}$$

Note that

$$\bar{\pi}>\hat{\pi}>\pi>\underline{\pi} \qquad \text{(A.21)}$$

for $p\in(0,1)$. The last two inequalities are straightforward. The first inequality holds if and only if

$$\lambda^2(1-p)-2\lambda+1<0$$

which is satisfied under our assumption (**??**), i.e. for all $\lambda\geq\lambda^*\equiv\frac{1-\sqrt{p}}{1-p}>\frac{1}{2}$.

Combining (**??**) and (**??**), we have that no effort after a bad signal dominates effort (when $\mathcal{P}=0$) if and only if

$$\begin{aligned}&\text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}])\\ &<\left(\text{prob}[\bar{s}]+\text{prob}[\underline{s}]p\right)u(\hat{E}[\tilde{\theta}])+\text{prob}[\underline{s}](1-p)EU\left(\tilde{R}=0\right)\end{aligned}$$

where

$$EU\left(\tilde{R}=0\right) \equiv \underline{\pi} u(\hat{E}[\tilde{\theta}]) + (1-\underline{\pi})u(\underline{\theta})$$

After collecting terms, we have

$$\begin{aligned}\text{prob}[\bar{s}]\left[u(E[\tilde{\theta}|\bar{s}]) - u(\hat{E}[\tilde{\theta}])\right] + \text{prob}[\underline{s}]\left[u(E[\tilde{\theta}|\underline{s}]) - EU\left(\tilde{R}=0\right)\right] \\ < p\text{prob}[\underline{s}]\left[u(\hat{E}[\tilde{\theta}]) - EU\left(\tilde{R}=0\right)\right]\end{aligned}$$

All the differences in the square brackets are positive. The first one due to (**??**), the second one due to the concavity of u, and the third one due to both the concavity of u and (**??**).

Rearranging, we arrive at

$$\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}\frac{u(E[\tilde{\theta}|\bar{s}]) - u(\hat{E}[\tilde{\theta}])}{u(\hat{E}[\tilde{\theta}]) - EU\left(\tilde{R}=0\right)} + \frac{u(E[\tilde{\theta}|\underline{s}]) - EU\left(\tilde{R}=0\right)}{u(\hat{E}[\tilde{\theta}]) - EU\left(\tilde{R}=0\right)} < p \tag{A.22}$$

It is clear that the left-hand side is strictly positive so that seller's effort dominates when p is small. The left-hand is, however, also strictly smaller than one so that no effort after a bad signal dominates when p is large.[13]

The condition

$$\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}\frac{u(E[\tilde{\theta}|\bar{s}]) - u(\hat{E}[\tilde{\theta}])}{u(\hat{E}[\tilde{\theta}]) - EU\left(\tilde{R}=0\right)} + \frac{u(E[\tilde{\theta}|\underline{s}]) - EU\left(\tilde{R}=0\right)}{u(\hat{E}[\tilde{\theta}]) - EU\left(\tilde{R}=0\right)} < 1$$

simplifies to

$$\text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}]) + \text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}]) < u(\hat{E}[\tilde{\theta}])$$

By concavity,

$$\text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}]) + \text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}]) < u(E[\tilde{\theta}])$$

and so the condition holds when

$$u(E[\tilde{\theta}]) \leq u(\hat{E}[\tilde{\theta}])$$

which is always true due to (**??**).

Hence, whenever $EU^{e=1}(\mathcal{P}=0) < EU^{e=1,e=0}(\mathcal{P}=0)$ holds, the privately optimal contract entails no effort after a bad signal for low levels of per unit pledgeable income $\mathcal{P}$. For levels of $\mathcal{P} \geq \hat{\mathcal{P}}$ where $\hat{\mathcal{P}}$ is given by $EU^{e=1}\left(\hat{\mathcal{P}}\right) = EU^{e=1,e=0}\left(\hat{\mathcal{P}}\right)$ and $A\hat{\mathcal{P}} < (\pi - \underline{\pi})\Delta\theta$, the optimal contract is the one with effort. For $A\mathcal{P} > (\pi - \underline{\pi})\Delta\theta$, the first-best is reached.

[13] Note that this inequality is evaluated at $\mathcal{P} = 0$ and $\mathcal{P}$ is a function of p. There is, however, an open set of parameters for which no effort after a bad signal dominates.

A.1 Proof of Lemma ??

Let μ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the participation and incentive-compatibility constraints (**??**) and (**??**), respectively. Furthermore, let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha \geq 0$ and $\alpha \leq 1$. The first-order conditions with respect to expected transfers $\bar{\tau}$, $\underline{\tau}$ and margin α are:

$$u'(E[\theta|\bar{s}] + \bar{\tau}) = \mu \tag{A.23}$$

$$u'(E[\theta|\underline{s}] + \underline{\tau}) = \mu + \frac{\mu_{\underline{s}}}{\text{prob}[\underline{s}]} \tag{A.24}$$

$$\mu_{\underline{s}} A (1 - \mathcal{P}) + \mu_0 = \mu \text{prob}[\underline{s}] A (R - 1) + \mu_1 \tag{A.25}$$

where $u'(E[\theta|\bar{s}] + \bar{\tau})$ and $u'(E[\theta|\underline{s}] + \underline{\tau})$ are marginal utilities conditional on the good and the bad signal, respectively.

Equation (**??**) implies that $\mu > 0$ (and the participation constraint binds). Since $\mu_1 \geq 0$, the right-hand side of equation (**??**) is strictly positive. Now, suppose $\mathcal{P} \geq 1$. Then, it must be that $\mu_0 > 0$ for the equation (**??**) to hold. Hence, $\alpha^* = 0$ and margins are not used for $\mathcal{P} \geq 1$.

Proof of Proposition ??

Substituting (**??**) and (**??**) into (**??**), we arrive at:

$$\frac{u'(E[\theta|\underline{s}] + \underline{\tau})}{u'(E[\theta|\bar{s}] + \bar{\tau})} = 1 + \frac{R - 1}{1 - \mathcal{P}} + \frac{\mu_1 - \mu_0}{u'(E[\theta|\bar{s}] + \bar{\tau}) \text{prob}[\underline{s}] (1 - \mathcal{P}) A} \tag{A.26}$$

First, we claim that for any optimal $\alpha \in [0, 1]$, $\mu_{\underline{s}} > 0$ and the incentive constraint after bad news is binding. For $\alpha = 0$, we are solving the same problem as in Section **??** and the claim follows from Lemma **??**. For $0 < \alpha \leq 1$, we have $\mu_0 = 0$ and equation (**??**) implies that $\frac{u'(E[\theta|\underline{s}]+\underline{\tau})}{u'(E[\theta|\bar{s}]+\bar{\tau})} > 1$. But then, by equations (**??**) and (**??**), it must be that $\mu_{\underline{s}} > 0$.

Since the incentive constraint after bad news is binding, we have

$$\underline{\tau} = \alpha A + (1 - \alpha) A\mathcal{P}$$

and, using the binding participation constraint,

$$\bar{\tau} = -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]} [\alpha AR + (1 - \alpha) A\mathcal{P}]$$

Second, we claim that the left-hand side of (**??**) is decreasing in α. This is because $\frac{\partial \underline{\tau}(\alpha)}{\partial \alpha} = A (1 - \mathcal{P}) > 0$, $\frac{\partial \bar{\tau}(\alpha)}{\partial \alpha} = -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]} A (R - \mathcal{P}) < 0$ and u' is decreasing.

Denote the left-hand side of (**??**) as $\varphi(\alpha)$. If $\varphi(0) < 1 + \frac{R-1}{1-\mathcal{P}}$, then $\varphi(\alpha) < 1 + \frac{R-1}{1-\mathcal{P}}$ for any $\alpha \in [0, 1]$ and so we must have $\mu_0 > 0$ and hence $\alpha^* = 0$. By the same logic, if $\varphi(1) > 1 + \frac{R-1}{1-\mathcal{P}}$, then $\mu_1 > 0$ and hence $\alpha^* = 1$. Otherwise, $\alpha^* \in (0, 1)$ is given by $\varphi(\alpha^*) = 1 + \frac{R-1}{1-\mathcal{P}}$.

A.2 Proof of Lemma ??

Let $\mu_{\bar{s}}$ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the incentive compatibility constraints (??) and (??), respectively, and let μ denote the multiplier on the participation constraint (??). Furthermore, let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha \geq 0$ and $\alpha \leq 1$, and let μ_2 and μ_3 be the Lagrange multipliers on the constraints $\alpha A \geq \tau(\underline{\theta}, \underline{s}, 0)$ and $\alpha A \geq \tau(\bar{\theta}, \underline{s}, 0)$, respectively. The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s}, R)$, $\tau(\underline{\theta}, \bar{s}, R)$, $\tau(\bar{\theta}, \underline{s}, R)$, $\tau(\underline{\theta}, \underline{s}, R)$, $\tau(\bar{\theta}, \underline{s}, 0)$, $\tau(\underline{\theta}, \underline{s}, 0)$, and α are:

$$\bar{u}'(\tau(\bar{\theta}, \bar{s}, R)) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.27}$$

$$\underline{u}'(\tau(\underline{\theta}, \bar{s}, R)) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.28}$$

$$\bar{u}'(\tau(\bar{\theta}, \underline{s}, R)) = \mu - \frac{\mu_{\underline{s}}}{p\text{prob}[\underline{s}]} \tag{A.29}$$

$$\underline{u}'(\tau(\underline{\theta}, \underline{s}, R)) = \mu - \frac{\mu_{\underline{s}}}{p\text{prob}[\underline{s}]} \tag{A.30}$$

$$\bar{u}'(\tau(\bar{\theta}, \underline{s}, 0)) = \mu + \frac{\mu_{\underline{s}}}{(1-p)\,\text{prob}[\underline{s}]} + \frac{\mu_3}{(1-\lambda)\,\pi\,(1-p)} \tag{A.31}$$

$$\underline{u}'(\tau(\underline{\theta}, \underline{s}, 0)) = \mu + \frac{\mu_{\underline{s}}}{(1-p)\,\text{prob}[\underline{s}]} + \frac{\mu_2}{(1-\pi)\,\lambda\,(1-p)} \tag{A.32}$$

$$\mu\text{prob}[\underline{s}]A\left[R - 1 - (1-p)\,\mathcal{P}\right] + \mu_1 = \mu_0 + A\mathcal{P}\mu_{\underline{s}} + A\left(\mu_2 + \mu_3\right) \tag{A.33}$$

where we use a shorthand $\bar{u}'(\tau(\bar{\theta}, \tilde{s}, \tilde{R}))$ to denote marginal utility in state $\bar{\theta}$ conditional on the signal $\tilde{s}$ and return $\tilde{R}$ and, similarly, $\underline{u}'(\tau(\underline{\theta}, \tilde{s}, \tilde{R}))$ to denote marginal utility in state $\underline{\theta}$ conditional on the signal $\tilde{s}$ and return $\tilde{R}$.

First, we claim that if $pR + B < 1$, then $\alpha^* = 1$. Note that we must have that $\mu > 0$ and the participation constraint binds. Otherwise, equation (??) cannot hold. Now, suppose $pR + B < 1$ or, equivalently, $R - 1 < (1-p)\,\mathcal{P}$. It follows from equation (??) that $\mu_1 > 0$ must hold since the right-hand side of (??) is non-negative. Hence, $\alpha^* = 1$.

Second, we claim that if $\alpha^* = 1$, risk-taking after bad news cannot be strictly optimal. For $\alpha^* = 1$, the entire balance sheet of the protection seller is put in the margin after bad news. It is thus ring-fenced from actions of the protection seller, making risk-management and risk-taking decisions equivalent. But then, the contract with risk-management and margins weakly dominates the contract with risk-taking and margins (strictly if in the optimal contract with risk-management and margins, the margin is smaller than 1).

Proof of Proposition ??

We know from the proof of Lemma **??** that the participation constraint binds and that $\alpha^* < 1$ and $\mu_1 = 0$ since if $\alpha^* = 1$, risk-taking cannot be strictly optimal. The proof of the Proposition proceeds in several steps.

First, we claim that the incentive constraint after bad news must be slack and $\mu_{\underline{s}} = 0$.

Suppose otherwise. Then, we have

$$\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)=(1-\alpha)\,A\mathcal{P}+\underline{\pi}\tau(\bar{\theta},\underline{s},0)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},0) \tag{A.34}$$

Since $\tau(\bar{\theta},\underline{s},0)\leq\alpha A$ and $\tau(\underline{\theta},\underline{s},0)\leq\alpha A$, we have

$$\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)\leq(1-\alpha)\,A\mathcal{P}+\alpha A$$

implying that transfers $\tau(\bar{\theta},\underline{s},R)$ and $\tau(\underline{\theta},\underline{s},R)$ satisfy the incentive compatibility condition that induces effort after bad news (**??**). Using the binding participation constraint, we get

$$\begin{aligned} &-\text{prob}[\underline{s}]\left[\alpha A\,(R-1)+(1-p)\left((1-\alpha)A\mathcal{P}+\underline{\pi}\tau(\bar{\theta},\underline{s},0)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},0)\right)\right] \\ =\;& \text{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)\right]+\text{prob}[\underline{s}]p\left[\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)\right]\end{aligned}$$

or, using (**??**) and simplifying,

$$\begin{aligned} -\text{prob}[\underline{s}]\alpha A\,(R-1)=\;&\text{prob}[\bar{s}]\left[\bar{\pi}\tau(\bar{\theta},\bar{s},R)+(1-\bar{\pi})\tau(\underline{\theta},\bar{s},R)\right] \\ &+\text{prob}[\underline{s}]\left[\underline{\pi}\tau(\bar{\theta},\underline{s},R)+(1-\underline{\pi})\tau(\underline{\theta},\underline{s},R)\right]\end{aligned}$$

so that the transfers in the equation above satisfy the participation constraint in the contract with margins and effort after both signals. Hence, inducing effort after both signals is feasible with these transfers. Using the same steps as in the proof of Proposition **??**, we show that the expected utility of the contract with margins and effort after both signals is at least as high as the expected utility of the contract with margins and risk-taking after bad news, $EU^{e=1,e=1}-EU^{e=1,e=0}\geq 0$, contradicting the optimality of the contract with risk-taking after bad news. Showing that $EU^{e=1,e=1}-EU^{e=1,e=0}\geq 0$ is equivalent to showing that

$$\pi(1-\lambda)\left[u(\bar{\theta}+\tau(\bar{\theta},\underline{s},R))-u(\bar{\theta}+\tau(\bar{\theta},\underline{s},0))\right]+(1-\pi)\lambda\left[u(\underline{\theta}+\tau(\underline{\theta},\underline{s},R))-u(\underline{\theta}+\tau(\underline{\theta},\underline{s},0))\right]$$

is non-negative. The expression in the first square bracket is non-negative by (**??**) and (**??**). The expression in the second square bracket is non-negative by (**??**) and (**??**). This completes the proof of the claim. We conclude that if risk-taking after bad news is optimal, then the incentive constraint after bad news must be slack in the optimal contract with margins and risk-taking.

Second, we claim that $\alpha A>\tau(\bar{\theta},\underline{s},0)$ and $\mu_3=0$. Suppose not, i.e. $\alpha A=\tau(\bar{\theta},\underline{s},0)$ and $\mu_3\geq 0$. Given the feasibility constraint on $\tau(\underline{\theta},\underline{s},0)$, we can either have $\alpha A=\tau(\bar{\theta},\underline{s},0)>\tau(\underline{\theta},\underline{s},0)$, or $\alpha A=\tau(\bar{\theta},\underline{s},0)=\tau(\underline{\theta},\underline{s},0)$. We first show that $\tau(\bar{\theta},\underline{s},0)>\tau(\underline{\theta},\underline{s},0)$ cannot hold. Suppose otherwise, so that $\mu_2=0$. Equations (**??**) and (**??**) imply that $\bar{u}'(\tau(\bar{\theta},\underline{s},0))\geq\underline{u}'(\tau(\underline{\theta},\underline{s},0))$ so that

$$\bar{\theta}+\tau(\bar{\theta},\underline{s},0)\leq\underline{\theta}+\tau(\underline{\theta},\underline{s},0)$$

or, equivalently,

$$\Delta\theta+\alpha A\leq\tau(\underline{\theta},\underline{s},0)$$

which contradicts $\tau(\underline{\theta},\underline{s},0)<\alpha A$. We next show that $\tau(\bar{\theta},\underline{s},0)=\tau(\underline{\theta},\underline{s},0)$ cannot hold either. Suppose otherwise, so that $\alpha A=\tau(\bar{\theta},\underline{s},0)=\tau(\underline{\theta},\underline{s},0)$. By (**??**) and (**??**), $\tau(\bar{\theta},\underline{s},R)\geq$

$\tau(\bar{\theta}, \underline{s}, 0)$, and by (**??**) and (**??**), $\tau(\underline{\theta}, \underline{s}, R) \geq \tau(\underline{\theta}, \underline{s}, 0)$ implying

$$\begin{aligned} \tau(\bar{\theta}, \underline{s}, R) &\geq \tau(\bar{\theta}, \underline{s}, 0) = \alpha A \geq 0 \\ \tau(\underline{\theta}, \underline{s}, R) &\geq \tau(\underline{\theta}, \underline{s}, 0) = \alpha A \geq 0 \end{aligned} \tag{A.35}$$

Since the participation constraint binds, we have

$$\begin{aligned} & \text{prob}[\bar{s}] \left[\bar{\pi}\tau(\bar{\theta}, \bar{s}, R) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s}, R)\right] + \text{prob}[\underline{s}]p \left[\underline{\pi}\tau(\bar{\theta}, \underline{s}, R) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s}, R)\right] \\ = \; & -\text{prob}[\underline{s}] \left[\alpha A (R-1) + (1-p)(\alpha A + (1-\alpha) A\mathcal{P})\right] \end{aligned}$$

The right-hand side of the expression above is negative. On the left-hand side, the second term in square bracket is non-negative by (**??**). We now show that the first term in square bracket is also non-negative, implying that the equation above cannot hold. We either have that the incentive constraint after good news binds or it is slack. If it binds, then the first term on the left-hand side (in square bracket) is equal to $A\mathcal{P} > 0$. If it is slack, then $\mu_{\bar{s}} = 0$ and $\tau(\bar{\theta}, \bar{s}, R) = \tau(\bar{\theta}, \underline{s}, R) \geq \alpha A$ while $\tau(\underline{\theta}, \bar{s}, R) = \tau(\underline{\theta}, \underline{s}, R) \geq \alpha A$ so that the expected transfer is non-negative. This completes the proof the claim.

Third, we show that the feasibility constraint $\alpha A \geq \tau(\underline{\theta}, \underline{s}, 0)$ must bind. Suppose otherwise, $\alpha A > \tau(\underline{\theta}, \underline{s}, 0)$ and $\mu_2 = 0$. Then, by equations (**??**) through (**??**), there is full insurance conditional on bad news, and expected transfers after bad news are equal to zero. But then, the incentive constraint after bad news (**??**) cannot be slack for any $\alpha \in [0,1]$. A contradiction. Hence, $\alpha A = \tau(\underline{\theta}, \underline{s}, 0)$ in the optimum.

Fourth, we claim that the incentive constraint after good news is slack, implying $\mu_{\bar{s}} = 0$. We prove the claim by characterizing the optimal contract and verifying that the incentive constraint after good news never binds. Equation (**??**) yields

$$0 \leq \mu_2 = \mu \text{prob}[\underline{s}] \left[R - 1 - (1-p)\mathcal{P}\right] - \frac{\mu_0}{A}$$

Replacing $R - 1 - (1-p)\mathcal{P}$ with $pR + B - 1$, and using the first-order conditions to substitute for μ and μ_2, we arrive at:

$$\frac{\underline{u}'(\tau(\underline{\theta}, \underline{s}, 0))}{\underline{u}'(\tau(\underline{\theta}, \underline{s}, R))} = 1 + \frac{pR + B - 1}{(1-p)(1-\underline{\pi})} - \frac{\mu_0}{\underline{u}'(\tau(\underline{\theta}, \underline{s}, R)) A (1-p) \text{prob}[\underline{s}] (1-\underline{\pi})} \tag{A.36}$$

Since $\mu_{\bar{s}} = 0$, marginal utilities are equalized across five states in which the seller does not default on the contract. We thus have $\tau(\bar{\theta}, \bar{s}, R) = \tau(\bar{\theta}, \underline{s}, R) = \tau(\bar{\theta}, \underline{s}, 0)$ and $\tau(\underline{\theta}, \bar{s}, R) = \tau(\underline{\theta}, \underline{s}, R)$. The optimal transfers are given by (using the participation constraint):

$$\tau(\underline{\theta}, \tilde{s}, R) = \frac{\pi \Delta\theta - \text{prob}[\underline{s}] (1-p) A\mathcal{P}}{1 - \text{prob}[\underline{s}] (1-\underline{\pi}) (1-p)} - \alpha^* A \frac{\text{prob}[\underline{s}] \left[pR + B - 1 + (1-\underline{\pi})(1-p)\right]}{1 - \text{prob}[\underline{s}] (1-\underline{\pi})(1-p)},$$

$\tau(\bar{\theta}, \tilde{s}, \tilde{R}) = \tau(\underline{\theta}, \tilde{s}, R) - \Delta\theta$, and $\tau(\underline{\theta}, \underline{s}, 0) = \alpha^* A$. Note that $\frac{\partial \tau(\underline{\theta}, \underline{s}, R)}{\partial \alpha} < 0$ while $\frac{\partial \tau(\underline{\theta}, \underline{s}, 0)}{\partial \alpha} = A > 0$, implying that the left-hand side of (**??**) is decreasing in α (since u' is decreasing).

Denote the left-hand side of (**??**) as $\phi(\alpha)$. If $\phi(0) < 1 + \frac{pR+B-1}{(1-p)(1-\underline{\pi})}$, then $\phi(\alpha) < 1 + \frac{pR+B-1}{(1-p)(1-\underline{\pi})}$ for any $\alpha \in [0,1]$ and so we must have $\mu_0 > 0$ and hence $\alpha^* = 0$. If $\phi(1) >$

$1+\frac{pR+B-1}{(1-p)(1-\underline{\pi})}$, then equation (**??**) cannot hold for any $\alpha \in [0,1)$ and the contract with margins and risk-taking after bad news cannot be optimal. Otherwise, $\alpha^* \in (0,1)$ is given by $\phi(\alpha^*) = 1+\frac{pR+B-1}{(1-p)(1-\underline{\pi})}$.

We now check that the incentive constraint after good news (**??**) is indeed slack. This is equivalent to:

$$A\mathcal{P}+\bar{\pi}\Delta\theta > \frac{\pi\Delta\theta - \text{prob}[\underline{s}]\,(1-p)\,A\mathcal{P}}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)} - \alpha^* A\frac{\text{prob}[\underline{s}]\,[pR+B-1+(1-\underline{\pi})\,(1-p)]}{1-\text{prob}[\underline{s}]\,(1-\underline{\pi})\,(1-p)}$$

Note that for any $\alpha^* \geq 0$ the inequality above always holds under our assumption (**??**), which ensures that condition (**??**) is satisfied.

Proof of Proposition ??

Consider the contract with risk-management after both signals. Suppose, contrary to the claim in the proposition, that there exists a contract $(\bar{\tau}_i', \underline{\tau}_i') \neq (\bar{\tau}_i, \underline{\tau}_i)$, $i = 1, \ldots, N$, which satisfies participation and incentive constraints of each protection seller and yields a higher utility for the protection buyer.

Since $\bar{\tau}_i' \leq A\mathcal{P}_i$ and $\underline{\tau}_i' \leq A\mathcal{P}_i$ with $\mathcal{P}_i = \frac{\mathcal{P}}{N}$ holds for each i, we have $\sum_{i=1}^{N} \bar{\tau}_i' \leq A\mathcal{P}$ and $\sum_{i=1}^{N} \underline{\tau}_i' \leq A\mathcal{P}$. Similarly, $E[\tau_i'] \leq 0$ for each i implies that $\sum_{i=1}^{N} E[\tau_i'] \leq 0$. Let $\sum_{i=1}^{N} \bar{\tau}_i' \equiv \bar{\tau}'$ and $\sum_{i=1}^{N} \underline{\tau}_i' \equiv \underline{\tau}'$. Then, we have that

$$\text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}]+\bar{\tau}') + \text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}]+\underline{\tau}') > \text{prob}[\bar{s}]u(E[\tilde{\theta}|\bar{s}]+\bar{\tau}) + \text{prob}[\underline{s}]u(E[\tilde{\theta}|\underline{s}]+\underline{\tau})$$

But this contradicts the optimality of $\bar{\tau}$ and $\underline{\tau}$ for $N = 1$.

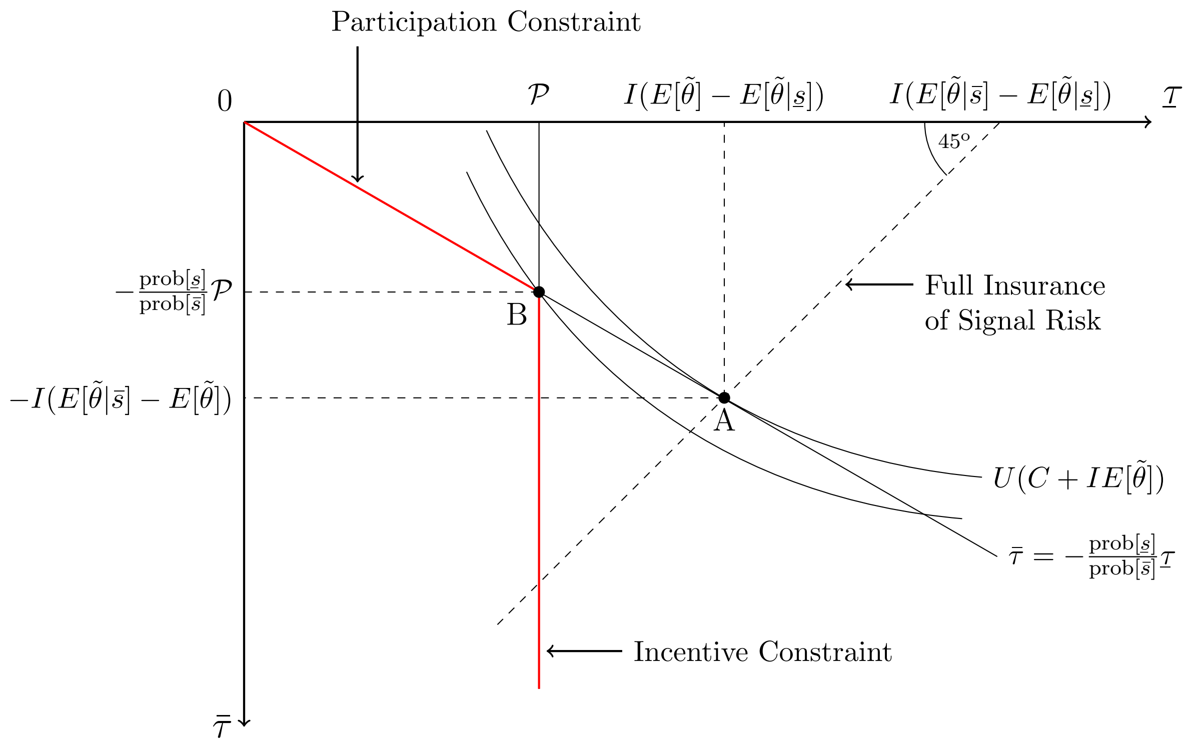

Figure 2: Optimal contracts when effort is observable (A) and when it is not, yet the protection seller exerts effort after a bad signal (B) (no counterparty risk)

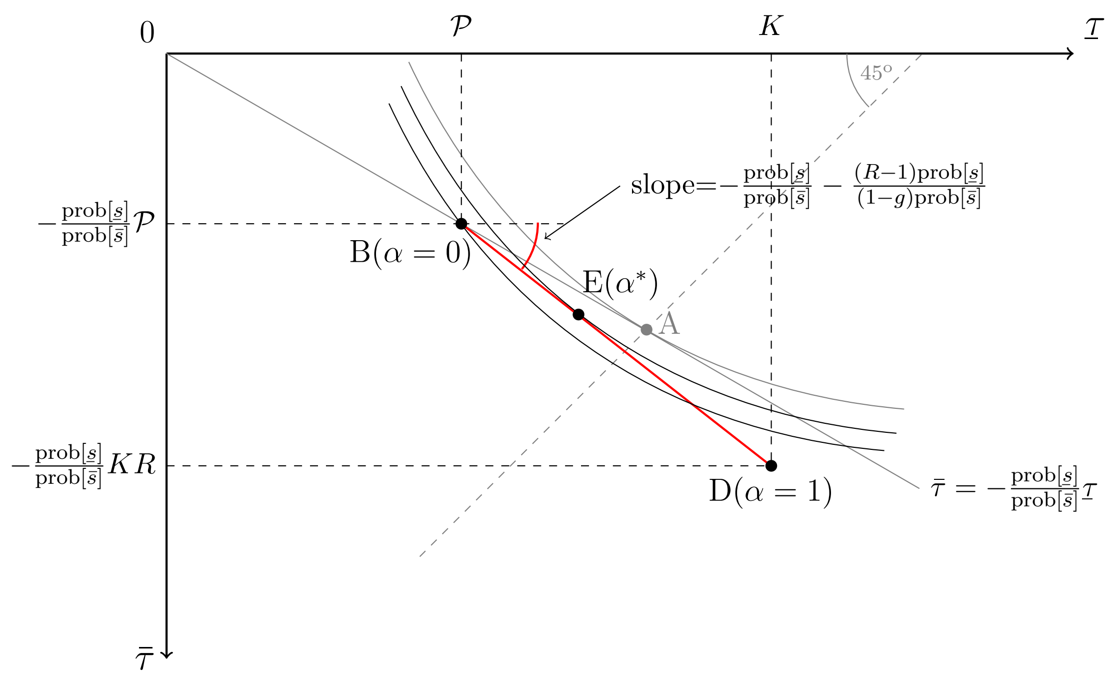

Figure 3: Margins with risk-management effort