Ambiguity and rollover risk: a possible explanation for market freezes?

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Abstract

The 2007-2008 financial crisis showed that the market for secured borrowing can easily break down when investors are not able to assess their own (or other agents') risk exposure because of the widespread ambiguity that affects the economic environment. In this paper, we develop a simple partial equilibrium model to recreate in a stylized framework some of the dynamics that might have contributed in determining the credit crunch. Our analysis shows that ambiguity and ambiguity aversion play a crucial role in generating freezes in the market for secured debt, since they directly impact on the market evaluation of the assets used as collateral. Interestingly, this result is driven by rollover risk and ambiguity only, and it does not rely on the existence of any friction or on the specific credit rating of the collateral asset.

When we consider policy actions aimed at maintaining markets' activity, our model suggests the importance of interventions designed to contain the perceived level of ambiguity. Furthermore, we argue that, during a financial crisis, only unambiguous policies can successfully restore investors' trust and trading. Finally, we show how a well-designed

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tradable insurance credits (TICs)-policy can mitigate the effects of ambiguity.

Ambiguity; Ambiguity aversion; Market freeze; Rollover risk.

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1 Introduction

The dramatic features of the 2007-2008 financial crisis clearly showed traditional models' inability to endogenously explain the causes underlying the credit crunch. Similarly, classic tenets of central banking seem to have been inappropriate, since, notwithstanding the efforts of the various policy makers, the financial crisis intensified over the Summer 2008, acutely hitting companies such as the U.S. government-sponsored enterprises (GSEs) Fannie Mae and Freddie Mac, the investment bank Lehman Brothers, and the insurance company American International Group (AIG).

From these simple considerations, it seems natural to focus our efforts on the developments of alternative frameworks that could account for the economic and financial challenges which originated during the crisis. In particular, following a growing body of research that views markets' complexity and the poor quality of available information as the main causes of investor's behaviors that are inconsistent with the predictions of standard models, in this paper we construct a simple partial equilibrium framework to recreate in a stylized setting some of the dynamics that characterized the last crisis, with specific emphasis on the role played by ambiguity and ambiguity aversion in determining freezes of the market for secured borrowing.[1] Apart from the recent events, the study of freeze-phenomena in the market for rollover debt is also of theoretical interest. Indeed, it is a well known result since Bester [3] that the use of high rating assets as collateral serves as a signalling tool against credit rationing in settings characterized by imperfect information, so that no borrower is denied a credit at the equilibrium. In this paper, we try to explain the contradictory evidence through a broader

[1] In what follows we will use the notions of 'ambiguity' and 'uncertainty' interchangeably, without special or technical meanings, unless otherwise stated.

theoretical framework, in which we account for ambiguity and ambiguity aversion.

Specifically, we consider a structured investment vehicle (SIV) that has to rise asset backed finance in accordance with an exogenously defined (short term) maturity structure, and we investigate how ambiguity (aversion) impacts on the evaluation of the collateral asset by potential lenders.[2] In the model, an exogenous random event can determine the occurrence of some unfamiliar (and therefore ambiguous) situation characterized by too vague an information level, that prevents agents from being sufficiently confident in their probabilistic assignments. The positive probability that the economy might experience some ambiguity seriously impact the evaluation of the assets used as collateral, determining the inability to rollover collateralized debt. Indeed, ambiguity averse traders do not subscribe any debt contract because they evaluate zero the borrowing capacity of collateral. Interestingly, no liquidation cost, or risk of fire sale are needed to derive this result.[3] Similarly, the particular credit rating of the collateral asset is not relevant. Given these preliminary findings, the role of interventions and regulations aimed at, not only solving, but especially preventing, the insurgence of ambiguous states seems to be essential. Furthermore, since ambiguity directly impacts assets' evaluation, a policy designed to prevent market freezes could be effective if it stabilizes the fair value of the asset. In this vein, a public insurance guarantee in the spirit of the tradable insurance credits (TICs) based proposal elaborated by Caballero and Kurlat [7] can contribute by rendering collateral assets insensitive to both ambiguity and rollover risk. Further, in line with what we observed during the last financial turnover, our analysis shows that any public action aimed at restoring the trading activity (meaning lenders' willingness to finance the SIV) has no effect if its content is not clear enough. In particular, if agents are not confident about the efficacy of the policy in the long term, and such an am-

[2] Since in the model we focus only on the evaluation problem of potential lenders, we will use the notions of 'lenders', 'agents', 'investors', 'individuals' and 'traders' interchangeably, without special or technical meanings, unless otherwise stated.

[3] By fire sale we mean (as it is in [1] and [2]) that the liquidation price of the asset is lower than its debt capacity.

biguity problem is extremely severe, then the freeze of the market cannot be avoided.

Compared to the previous literature on credit rationing, our framework is less demanding since the main result is driven by ambiguity only, and we do not make any use of frictions such as asymmetric information and/or transaction costs. Our analysis is mainly inspired by a previous version [1] of a recent work [2] by Acharya, Gale and Yorulmazer (2010), in which the debt capacity of a finitely lived asset is analyzed in a framework characterized by *i)* short-term debt, *ii)* risk of fire sale in case of borrower's default, and *iii)* a "pessimistic" information structure.[4] Nevertheless, our model is only stimulated by [1], since it significantly differs in terms of focus and provided results. In particular, [1] focuses on the joint effects of rollover and liquidation risks on the evaluation of the borrowing capacity of the collateral asset. In our model, we ignore (without excluding) liquidation risk, and we directly analyze the impact of ambiguity on the fundamental value of the asset, that in our setting corresponds to its borrowing capacity. In [1] instead, these two quantities do not always coincide, leading to the natural problem of justifying why the price of the asset should be exogenously fixed at the borrowing capacity and not at the fundamental value as it is usually the case. Technically both papers rely on a two-states Markov chain, characterized by a particular relationship between the state-probabilities. However, while in [1] such a relation is exogenously assumed, in our model it follows endogenously from the particular preference specification. Indeed, in [1], the specific path of the borrowing capacity that generates the freeze originates if the probability of receiving good news when the economy is in a low state is sufficiently low with respect to the one of receiving bad news during a high state, meaning that these two probabilities satisfy a specific inequality that also involves the liquidation cost parameter. In our framework, initial assumptions are less demanding since we do not treat the two states of the economy (that is, with or without ambiguity) asymmetrically, and we simply assume that, once they originate, they (equally) tend more to be persistent

[4] In [2], instead, the result is no longer driven by the pessimistic information structure, but by the lower frequency of news' arrivals with respect to the one of rollovers.

over time than to change. Therefore, apart from the relevance shown during the financial turmoil, ambiguous preferences have the advantage of allowing for less demanding initial conditions, while standard literature needs to assume the existence of frictions (for example the liquidation cost in [1]) and/or of particular relationships among the state probabilities in order to generate the collapse of trading.

Caballero and Krishnamurthy [6] consider a model of optimal intervention in a flight to quality episode under ambiguity. Specifically, they show that, when an uncertainty-shock limits the aggregate liquidity, ambiguity averse agents fear that there won't be enough liquidity available in case they suddenly need it. Under these circumstances, agents' willingness to make risky investments is reduced, and capital is moved away towards the safest possible vehicles. As [6] does, our paper also tries to explain financial crises through ambiguity and ambiguity aversion. In addition, we assume that uncertainty is exacerbated by some exogenous event, so that it is the *possibility* (that is, a positive probability) that the economy *will experience* an uncertain situation that determines the crisis. However, the focus of the analysis is different: in [6] ambiguity impacts in a market perspective, so that agents put in act protective actions that inevitably lead to flights to quality. In particular, each investor considers his own worst possible scenario as the effective one, but the aggregate resulting scenario is actually impossible, so that the economy is left over-exposed to risk. Our model is supplementary to [6]'s analysis, since ambiguity acts directly at the individual level, affecting the (individual) evaluation of random payoffs.

The paper proceeds as follows: Section 1 briefly motivates our analysis of ambiguity in markets for collateralized debt in light of the last financial crisis, Section 2 characterizes the decision theoretic framework and the economic set up. Section 3 explicitly solves the model. Section 4 concludes. All proofs are collected in the Appendix.

2 Ambiguity, collateralized debt markets and the crisis

The focus on the market for rollover debt and on ambiguity seems to be a good starting point to better understand the factors that determined the recent collapse experienced in vast majority of financial markets. Indeed, financial researchers (among the others, see for example Diamond and Rajan [10]) generally agree that the crisis had its origin in the misallocation of resources to real estate, financed through the issuance of complicated financial products that were largely purchased by market based institutions, whose funding needs were satisfied through short term borrowing in capital markets. The short term nature of much of these liabilities was mainly due to the high level of complexity and uncertainty induced by the development of structured finance, and, in particular, by the extensive use of the securitization process, intended as the practice of reparcelling and selling mortgage loans, in order to disperse risks. With the boom of the housing market, the securitization process expanded dramatically, and its repetition originated complicated exotic securities, most of which were erroneously considered as virtually risk-free, and certified as such by rating agencies. When house prices started to decline, the evaluation complexity and the exaggerated riskiness of these instruments became clear, generating an immediate increase of the risk premia demanded for financing institutions on a long term basis, while, for short-term claims, the excessive risk was compensated by the option to forgo the investment earlier. Therefore, financial and non-financial institutions expanded the issuance of short-term contracts, without considering the possibility of becoming illiquid and unable to rollover the debt. On the contrary, with the continuous drop in house prices, mortgage defaults turned out to be highly relevant and mortgage backed securities fell in value, which made it impossible to price them using standard techniques. They were hard to borrow against, even short term, and rollover became virtually impossible, leading to a proper freeze of the market and a collapse of liquidity. A concrete example of this process was the worldwide collapse in the market for Asset Backed Commercial Paper (ABCP) in the Summer

2007, whose decline was estimated to have reached $3000bn between early August and early November in the U.S. market alone (see [8]). Another relevant case was represented by Bear Stearns' bankruptcy, whose failure, according to the Security and Exchange Commission's Chairman Christopher Cox ([9]), was mainly imputable to the bank's inability to issue short term debt backed by assets with a relatively high credit rating.

Ambiguity and ambiguity aversion seem to be obvious side-effects which originated from the complexity and the sophistication of the valuation techniques of the available financial instruments. More recent features of the financial turmoil, such as the wide spread of Greeks bond over German ones, also seem to be more imputable to ambiguity rather than to traditional market frictions (such as information asymmetry) advocated in the standard literature. Indeed, the introduction of ambiguity in financial market theory is relatively recent, since traditional finance typically assumes that agents are (subjective) expected utility ((S)EU) maximizers, so that they evaluate alternative investment opportunities by simply confronting the respective expected utility values, computed through a unique probability distribution, which might be objectively given or subjectively derived. Experimental works in finance and in decisions contradict (S)EU predictions, in particular, one of the most common violations is represented by Ellsberg's paradox [13] that provides experimental evidence of agents' inability to derive a unique probability distribution over the reference state space when the economic environment is perceived as uncertain. In the context of a simple investment problem, if the agent has too little information to derive a unique prior, not only will the payoff of any asset be uncertain, but also its expected value, which will be compatible with a set of equally plausible probability distributions. After [13], uncertain environments have became known as *ambiguous* and the general "dislike" for them as *ambiguity aversion.* Motivated by Ellsberg's findings, researchers in decisions have started to elaborate a new class of preferences to accommodate ambiguity. More recently, these models have been introduced in standard macroeconomic and finance contests, with the aim of achieving a better representation of reality. Indeed, a wide body of literature has been dedicated to the absence of trade under ambiguity in general equilibrium models a' la Lucas.[5] Recently, ambiguity has also been applied in the analysis of market microstructure and regulation, to investigate the behavior of traders and intermediaries from a microeconomic perspective (Easley and O'Hara [11], [12]).

3 Setting

In this Section we introduce the setting that will constitute the framework of our analysis. First, we describe the financial market under consideration. Next, we discuss how ambiguity might be relevant in such a setup, and finally we move to a complete characterization of agents' preferences.

3.1 The financial market

A structured investment vehicle (SIV) is set up at time 0 by some unmodeled financial institution. The SIV holds a collateral asset with maturity 1, and terminal value that can be either 0, in case of default (say event L), or $V > 0$, in case of success (say event H). For simplicity, the current yield of the asset and the riskfree rate are set to zero, and the market is risk neutral. There are no frictions, such as information asymmetries, transaction, and/or liquidation costs. The SIV has to rise asset-backed finance, by issuing short term debt with fixed maturity $0 < \tau < 1$. The maturity structure of the debt is taken as given and it cannot be modified. In particular, the debt is rolled over N times, where N is such that: $(N + 1)\tau = 1$; $t_n = n\tau$, $n = 0, 1, ..., N + 1$, is the date at which the n-th rollover occurs.

As it is usual in banking theory, we define the *debt* (or *borrowing*) *capacity* of a risky asset as the maximum amount that can be borrowed, using only the asset as collateral. The market for short term debt is said to *freeze* if the borrowing capacity of the risky asset drops to zero in any state at any date, since it is sufficient for the economy to switch to that particular state on that date to render the SIV unable to borrow any further. In our model, the fundamental value of the asset and its borrowing capacity always

[5] Two pioneering examples are Epstein and Wang (1994) and Mukerji and Tallon (2001).

coincide. However, we explicitly refer to the borrowing capacity since the implicit assumption beyond the idea of "market freeze" described above is that, if the SIV has to liquidate the asset at any date, this will be purchased by another institution that has to rise asset-backed finance as well and that cannot simply buy and hold the asset until the conditions of the market return possibly favorable to liquidate.

3.2 Ambiguity

In the following the market starts from some initial familiar ambiguity-free situation at time 0, say $s_0 = F$. As time passes, some unmodeled shock (for example the failure of some unrelated market agent) that has the externality effect to rise the perceived level of uncertainty can occur with probability $1 - q$. As a result, ambiguity arises and the market moves to an unfamiliar ambiguous state U. In such a state, the fair value of the collateral becomes ambiguous, in particular the probability of not-default is possibly lowered to some level which cannot be precisely estimated because of uncertainty. Further, investors are not even able to derive a unique prior over the possible future evolution of the economy (namely if it will stay into the unfamiliar state or switchback to the familiar one), and they know that a return to normality does not prevent the insurgence of further uncertainty in the future. This lack of accurate information is reflected in agents' preferences that are assumed to display ambiguity aversion. In particular, to allow for ambiguity and ambiguity aversion, we use Schmeidler (1989)'s representation (the so called Choquet Expected Utility, or briefly CEU, model) for preferences.

3.3 Ambiguity and preferences: the CEU model

In his seminal paper [16], Schmeidler criticizes the standard paradigm of (subjective) expected utility according to which, if the set of possible states of the word can be partitioned into K equiprobable events, each of them is assigned probability $1/K$, no matter what the quality and/or the quantity of information that has induced that particular assignment was. In general, he notices that the probability attached to an uncertain event does not reflect the heuristic amount of information that led to that specific assessment. Motivated by this consideration, he suggests the use of non-additive probabilities (meaning that they not add up to 1), or capacities, in order to allow for transmission or recording of information that additive probabilities cannot represent. Formally, given the reference state space $\Omega = \{s_1, s_2, ..., s_K\}$, with corresponding algebra $\mathcal{F}$, a capacity υ is a real valued function $\upsilon : \mathcal{F} \rightarrow [0, 1]$ such that: 1) $\upsilon(\varnothing) = 0$, $\upsilon(\Omega) = 1$ and $\upsilon(E) \geq \upsilon(F)$ $\forall E, F \in \mathcal{F}$ $s.t.$ $F \subseteq E$; 2) $\sum_{s \in \Omega} \upsilon(s) \leq 1$. Hence, the usual additive probabilities are simply a particular capacity assignment for which 2) is strengthen to $\sum_{s \in \Omega} \upsilon(s) = 1$. In general, each state $s_k \in \Omega$ is assigned a capacity $\upsilon(s_k)$ and, to measure the amount of available information (and consequently the ambiguity level), Schmeidler proposes the index $A(\upsilon) = 1 - \sum_{s_k \in S} \upsilon(s_k)$. A lower index $A(\upsilon)$ indicates more precise information or, equivalently, lower ambiguity. In particular, $A(\upsilon) = 0$ corresponds to the (S)EU case, without ambiguity and with additive probabilities.

The main novelty in Schmeidler's model is that it accounts also for phenomena that do not occur when risk only is considered, such as the violation of (S)EU theory described by Ellsberg. Weakening the standard axioms of (S)EU, Schmeidler develops a preferences' representation based on non-additive probabilities, that allows for the fact that agents may not be fully confident on their probability assignment over uncertain events. More specifically, Schmeidler's axiomatization implies (and is equivalent to) the existence of a capacity over the reference state space, and a functional representation for preferences based on the Choquet integral. Therefore, CEU preferences are characterized by a standard utility index u, which, as usual, reflects attitudes towards risk, and by a capacity υ over the reference state space. When the capacity υ is convex, that is, when for any two events E and F in $\mathcal{F}$, $\upsilon(E) + \upsilon(F) \leq \upsilon(E \textit{ and } F \textit{ both occur}) + \upsilon(E \textit{ or } F \textit{ or both occur})$, the Choquet integral reduces to the *minimum* of a standard integral over a particular set $C(\upsilon)$, also referred to as the *core* of υ. For a given capacity υ, its core is uniquely determined since it is the set of additive probability distributions that eventwise dominate υ. Furthermore, convexity implies (and is equivalent to) ambiguity aversion in the agent's preferences.[6] $C(\upsilon)$ is interpreted as the set of effective priors considered by the agent, and ambiguity is reflected in its multivalued nature. The decision maker expresses ambiguity aversion by assigning higher probabilities to unfavorable states, as reflected in the minimization over $C(\upsilon)$.[7]

After Schmeidler's seminal paper, a growing body of the decision theoretic literature has been dedicated to ambiguity and ambiguity aversion, and many criticisms to the CEU model have been suggested. In Schmeidler's representation, ambiguity aversion coincides with convexity of the capacity; Epstein [14] and Ghirardato and Marinacci [15] have discussed this notion, shoving that convexity is neither necessary nor sufficient to lead to a Ellsberg's type ordering among bets. In this paper, we consider only pairs of mutually exclusive events, so that convexity is trivially satisfied because of the specific feature of the problem under consideration. Further, as it will be clear, the agents who populate the model behave consistently with an intuitive idea of aversion to ambiguity and the choice-criterion implied by the CEU model. Therefore, we believe that the following analysis cannot be criticized in light of the results in [14] and [15].

4 The model

Let us denote by s_n, $s_n = F, U$, the generic state occurred at time t_n. At the penultimate date, if the economy is in state $s_N = U$, ambiguity impacts

[6] Ambiguity aversion implies that the agent prefers to bet on lottery with known rather than unknown probabilities.

[7] For concreteness, let us assume that a CEU agent characterized by utility index u is evaluating two alternative bets, say f or g, with payoff f_s and g_s, respectively, depending on the occurrence of a particular state s. There are only two possible realizations of s, say B or B^c, and beliefs are represented by a capacity $\upsilon = \{\upsilon(B), \upsilon(B^c)\}$. Hence, f is preferred over g if and only if $\min\limits_{\mu \in C(\upsilon)} [\mu \times u(f_B) + (1-\mu) \times u(f_{B^c})] \geq \min\limits_{\mu \in C(\upsilon)} [\mu \times u(g_B) + (1-\mu) \times u(g_{B^c})]$. Notice that, by definition of $C(\upsilon)$:

$$C(\upsilon) = \{\mu \in [0,1]\,,\ \mu \geq \upsilon(B),\ 1-\mu \geq \upsilon(B^c)\}\,,$$

the agent acts as if he were able to establish for each event s only the minimal probability of occurrence $\upsilon(s)$.

on the evaluation of the collateral asset, and the market is not able to derive a unique probability distribution for the two possible payoff's realizations "success" (H) and "default" (L). Agents assign to the occurrences H and L capacities $w(H)$ and $w(L)$, respectively, and evaluate the random (and ambiguous) payoff according to the CEU model. Viceversa, in the familiar state $s_N = F$, the market sets the probability of success to p, and to $1-p$ the one of default. Therefore, any time t_{N+1} contingent payoff $x = \{x(H), x(L)\}$ is evaluated at time t_N as $E_{t_N}[x|\, s_N = F] = px(H) + (1-p)\,x(L)$ or $E_{t_N}[x|\, s_N = U] = \min\limits_{(\mu,1-\mu)\in C(w)} \{\mu x(H) + (1-\mu)\,x(L)\}$.[8]

Agents' perception of the unfamiliar state with respect to the familiar one can be modeled by characterizing the relationship between the probability distribution $(p, 1-p)$ and the capacity assignment $w = (w(H), w(L))$. Specifically, if the economy is perceived as generally healthy, the familiar state is considered at least potentially better than some unknown situation, so that $p > w(H)$. Notice that this assumption does not imply any form of pessimism: indeed, by definition of core, $w(H)$ is the minimal considered probability of the occurrence of state H, which is reasonably lower than p, if the ambiguous situation is not certainly better than the current one. However, in practise, $C(w)$ is the set of effective priors considered by ambiguity averse agents, and, by definition, it includes also distributions for which the probability of the occurrence H is higher in value than p.

Ambiguity affects also agents' beliefs about the possible evolution of the economy (namely, the next period state). In particular, we denote by $v(U)$ and $v(F)$ the two next-period contingent state capacities assigned to the events $(s_{n+1} = U|\, s_n = U)$ and $(s_{n+1} = F|\, s_n = U)$, respectively. Hence, if the unfamiliar state ever occurs, agents believe that the minimal probability of a switch to (staying in) the familiar (unfamiliar) state in the next period is $v(F)$ $(v(U))$. Denoting by $C'(v)$ a particular subset of the core of v, whose construction will be explained below, any time $t_{n+1} \neq t_{N+1}$ contingent payoff

[8] $C(w)$ is the core of w.

$y = \{y(F), y(U)\}$ is evaluated at time t_n as:

$$E_{t_n}[y|\, s_n = F] = qy(F) + (1-q)\, y(U) \tag{1}$$

$$E_{t_n}[y|\, s_n = U] = \min_{(\mu,1-\mu)\in C'(v)} \{\mu y(F) + (1-\mu)\, y(U)\} \tag{2}$$

Next, we assume that states of the market are believed more to be persistent over time than to change, so that $q > 1-q$ and $q > v(F)$.[9]

The persistency assumption is less demanding than how it could appear at first sight. First of all, notice that the reinforcing loop between market liquidity and funding liquidity described in Brunnermeier and Pedersen [5] is likely to generate a mechanism of state persistence (and hence to affect agents' belief in this sense). In particular, high (low) liquidity at time t_n eases (imposes) funding restrictions at time t_{n+1} by improving (reducing) collateral values and lowering (increasing) margins. Finally, at time t_{n+1}, the availability (lack) of funding enhances (hinders) trading and market liquidity, and so on and so forth for the subsequent periods. Second, in our model the economy starts off from the familiar state at time zero, hence, if we are introducing any bias, this will be in favour of the familiar state and not of the unfamiliar one as it would be in a pessimistic fashion.

To see why the inequality $q > v(F)$ must hold, suppose instead that $q \le v(F)$. Hence, the set of effective priors considered by the agents under the unfamiliar state is made of all probability distributions $(\mu(F), \mu(U))$, such that $\mu(F) \ge v(F) \ge q$. Moreover, $\mu(U) = 1 - \mu(F) \le 1 - q$. Since $q \ge 1-q$, it thus follows that $\mu(F) \ge (q \ge)\, \mu(U)$, which contradicts the state-persistency assumption. Lemma 1 reports a result that will be useful in the subsequent analysis.

Lemma 1 *Consider a time t_N contingent payoff $x = \{x(U), x(F)\}$, and denote its contingent evaluation at time t_n, $E_{t_n}[x|\, s_n = i]$, $i = F, U$, by $E_{t_n}[x|\, i]$. If there exists n^*, $1 \le n^* \le N$, s.t. $E_{t_{n^*}}[x|\, U] \le E_{t_{n^*}}[x|\, F]$, then $\forall n < n^*$:*

[9] By definition of $C(v)$, this assumption also implies that $v(F) \le 0.5$ and $v(U) \ge 0.5$. However, these inequalities do not absolutely affect the results below.

1) $E_{t_n}[x|U] < E_{t_n}[x|F]$*;*
2) $E_{t_n}[x|U] = v(F) E_{t_{n+1}}[x|F] + (1 - v(F)) E_{t_{n+1}}[x|U]$.

Hence, if at any rollover date t_{n^*} the occurrence of the unfamiliar state (weakly) hurts valuation, then the unfamiliar state (strictly) reduces valuation at all times that precede t_{n^*}. Further, assuming the existence of n^* as specified above, part 2 provides us with a computational algorithm for the conditional CEU-value of the payoff. In particular, if the unfamiliar state realizes at any date before t_{n^*}, the effective probability considered at that date is the one that attaches probability $v(F)$ to the familiar state, and probability $1 - v(F)$ to the unfamiliar one.[10]

Denoting by B_n^i the debt capacity of the asset evaluated at time t_n, if $s_n = i$, by definition, $B_n^i = \max_D E_{t_n}[D|s_n = i]$, where D is the face value of the debt issued at t_n. What is crucial in our framework is the way expectations are derived. Specifically, if $s_n = F$, standard rules apply, and agents compute expected values by averaging contingent payoffs with probability weights q and $1 - q$. If $s_n = U$, expectations are evaluated according to Schmeidler's model where the set of effective prior considered by each agent is:[11]

$$C'(v) = C(v) \diagup \{\mu \in \Delta(\{U, F\}),\ \mu(F) \geq \mu(U)\}$$

Assuming $p > w(H)$, we proceed backward in time and, at the penultimate date t_N , the face value of the debt issued at t_N, D, cannot be larger than V, otherwise the SIV will default for sure, since potential risk neutral lenders who try to break even in expectation would never subscribe such a contract. In practice, the maximum amount that can be borrowed by the SIV at time t_N, that is, its borrowing capacity at t_N, is $B_N^F = pV$, or $B_N^U = w(H)V$, and $B_N^F > B_N^U$. Going back one period at date t_{N-1}, $s_{N-1} = F$ or $s_{N-1} = U$. In both cases, to avoid default, the face value of the debt issued at t_{N-1}, D, cannot be larger than B_N^F. Hence, using (1) and

[10] Notice that, since the events $s_n = F$ and $s_n = U$ are mutually exclusive, the evaluation of a payoff with highest realization in state F is not affected by the specific $v(U)$.
[11] This definition follows from the state persistency assumption.

(2), the evaluation of D at time t_{N-1} is:

$$
\begin{aligned}
E_{t_{N-1}}\left[D|\, s_{N-1}=F\right] &= \begin{cases} (1-q)\,B_N^U+qD & B_N^U<D\leq B_N^F \\ D & B_N^U\geq D \end{cases} \\
E_{t_{N-1}}\left[D|\, s_{N-1}=U\right] &= \begin{cases} (1-v(F))\,B_N^U+v(F)D & B_N^U<D\leq B_N^F \\ D & B_N^U\geq D \end{cases}
\end{aligned}
$$

By definition of B_{N-1}^i, $B_{N-1}^F=\max\left\{B_N^U,(1-q)\,B_N^U+qB_N^F\right\}$ and $B_{N-1}^U=\max\left\{B_N^U,(1-v(F))\,B_N^U+v(F)B_N^F\right\}$. Notice that $B_{N-1}^F>B_{N-1}^U$, so that usual iteration methods lead to the following characterization of the path of the borrowing capacity:

Theorem 2 *$B_N^F=pV$ and $B_N^U=w(H)V$. For any $n<N$:*

$$
\begin{aligned}
B_{n-1}^F &= \max\left\{B_n^U,(1-q)\,B_n^U+qB_n^F\right\} \qquad (3)\\
B_{n-1}^U &= \max\left\{B_n^U,(1-v(F))\,B_n^U+v(F)B_n^F\right\} \qquad (4)
\end{aligned}
$$

Additional properties of the conditional borrowing capacities are provided by the following Proposition.

Proposition 3 *For any $n\leq N$ the conditional borrowing capacities satisfy: 1) $B_n^U<B_n^F$; 2) $B_{n-1}^F<B_n^F$; 3) $B_{n-1}^U>B_n^U$.*

The first inequality assures that at any rollover date t_n the debt capacity is higher if the familiar state realizes. The second inequality has intuitive meaning: as the time to maturity of the asset approaches, the borrowing capacity evaluated under the familiar state increases, since the probability of not incurring into the unfamiliar one (q^{N-n}) also increases, and this has a positive effects on the evaluation by property 1 of the Proposition. A similar argument applies to the last inequality: as the time to maturity of the asset approaches, the minimal probability of not incurring into the familiar state ($v^{N-n}(U)$) increases, reducing the evaluation of the borrowing capacity of the asset. Using property 2 and the definitions provided in Theorem 4 , we get $B_{n-1}^F=(1-q)\,B_n^U+qB_n^F$ and $B_{n-1}^U=(1-v(F))\,B_n^U+$

$v(F)B_n^F$. Further, since $B_n^U < B_n^F$, the hypothesis of Lemma 1 is satisfied, hence the financial problem under uncertainty can be transformed into an ambiguity-free setting characterized by probability distributions $(q, 1-q)$ and $(v(F), 1-v(F))$.

4.1 The "freeze-result"

The effect of an increase in the number of rollovers can be easily analyzed by reducing the length of each contract τ or, equivalently, of each time interval. Clearly, the occurrence of the unfamiliar state at the penultimate date (which is an exogenous event) should not be influenced by the number of rollovers. Specifically, to not vary the probability of realization of the unfamiliar state at the penultimate date, it is sufficient to choose $\alpha, \beta > 0$, such that $1 - v(\tau) = e^{-\alpha\tau}$, $q(\tau) = e^{-\beta\tau}$, and $\lim\limits_{\tau \to 0} v(\tau) = 0$ and $\lim\limits_{\tau \to 0} q(\tau) = 1$, where $v(\tau)$ is the capacity $v(F)$, expressed as a function of τ. Further, to satisfy the state-persistency assumption, we also require $q(\tau) > v(\tau)$. $B_n^i(\tau)$ denotes the borrowing capacity at date t_n, if $s_n = i$, $i = F, U$. Theorem 4 assures that $B_n^F(\tau)$ is bounded below away from 0, even if the number of rollovers becomes infinite. Hence, it is always positive, no matter how small τ can be.

Theorem 4 $B_n^F(\tau) \geq q(\tau)^{N-n}\hat{V} \;\; \forall \; n, \tau$, *where* $\hat{V} = (1-q)\, w(H)V + qpV$, $\tau = 1/N$. *As* $\tau \to 0$ *and* $n\tau \to t$, $B_n^F(\tau) \geq e^{-\beta(1-t)}\hat{V}$

As discussed in Proposition 3, the path of B_n^U is decreasing over time. Therefore, to prove that $\lim\limits_{\tau \to 0} B_n^U(\tau) = 0, \forall n$ (or, equivalently, that the market freezes), it is sufficient to discuss the limit for the case $n = 1$, that is:

Theorem 5 $\lim\limits_{\tau \to 0} B_1^U(\tau) = 0$

Apart from the limit result of Theorem 5, it is reasonable to identify the freeze with the condition $\exists t_n : B_n^U \leq \bar{B}$ for some given threshold level $\bar{B}$, meaning that the backed debt contracts will not be subscribed by any investor if their face value is lower than $\bar{B}$. Since the path of B_n^U is decreasing over time, for any given $\bar{B}$, it is sufficient to derive conditions under which

$B_1^U \leq \bar{B}$. From (4) it is immediate to notice that B_1^U decreases as the credit rating of the asset deteriorates (that is, as p declines) and as the ambiguity problem becomes more severe. Specifically, the more likely is the economy to fall into the unfamiliar state (that is, the lower q is), and the higher is the ambiguity concerning either the future evolution of the economy (reflected by lower $v(F)$s), or the effects of the exogenous shock on the asset (reflected by lower $w(H)$s), the lower is B_1^U, so that the freeze is easier to occur, even abstracting from the limit case.

Notice that if we have considered an equivalent (S)EU framework, that is, without ambiguity, the freeze result would have not followed from the assumptions specified above. In particular, denoting by $(q', 1-q')$ the probability distribution over the two events $\{ s_{n+1} = i | s_n = U\}$, $i = U, F$, where the U state should be considered as a recession state, the state persistency assumption would have implied $1 - q' > q'$, which is not sufficient to generate a time decreasing path in the conditional borrowing capacity B_n^U and, correspondingly, the freeze result.

4.2 The effects of ambiguity

Next, we characterize the effects of increases in the perceived ambiguity. Specifically, we consider two financial markets characterized by capacities $\tilde{v}$ and v, respectively. In particular, to allow for ambiguity comparisons, we assume that the assignments $\tilde{v}$ and v are such that $\tilde{v}(U) = \tilde{v}(F) = \tilde{v} > v(F) = v(U) = v$, so that $\tilde{v} > v \Rightarrow A(\tilde{v}) = 1 - 2\tilde{v} < 1 - 2v = A(v)$. Hence, the market with capacity v suffers from a more severe ambiguity problem.[12] Let us denote by B_n^{Fi} and B_n^{Ui}, $i = \tilde{v}, v$, the state dependent borrowing capacities in the two financial markets at time t_n.

Proposition 6 *Suppose $\tilde{v} > v$, then for any $n < N$: 1) $\left|B_n^{U\tilde{v}} - B_n^{F\tilde{v}}\right| < \left|B_n^{Uv} - B_n^{Fv}\right|$ and $\left|B_{n-1}^{F\tilde{v}} - B_n^{F\tilde{v}}\right| < \left|B_{n-1}^{Fv} - B_n^{Fv}\right|$; 2) $B_n^{F\tilde{v}} - B_n^{Fv} > 0$ and $B_n^{U\tilde{v}} - B_n^{Uv} > 0$.*

[12] This ambiguity-comparison is in accordance to Dow and Werlang (1992).

Hence, the marginal benefit from not incurring into the unfamiliar state at any date is higher, the higher is the ambiguity level. Similarly, as the expiration date approaches, the marginal benefit provided by an increase in the probability of not incurring into the unfamiliar state becomes lower.[13] Finally, the last result shows that at any date higher levels of ambiguity reduce both conditional borrowing capacities.

4.3 Trying to maintain the market active

4.3.1 Tradable insurance credits

Next, we discuss how a policy response based on public insurance can successfully affect agents' evaluation. However, given the simplicity of our framework, we will not consider important aspects such as the pricing of this particular insurance contract, enforcement and moral hazard issues that could possibly arise. Similarly, we implicitly assume that the Central Bank has resources enough to deliver on this insurance if it has to. What we have in mind is the kind of policy action suggested by Caballero and Kurlat in [7].[14] In their proposal, the Central Bank issues tradable insurance credits (TICs) that would entitle their holder to attach a central bank guarantee to the assets used as collateral. In our model convertibility works as follows: after the occurrence of two (not necessarily consecutive) unfamiliar states, the TICs can be attached to a specific asset. However, whenever two consecutive familiar states realize, the economy is considered fully recovered and the TICs are no longer related with the asset. Nevertheless, the credits stay in the bank's holdings so that it will be possible to re-attach the TICs to the assets in case of future insurgence of two unfamiliar states. During normal times (that is, before the insurgence of the first unfamiliar state, or after the occurrence of two familiar ones), the TICs are not convertible and the Central Bank can buy or sell them at a price established by the market.[15] The Central Bank is also responsible for establishing the TIC/assets ra-

[13] Such a probability is q^{N-n} in both markets.
[14] We refer the Reader to [7] for an interesting analysis of this proposal.
[15] TICs' pricing is not considered here.

tio which should be compulsorily maintained for leveraged institution. The main characterizing features of TICs are described in [7], in particular: "*the TIC-policy is not a conventional insurance policy in the sense that an insurance policy exchanges a fee during normal times for a cash injection during crises. Rather the TIC-policy is an "insurance-squared policy": for a fee, it ensures that financial institutions will have access to insurance for their assets during systemic crises*", further "*TICs are equivalent to CDS during systemic crises but not during normal times. That is, TICs are contingent-CDS. They become activated only when a systemic crisis arises.*" (Caballero and Kurlat, [7]). In our framework, TICs work because they stabilize the borrowing capacity of the collateral, by rendering it insensitive to both ambiguity and rollover risk, as it is immediately revealed by a simple analysis of the event tree concerning the states-transition of the economy. Specifically, let us assume that the TIC-insurance guarantees that (after conversion) in case of default of the asset the Central Bank will pay out αV, where α is chosen by the Central Bank itself. Hence, for an asset which is collateral to a debt contract that can be rolled-over N times, the 2^{N-4} sub-trees that originates at the $N-4th$ node, are all identical apart from the initial node, thanks to the TIC-insurance. For the $N-4th$ node there are only two occurrences: U or F, hence, it suffices to choose α such that $B_{N-4}^{U} = B_{N-4}^{F}$, to render the borrowing capacity of the asset insensitive to both ambiguity and rollover risk.

As expected, α is higher the higher is the effect $(p - w(H))$ of ambiguity on the credit rating of the asset. Similarly, α also increases as the level of perceived ambiguity $(q - v(F))$ about the future evolution of the economy increases. Viceversa, α reduces the more optimistic agents are, that is, the higher $v(U)$ is (See tables 1, 2, 3, 4 and 5 in the Appendix). Finally, the TIC-policy has the further advantage that it commits the Central Bank to offer the insurance coverage against default only in 9/16 out of the 2^{N} possible scenarios that could realize at time t_N.

4.3.2 Ambiguous policy

Next, we consider a slight variation of the previous framework. In particular, we assume that the economy is currently (that is, at time 0, under the familiar state) experiencing a deep long-lasting crisis, so that the final familiar state is believed to be worse than an unfamiliar one induced by some strong policy intervention. When agents evaluate some unmodeled action that could potentially produce a switch to an unfamiliar (better) state, $(1-q)$ can be interpreted as the probability of success of the policy in the short period. However, the action's contents might be not completely transparent, in the sense that it might be unclear whether, in case of temporary success, the positive effects will be only transitory or permanent, so that the probabilities of switching back to the familiar state (i.e. the crisis) or of staying into the unfamiliar (better) one cannot be uniquely derived. However, if, despite the mediation, the economy switches back to the familiar state, there will be again a possibility for a novel action with same probability of success. Hence, if the policy has failed to maintain stability, the probability distribution over the possible realizations of the asset's payoff will be $(p, 1-p)$. Instead, if the intervention has been successful, the economy will be in some unfamiliar state that can only be better than the current one. As before, this perception can be characterized through the relationship between the probability distribution $(p, 1-p)$ and the capacity $w = (w(H), w(L))$. Specifically, in this case w is such that $w(H) > p$. Notice that this requirement precisely reflects the feeling that "nothing can be worse than the current situation". In fact, according to the CEU model, agents consider all the distributions for which the probability of success is greater than $w(H)$, and, consequently, than p.

To show the positive effects of a possible policy intervention, we repeat the analysis of the previous setting. In particular, the maximum amount that can be borrowed by the SIV at time t_N is $B_N^F = pV$ or $B_N^U = w(H)V$. Notice that now $B_N^F < B_N^U$, hence, at t_{N-1}, the face value of the debt cannot be larger than B_N^U. Using (1) and (2), it thus follows that:

$$
\begin{aligned}
E_{t_{N-1}}\left[D|\, s_{N-1}=F\right] &= \begin{cases} (1-q)\,B_N^U+qD & B_N^F<D\leq B_N^U \\ D & B_N^F\leq D \end{cases} \\
E_{t_{N-1}}\left[D|\, s_{N-1}=U\right] &= \begin{cases} (1-v(U))\,B_N^F+v(U)D & B_N^F<D\leq B_N^U \\ D & B_N^F\leq D \end{cases}
\end{aligned}
$$

Using the definition of $B_{N-1}^{(\cdot)}$, we get $B_{N-1}^F=\max\left\{B_N^F,(1-q)\,B_N^U+qB_N^F\right\}$ and $B_{N-1}^U=\max\left\{B_N^F,(1-v(U))\,B_N^F+v(U)B_N^U\right\}$. In this framework, we are no longer able to characterize the relation between B_{N-1}^F and B_{N-1}^U, since $v(U)$ cannot be bounded any further (unless under misleading and behaviorally unfounded assumptions), apart from the usual restriction $0\leq v(U)\leq 1-v(F)$. Therefore, in general, it is not possible to proceed recursively as in the previous case. More specifically, we can only characterize the paths of the conditional borrowing capacities as we did in Theorem 4, whose analog is the following:

Theorem 7 *$B_N^F=pV$ and $B_N^U=w(H)V$.*
$B_{N-1}^F=\max\left\{B_N^F,(1-q)\,B_N^U+qB_N^F\right\}$ and
$B_{N-1}^U=\max\left\{B_N^F,(1-v(U))\,B_N^F+v(U)B_N^U\right\}$.
For $n<N-1$, setting $B_n^{\min}=\min\left\{B_n^U,B_n^F\right\}$:

$$
\begin{aligned}
B_{n-1}^F &= \max\left\{B_n^{\min},(1-q)\,B_n^U+qB_n^F\right\} \\
B_{n-1}^U &= \max\left\{B_n^{\min},\min_{(\mu,1-\mu)\in C(v)}\mu B_n^U+(1-\mu)\,B_n^F\right\}
\end{aligned}
$$

In conclusion, there exist fundamentals for which B_n^F and B_n^U are positive at any date, so that credit is not in principle rationed. Despite this result, we now show that if agents are not confident about the effectiveness of the policy in the long term (meaning if it will succeed in avoiding a return to the familiar-crisis state), and such ambiguity problem is extremely severe, then the freeze of the market cannot be avoided.

Holding $v(F)$ fixed, higher degrees of ambiguity correspond to lower values of $v(U)$ (the minimal probability of the policy's effectiveness for at least two consecutive periods). If $v(U) < 1 - q$, it can be shown that $B^F_{N-1} > B^U_{N-1}$, and the entire analysis underlying Theorem 4 can be repeated, since the recursive arguments used above do not rely on the initial (for the procedure, final for the financial problem of reference) point. Hence, under these circumstances, even if agents believe that nothing can be worse than the familiar state, the market ends up in a freeze, as it is formalized in the following Proposition.

Proposition 8 *Suppose $wV > pV$ and $v(U) < (1-q)$ then: 1) $B^F_N = pV$ and $B^U_N = w(H)V$, $B^F_{N-1} = \max\left\{B^F_N, (1-q)\, B^U_N + qB^F_N\right\}$ and $B^U_{N-1} = \max\left\{B^F_N, (1-v(U))\, B^F_N + v(U)B^U_N\right\}$; $B^F_{n-1} = \max\left\{B^U_n, (1-q)\, B^U_n + qB^F_n\right\}$ and $B^U_{n-1} = \max\left\{B^F_n, (1-v(U))\, B^F_n + v(U)B^U_N\right\}$, $\forall n < N-1$; 2)$B^U_n < B^F_n$, $B^F_{n-1} < B^F_n$ and $B^U_{n-1} \geq B^U_n$, $\forall n < N-1$; 3) Theorem* 4 *and Theorem* 5 *hold.*

The previous discussion seems to provide a rationale for some of the public interventions operated during the crisis, and also a possible motivations for their effectiveness or failure. For example, the accommodative monetary policy that characterized the period between Summer 2007 and the end of 2008, and, in particular, the liquidity support announcements during the period June 2007 - September 2008, can be interpreted as strong and reliable messages from the policy maker in which goals and effective tools were clearly communicated, so that ambiguity had been maintained fairly low and, in the model, $v(U)$, the minimal probability of effectiveness of the action in restoring stability for at least two consecutive periods, increased.

Other facilities implying a direct intervention of the central authority as a market maker or liquidity provider in the troubled markets [16] did not always succeed. In accordance to the possible motivation provided in our framework, also ongoing discussions among academics and practitioners suggest that unsuccessful results are probably due to the fact that, since these

[16] The Term Auction Facility (TAF), the Single Tranche Open Market Operation Program, the Term Securities Lending Facility (TSLF), the Primary Dealer Credit Facility (PDCF), the Commercial Paper Funding Facility (CPFF) and the Term Asset-Backed Securities Loan Facility (TALF) are some examples.

facilities were newly introduced, and announced as short-term, with only the possibility for extensions in the future, they were perceived as generally ambiguous. Even more serious is probably the ambiguity problem generated by the Troubled Asset Relief Program (TARP), according to which the US Department of the Treasury was allowed to purchase or insure "troubled assets". First of all the criteria for participation were very unclear, both in terms of institutions to be supported and assets to be purchased. Second, uncertainty further increased, since the government had never properly disclosed the amount of the operation and its actual recipients. Finally, the use of the received money was not properly supervised or clarified. As a result, the ambiguity perceived by taxpayers simply increased, determining the unsatisfactory performance of the program.

4.4 Policy suggestions

In this Subsection we derive from the previous analysis some simple policy suggestions. In doing so, we have to keep in mind that our framework is obviously extremely rudimentary, and, most importantly, that it only deals with one side of the market. Hence we refrain from judging the effectiveness and the feasibility of such interventions in the real world. Similarly, we do not evaluate possible side effects that could occur, including moral hazard issues. Our goal here is simply to derive a link between the mathematical relationships that technically determine our freeze result and the policy implications that could potentially determine their failure.

4.4.1 Trying to prevent the crisis

In our framework, the freeze is mainly determined by three factors: *i)* systemic risk renders the market sensitive to spillover effects that might determine an increase in the perceived ambiguity with positive probability (i.e. $q \neq 1$ and $v(F) < q$); *ii)* individual adverse circumstances might negatively feedback on assets' evaluation ($w(H) < p$); *iii)* due to the complexity of structured finance, financial institutions are simultaneously lenders and borrowers.

Addressing systemic risk

The high level of interconnections among different market's players calls for a system-wide orientation of the regulatory framework to limit the risk of distress-contagion episodes and externalities costs that can lead to significant losses for the whole economy. The macroprudential approach is now widely supported among researchers and policy makers as a powerful tool to measure and, possibly, reduce system-wide risks. To enhance financial stability, macroprudential regulation and supervision prescribe to identify (and focus on) institutions that are systemically important, since their individual adverse circumstances affect the entire financial system. This requires the expansion of the scope of regulation also to the shadow banking system, and generally to institutions, instruments and markets that are outside the boundaries of the current framework. On the other hand, it is also necessary to globally reinforce all markets participants to render them less prone to be affected by externalities that originate from individual idiosyncratic distresses. One important issue to be urgently addressed concerns capital requirements, as an adequate level of capital is a key ingredient to strengthen individual resilience to market liquidity episodes. In particular, the crisis has shown that the intensive use of the securitization process has dramatically increased the level of systemic risk, therefore, resecuritizations warrant higher capital charges. In addition, the institution of central counterparties aimed at facilitating the clearing process might also contribute to insulate the overall market form the troubles of any single participant. In the context of our model, policy actions aimed at either preventing systematically important institutions to fail, or at reducing market externalities deriving from individual liquidity problems, correspond to a lower probability of occurrence of the unfamiliar state U (or, equivalently, to a higher q), with positive effects on the evaluation of the two contingent borrowing capacities that result higher at any rollover-date. In the limit case q approaches to 1, so that ambiguity disappears and credit is not rationed at all.

Addressing ambiguity

Market supervision aimed at reducing uncertainty and at restoring investors' trust and confidence into financial institutions is at the basis of the limitation of risk of systemic contagions. Indeed, one of the main amplifiers of the credit crunch has been precisely the lack of transparency concerning (in particular) structured products, that has led to a massive reduction in investment levels. Relevant information should be publicly available, and higher standardization and coordination should be enhanced. In our framework, any attempt to improve disclosure -possibly also with the development of central counterparties to foster transparency- translates into a reduction of the probability of the unfamiliar state to realize (equivalently, an increase in q).

Apart from market supervision, regulators and/or policy makers should be active participants in case of unexpected accidents. A regulator that credibly conveys the message that it commits itself to ensure stability of the financial system through recapitalization of entities that have a possibility of survival, and the merger of those that have not, would address precisely this necessity. Specifically, agents would perceive this granted intervention as a greater probability of restoring the familiar state, if the unfamiliar one ever occurs. Hence, they would increase the value of the capacity $v(F)$, the minimal probability of switching from the unfamiliar state to the familiar one. In particular, provided that the message is strong enough, $v(F) \geq q$.[17] In our model this is what it is needed to prevent Lemma 1 property 1 to hold. As a consequence, the implication $\exists t_n : B_n^U < B_n^F \rightarrow B_m^U < B_m^F$, $\forall t_m < t_n$ (which is at the basis of the freeze result) fails, as it is shown in the following Lemma.

Lemma 9 *Suppose $v(F) \geq q$, and consider a time t_N contingent payoff $x = \{x(U), x(F)\}$. Denote its evaluation at time t_n by $E_{t_n}[x|\, i]$, $i = F, U$. If there exists n^*, $1 \leq n^* \leq N$, such that $E_{t_{n^*}}[x|\, U] \leq E_{t_{n^*}}[x|\, F]$, then $\forall n < n^*$, $\forall i \in N$ s.t. $1 \leq m \leq \frac{n^*}{2}$: $E_{t_n}[x|\, U] \geq E_{t_n}[x|\, F]$, for $n = n^* - (2m-1)$, and $E_{t_n}[x|\, U] \leq E_{t_n}[x|\, F]$, for $n = n^* - 2m$.*

As it is for q, any rise in $v(F)$ increases the evaluation of the two contingent borrowing capacities at any rollover-date; in the limit case ($v(F) \geq q$), the freeze can be in principle prevented.

[17] This inequality implies that agents do not longer believe in state-persistence.

Addressing assets' sensitivity

The complexity and the uncertainty surrounding assets evaluation techniques, together with the procyclicality added by the fair value accounting conventions, have increased price volatility, and, in general, the market liquidity risk. Regulators should necessarily deal with uncertainty, in particular, they should enhance transparency by reducing the complexity in accounting standards. Similarly, they should provide accurate guidance for mark-to-market valuation and regulate information disclosure on the availability and the standardization of variance and historical data of prices. With this specific aim authorities (for example The International Organization of Securities Commissions) are now introducing legislation aimed at encouraging securitizers to disclose more information on their own portfolios and risk retentions. Finally, also credit rating agencies should be involved in the process of disclosure improvement. Apart from enhancing information quality, it is also important to avoid fire sales as a reaction to idiosyncratic events. On the one hand, the accumulation of adequate capital buffers would render financial institutions more prepared against averse events, so that they would not be forced to liquidate assets at extremely unfavorable prices. On the other hand, particular market clearing infrastructures could act as "absorber" for the failure of individual market participants, rendering assets prices less sensitive to individual circumstances. Reducing assets' sensitiveness to idiosyncratic risk would lower the degree of uncertainty surrounding their evaluation in case any unfamiliar event occurs. In our model, this translates into a narrower core of w, $C(w)$, or, equivalently, in an increase of $w(H)$. Consequently, contingent borrowing capacities are higher at any date. In the limit case $w(H) \to p$, so that ambiguity has no effect.

Addressing counterparties' risk

The development of structured finance has contributed to the virtual abolishment of any specific boundary between the lending and the borrowing sector, so that most financial institutions are simultaneously lenders and borrowers. In our model, this complex architecture is mirrored in the assumption that (in case of liquidation) the collateral has to be purchased by a secondary SIV that has to rise asset backed finance as well. The institution of a clearinghouse, or another central counterparty, could assure that all transactions are concluded, so that liquidity disappearance and prices' collapse would be limited. Similarly, (securitization) products' simplification and standardization could both improve liquidity and reduce valuation challenges, by enhancing transparency and investors' understanding of the underlying risks.

5 Conclusion and discussion

In this paper, we have recreated in a stylized market some of the dynamics that might have contributed in determining the 2007-2008 financial crisis. In particular, we have discussed how ambiguity and ambiguity aversion affect agents' willingness to trade in the market for secured borrowing, and the consequent importance of policies designed to contain the perceived level of ambiguity. Specifically, we have shown that any public intervention has no effect if its content is not clear enough: indeed, if agents are not confident about the effectiveness of the policy in the long term (meaning if it will succeed in avoiding a return to the crisis state), and such ambiguity problem is extremely severe, then the freeze of the market cannot be solved. In our model, the TIC-policy recently proposed by Caballero and Kurlat successfully mitigates the effects of ambiguity, by rendering the borrowing capacity of the collateral insensitive to both ambiguity and rollover risk.

We recognize that our freeze result could be obtained from different behavioral and/or informational assumptions. However, we believe that ambiguous preferences allow for possibly the least demanding set of initial assumptions among the existing frameworks in the literature, since exogenous frictions, such as liquidation costs and information asymmetries, are not needed. Similarly, all the particular relationships among the variables of the model endogenously follow from the functional representation of preferences. With respect to the standard behavioral finance's assumptions based on individuals' irrationality, the introduction of ambiguity in finance theory seems to have the advantage of providing a unified framework that could be possibly used to address alternative financial puzzles, while behavioral finance has often the tendency to elaborate different theories to explain alternative anomalies.

The crucial role played by ambiguity and ambiguity aversion in the 2007-2008 crisis has been now widely recognized. In addition, also more recent features of the financial turnover seem to be imputable to ambiguity (aversion). In particular, the current wide spread of the Greek bonds over the German ones cannot only be due to "traditional frictions" (such as asymmetric information), and academics and practitioners seem to agree that one of the driving factors for the Greek crisis is indeed ambiguity. Hence, we strongly believe that the study of financial markets' behavior under ambiguity could be extremely useful, not only as far as theory is concerned, but especially also at policy level. We are aware that quantifying the level of ambiguity in practice is a difficult task, however we also believe that the dispersion of agents' beliefs (for example measured by the dispersion of analysts' forecast) could work as a preliminary good approximation.

Our framework is clearly too simple to be used for policy-design and/or simulations, however we believe that it represents a preliminary step in this direction, so that it might be definitely worth it to extend it to more complete set-ups that could possibly provide a more accurate model for the complexity of financial markets. In particular these efforts need to be pursued in at least two directions. First of all, in our model the particular maturity structure of the debt (that is, short term financing) is exogenously given, however it is also necessary to understand why SIVs and similar institutions have relied so much on short term funding. In other words, we have to consider also the evaluation problem of the other side of the market to determine the conditions under which short term financing results optimal for borrowers. In this sense an important contribution is provided in Brunnermeier and Oehmke [4], who show that short maturity debt contracts are preferred by financial institutions, when the interim information is mostly about the probability of default of the collateral asset, rather than about the recovery in default. A result that seems promising also in light of our ambiguity application. Further, more structure should be added to discuss optimality, (in)efficiency of the equilibrium in a social perspective, and the consequent role of regulations and policy implications.

Proofs

In this Appendix we use the following notation: for any random variable x, we set $E_{t_n}[x|\,i] = E_{t_n}[x|\,s_n = i]$, $0 \le n \le N$, and $i = F, U$.

Lemma 1. Suppose $\exists n^*$, $1 \le n^* \le N$, as required. By (1) and (2), $E_{t_{n^*-1}}[x|\,F] = qE_{t_{n^*}}[x|\,F] + (1-q)\,E_{t_{n^*}}[x|\,U] \ge$

$E_{t_{n^*-1}}[x|\,U] = \min_{(\mu,1-\mu)\in C(v)} \left\{\mu E_{t_{n^*}}[x|\,F] + (1-\mu)\,E_{t_{n^*}}[x|\,U]\right\}.$

Hence:

$E_{t_{n^*-1}}[x|\,U] = v\,(F)\,E_{t_{n^*}}[x|\,F] + (1 - v\,(F))\,E_{t_{n^*}}[x|\,U] < E_{t_{n^*-1}}[x|\,F]$. Therefore, n^*-1 satisfies the initial hypothesis, so that the argument follows by induction. The second implication follows from property 1. ■

Theorem 2. At t_{n-1}, $D \le \max\left\{B_n^U, B_n^F\right\}$ must hold. Let $B_n^{\max} = \max\left\{B_n^U, B_n^F\right\}$ and $B_n^{\min} = \min\left\{B_n^U, B_n^F\right\}$.

Denoting by $\mathbf{1}$ the indicator function for the event $\left\{B_n^{\max} = B_n^F\right\}$, if $B_n^{\min} < D \le B_n^{\max}$:

$$E_{t_{n-1}}[D|\,F] = \left[(1-q)\,B_n^U + qD\right]\mathbf{1} + (1-\mathbf{1})\left[qB_n^F + (1-q)\,D\right]$$

otherwise $E_{t_{n-1}}[D|\,F] = D$. Similarly, if $B_n^{\min} < D \le B_n^{\max}$,

$$E_{t_{n-1}}[D|\,U] = \left[(1-v(F))\,B_n^{\min} + v(F)D\right]$$

otherwise $E_{t_{n-1}}[D|\,U] = D$.

$B_{n-1}^i = \max\limits_{D \le B_n^F} E_{t_{N-1}}[D|\,i]$, hence

$$B_{n-1}^F = \max\left\{B_n^{\min}, \left[(1-q)\,B_n^U + qB_n^F\right]\right\}$$

$$B_{n-1}^U = \max\left\{B_n^{\min}, (1-v(F))\,B_n^{\min} + v(F)B_n^{\max}\right\}.$$

Since $B_{N-1}^F > B_{N-1}^U$, by Lemma 1, $B_{N-2}^F > B_{N-2}^U$. By definition of $E_{t_{n-1}}[D|\,i]$, and iteratively using Lemma 1, $B_n^F > B_n^U$, $\forall n$, and the definitions of B_{n-1}^F and B_{n-1}^U follow. ■

Proposition 3 . By Lemma 1, $B^F_{N-2} > B^U_{N-2}$. Using iteratively the definition of $E_{t_{n-1}}[D|\, i]$ and Lemma 1, $B^F_n > B^U_n$, $\forall n$. The second property follows from the first and (3). $B^U_{n-1} \geq B^U_n$ follows from property 1, using (4) and Lemma 1. ■

Theorem 4. Use Proposition 6 in [1], replacing $N+1$ by N and V by $\hat{V}$. ■

Theorem 5. By definition, $B^U_1(\tau) = e^{-\alpha\tau} B^U_2(\tau) + (1 - e^{-\alpha\tau}) B^F_2(\tau)$

By Property 3 of Proposition 3, $B^U_2(\tau) < B^U_1(\tau)$, hence $\exists\tilde{\lambda} < 1$ such that $B^U_2(\tau) = \tilde{\lambda} B^U_1(\tau)$.

By continuity of real numbers, $\exists\lambda^*, \tilde{\lambda} < \lambda^* < 1$ such that:

$B^U_1(\tau) = e^{-\alpha\tau}\tilde{\lambda} B^U_1(\tau) + (1 - e^{-\alpha\tau}) B^F_2(\tau) < ...$

$... < e^{-\alpha\tau}\lambda^* B^U_1(\tau) + (1 - e^{-\alpha\tau}) B^F_2(\tau)$

hence $B^U_1(\tau) < \frac{1-e^{-\alpha\tau}}{1-\tilde{\lambda}e^{-\alpha\tau}} B^F_2(\tau)$.

$B^F_2(\tau)$ is bounded by Theorem 4, $e^{-\alpha\tau} \to 1$ as $\tau \to 0$, hence $\underset{\tau\to 0}{lim} B^U_1(\tau) = 0$. ■

Proposition 6. Since $B^{s\tilde{v}}_N = B^{sv}_N$ $(s = F, U)$, $\left|B^{Fi}_n - B^{Ui}_n\right| = (q - i)\left|B^{Fi}_{n+1} - B^{Ui}_{n+1}\right|$. $\left|B^{Fv}_{N-1} - B^{Uv}_{N-1}\right| > \left|B^{F\tilde{v}}_{N-1} - B^{U\tilde{v}}_{N-1}\right|$ $\quad i = \tilde{v}, v$, since $(q - v) > (q - \tilde{v})$. Hence:

$$\left|B^{Fv}_{N-2} - B^{Uv}_{N-2}\right| > \left|B^{F\tilde{v}}_{N-2} - B^{U\tilde{v}}_{N-2}\right|.$$

Repeating this argument, the property follows. The second property derives from the first and $\left|B^{Fi}_{n-1} - B^{Fi}_n\right| = (1 - q)\left|B^{Ui}_n - B^{Fi}_n\right|$. The last property follows from $\left[B^{Fi}_n, B^{Ui}_n\right]' = \left([\mathbf{q}, \mathbf{v}]'\right)^{N-n} [pV, w(H)V]'$, where $\mathbf{q} = [q, 1 - q]$ and $\mathbf{v} = [v(F), 1 - v(F)]$. ■

Theorem 7. For $n \neq N, N-1$ use the proof of Theorem 4 , without $B^U_n \leq B^F_n$. ■

Lemma 9. Suppose $\exists n^*$, $1 \leq n^* \leq N$, as required, hence:

$$E_{t_{n^*-1}}[x|\, F] = qE_{t_{n^*}}[x|\, F] + (1 - q)\, E_{t_{n^*}}[x|\, U]$$

$E_{t_{n^*-1}}[x|\, U] = \min\limits_{(\mu,1-\mu)\in C(v)} \left\{\mu E_{t_{n^*}}[x|\, F] + (1 - \mu)\, E_{t_{n^*}}[x|\, U]\right\}$. (1), $E_{t_{n^*}}[x|\, U] \leq E_{t_{n^*}}[x|\, F]$, $v(F) > q$ imply $E_{t_{n^*-1}}[x|\, U] > E_{t_{n^*-1}}[x|\, F]$.

Consider $n^* - 2$, then

$$E_{t_{n^*-2}}[x|F] = qE_{t_{n^*-1}}[x|F] + (1-q)E_{t_{n^*-1}}[x|U]$$

and $E_{t_{n^*-2}}[x|U] = \min_{(\mu,1-\mu)\in C(v)} \left\{\mu E_{t_{n^*-1}}[x|F] + (1-\mu)E_{t_{n^*-1}}[x|U]\right\}$. Hence, (1), $E_{t_{n^*-1}}[x|U] > E_{t_{n^*-1}}[x|F]$, $(1 - v(U)) > q$ imply $E_{t_{n^*-2}}[x|U] < E_{t_{n^*-2}}[x|F]$. Replacing n^* by $n^* - 2$, the result follows by induction. ■

Tables

$\boldsymbol{\alpha}$	**0.27**	**0.31**	**0.12**
$\mathbf{w}\,(H)$	**0.50**	**0.40**	**0.70**
p	0.80	0.80	0.80
$v\,(F)$	0.30	0.30	0.30
q	0.80	0.80	0.80
$v\,(U)$	0.60	0.60	0.60
V	1000	1000	1000

Table 1: Sensibility to ambiguity on credit rating.

$\boldsymbol{\alpha}$	**0.27**	**0.16**	**0.41**
$w\,(H)$	0.50	0.50	0.50
$\mathbf{p}$	**0.80**	**0.70**	**0.90**
$v\,(F)$	0.30	0.30	0.30
q	0.80	0.80	0.80
$v\,(U)$	0.60	0.60	0.60
V	1000	1000	1000

Table 2: Sensibility to credit rating.

$\boldsymbol{\alpha}$	**0.27**	**0.34**	**0.19**
$w\,(H)$	0.50	0.40	0.70
p	0.80	0.80	0.80
$\mathbf{v}\,(F)$	**0.30**	**0.20**	**0.40**
q	0.80	0.80	0.80
$v\,(U)$	0.60	0.60	0.60
V	1000	1000	1000

Table 3: Sensibility to ambiguity on transition's believes.

$\boldsymbol{\alpha}$	**0.27**	**0.14**	**0.39**
$w(H)$	0.50	0.50	0.50
p	0.80	0.70	0.90
$v(F)$	0.30	0.30	0.30
$\mathbf{q}$	**0.80**	**0.70**	**0.90**
$v(U)$	0.60	0.60	0.60
V	1000	1000	1000

Table 4: Sensibility to transition's believes.

$\boldsymbol{\alpha}$	**0.27**	**0.28**	**0.23**
$w(H)$	0.50	0.40	0.70
p	0.80	0.80	0.80
$v(F)$	0.30	0.20	0.40
q	0.80	0.80	0.80
$\mathbf{v}(U)$	**0.60**	**0.50**	**0.70**
V	1000	1000	1000

Table 5: Sensibility to optimism.

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