Speculation and Hedging in Segmented Markets

Itay Goldstein, Yan Li, Liyan Yang*

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Abstract

We analyze a model with segmented markets, where sophisticated traders – e.g., hedge funds – trade both a simple and a complex asset, while simple traders – e.g., retail investors – only have access to the simple asset. This corresponds to real world cases, where individual traders only trade stocks or indexes, while hedge funds and other sophisticated investors trade assets like derivatives, convertible bonds, etc. We analyze the implications of segmentation, sophistication, and complexity for price informativeness and the cost of capital. The different trading motives of the different traders (due to their different opportunities) imply that adding more informed traders may reduce price informativeness. This also provides a source for strategic complementarities in information production leading to multiple equilibria and price jumps.

Keywords: Speculation, Hedging, Market Segmentation, Price Informativeness, Information Acquisition, Asset Prices

JEL Classifications: G14, G12, G11, D82

*Itay Goldstein: Department of Finance, Wharton School, University of Pennsylvania, Philadelphia, PA 19104; Email: itayg@wharton.upenn.edu; Tel: 215-746-0499. Yan Li: Department of Finance, Fox School of Business, Temple University, Philadelphia, PA 19122; Email: liyanlpl@temple.edu; Tel: 215-204-4148. Liyan Yang: Department of Finance, Joseph L. Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario M5S 3E6; Email: liyan.yang@rotman.utoronto.ca; Tel: 416-978-3930.

1 Introduction

What is the effect of hedge funds on the efficiency of financial markets? How does trading of complex securities affect the informativeness of prices in financial markets? How are prices and trading behavior affected by market segmentation? These and other related questions are key to understanding modern financial markets which increasingly feature sophisticated traders, complex assets, and segmentation.

In this paper, we study a model that is motivated by these features of modern financial markets. In our model, there are two types of (rational) traders – simple (e.g., individuals) and sophisticated (e.g., financial institutions, hedge funds) – and two types of assets – simple (e.g., stocks, indexes) and complex (e.g., derivatives, convertible bonds). Markets are segmented, such that simple traders can only trade the simple asset (or they find it too costly to trade the complex asset), while sophisticated traders can trade both types of assets. In this framework, we analyze the trading behavior and price determination in both markets. We provide results on the effect of the size of the sophisticated-traders population and the attractiveness of the complex asset on the informativeness of the price system, the cost of capital, and the incentives of traders to produce information.

A key feature of our model is that sophisticated traders may end up trading the different assets for two different purposes: speculating and hedging. At the same time, simple traders, who are more limited in their trading opportunities, trade the simple asset for speculative purposes. This may lead to a situation where the trading behavior of the different types of traders responds differently to information, and so the informativeness and efficiency of the price system are reduced. As a result, the presence of sophisticated traders, complex assets, and market segmentation might have negative consequences.

Our model corresponds to many real-world examples. Let us describe a few of them. The hedge fund industry is one of the fastest-growing sectors of the economy.[1] One common trading strategy of hedge funds is the convertible-bond arbitrage strategy. A convertible bond

[1] According to Hedge Fund Research, hedge funds today manage $1.7 trillion in assets, compared to just 38 billion in 1990. Undoubtedly, hedge funds have substantial market influences, which can be further magnified due to leverage. Hedge funds also dominate certain special markets, such as trading in derivatives with high-yield ratings and distressed debt.

is a bond that can be converted into the issuing company's stock in the future.[2] Because of the complexity of convertible bonds, they are difficult for retail investors to navigate. With their superior physical and human capital, hedge funds have become the dominant players in this important market.[3] When a hedge fund has favorable information about a company, its common strategy is to buy the company's convertible bonds in hope of exchanging them for stocks when the stock price rises in the future, and at the same time, to short stocks of the same company to hedge itself.[4] Hence, while retail investors may trade stocks for speculative reasons, hedge funds may trade them to hedge their positions in convertible bonds, and the scenario of our model ensues.

Another scenario that relates to our model is the burgeoning hedging activity through the wide use of derivatives. It is well known that put option writers routinely short the underlying stocks to hedge their long positions in the options. Similarly, recent Wall Street Journal articles have brought to the spotlight the practice of using credit-default swaps in mortgage-backed securities and municipal bonds. Credit-default swaps have been created to protect mortgage-backed securities or bond holders if the issuer defaults. In essence, credit-default swaps allow investors to short-sell the underlying assets for hedging purposes.[5] Again, it is usually the more sophisticated traders who trade multiple assets, some of them complex, while the simple traders tend to shy away from them. Hence, this is similar to our model.

More generally, our model appeals to the broad hedging activity that entrepreneurs engage in. Since a lot of their human capital is invested in their firms, they may try to hedge this firm-specific risk by short-selling the firm's stock or the stocks of other firms in the

[2] The convertible bond market has been a growing and important market. In 2007, the new issuance of convertible bonds was 76.4 billion, compared to 187.5 billion of new stock issuance.

[3] Currently 70% of convertible bonds are owned by hedge funds.

[4] An interesting anecdote occurred in 2005, when many hedge funds had long positions in General Motors (GM) convertible bonds and short positions in GM stocks. They suffered huge losses when a billionaire investor tried to buy GM stock and at the same time its debt was being downgraded by credit-ratings agencies.

[5] The famous Goldman Sachs case involves using Credit Default Swaps to short the mortgage-backed securities. Credit-default swaps have long been used for corporations, but only became available on municipal debt over the past few years. Now, investors can short-sell the bonds of more than a dozen states as well as towns and cities, toll bridges, highways and tunnels. In the past three years Wall Street investment banks sold $43 billion worth of California state bonds. During that period, they traded $27.5 billion worth of credit default swaps. A recent article in Wall Street Journal (May 14, 2010) talks about states having to pay a lower price to sell their bonds due to the shorting activity via CDS.

same industry. Hence, like the sophisticated traders in our model, their actions may be interpreted as taking speculative positions in their human capital (which is a complex asset) while short selling related stocks (which are simple assets). At the same time, other traders in the economy have access only to the traded stocks, and so they use them for speculative trading. This corresponds to the market segmentation of our model.

Our model is based on the classic paper of Grossman and Stiglitz (1980). We have two classes of traders: sophisticated traders and simple traders. They are heterogenous with regard to their investment opportunities and information. All traders can trade a simple asset and a riskless asset in the financial markets, but only sophisticated traders can trade a complex asset such as convertible bonds, derivatives or human capital as in the aforementioned examples. All traders observe the price of both assets.[6] The simple asset and the complex asset share a common fundamental component and sophisticated traders may use the simple asset to hedge their investments in the complex asset (or vice versa). Before entering the financial market, simple traders can collect private information about the common fundamental at some cost, while sophisticated traders are endowed with private information.

We solve the model in closed form and characterize how the prices of the two assets are determined. We further analyze how the cost of capital and price informativeness of these two assets depend on interesting model parameters, such as the number of sophisticated traders and the profitability of speculative positions in the complex asset. The results depend crucially on the trading behavior of sophisticated traders. More specifically, sophisticated traders trade the simple risky asset for two reasons: speculating based on superior information about the simple asset's payoff, and hedging their investment in the complex asset. Depending on the strength of these two motives, our model generates very different results regarding the cost of capital and price informativeness. Of particular interest to us is the case where the hedging motive in the simple asset is strong. In this case, sophisticated traders trade very differently from simple traders and tend to reduce the informativeness of the price and increase the cost of capital.

[6] Such prices tend to be easily available even for people who don't actively trade the assets.

We further study the incentive of simple traders to collect information regarding the fundamental of the simple asset. Most of the existing literature predicts that when more investors are informed, the value of the information is reduced, and investors have less incentive to gather information, resulting in strategic substitution in learning.[7] In our model, however, learning complementarities can naturally arise. That is, as more simple traders become informed, information becomes more valuable, and uninformed simple traders have a stronger incentive to collect it, generating strategic complementarity in information acquisition. The intuition is as follows. Suppose that the fundamental of the two assets is strong. If sophisticated investors can better explore the trading opportunities in the complex asset, they will increase their investment in the complex asset and decrease their investment in the simple asset (due to hedging). When the price informativeness of the simple asset is determined mainly by the sophisticated traders' hedging-motivated trading, raising the number of informed simple traders will raise their speculative demand, making the two offsetting forces – from simple traders and sophisticated traders – more balanced. This, in turn, will make the price less responsive to changes in the signal, so that uninformed simple traders have a more difficult time gleaning information from prices. The resulting learning complementarities can generate multiplicity of equilibria and excess volatility in prices.[8]

Our paper is related to papers featuring hedging motivated trading in financial markets. Glosten (1989), Spiegel and Subrahmanyam (1992), Dow and Rahi (2003), Goldstein and Guembel (2008) and Kyle, Ou-Yang and Wei (2010), among others, study Kyle (1985) type models with endogenous noise trading generated from risk-averse uninformed hedgers who hedge their endowment risk optimally. Similar formulations of hedging motives also appear in Grossman-Stiglitz (1980) type models, for example, Duffie and Rahi (1995), Lo, Mamaysky and Wang (2004), Watanabe (2008), Biais, Bossaerts and Spatt (2010) and Huang and Wang (2010). In all these papers, hedgers' endowments are assumed to be correlated

[7] In particular, Grossman and Stiglitz (1980, p394) formulated the following two conjectures about price informativeness and strategic learning: "Conjecture 1: The more individuals who are informed, the more informative is the price system... Conjecture 2: The more individuals who are informed, the lower the ratio of the expected utility of the informed to the uninformed."

[8] Recently, different papers derive complementarities in financial markets based on other forces. For example, see Barlevy and Veronesi (2000), Ganguli and Yang (2009), Garcia and Strobl (2010), Goldstein, Ozdenoren, and Yuan (2010), and Mele and Sangiorgi (2010).

with the performance of some underlying tradable asset and hence they have an incentive to use the asset to hedge their endowment shocks. The hedging-motivated trading in this literature is mainly a device to prevent fully revealing prices and/or to complete the model (by endogenizing noise trading). In contrast, in our paper, the hedging-motivated trading on the (simple) asset does not come from the passive endowment shocks, but instead comes from the active trading from another related (complex) asset. This creates the inherent link between speculation and hedging, which is at the core of our model. This channel has strong empirical motivation (partly discussed above) and is particularly suitable for analyzing how different trading opportunities affect asset prices and information acquisition.

Our paper is also related to papers studying multiple assets in (noisy) rational expectations equilibrium settings. Admati (1985) is the first to analyze the properties of noisy rational expectations equilibria for a class of economies with many risky assets. Watanabe (2008) and Biais, Bossaerts and Spatt (2010) extend Admati's model to an overlapping generation setting to study the effect of asymmetric information and supply shocks on portfolio choice, return volatility and trading volume. Yuan (2005) introduces borrowing constraints into a two-asset model and shows how trading can cause contagion across two fundamentally independent markets. Veldkamp (2006) introduces markets for information and generates high price covariance within a rational expectations framework. Nieuwerburgh and Veldkamp (2009, 2010) show that the interactions between the multi-asset portfolio problem and the information acquisition problem help to explain the home-bias puzzle and the under-diversification puzzle. All the above-mentioned papers assume that all investors have equal access to the same investment vehicles, unlike the market-segmentation scenarios which are the focus of our paper. We demonstrate in Section 6.2 that this segmentation is key to our results.

Finally, there are previous papers that analyzed different notions of segmentation in information-based models. For example, see Chowdry and Nanda (1991) and Madhavan (1995). They consider cases with multiple markets, where the information from one market may not be available to traders in the other market. In contrast, our notion of segmentation is that of different trading opportunities for different traders, and we do allow for information flows across markets.

The remainder of the paper is organized as follows. Section 2 describes the model analyzed in the paper. In Section 3, we solve for the trading behavior and prices in the two markets. Section 4 analyzes the effect of sophisticated traders and complex asset (in a segmented market) on price informativeness and the cost of capital. In Section 5, we solve for the information acquisition decision, and show that complementarities will sometimes arise in equilibrium. Section 6 discusses different settings to better understand the ingredients behind our main results. Section 7 concludes. All proofs are relegated to the appendix.

2 The Model

2.1 Environment

Consider an economy with a single good that can be either consumed or invested. Time is discrete and has three dates ($t = 0, 1, 2$). At date 1, a competitive financial market opens, and three assets — one riskless asset, one simple risky asset and one complex risky asset — are traded at prices 1, $\tilde{P}$ and $\tilde{Q}$, respectively, in the market.[9] At date 2, the riskless asset pays one unit of good, the simple risky asset pays a normally distributed random variable $\tilde{v}$, and the complex risky asset pays a normally distributed random variable $\tilde{\kappa}$. The riskless asset is in unlimited supply and both risky assets have limited supply. Assume that the simple risky asset has a supply of $\bar{x}_v > 0$ and that the complex risky asset has a supply of $\bar{x}_\kappa > 0$. As we will specify below, the payoffs of the two risk assets are assumed to be correlated.

There are two classes of rational traders in the economy: sophisticated traders (of mass $\mu > 0$) and simple traders (of mass $z > 0$). Traders derive their expected utility only from their date 2 consumption; they have constant-absolute-risk-aversion (CARA) utility functions over consumption c: $-e^{-\gamma c}$, where γ is the risk-aversion parameter. The risk aversion parameter for sophisticated traders is denoted $\gamma_H > 0$ and that for simple traders is denoted $\gamma_S > 0$.

[9] Throughout the paper, a tilde ($\tilde{}$) always signifies a random variable.

The terms "sophisticated" and "simple" capture the fact that different traders have different investment opportunity sets. Specifically, at date 1, sophisticated traders can trade all three assets, while simple traders can only trade the simple risky asset and the riskless asset. That is, only sophisticated traders have the "know-how" for trading the complex asset. We can think of the sophisticated traders as financial institutions, such as hedge funds, and interpret the simple traders as individuals. Assume that all traders can observe both prices $\tilde{P}$ and $\tilde{Q}$. This can be justified, given that nowadays investors can easily obtain this kind of price information via internet (while, at the same time, unsophisticated traders tend to stay away from trading in complex assets).

In both markets, there are noise traders, who trade for exogenous liquidity reasons. We use $\tilde{n}_v \sim \mathcal{N}(0, \sigma_{nv}^2)$ (with $\sigma_{nv} > 0$) to denote noise trading in the simple risky asset market and $\tilde{n}_\kappa \sim \mathcal{N}(0, \sigma_{n\kappa}^2)$ (with $\sigma_{n\kappa} > 0$) to denote noise trading in the complex risky asset market. We assume that $\tilde{n}_v$ is independent of $\tilde{n}_\kappa$, which is reasonable given that the two markets are segmented. Note that our results do not depend on the size of σ_{nv} relative to $\sigma_{n\kappa}$.

2.2 Asset Payoffs and Information Structure

At date 0, rational traders can purchase data that is useful in forecasting the payoffs $\tilde{v}$ and $\tilde{\kappa}$ of the risky assets. If they do so, the signal they receive is $\tilde{\theta}$, which can be thought of as the fundamental of the assets. The payoffs of the risky assets are then:

$$\begin{cases} \tilde{v} = \bar{v} + \tilde{\theta} + \tilde{\varepsilon}, \\ \tilde{\kappa} = \bar{\kappa} + A\tilde{\theta} + \tilde{\eta}, \end{cases} \tag{1}$$

where $\bar{v}$ and $\bar{\kappa}$ are the priors, and $\tilde{\varepsilon}$ and $\tilde{\eta}$ are residual noise terms conditional on the signal $\tilde{\theta}$. We assume that the noise terms and the signal are normally distributed: $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\tilde{\eta} \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$, and $\tilde{\theta} \sim \mathcal{N}(0, \sigma_\theta^2)$ ($\sigma_\varepsilon, \sigma_\eta, \sigma_\theta > 0$). The two noise terms $(\tilde{\varepsilon}, \tilde{\eta})$ are independent of the fundamental $\tilde{\theta}$, but are correlated with one another with the coefficient $\rho \in (0, 1)$. The parameter A is greater than 0; it represents the sensitivity of the complex asset's payoff to the signal (the sensitivity of the simple asset's payoff is normalized to one).

Our model is meant to capture a situation where two correlated assets are traded in segmented markets. Segmentation is represented by the fact that some traders have access only to one of the two markets. As mentioned above, we assume correlation across assets in fundamentals and in noise terms, and lack of correlation between each noise term and the fundamental. This generates the link between speculation and hedging, which is central to our mechanism. This structure can be justified by thinking of the "fundamentals" of the two assets as the result of estimation from the data using an OLS regression. Since the payoffs on both assets are correlated, their estimated "fundamentals" as well as residual noise terms will be correlated, while, at the same time, the noise terms will be independent of the estimated fundamentals. Note that, for simplicity, we assume that the fundamentals of the two assets are captured by a single random variable $\tilde{\theta}$, and are thus perfectly correlated. Our results are robust to a more general assumption that they are only imperfectly correlated.

Finally, we assume that sophisticated traders are superior to simple traders in collecting data. Specifically, sophisticated traders can collect data at no cost, while simple traders have to spend a cost $\tau > 0$ to acquire the data and hence the signal $\tilde{\theta}$.[10] A simple trader is called *informed* if she chooses to acquire the signal $\tilde{\theta}$ and *uninformed* otherwise. Like Grossman and Stiglitz (1980), at date 1, the asset prices $\tilde{P}$ and $\tilde{Q}$ will partially reveal the signal $\tilde{\theta}$ through the trading of the informed simple traders and the sophisticated traders. The uninformed simple traders can extract information about $\tilde{\theta}$ from observing prices. Of course, informed traders also observe prices, but this extra price information is redundant in forecasting $\tilde{\theta}$ given that they know $\tilde{\theta}$ perfectly.

2.3 Timeline

The timeline of the model is as follows. At date 0, simple traders choose whether or not to acquire the signal $\tilde{\theta}$ at cost $\tau > 0$. Sophisticated traders costlessly observe $\tilde{\theta}$. At date 1, the financial market opens. Informed and uninformed simple traders trade the riskless asset and the simple risky asset at prices 1 and $\tilde{P}$, respectively. Sophisticated traders trade the riskless

[10] The result remains unchanged if the sophisticated traders also have to spend a cost, although lower than τ, to acquire the signal $\tilde{\theta}$; but this assumption will make the analysis messier.

asset, the simple risky asset and the complex risky asset at prices 1, $\tilde{P}$ and $\tilde{Q}$, respectively. Noise traders trade $\tilde{n}_v$ in the simple risky asset and $\tilde{n}_\kappa$ in the complex risky asset. At date 2, payoffs are received. All rational traders consume.

To summarize, $(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\eta}, \tilde{n}_v, \tilde{n}_\kappa)$ are underlying random variables which characterize the economy. They are all independent of each other, except that $\tilde{\varepsilon}$ and $\tilde{\eta}$ are positively correlated with each other. The tuple

$$\mathcal{E} = (\mu, z, \gamma_H, \gamma_S, \tau, \bar{v}, \bar{\kappa}, \sigma_\theta, \sigma_\varepsilon, \sigma_\eta, \rho, A, \sigma_{nv}, \sigma_{n\kappa})$$

defines an economy.

3 Trading and Prices

We start by analyzing trading behavior and prices in the financial market, given the fraction of informed traders λ. The equilibrium concept that we use is the rational expectations equilibrium (REE), as in Grossman and Stiglitz (1980). In equilibrium, traders trade to maximize their expected utility given their information set, where sophisticated traders and informed simple traders know $\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right)$, while uninformed simple traders know $\left(\tilde{P}, \tilde{Q}\right)$. Prices of the complex and simple assets are set to clear the markets. We now turn to a detailed derivation of the equilibrium.

3.1 Price Functions

The trading of the sophisticated traders and that of the informed simple traders are affected by the information set $\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right)$, while the uninformed individual traders' trading is affected by the information set $\left(\tilde{P}, \tilde{Q}\right)$. In the simple-asset market, noise traders demand $\tilde{n}_v$. Hence, the price of the simple risky asset is a function of $\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_v\right)$:

$$\tilde{P} = P\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_v\right).$$

Similarly, the price of the complex risky asset is a function of $\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_\kappa\right)$:

$$\tilde{Q} = Q\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_\kappa\right).$$

Combining $\tilde{P} = P\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_v\right)$ and $\tilde{Q} = Q\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_\kappa\right)$ and solving for $\tilde{P}$ and $\tilde{Q}$, we expect that both prices are functions of $\left(\tilde{\theta}, \tilde{n}_v, \tilde{n}_\kappa\right)$.

We are interested in equilibria where $P\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_v\right)$ and $P\left(\tilde{\theta}, \tilde{P}, \tilde{Q}, \tilde{n}_v\right)$ are linear functions:

$$\begin{aligned}
\tilde{P} &= a_0 + a_\theta \tilde{\theta} + a_v \tilde{n}_v + a_\kappa \tilde{n}_\kappa, \\
\tilde{Q} &= c_0 + c_\theta \tilde{\theta} + c_v \tilde{n}_v + c_\kappa \tilde{n}_\kappa,
\end{aligned}$$

where the coefficients are endogenously determined.

We first examine the decisions of the sophisticated traders and the informed simple traders, which in turn determine the information content in prices $\tilde{P}$ and $\tilde{Q}$ through the order flow in the simple asset and the complex asset. We then solve for the decisions of the uninformed simple traders, and finally we use the market clearing condition to find the coefficients in the price functions. Given the CARA-normal setup, we will use the feature that information revealed in equilibrium by order flow and information revealed by asset prices are equivalent (e.g., Romer, 1993; Vives, 1995).

3.2 Traders' Demand

3.2.1 Sophisticated traders

Sophisticated traders have information $\mathcal{F}_H = \{\tilde{\theta}, \tilde{P}, \tilde{Q}\}$. They choose investment in the simple risky asset Z_H and in the complex risky asset K to maximize $E[-e^{-\gamma_H[(\tilde{v}-\tilde{P})Z_H+(\tilde{\kappa}-\tilde{Q})K]}|\mathcal{F}_H]$, where $E\left[\cdot|\, \mathcal{F}_H\right]$ is the expectation operator conditional on the information set $\mathcal{F}_H$.

Given the assumptions of CARA preferences and normal distributions, their optimal investments are

$$K\left(\tilde{\theta},\tilde{P},\tilde{Q}\right) = \frac{1}{\gamma_H}\frac{VAR\left(\tilde{v}|\mathcal{F}_H\right)\left[E\left(\tilde{\kappa}|\mathcal{F}_H\right)-\tilde{Q}\right]-COV\left(\tilde{v},\tilde{\kappa}|\mathcal{F}_H\right)\left[E\left(\tilde{v}|\mathcal{F}_H\right)-\tilde{P}\right]}{\left|VAR\left(\tilde{v},\tilde{\kappa}|\mathcal{F}_H\right)\right|} \tag{2}$$

$$= \frac{1}{\gamma_H}\frac{\bar{\kappa}+A\tilde{\theta}-\tilde{Q}}{\left(1-\rho^2\right)\sigma_\eta^2}-\frac{1}{\gamma_H}\frac{\rho\left(\bar{v}+\tilde{\theta}-\tilde{P}\right)}{\left(1-\rho^2\right)\sigma_\eta\sigma_\varepsilon}, \tag{3}$$

$$Z_H(\tilde{\theta},\tilde{P},\tilde{Q}) = \frac{1}{\gamma_H}\frac{VAR\left(\tilde{\kappa}|\mathcal{F}_H\right)\left[E\left(\tilde{v}|\mathcal{F}_H\right)-\tilde{P}\right]-COV\left(\tilde{v},\tilde{\kappa}|\mathcal{F}_H\right)\left[E\left(\tilde{\kappa}|\mathcal{F}_H\right)-\tilde{Q}\right]}{\left|VAR\left(\tilde{v},\tilde{\kappa}|\mathcal{F}_H\right)\right|} \tag{4}$$

$$= \frac{1}{\gamma_H}\frac{\bar{v}+\tilde{\theta}-\tilde{P}}{\left(1-\rho^2\right)\sigma_\varepsilon^2}-\frac{1}{\gamma_H}\frac{\rho\left(\bar{\kappa}+A\tilde{\theta}-\tilde{Q}\right)}{\left(1-\rho^2\right)\sigma_\eta\sigma_\varepsilon}, \tag{5}$$

where $VAR\left(\cdot|\mathcal{F}_H\right)$ and $COV\left(\cdot,\cdot|\mathcal{F}_H\right)$ are the conditional variance and covariance operators respectively, and $|\cdot|$ is the determinant operator.

In equations (3) and (5), the first term represents speculation-based trading and the second term represents hedging-motivated trading. For example, take equation (3). A sophisticated trader demands more of the complex asset when $\bar{\kappa}+A\tilde{\theta}-\tilde{Q}$ is high, and so the expected value of the asset is high relative to its price. But, he demands less of the complex asset when $\bar{v}+\tilde{\theta}-\tilde{P}$ is high (and the correlation between the two assets is high) because then holding the simple asset becomes more profitable, and so he reduces his demand of the complex asset to hedge his position in the simple asset.

3.2.2 Informed simple traders

Informed simple traders have information set $\mathcal{F}_I=\{\tilde{\theta},\tilde{P},\tilde{Q}\}$. They choose simple risky asset holdings Z_I to maximize $E[-e^{-\gamma_S[Z_I(\tilde{v}-\tilde{P})]}|\mathcal{F}_I]$. Given the assumptions of CARA preference and normal distributions, their optimal holdings are

$$Z_I(\tilde{\theta},\tilde{P},\tilde{Q})=\frac{E\left(\tilde{v}|\mathcal{F}_I\right)-\tilde{P}}{\gamma_S VAR\left(\tilde{v}|\mathcal{F}_I\right)}=\frac{\bar{v}+\tilde{\theta}-\tilde{P}}{\gamma_S\sigma_\varepsilon^2}. \tag{6}$$

3.2.3 Uninformed simple traders

Uninformed simple traders observe only the realizations of prices, $\tilde{P}$ and $\tilde{Q}$, so their information set is $\mathcal{F}_U = \{\tilde{P}, \tilde{Q}\}$. They choose stock holdings in the simple asset Z_U to maximize $E[-e^{-\gamma_S[Z_U(\tilde{v}-\tilde{P})]}|\mathcal{F}_U]$. The demand of the uninformed individual traders for the simple asset is

$$\frac{E(\tilde{v}|\tilde{P}, \tilde{Q}) - \tilde{P}}{\gamma_S VAR(\tilde{v}|\tilde{P}, \tilde{Q})}. \tag{7}$$

Using the analysis of the behavior of sophisticated traders and informed simple traders, the two risky asset prices $\tilde{P}$ and $\tilde{Q}$ are equivalent to two public signals $\tilde{s}_\kappa$ and $\tilde{s}_v$ to the uninformed simple traders. Recall that, given the CARA-normal setup, information revealed in equilibrium by order flow and information revealed by asset prices are equivalent. In the complex-asset market, combining the total order flow for the complex risky asset $\mu K\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right) + \tilde{n}_\kappa$ (which is equal to $\bar{x}_\kappa$ by the market clearing condition) and the asset prices $\tilde{P}$ and $\tilde{Q}$ generates the following public signal to the uninformed simple traders:

$$\tilde{s}_\kappa = \tilde{\theta} + b_\kappa^{-1}\tilde{n}_\kappa, \text{ with } b_\kappa = \frac{\mu\left(A/\sigma_\eta - \rho/\sigma_\varepsilon\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\eta}. \tag{8}$$

Similarly, the order flow in the simple risky asset $\mu Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q}) + \lambda Z_I(\tilde{\theta}, \tilde{P}, \tilde{Q}) + \tilde{n}_v$ (which is $\bar{x}_v$ minus the uninformed traders' demand) and asset prices $\tilde{P}$ and $\tilde{Q}$ form another public signal:

$$\tilde{s}_v = \tilde{\theta} + b_v^{-1}\tilde{n}_v, \text{ with } b_v = \frac{\mu\left(1 - A\rho\sigma_\varepsilon/\sigma_\eta\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S\sigma_\varepsilon^2}. \tag{9}$$

Hence, by equations (8) and (9), we have

$$VAR(\tilde{v}|\tilde{P}, \tilde{Q}) = VAR(\tilde{\theta}|\tilde{s}_\kappa, \tilde{s}_v) + \sigma_\varepsilon^2, \tag{10}$$

and

$$E(\tilde{v}|\tilde{P}, \tilde{Q}) = \bar{v} + VAR(\tilde{\theta}|\tilde{s}_\kappa, \tilde{s}_v)\left(b_v^2\sigma_{nv}^{-2}\tilde{s}_v + b_\kappa^2\sigma_{n\kappa}^{-2}\tilde{s}_\kappa\right), \tag{11}$$

where

$$VAR(\tilde{\theta}|\tilde{s}_\kappa, \tilde{s}_v) = \left(\sigma_\theta^{-2} + b_v^2\sigma_{nv}^{-2} + b_\kappa^2\sigma_{n\kappa}^{-2}\right)^{-1}. \tag{12}$$

Plugging (10) and (11) in (7), we get the demand of uninformed traders.

3.3 Market Clearing

Finally, in equilibrium the sum of demands has to equal the supply in both the complex and simple assets. For the complex asset, this implies:

$$\mu K\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right) + \tilde{n}_\kappa = \bar{x}_\kappa, \tag{13}$$

while for the simple asset:

$$\mu Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q}) + \lambda Z_I(\tilde{\theta}, \tilde{P}, \tilde{Q}) + (z - \lambda)\, Z_U(\tilde{P}, \tilde{Q}) + \tilde{n}_v = \bar{x}_v. \tag{14}$$

Plugging (2)-(7) into (13)-(14), and solving for the prices $\tilde{P}$ and $\tilde{Q}$, we prove the following proposition.

Proposition 1 *For any given $\lambda > 0$, there exists a unique linear REE in which,*[11]

$$\begin{aligned}\tilde{P} &= a_0 + a_\theta \tilde{\theta} + a_v \tilde{n}_v + a_\kappa \tilde{n}_\kappa, \\ \tilde{Q} &= c_0 + c_\theta \tilde{\theta} + c_v \tilde{n}_v + c_\kappa \tilde{n}_\kappa.\end{aligned}$$

The coefficients a_0, a_θ, a_v, a_κ, c_0, c_θ, c_v, and c_κ are given as a function of the exogenous parameters of the model in the proof in the appendix.

[11] Although all the variables here depend on λ, for simplicity of notation, we do not express the dependence explicitly.

4 Complex Assets, Sophisticated Traders, and Price Informativeness

4.1 Price Informativeness

The uniqueness of the setup studied in our paper stems from the existence of a complex asset and sophisticated traders, and the assumption that only the sophisticated traders can trade the complex asset. In this section, we study the implications that these features have for the informativeness of the price system. In particular, we focus on the effect of the parameters μ and A. Recall that μ captures the size of the sophisticated-traders population, while A is the sensitivity of the complex asset to the information. This is associated with the advantage held by the sophisticated traders and hence with the level of complexity of the complex asset. Hence, these two parameters are key in studying the implications of having sophisticated traders and complex assets.

In our measurement of price informativeness we try to capture the amount of uncertainty about $\tilde{\theta}$ that is reduced by observing the two prices $\tilde{P}$ and $\tilde{Q}$. This is similar to the amount of information gleaned by the uninformed traders from the price. Hence, we define price informativeness as:

$$PI = \frac{VAR(\tilde{\theta})}{VAR(\tilde{\theta}|\tilde{P},\tilde{Q})} - 1 = b_v^2 \frac{\sigma_\theta^2}{\sigma_{nv}^2} + b_\kappa^2 \frac{\sigma_\theta^2}{\sigma_{n\kappa}^2}, \tag{15}$$

where the second equality follows from equations (8) and (9) and after applying Bayes' rule. This concept is consistent with using h to measure the informativeness of a signal $\tilde{\theta} + \tilde{n}/h$.

In (15), the two endogenous parameters b_v^2 and b_κ^2 (that depend on μ and A) are taken from (8) and (9):

$$\begin{aligned} b_v &= \frac{\mu\left(1 - A\rho\sigma_\varepsilon/\sigma_\eta\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S \sigma_\varepsilon^2}, \\ b_\kappa &= \frac{\mu\left(A/\sigma_\eta - \rho/\sigma_\varepsilon\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\eta}. \end{aligned}$$

They represent the information injected by the sophisticated and the informed simple traders into the price system. In particular, $\frac{\mu(1-A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H(1-\rho^2)\sigma_\varepsilon^2}$ in b_v represents the trading of sophisticated traders in the simple asset, while $\frac{\lambda}{\gamma_S \sigma_\varepsilon^2}$ represents the trading of informed simple traders. The key insights of our model stem from the fact that these two terms may have opposite signs, indicating that, in response to $\tilde{\theta}$, sophisticated traders and informed simple traders are trading the simple asset in opposite directions. This will happen when the hedging-based trading of sophisticated traders in the simple asset dominates their speculative-based trading in this asset (recall that simple traders only engage in speculative-based trading since they only trade one asset). The fact that different traders trade in opposite directions reduces the ability to infer information from the price. This may generate some interesting implications, e.g., that increasing the size of the informed-traders population (either the sophisticated ones or the informed simple ones) may reduce the informativeness of the price system. This is against the implications of traditional models, where having more informed traders is beneficial for informativeness.

4.1.1 The effect of μ

We first turn to explore the effect of the size μ of the sophisticated-traders population on price informativeness. This is summarized in the following proposition.

Proposition 2 *If $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$ and μ is sufficiently small, then $\frac{\partial PI}{\partial \mu} < 0$, i.e., price informativeness decreases in the size of the sophisticated-traders population. Otherwise, $\frac{\partial PI}{\partial \mu} > 0$.*

As the proposition shows, increasing the size of the sophisticated-traders population will have a negative effect on price informativeness when two conditions hold. The first condition is that, in response to information $\tilde{\theta}$, the sophisticated traders are trading the simple asset in opposite direction to the informed simple traders (i.e., $\frac{\partial Z_H(\tilde{\theta},\tilde{P},\tilde{Q})}{\partial \tilde{\theta}} < 0$ and $\frac{\partial Z_I(\tilde{\theta},\tilde{P},\tilde{Q})}{\partial \tilde{\theta}} > 0$). This occurs when

$$(A\rho\sigma_\varepsilon/\sigma_\eta) > 1, \tag{16}$$

and so the hedging-motivated trade by the sophisticated traders is sufficiently strong. It is useful to understand in detail when this condition holds, as there are four parameters that go into it. First, this condition is more likely to hold when A is large, since A captures the sensitivity of the complex asset to information, and so when it is large, sophisticated traders are more likely to take a speculative position in the complex asset and a hedging position in the simple asset. Second, the condition is more likely to hold when the correlation ρ between the noise terms of the two assets is larger, as then hedging plays a greater role. Third, the condition is more likely to hold when σ_ε is larger, and so there is more noise in the payoff of the simple asset. In this case, the simple asset becomes more suitable for hedging than for speculating. Fourth, for a similar reason, the condition is more likely to hold when σ_η is smaller. Overall, condition (16) is key in our paper, as it summarizes when sophisticated traders trade the simple asset based on the signal $\tilde{\theta}$ primarily for hedging rather than for speculative purposes.

The second condition for μ to have a negative effect on price informativeness is that μ be sufficiently small. When μ is large, trading by the sophisticated traders is dominant for the determination of price informativeness, and so when there is more of it, price informativeness increases. But, when μ is small, the trading by the informed simple traders becomes more important and it is the one that determines price informativeness, and so the fact that more sophisticated traders trade against the informed simple traders reduces the overall informativeness.

In summary, under the above mentioned conditions, our model predicts that increasing the presence of sophisticated traders, e.g., hedge funds, in financial markets will reduce price informativeness. These traders have superior investment opportunities, and hence use simple assets for hedging purposes. Since simple traders use simple assets for speculative purposes, the two end up trading in different directions in response to $\tilde{\theta}$, and reduce price informativeness.

4.1.2 The effect of A

Next, we look at the impact of A on the informativeness of the price system. This is summarized in the following proposition.

Proposition 3 *For a sufficiently small A, $\frac{\partial PI}{\partial A} < 0$, i.e., the informativeness of the price system decreases in the sensitivity of the complex asset to information. For a sufficiently large A, $\frac{\partial PI}{\partial A} > 0$.*

According to the proposition, an increase in the sensitivity of the complex asset to the signal $\tilde{\theta}$ may lead to a reduction in the overall informativeness of the price system. First, note that in a model with one asset and no endogenous hedging, this would not be the case. In such a model, when the asset is more sensitive to the signal, traders trade more aggressively on the signal, leading to greater informativeness. But, in our model, this intuition might get reversed. In general, when A is low, in response to the signal $\tilde{\theta}$, both types of traders use the simple asset for speculation, and sophisticated traders use the complex asset for hedging. Then, prices are relatively informative. But, then, an increase in A induces sophisticated traders to start speculating more with the complex asset and hedging more with the simple asset, and this goes against the direction of trade of simple traders and reduces price informativeness. Hence, the implication is that having a more complex asset, i.e., one that provides greater return on information, might reduce overall price informativeness.

4.2 Cost of Capital

One important implication of price informativeness is its effect on the cost of capital. Thinking about the simple asset (which may be thought of as a stock issued by a firm), we define the cost of capital as follows (see, e.g., Easley and O'Hara (2004)):

$$CC = E(\tilde{v} - \tilde{P}). \tag{17}$$

That is, this is the expected difference between the cash flow generated by the security and its price, which is due to the risk taken by the traders who hold the security.

From the proof of Proposition 1, we then know that

$$CC = \frac{\bar{x}_v + (\rho\sigma_\eta/\sigma_\varepsilon)\,\bar{x}_\kappa}{L}, \tag{18}$$

where L is an endogenous coefficient as defined in the proof of Proposition 1:

$$L = \left(\frac{\mu}{\gamma_H} + \frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2} + \frac{z-\lambda}{\gamma_S}\frac{1}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}. \tag{19}$$

The first term on the RHS of (19) captures the residual risk borne by the informed traders – i.e., sophisticated and informed simple traders – due to the uncertainty about $\tilde{\varepsilon}$. The second term captures the risk borne by the uninformed simple traders, who face additional risk due to not knowing $\tilde{\theta}$ (and trying to infer it from the price). Combining (19) with (18), we can see that the cost of capital increases in the risk that traders are exposed to per unit of the security and in the supply of the security (which increases the total amount of risk that has to be absorbed). It decreases in the size of the traders population, since when there are more traders, the risk can be shared more broadly.[12]

The implications of price informativeness are then clear. From (15), (18), and (19) and since $VAR(\tilde{v}|\tilde{P},\tilde{Q}) = VAR(\tilde{\theta}|\tilde{P},\tilde{Q}) + \sigma_{\varepsilon}^2$, an increase in price informativeness leads to a decrease in the cost of capital, as uninformed simple traders are exposed to less risk when they observe more information in the price. Then, the following corollary immediately follows from Proposition 3.

Corollary 1 *For a sufficiently small A, $\frac{\partial CC}{\partial A} > 0$, i.e., the cost of capital increases in the sensitivity of the complex asset to information. For a sufficiently large A, $\frac{\partial CC}{\partial A} < 0$.*

Similarly, we can analyze the effect of the size μ of the sophisticated traders population on the cost of capital. This is, however, not so obvious, since μ affects the cost of capital not only via the informativeness of the price. There is also a direct effect, by which an increase in the size of the sophisticated traders population implies that the risk can be spread more widely, and so the cost of capital decreases. Still, we can prove the following proposition.

Proposition 4 *If μ is sufficiently small and either A is sufficiently large or σ_η is sufficiently small, then $\frac{\partial CC}{\partial \mu} > 0$, i.e., the cost of capital increases in the size of the sophisticated-traders population. Otherwise, $\frac{\partial CC}{\partial \mu} < 0$.*

The intuition is simple. We know that, by Proposition 2, when μ is small and when sophisticated traders use the simple asset primarily for hedging purpose, an increase in the size of the sophisticated traders population reduces price informativeness, and this has a

[12] This is only the direct effect. As we already know, the size of the traders population affects the informativeness of the price, and hence has an additional indirect effect on the cost of capital.

positive effect on the cost of capital. This effect will dominate the direct negative effect of μ on the cost of capital when traders have a strong incentive to speculate in the complex asset and hedge with the simple asset, i.e., when either A is sufficiently large or σ_η is sufficiently small.

5 Learning Complementarities, Multiplicity and Price Jumps

5.1 Information Market and Learning Complementarities

We now go back to date 0 and analyze the choice made by simple traders on whether to pay the cost τ and become informed or not. An argument similar to that of Grossman and Stiglitz (1980) shows that for a given fraction λ of simple traders that choose to purchase the signal $\tilde{\theta}$, the expected net benefit from purchasing information to a potential purchaser is:

$$B\left(\lambda\right)=\sqrt{\frac{VAR(\tilde{v}|\tilde{P},\tilde{Q})}{VAR(\tilde{v}|\tilde{\theta},\tilde{P},\tilde{Q})}}-e^{\gamma_S\tau}=\sqrt{\frac{VAR(\tilde{v}|\tilde{P},\tilde{Q})}{\sigma_\varepsilon^2}}-e^{\gamma_S\tau}. \tag{20}$$

That is, the trader benefits more from acquiring information when the variance of the asset's cash flow conditional on the information in the pricing system ($VAR(\tilde{v}|\tilde{P},\tilde{Q})$) is significant relative to the variance conditional on knowing the fundamental $\tilde{\theta}$ ($VAR(\tilde{v}|\tilde{\theta},\tilde{P},\tilde{Q})=\sigma_\varepsilon^2$).

Analyzing the benefit function $B\left(\cdot\right)$ enables us to determine the equilibrium fraction of informed simple speculators, λ^*. If $B\left(0\right)<0$, i.e., a potential buyer does not benefit from becoming informed when no simple traders are informed, then there exists an equilibrium in the information market with $\lambda^*=0$, i.e., where no one purchases information. If $B\left(z\right)>0$, a potential buyer is strictly better off by being informed when all other simple traders are also informed. Then, there is an equilibrium where all simple traders are informed, i.e., $\lambda^*=z$. Given an interior fraction of informed simple traders ($0<\lambda^*<z$), if every potential buyer is indifferent between becoming informed and remaining uninformed, i.e., $B\left(\lambda^*\right)=0$, then that fraction λ^* represents an equilibrium fraction of informed traders.

Our focus here is on a particular feature of the benefit function $B\left(\cdot\right)$. We are interested in its shape, which determines whether information acquisition is a strategic complement or substitute. As we show later, this has implications for equilibrium multiplicity and the potential for price jumps. If $B(\cdot)$ is increasing (decreasing) at $\lambda \geq 0$, then information acquisition is a strategic complement (substitute) at that fraction λ. Under strategic complementarity (substitutability), speculators' incentive to acquire information increases (decreases) in the fraction of informed simple traders. Formally, we have the following definition:

Definition 1 ***[Strategic Complement/Substitute in Information Acquisition]***
If $B'(\lambda) > 0$, then learning is a strategic complement at λ, and if $B'(\lambda) < 0$, then learning is a strategic substitute at λ.

The traditional Grossman-Stiglitz framework exhibits strategic substitutes in information acquisition, as an increase in the proportion of speculators who become informed implies that the price becomes more informative, and so the incentive to produce information, for agents who observe the price, decreases. As we will show, our model with segmented markets may generate an opposite force that gives rise to strategic complementarities in information acquisition. This has important implications, as strategic complementarities lead to multiple equilibria, which are sometimes interpreted as a source of instability in financial markets.

By equation (20), we know that

$$B'(\lambda) > 0 \text{ iff } \frac{\partial VAR(\tilde{v}|\tilde{P},\tilde{Q})}{\partial \lambda} > 0.$$

Given the definition of $VAR(\tilde{v}|\tilde{P},\tilde{Q})$,

$$VAR(\tilde{v}|\tilde{P},\tilde{Q}) = VAR(\tilde{\theta}|\tilde{P},\tilde{Q}) + \sigma_{\varepsilon}^2,$$

and the price-informativeness PI,

$$PI = \frac{VAR(\tilde{\theta})}{VAR(\tilde{\theta}|\tilde{P},\tilde{Q})} - 1,$$

we know that

$$\frac{\partial VAR(\tilde{v}|\tilde{P},\tilde{Q})}{\partial \lambda} > 0 \text{ iff } \frac{\partial PI}{\partial \lambda} < 0.$$

That is, the condition for strategic complementarities in information acquisition is that an increase in the fraction of informed simple traders makes the price system less informative, thereby increasing the incentive of the uninformed traders to become informed. The following proposition uses this observation to derive the ranges of parameters for which our model exhibits strategic complementarities vs. substitutes.

Proposition 5 *Let* $D = \frac{\mu\gamma_S(A\rho\sigma_\varepsilon/\sigma_\eta - 1)}{(1-\rho^2)\gamma_H}$.
(a) If $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$ *(i.e., if* $D > 0$*), then information acquisition is a complement in the region* $[0, \min(D, z)]$ *and a substitute in the region* $[\min(D, z), z]$.
(b) If $(A\rho\sigma_\varepsilon/\sigma_\eta) \leq 1$ *(i.e., if* $D \leq 0$*), then information acquisition is a substitute in the region* $[0, z]$.

The intuition is as follows. In response to the signal $\tilde{\theta}$, informed simple traders trade the simple asset for speculative reasons. At the same time, sophisticated traders trade it both for hedging and for speculative reasons. When $A\rho\sigma_\varepsilon/\sigma_\eta > 1$, their trading of the simple asset is mainly for hedging reasons, and hence in this case, the trades of the informed simple traders respond to $\tilde{\theta}$ in opposite direction to those of the sophisticated traders. Then, as long as the mass of informed simple traders is not very large (i.e., below D), the informativeness of the price is driven by the trades of the sophisticated traders, and adding more informed simple traders, who trade in the opposite direction, reduces the informativeness. This leads to the complementarity in information acquisition, as having more informed simple traders, increases the incentive of other simple traders to become informed.

Note that the intuition is very related to that behind Proposition 2. There, increasing the mass of sophisticated traders might have reduced the informativeness of the price, due to the fact that they were trading against the dominant force of the simple traders. Here, increasing the mass of informed simple traders might reduce the informativeness of the price, since these traders trade against the dominant force of the sophisticated traders. The additional implication is that this leads to complementarities in information acquisition, as then simple traders find it more beneficial to acquire information.

Interestingly, the proposition provides implications as to when complementarities are more likely to arise. This is the case when the sensitivity of the complex asset to information (A) is higher and when the mass of sophisticated traders (μ) is higher. Hence, complementarities will occur when there are more sophisticated financial institutions (hedge funds) with strong investment opportunities that are not accessible to the large public.

5.2 Multiplicity and Price Jumps

We now explore the implications of learning complementarities for equilibrium outcomes in the information market. The following proposition characterizes equilibrium outcomes and reveals that a necessary condition for multiple equilibria in our model is the presence of learning complementarities.

Proposition 6 *If $B(0) < 0$ and $\max_{\lambda \in [0,z]} B(\lambda) > 0$, there will be three equilibria in the information market: $\lambda^* = 0$, $\lambda^* = \lambda_1$, and $\lambda^* = \lambda_2$, where $\lambda_1, \lambda_2 \in (0, z]$ and $\lambda_1 < \lambda_2$, and $\lambda^* = 0$ and $\lambda^* = \lambda_2$ are stable equilibria. Otherwise, there is a unique (stable) equilibrium.*

As the proposition shows, for multiple equilibria to arise, the net benefit from information production must be negative at $\lambda = 0$ and positive at some $\lambda > 0$. Hence, there must be a region where the net benefit from information production is increasing in the mass of agents λ who choose to produce information, i.e., where strategic complementarities exist. We know from Proposition 5 that this happens only when $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$. Note that the condition of $B(0) < 0$ for multiplicity in Proposition 6 indicates that when there are multiple information market equilibria, $\lambda^* = 0$ is always one of them. Moreover, it is a stable equilibrium. The other stable equilibrium can be either an interior equilibrium (as Figure 1 illustrates) or the whole population of simple traders, $\lambda^* = z$. In addition, there is another equilibrium where $\lambda^* > 0$ which is not stable.

Figure 1 illustrates the possibility of multiple equilibria in the information market in the model. The parameter values are set as $\gamma_H = \gamma_S = 2$, $z = 1$, $\mu = 0.5$, $\rho = 0.8$, $A = 2$, $\tau = 0.1158$, and $\sigma_\theta = \sigma_\varepsilon = \sigma_\eta = \sigma_{nv} = \sigma_{n\kappa} = 1$. There are three information equilibria, where the corresponding equilibrium fractions of informed speculators are $\lambda^* = 0$, 0.73, and 0.94. Among these equilibria, the boundary equilibrium ($\lambda^* = 0$) and the larger interior equilibrium ($\lambda^* = 0.94$) are stable, while the other interior equilibrium ($\lambda^* = 0.73$)

is unstable.

FIGURE 1 GOES HERE

The literature (e.g., Barlevy and Veronesi, 2000; Mele and Sangiorgi, 2010) has used switches between these different (stable) equilibria to explain crashes/rebounds (large movements in stock prices) and excess volatility in financial markets. We illustrate this idea in Figure 2 by conducting comparative static analysis with respect to information acquisition cost τ.[13]

FIGURE 2 GOES HERE

In Figure 2, we set $\gamma_H = \gamma_S = 2$, $z = 1$, $\mu = 0.5$, $\rho = 0.8$, $A = 2$, $\bar{v} = 100$, $\bar{x}_v = \bar{x}_\kappa = 10$ and $\sigma_\theta = \sigma_\varepsilon = \sigma_\eta = \sigma_{nv} = \sigma_{n\kappa} = 1$.[14] The left panel of the figure depicts the equilibrium fractions of informed simple traders λ^* against the information acquisition cost τ, while its right panel depicts expected stock prices $E(\tilde{P})$ against τ. The cost of information τ represents a measure of the easiness of collecting information: a proliferation of sources of information about financial markets leads to easier access to information and corresponds to a low value of τ. As τ falls to slightly below 0.116, for example, the number of information market equilibria jumps from one ($\lambda^* = 0$) to 3 ($\lambda^* = 0$ or $\lambda^* \in (0, 1]$) in Figure 2(a). If the information market coordinates in such a way that it always ends up with a stable equilibrium with the largest fraction of informed traders, then in Figure 2(b) there may be a sharp rise in the average price from 68.7 to 75.1, as the information cost drops slightly below 0.116. In addition, as the market switches between different equilibria, stock prices exhibit more volatility than do the underlying fundamentals, leading to excess volatility.

5.3 Comparative Statics

We close this section by discussing some results on the impact of A (the sensitivity of the complex asset to the information) and μ (the size of the sophisticated-traders population) on the equilibrium fraction of simple traders λ^* who produce information. The following

[13] The same idea can be illustrated using comparative static analysis with respect to other parameters.

[14] The results are robust to the choice of parameter values, as long as $(A\rho\sigma_\varepsilon\sigma_\eta) > 1$.

proposition collects some results we can prove analytically in our framework.

Proposition 7 *(i). For a sufficiently large A or μ, the unique equilibrium fraction of informed simple traders is* 0, *i.e.,* $\lambda^* = 0$.

(ii). For a sufficiently small A, the unique equilibrium fraction of informed simple traders (weakly) increases in the sensitivity of the complex asset to information, i.e., $\frac{\partial \lambda^*}{\partial A} \geq 0$.

(iii). For a sufficiently small μ, if $(A\rho\sigma_\varepsilon/\sigma_\eta) \leq 1$, the unique equilibrium fraction of informed traders (weakly) decreases in the size of the sophisticated-traders population, i.e., $\frac{\partial \lambda^*}{\partial \mu} \leq 0$*; if $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$, the maximum equilibrium fraction of informed simple traders (weakly) increases in the size of the sophisticated-traders population, i.e.,* $\frac{\partial \lambda^*}{\partial \mu} \geq 0$.

The combination of these results reveals non-monotone relations between the equilibrium proportion of informed simple traders λ^* and the parameters A or μ. Regarding A, which represents the complexity of the complex asset, we learn from part (ii) of the proposition that, when it is small, the proportion of informed simple traders increases in it. This follows from Proposition 3, according to which, for small values of A, an increase in A decreases the price informativeness. Hence, this strengthens the incentive of simple traders to gather information. Yet, we see in part (i) of the proposition that when A is sufficiently large, asset prices reveal the information $\tilde{\theta}$ to a great extent through the trading activity of sophisticated traders, which in turn leaves little incentive for simple traders to become informed. As a result, no one chooses to acquire information in equilibrium. Similarly, the results about μ, contained in parts (i) and (iii) of the proposition, are consistent with the insights from Proposition 2. Interestingly, for small values of μ, if $(A\rho\sigma_\varepsilon\sigma_\eta) > 1$, an increase in the size of the sophisticated-traders population encourages more information production by simple traders, as it leads to a decrease in price informativeness. This is related to the strategic complementarities identified in the previous subsections.

6 Alternative Settings

To demonstrate the novelty of our proposed theoretical mechanism, we now show that our main results – originating from traders reacting to the same information in opposite directions

– do not follow from more traditional theoretical settings. The first setting we analyze is an otherwise identical model except that the uninformed traders can only observe the price of the simple risky asset, $\tilde{P}$, and cannot observe the complex asset price $\tilde{Q}$. This can be thought of as complete segmentation, in that the price in the complex market is not revealed to people outside the market. The second setting is a case with no segmentation at all, as it assumes that all investors can trade both risky assets.

We show that in both these settings, the confusion, due to traders trading in different directions, does not arise, and hence there are no complementarities. Other results, such as the decrease in informativeness due to an increase in the mass of sophisticated traders, also do not arise. Hence, our theoretical mechanism relies on the setting we describe, which is empirically plausible, where some traders cannot (or find it costly to) trade in certain markets, but yet observe the prices in these markets.

6.1 Uninformed Traders Cannot Observe the Price of the Complex Asset

Now, suppose that the uninformed traders can only observe the price of the simple asset $\tilde{P}$. Following the steps in our main model, we now derive the signal that uninformed traders extract from the price using the orders of the informed traders and noise traders:

$$\mu Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q}) + \lambda Z_I(\tilde{\theta}, \tilde{P}) + \tilde{n}_v, \tag{21}$$

where by equations (5) and (6),

$$\begin{aligned} Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q}) &= \frac{1}{\gamma_H}\frac{\bar{v} + \tilde{\theta} - \tilde{P}}{(1-\rho^2)\,\sigma_\varepsilon^2} - \frac{1}{\gamma_H}\frac{\rho\left(\bar{\kappa} + A\tilde{\theta} - \tilde{Q}\right)}{(1-\rho^2)\,\sigma_\eta\sigma_\varepsilon}, \\ Z_I(\tilde{\theta}, \tilde{P}, \tilde{Q}) &= \frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\gamma_S\sigma_\varepsilon^2}. \end{aligned}$$

Now, using equation (30) in the appendix (which is obtained from the market clearing condition in the complex asset), we rewrite $Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q})$ in terms of $\tilde{P}$ and the noise in the complex asset market $\tilde{n}_\kappa$ as follows:

$$Z_H(\tilde{\theta}, \tilde{P}, \tilde{Q}) = \frac{1}{\gamma_H} \frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\sigma_\varepsilon^2} - \frac{\rho\left(\bar{x}_\kappa - \tilde{n}_\kappa\right)\sigma_\eta}{\mu\sigma_\varepsilon}.$$

Thus, (21) becomes:

$$\begin{aligned} & \mu\left[\frac{1}{\gamma_H} \frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\sigma_\varepsilon^2} - \frac{\rho\left(\bar{x}_\kappa - \tilde{n}_\kappa\right)\sigma_\eta}{\mu\sigma_\varepsilon}\right] + \lambda\frac{1}{\gamma_S}\frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\sigma_\varepsilon^2} + \tilde{n}_v \\ = & \frac{\mu}{\gamma_H}\frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\sigma_\varepsilon^2} - \frac{\rho\left(\bar{x}_\kappa - \tilde{n}_\kappa\right)\sigma_\eta}{\sigma_\varepsilon} + \frac{\lambda}{\gamma_S}\frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\sigma_\varepsilon^2} + \tilde{n}_v, \end{aligned}$$

which is equivalent to the following signal to the uninformed traders:

$$\tilde{s}_P = \tilde{\theta} + \left(\frac{\mu}{\gamma_H\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right)^{-1}\left(\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\tilde{n}_\kappa + \tilde{n}_v\right). \tag{22}$$

Since this is the only price signal that is available to the uninformed traders, the overall informativeness is then given by:

$$PI = \frac{VAR(\tilde{\theta})}{VAR(\tilde{\theta}|\tilde{P})} - 1 = \left(\frac{\mu}{\gamma_H\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right)^2 \frac{\sigma_\theta^2}{VAR\left(\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\tilde{n}_\kappa + \tilde{n}_v\right)}. \tag{23}$$

It is clear that this is increasing in μ and in λ, so an increase in the mass of informed (either sophisticated or simple) traders is always beneficial to informativeness, and there is no strategic complementarity, just like in the traditional Grossman-Stiglitz framework.

The reason is the following. When $\tilde{Q}$ is unobservable to the uninformed traders, they cannot use the two separate signals $\tilde{s}_v$ and $\tilde{s}_\kappa$ in our main setting to infer $\tilde{\theta}$. They only observe one signal from the price $\tilde{P}$, or equivalently, the total order flow in the simple asset, and from that they try to infer the fundamental. Given that they do not observe the price of the complex asset $\tilde{Q}$, they form expectations on it based on the order flow in the simple asset and use this to infer $\tilde{\theta}$. In equilibrium, $\tilde{Q}$ adjusts in a way such that the order flow from sophisticated traders in the simple asset and that from informed simple traders react to the information $\tilde{\theta}$ in the same direction. This shuts down the mechanism generating learning complementarities in the main setting (where uninformed traders observe $\tilde{Q}$ and get confused by the sophisticated traders' hedging-based trades and informed simple traders' speculation-based trades).

The direct implication is that the destabilizing effects of sophisticated traders – in terms of generating complementarities and multiplicity – only arise when simple uninformed traders can observe the price of complex assets.

6.2 Simple Traders Can Trade the Complex Asset

Suppose everyone can trade the two risky assets and can observe the two prices. The demand functions of the sophisticated traders for both assets are still given by equations (3) and (5). A similar argument can also deliver the demand functions of the informed simple traders as follows:

$$K_I\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right) = \frac{1}{\gamma_S} \frac{\bar{\kappa} + A\tilde{\theta} - \tilde{Q}}{\left(1-\rho^2\right)\sigma_\eta^2} - \frac{1}{\gamma_S} \frac{\rho\left(\bar{v} + \tilde{\theta} - \tilde{P}\right)}{\left(1-\rho^2\right)\sigma_\eta \sigma_\varepsilon}, \tag{24}$$

$$Z_I(\tilde{\theta}, \tilde{P}, \tilde{Q}) = \frac{1}{\gamma_S} \frac{\bar{v} + \tilde{\theta} - \tilde{P}}{\left(1-\rho^2\right)\sigma_\varepsilon^2} - \frac{1}{\gamma_S} \frac{\rho\left(\bar{\kappa} + A\tilde{\theta} - \tilde{Q}\right)}{\left(1-\rho^2\right)\sigma_\eta \sigma_\varepsilon}. \tag{25}$$

The order flow from informed and noise traders in the complex asset is:

$$\mu K\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right) + \lambda K_I\left(\tilde{\theta}, \tilde{P}, \tilde{Q}\right) + \tilde{n}_\kappa,$$

which is therefore equivalent to a signal:

$$\tilde{\theta} + \beta_\kappa^{-1}\tilde{n}_\kappa, \text{ with } \beta_\kappa = \left(\frac{\mu}{\gamma_H} + \frac{\lambda}{\gamma_S}\right) \frac{A - \rho\sigma_\eta/\sigma_\varepsilon}{\left(1-\rho^2\right)\sigma_\eta^2}. \tag{26}$$

The order flow from informed and noise traders in the simple asset is:

$$\left(\frac{\mu}{\gamma_H} + \frac{\lambda}{\gamma_S}\right)\left(\frac{\tilde{\theta}}{\left(1-\rho^2\right)\sigma_\varepsilon^2} - \frac{\rho A\tilde{\theta}}{\left(1-\rho^2\right)\sigma_\eta \sigma_\varepsilon}\right) + \tilde{n}_v,$$

which is equivalent to a signal:

$$\tilde{\theta}+\beta_v^{-1}\tilde{n}_v, \text{ with } \beta_v=\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1-A\rho\sigma_\varepsilon/\sigma_\eta}{\left(1-\rho^2\right)\sigma_\varepsilon^2}. \tag{27}$$

Then, the overall informativeness of the price system to uninformed traders is:

$$\begin{aligned} PI &= \frac{VAR(\tilde{\theta})}{VAR(\tilde{\theta}|\tilde{P},\tilde{Q})}-1=\left(\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1-A\rho\sigma_\varepsilon/\sigma_\eta}{\left(1-\rho^2\right)\sigma_\varepsilon^2}\right)^2\frac{\sigma_\theta^2}{\sigma_{nv}^2}+ \\ &\left(\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{A-\rho\sigma_\eta/\sigma_\varepsilon}{\left(1-\rho^2\right)\sigma_\eta^2}\right)^2\frac{\sigma_\theta^2}{\sigma_{nk}^2}. \end{aligned} \tag{28}$$

This is again increasing in μ and in λ, so an increase in the mass of informed traders is always beneficial to informativeness, and there is no strategic complementarity, just like in the traditional Grossman-Stiglitz framework.

The intuition is clear since when everyone can trade both assets, there is no confusion caused by different types of traders trading for different reasons and in different directions in response to information. Hence, this experiment highlights the importance of segmented markets and heterogeneous trading opportunities for our results.

7 Conclusion

We analyze a model where simple traders and sophisticated traders trade in segmented markets. Sophisticated traders trade both a simple and a complex asset, while simple traders have access only to the simple asset. The presence of sophisticated traders and complex assets in this segmented setting may have adverse implications for price informativeness. The key intuition is that sophisticated traders have different trading motives in the simple asset than simple traders, since they sometimes use it for hedging purposes. The diversity of trading motives may reduce the informativeness of the price and increase the cost of capital. This may also lead to complementarities in information production, as having more simple traders trade against the sophisticated traders reduces the informativeness of the price, and induces more traders to produce information.

Our model sheds some light on the nature of modern financial markets, where differences in investor sophistication, complexity of assets, and investment opportunities are very common. We provide a first step in analyzing the implications of these features for market efficiency and trading behavior. More research is required to fully understand various market structures motivated by these features.

Appendix

Proof of Proposition 1:

Using (13) and (3), we have:

$$\frac{1}{\gamma_H}\frac{\rho\left(\bar{\kappa}+A\tilde{\theta}-\tilde{Q}\right)}{\left(1-\rho^2\right)\sigma_\eta\sigma_\varepsilon}=\frac{\rho\left(\bar{x}_\kappa-\tilde{n}_\kappa\right)\sigma_\eta}{\mu\sigma_\varepsilon}+\frac{1}{\gamma_H}\frac{\rho^2\left(\bar{v}+\tilde{\theta}-\tilde{P}\right)}{\left(1-\rho^2\right)\sigma_\varepsilon^2}. \tag{29}$$

Plugging the above expression into the sophisticated traders' demand for the simple asset (equation (5)), we have

$$\begin{aligned} Z_H(\tilde{\theta},\tilde{P},\tilde{Q}) &= \frac{1}{\gamma_H}\frac{\bar{v}+\tilde{\theta}-\tilde{P}}{\left(1-\rho^2\right)\sigma_\varepsilon^2}-\frac{1}{\gamma_H}\frac{\rho\left(\bar{\kappa}+A\tilde{\theta}-\tilde{Q}\right)}{\left(1-\rho^2\right)\sigma_\eta\sigma_\varepsilon} \\ &= \frac{1}{\gamma_H}\frac{\bar{v}+\tilde{\theta}-\tilde{P}}{\sigma_\varepsilon^2}-\frac{\rho\left(\bar{x}_\kappa-\tilde{n}_\kappa\right)\sigma_\eta}{\mu\sigma_\varepsilon}. \end{aligned} \tag{30}$$

Plugging the above expression of Z_H in equation (30), the expression of Z_I in equation (6), and the expression of Z_U in equations (7) and (11), into the market clearing condition of the simple asset, (14), we can solve for the price $\tilde{P}$ as follows:

$$\begin{aligned}
&\left[\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2}+\frac{z-\lambda}{\gamma_S}\frac{1}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}\right]\tilde{P}\\
=\ &\left[\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2}+\frac{z-\lambda}{\gamma_S}\frac{1}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}\right]\bar{v}-\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\bar{x}_\kappa-\bar{x}_v\\
&\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2}\tilde{\theta}+\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}\left(b_v^2\sigma_{nv}^{-2}+b_\kappa^2\sigma_{n\kappa}^{-2}\right)\tilde{\theta}\\
&+\left(\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}b_v\sigma_{nv}^{-2}+1\right)\tilde{n}_v+\left(\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}b_\kappa\sigma_{n\kappa}^{-2}+\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\right)\tilde{n}_\kappa.
\end{aligned}$$

Then, substituting the expression of $\tilde{P}$ into equation (29), we can solve for the expression of $\tilde{Q}$. With a bit more algebra, we get the following coefficients for the price equations in the body of the proposition:

$$\begin{aligned}
a_0 &= \bar{v}-\frac{\bar{x}_v+(\rho\sigma_\eta/\sigma_\varepsilon)\,\bar{x}_\kappa}{L},\\
a_\theta &= \frac{1}{L}\left[\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2}+\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}\left(b_v^2\sigma_{nv}^{-2}+b_\kappa^2\sigma_{n\kappa}^{-2}\right)\right],\\
a_v &= \frac{1}{L}\left[\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}b_v\sigma_{nv}^{-2}+1\right],\\
a_\kappa &= \frac{1}{L}\left[\frac{z-\lambda}{\gamma_S}\frac{VAR(\tilde{\theta}|\tilde{s}_\kappa,\tilde{s}_v)}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}b_\kappa\sigma_{n\kappa}^{-2}+\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\right],
\end{aligned}$$

$$\begin{aligned}
c_0 &= \bar{\kappa}+\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\left(a_0-\bar{v}\right)-\frac{\left(1-\rho^2\right)\sigma_\eta^2\gamma_H}{\mu}\bar{x}_\kappa,\\
c_\theta &= A+\frac{\rho\sigma_\eta}{\sigma_\varepsilon}\left(a_\theta-1\right),\\
c_v &= \frac{\rho\sigma_\eta}{\sigma_\varepsilon}a_v,\\
c_\kappa &= \frac{\rho\sigma_\eta}{\sigma_\varepsilon}a_\kappa+\frac{\left(1-\rho^2\right)\sigma_\eta^2\gamma_H}{\mu},
\end{aligned}$$

with $b_\kappa=\frac{\mu(A/\sigma_\eta-\rho/\sigma_\varepsilon)}{\gamma_H(1-\rho^2)\sigma_\eta}$, $b_v=\frac{\mu(1-A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H(1-\rho^2)\sigma_\varepsilon^2}+\frac{\lambda}{\gamma_S\sigma_\varepsilon^2}$, $L=\left(\frac{\mu}{\gamma_H}+\frac{\lambda}{\gamma_S}\right)\frac{1}{\sigma_\varepsilon^2}+\frac{z-\lambda}{\gamma_S}\frac{1}{VAR(\tilde{v}|\tilde{P},\tilde{Q})}$ and

$VAR(\tilde{v}|\tilde{P},\tilde{Q}) = \left(\sigma_\theta^{-2} + b_v^2\sigma_{nv}^{-2} + b_\kappa^2\sigma_{n\kappa}^{-2}\right)^{-1} + \sigma_\varepsilon^2.$

Finally, after we obtain the two price functions, it is easy to show that the information content in prices is indeed equivalent to the information content in the order flows (i.e., $\left\{\tilde{P},\tilde{Q}\right\} = \{\tilde{s}_\kappa, \tilde{s}_v\}$), which therefore supports the claim of Vives (1995) and demonstrates that the two price functions we find are REE price functions. QED.

Proof of Proposition 2:

We know from (15) that:

$$\frac{\partial PI}{\partial \mu} = \frac{\partial b_v^2}{\partial \mu}\frac{\sigma_\theta^2}{\sigma_{nv}^2} + \frac{\partial b_\kappa^2}{\partial \mu}\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2}.$$

Then, given the definitions of b_v^2 and b_κ^2 in (8) and (9), we get:

$$\begin{aligned}
\frac{\partial PI}{\partial \mu} &= 2b_v\frac{\sigma_\theta^2}{\sigma_{nv}^2}\frac{\partial b_v}{\partial \mu} + 2b_\kappa\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2}\frac{\partial b_\kappa}{\partial \mu} \\
&= 2\mu\left[\frac{(1 - A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2}\right]^2\frac{\sigma_\theta^2}{\sigma_{nv}^2} \\
&\quad +2\mu\left[\frac{(A/\sigma_\eta - \rho/\sigma_\varepsilon)}{\gamma_H\left(1-\rho^2\right)\sigma_\eta}\right]^2\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2} \\
&\quad +\frac{2\lambda}{\gamma_S\sigma_\varepsilon^2}\frac{\sigma_\theta^2}{\sigma_{nv}^2}\frac{(1 - A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2}.
\end{aligned}$$

Thus, $\frac{\partial PI}{\partial \mu} < 0$ if and only if

$$\mu\left[1 + \left(\frac{(A/\sigma_\eta - \rho/\sigma_\varepsilon)\,\sigma_\varepsilon^2}{(A\rho\sigma_\varepsilon/\sigma_\eta - 1)\,\sigma_\eta}\right)^2\frac{\sigma_{nv}^2}{\sigma_{n\kappa}^2}\right] < \frac{\gamma_H\lambda\left(1-\rho^2\right)}{\gamma_S\left(A\rho\sigma_\varepsilon/\sigma_\eta - 1\right)},$$

which requires that $A\rho\sigma_\varepsilon/\sigma_\eta > 1$ and μ is small. QED.

Proof of Proposition 3:

We know from (15) that:

$$
\begin{aligned}
\frac{\partial PI}{\partial A} &= \frac{\partial b_v^2}{\partial A}\frac{\sigma_\theta^2}{\sigma_{nv}^2} + \frac{\partial b_\kappa^2}{\partial A}\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2} \\
&= 2b_\kappa \frac{\mu/\sigma_\eta}{\gamma_H\left(1-\rho^2\right)\sigma_\eta}\frac{\sigma_\theta^2}{\sigma_{nv}^2} + 2b_v \frac{-\mu\rho\sigma_\varepsilon/\sigma_\eta}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2}\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2} \\
&= 2\sigma_\theta^2 \frac{\mu/\sigma_\eta}{\gamma_H\left(1-\rho^2\right)}\left[\frac{b_\kappa}{\sigma_\eta\sigma_{nv}^2} - \frac{b_v\rho}{\sigma_\varepsilon\sigma_{n\kappa}^2}\right] \\
&= 2\sigma_\theta^2 \frac{\mu/\sigma_\eta}{\gamma_H\left(1-\rho^2\right)}\left[\frac{\frac{\mu\left(A\rho\sigma_\varepsilon/\sigma_\eta-\rho^2\right)}{\gamma_H(1-\rho^2)\rho\sigma_\varepsilon\sigma_\eta}}{\sigma_\eta\sigma_{nv}^2} - \frac{\left(\frac{\mu(1-A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H(1-\rho^2)\sigma_\varepsilon^2}+\frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right)\rho}{\sigma_\varepsilon\sigma_{n\kappa}^2}\right].
\end{aligned}
$$

It follows that $\frac{\partial PI}{\partial A} < 0$ for A sufficiently small, and $\frac{\partial PI}{\partial A} > 0$ for A sufficiently large. QED.

Proof of Proposition 4:

We take derivative of L with respect to μ at $\mu = 0$:

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \mu}\right|_{\mu=0} &= \frac{1}{\gamma_H\sigma_\varepsilon^2} + \frac{z-\lambda}{\gamma_S}\frac{\left(\sigma_\theta^{-2}+b_v^2\sigma_{nv}^{-2}+b_\kappa^2\sigma_{n\kappa}^{-2}\right)^{-2}}{VAR^2(\tilde{v}|\tilde{P},\tilde{Q})}\frac{2\lambda/\sigma_{nv}^2}{\gamma_S\sigma_\varepsilon^2}\frac{\left(1-A\rho\sigma_\varepsilon/\sigma_\eta\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2} \\
&= \frac{1}{\gamma_H\sigma_\varepsilon^2} + \left\{\frac{1}{\gamma_H\sigma_\varepsilon^2}\frac{z-\lambda}{\gamma_S}\left[\frac{\left(\sigma_\theta^{-2}+\left(\frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right)^2\sigma_{nv}^{-2}\right)^{-1}}{\left(\sigma_\theta^{-2}+\left(\frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right)^2\sigma_{nv}^{-2}\right)^{-1}+\sigma_\varepsilon^2}\right]^2\frac{2\lambda/\sigma_{nv}^2}{\gamma_S\sigma_\varepsilon^2}\right\}\frac{\left(1-A\rho\sigma_\varepsilon/\sigma_\eta\right)}{\left(1-\rho^2\right)}.
\end{aligned}
$$

It is clear that the above expression is negative when either A is large enough or σ_η is small enough. Hence, in this case, and when μ is sufficiently small, $\frac{\partial CC}{\partial \mu} > 0$. When μ is large, $\frac{\partial PI}{\partial \mu} > 0$, and then it is clear that $\frac{\partial CC}{\partial \mu} < 0$. QED.

Proof of Proposition 5:

Given that

$$
\begin{aligned}
PI &= b_v^2\frac{\sigma_\theta^2}{\sigma_{nv}^2} + b_\kappa^2\frac{\sigma_\theta^2}{\sigma_{n\kappa}^2}, \\
b_v &= \frac{\mu\left(1-A\rho\sigma_\varepsilon/\sigma_\eta\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S\sigma_\varepsilon^2}, \\
b_\kappa &= \frac{\mu\left(A/\sigma_\eta-\rho/\sigma_\varepsilon\right)}{\gamma_H\left(1-\rho^2\right)\sigma_\eta},
\end{aligned}
$$

we know

$$\frac{\partial PI}{\partial \lambda} \propto \frac{\partial b_v^2}{\partial \lambda} = 2b_v \frac{1}{\gamma_S \sigma_\varepsilon^2}.$$

Clearly, $\frac{\partial b_v^2}{\partial \lambda} < 0$ iff $b_v < 0$, i.e., iff $\frac{\mu(A\rho\sigma_\varepsilon/\sigma_\eta - 1)}{\gamma_H(1-\rho^2)\sigma_\varepsilon^2} > \frac{\lambda}{\gamma_S \sigma_\varepsilon^2} \iff \lambda < \frac{\mu\gamma_S(A\rho\sigma_\varepsilon/\sigma_\eta - 1)}{(1-\rho^2)\gamma_H}$. Then, the result stated in the proposition follows directly. QED.

Proof of Proposition 6:

When $(A\rho\sigma_\varepsilon/\sigma_\eta) \leq 1$, function B is decreasing in λ and there is a unique equilibrium in the information market. This is consistent with Proposition 6: the conditions $B(0) < 0$ and $\max_{\lambda \in [0,z]} B(\lambda) > 0$ can never be satisfied simultaneously, and as a result, uniqueness is guaranteed by the proposition.

Consider the case of $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$. In this case, the proof of Proposition 5 demonstrates that function B first increases with λ and then decreases with λ. We have the following three possibilities to discuss.

Suppose $B(0) < 0$ and $\max_{\lambda \in [0,z]} B(\lambda) > 0$. Then function B crosses zero either once or twice in the range of $[0, z]$, depending on the sign of $B(z)$. If $B(z) < 0$, then function B crosses zero twice in the range of $[0, z]$, say, at the values of $\lambda_1 \in (0, z)$ and $\lambda_2 \in (\lambda_1, z)$, implying that there are three information market equilibria: $\lambda^* = 0$, λ_1 and λ_2. Among these three equilibria, 0 and λ_2 are stable, while λ_1 is unstable. If $B(z) \geq 0$, then function $B(\lambda)$ crosses zero once in the range of $[0, z]$, say, at the value of $\lambda_1 \in (0, z)$, and there are still three information market equilibria: 0, λ_1 and z. $\lambda^* = 0$ and $\lambda^* = z$ are stable equilibria, while the interior equilibrium λ_1 is unstable.

Suppose $B(0) > 0$. If, in addition, $B(z) \geq 0$, then $B(\lambda) > 0$ for all values of λ in the range of $(0, z)$, so that there is a unique information market equilibrium – $\lambda^* = z$ – and it is stable. If $B(z) < 0$, then B crosses zero once in the range of $[0, z]$ and this value forms the unique equilibrium. (If $B(z) = 0$, then besides the stable aquarium, $\lambda^* = 0$ is also an equilibrium, but it is unstable.)

Suppose $\max_{\lambda \in [0,z]} B(z) < 0$. Then the only equilibrium is $\lambda^* = 0$ and it is stable. (If $\max_{\lambda \in [0,z]} B(z) = 0$, then besides 0, the value of λ which achieves the maximum is also an equilibrium, but again, it is unstable.) QED.

Proof of Proposition 7:

(i). By the definition of B (equation (20)) and equations (8), (9), (10) and (12), we have:

$$B(\lambda) = \sqrt{\frac{1}{\left(\sigma_\theta^{-2} + \left[\frac{\mu(1-A\rho\sigma_\varepsilon/\sigma_\eta)}{\gamma_H(1-\rho^2)\sigma_\varepsilon^2} + \frac{\lambda}{\gamma_S\sigma_\varepsilon^2}\right]^2 \sigma_{nv}^{-2} + \left[\frac{\mu(A/\sigma_\eta-\rho/\sigma_\varepsilon)}{\gamma_H(1-\rho^2)\sigma_\eta}\right]^2 \sigma_{n\kappa}^{-2}\right)\sigma_\varepsilon^2} + 1} - e^{\gamma_S\tau}. \quad (31)$$

Thus, as $A \to \infty$ or $\mu \to \infty$, $B(\lambda) \to 1 - e^{\gamma_S\tau} < 0$ for any $\lambda \in [0, z]$. As a result, the information market has a unique equilibrium, which is $\lambda^* = 0$.

(ii). When A is sufficiently small, then $(A\rho\sigma_\varepsilon/\sigma_\eta) \leq 1$. By Proposition 5, learning is a substitute and hence there is a unique equilibrium in the information market. By the definitions of B (equation (20)) and price-informativeness PI (equation (15)), we have

$$B(\lambda) = \sqrt{\frac{\sigma_\theta^2/\sigma_\varepsilon^2}{PI + 1}} - e^{\gamma_S\tau}, \quad (32)$$

which means that function B is negatively related to PI. By Proposition 3, $\frac{\partial PI}{\partial A} < 0$. Thus, an increase in A will shift function B up. If the equilibrium fraction λ^* of informed simple traders is determined by $B(\lambda^*) = 0$, then function B is strictly decreasing at the point λ^*, and as a result, λ^* will increase in A through the upward shifting of function B. If λ^* is equal to 0 (when $B(0) < 0$) or z (when $B(z) > 0$), then a small upward shift of function B will not change λ^*.

(iii). Suppose μ is sufficiently small. If in addition, $(A\rho\sigma_\varepsilon/\sigma_\eta) \leq 1$, then by Proposition 2, $\frac{\partial PI}{\partial \mu} > 0$; an argument similar to the above paragraph shows that $\frac{\partial \lambda^*}{\partial \mu} \leq 0$. If $(A\rho\sigma_\varepsilon/\sigma_\eta) > 1$, there might be multiple equilibria in the information market and when this happens, we focus on the largest equilibrium fraction of informed simple traders, and denote it as $\lambda^*_{\max}$. By equation (32) and Proposition 2, we know that an increase in μ will shift function B up. If $\lambda^*_{\max} = z$, then this upward shift will make $\lambda^*_{\max}$ stay at z. If $\lambda^*_{\max} < z$, then we have two possible cases to consider. First, if $B(\lambda^*_{\max})$ is strictly decreasing at the point $\lambda^*_{\max}$, then an increase in μ will increase $\lambda^*_{\max}$ through shifting upward function B. Second, if $B(\lambda^*_{\max})$ is flat at the point $\lambda^*_{\max}$, then an increase in μ will shift the function B up and $\lambda^*_{\max}$ will increase by a discrete amount. QED.

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Figure 1 Learning Complementarities and Multiple Equilibria

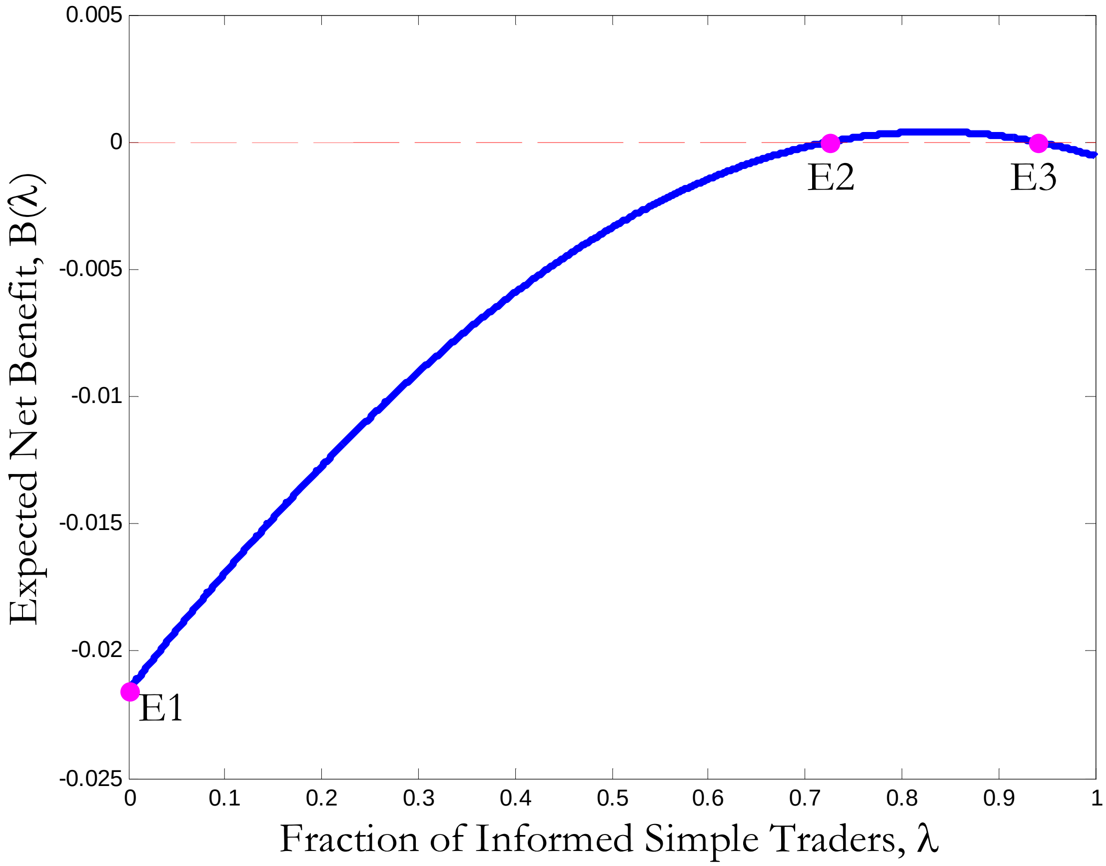

Figure 1 shows the possibility of strategic complementarities in information acquisition and the resulting multiple equilibria in the information market. Under the parameter configuration $\gamma_H=\gamma_S=2$, $z=1$, $\mu=0.5$, $\rho=0.8$, $A=2$, $\tau=0.1158$, and $\sigma_\theta=\sigma_\varepsilon=\sigma_\eta=\sigma_{n\nu}=\sigma_{n\kappa}=1$, there are three equilibrium fractions of informed speculators: $\lambda^*\in\{0, 0.73, 0.94\}$.

Figure 2 Multiplicity and Price Jumps

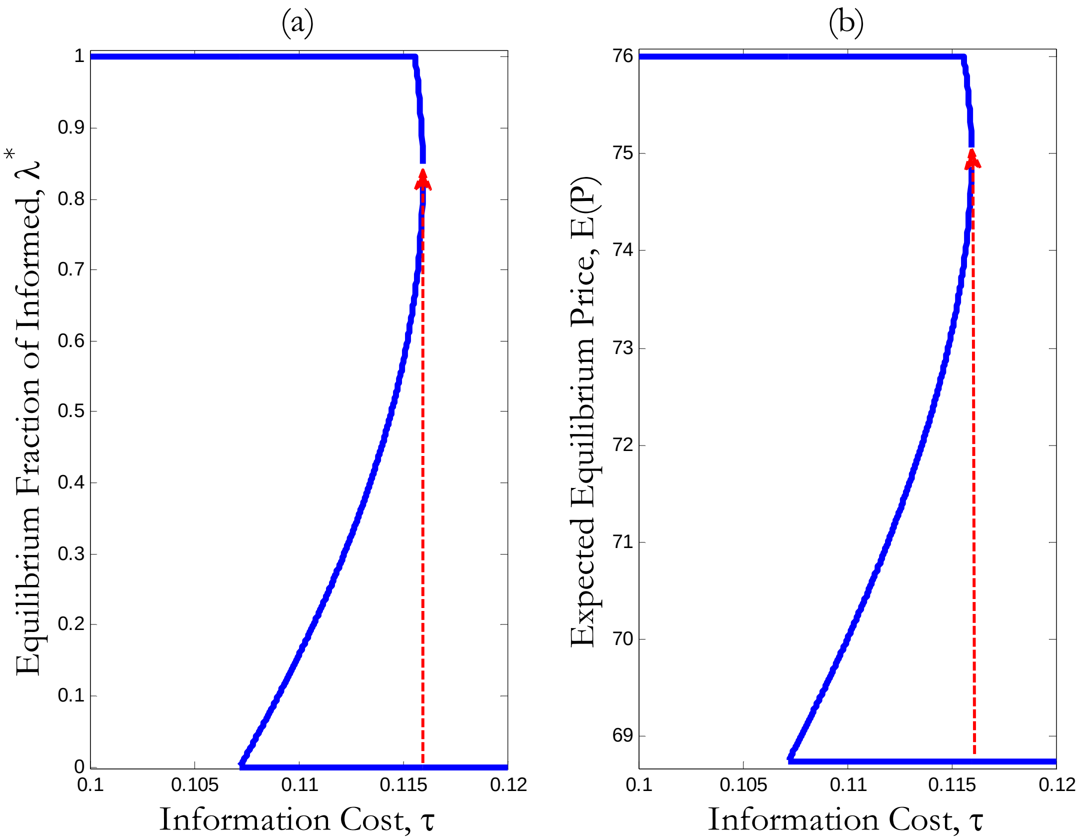

Figure 2 shows the implications for stock prices of changing the information cost τ. The parameters take values: $\gamma_H=\gamma_S=2$, z=1, μ=0.5, ρ=0.8, A=2, $\sigma_\theta=\sigma_\varepsilon=\sigma_\eta=\sigma_{nv}=\sigma_{n\kappa}=1$, $\bar{v}=100$, and $\bar{x}_v = \bar{x}_\kappa=10$.