Strategic Investments, Technological Uncertainty, and Expected Return Externalities*

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Abstract

We study the effect of competition in technological innovation on asset prices. Using a model of an innovation race in which firms face both technological and market-wide uncertainty when they exercise innovation options, we show that a firm's investment in innovation imposes an expected return "externality" on its rivals. In equilibrium, a firm's expected return *decreases* when the firm invests but it *increases* when the rivals invest. Furthermore, the model predicts that a firm's expected return increases as the firm falls behind in the race. We test this unique cross sectional prediction using an economy-wide panel on patenting activity of firms in the U.S. from 1976 to 2006 and find that the prediction is strongly supported in the data. Our analysis suggests that strategic considerations in investments are an important underpinning of the cross-section of returns.

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1 Introduction

The competitive structure of innovation is the most important driving force of firms' R&D investment decisions and, ultimately, of a society's rate of technological progress and economic growth.[1] While competition in innovation has received large attention in economics, little is known about whether and how it impacts asset prices. Investment-based asset pricing models do emphasize the connection between a firm's investment policy and risk, but do not study investment in a strategic setting.[2] If competition affects how firms invest, and if a firm's investment impacts its risk, then we should expect competition to affect the firm's cost of capital and, ultimately, the cross-section of expected returns.

In this paper, we show that strategic interactions between firms' investment decisions affect the cross-section of expected returns because a firm's investment impacts not only its own systematic risk (beta) but also that of its rival. We refer to this interconnectedness between investment and risk across competing firms as expected return "externalities." To assess the importance of such externalities, we compile an economy-wide sample of competitive innovation races over the past three decades and show that a firm's standing in the race is a robust determinant of its beta, as predicted by our theory.

We develop a parsimonious model in which two firms decide when to make an irreversible investment to acquire a monopoly rent—a patent. The patent's value is subject to market-wide systematic risk, and each firm's innovation process is subject to a firm-specific technological risk. The presence of technological risk implies that when a firm invests in innovation, it does not immediately make a discovery and thus does not preclude the other firm from investing at a later time. This allows for the existence of a "cross-section" of firms in which one firm has invested (the leader) while the other has not (the follower). Market-wide risk and irreversibility induce firms to delay their investments, while the patent's winner-takes-all nature tends to erode the option value of waiting. We characterize the equilibrium investment strategies of the competing firms and derive implications for the dynamics of each firm's beta during the innovation race. In

[1] A large body of work on the economics of innovation (see the seminal work of **?**; reviews by **?** or **?**; and analysis by **?** or **?**) and industrial organization (see **?**) shows how the competitive structure of the innovation process affects investment and growth.

[2] The foundations of investment-based asset pricing date back to **?** and have recently received renewed attention starting with the seminal works of **??** and **?**.

equilibrium, a firm's efficiency in innovation determines whether it takes the leader or follower role in the race and its expected return.

We show that, the follower's beta is a function of the leader's innovation success rate—the probability of the leader making a discovery and winning the race—despite the fact that success at discovery is idiosyncratic. This occurs because the leader's innovation success rate affects the leverage of the follower's investment option and hence alters its beta. The model features two types of equilibria: simultaneous equilibria, in which both firms invest at the same time, and leader-follower equilibria, in which the firm with the highest innovation success rate is the first to invest in innovation. When leader-follower equilibria arise, the leader's beta is always smaller than that of the follower and this result generalizes to innovation races with multiple firms.

We test this prediction using a comprehensive firm-level panel of patent application filing and patent award events in the U.S. over 1976-2006 which we create by combining information from five sources: (i) the NBER Patent Data Project, (ii) the Worldwide Patent Statistical Database compiled by the European Patent Office, (iii) the CRSP/Compustat Merged Database, (iv) the CRSP Daily and Monthly Stock Files, and (v) the TAQ database. The key advantage of this dataset is that it allows us to track innovation activity by both the technology fields of innovation and by individual firms over time with links to firm characteristics. We use these two distinct features to define which firms are active in innovation at any point in time and to empirically identify an innovation race. Our dataset is ideal for testing the theory because firms engaged in innovation races in our data compete for monopoly rents which explicitly derive from patents' exclusive rights granted to inventors, as assumed in the model.

We find a strong support for the model's prediction. The equity beta of a firm depends on how its recent patenting success compares to that of the other firms in the race: A firm's beta is lower the closer the firm is to the leading position in the race. This relationship is statistically and economically significant, holds using four alternative measures of firm beta at both monthly and annual data frequencies, and is robust to the fact that some firms are active in multiple races at the same time. We also show that employing alternative definitions of the innovation race, obtained by varying the length of the time over which we measure the race or by broadening/narrowing races' field of innovation, has no impact on the results. Since the firms we identify as active in innovation races account for about 40% of the total U.S. market capitalization over our sample period, our analysis suggests that competition in innovation has an economy-wide implications for the cross-section of expected returns.

Our theoretical analysis combines strategic investing, studied by the industrial organization literature on patent races,[3] with the concepts of investment irreversibility and risk studied by the real options literature in finance.[4] Our model is closely related to **?** who compares optimal timing of investment when two identical firms compete in R&D under different competitive structures. We generalize Weeds' model to allow for heterogeneity between the competing firms and derive explicit closed-form characterizations of the dynamics of the rival firms' risk. More broadly, our paper follows the seminal work of **??** in analyzing the effect of optimal investment decisions on asset prices. The closest papers from this literature are **?** and **?**. **?** derives the risk premia dynamics of two firms engaged in a multi-stage R&D game and numerically documents that risk premia increase when a firm lags behind. We abstract away from the multi-stage nature of competition in innovation and are able to derive closed-form solutions for the dynamics of beta in the innovation race. **?** analyze the risk dynamics of firms that compete in quantities in a product market, and have options to expand and contract production. They show that expected returns of competing firms are indirectly linked through the product market clearing condition and depend on the heterogeneity in the firms' cost structure. In contrast, we show that when firms' productivities are subject to idiosyncratic technological uncertainty, like in the context of competition in innovation, a firm's beta depends *directly* on the productivity of its rival, and this leads to expected return externalities. Finally, in contrast to all papers in this area, we test the predictions of our theory empirically.

Our empirical work relates both to the literature in economics that uses patent data to study firm performance and to the more recent literature in finance that links aggregate technology factors to asset prices. The performance literature documents a positive link between stock market valuation and patents (e.g., **??**) and between stock market valuation and patent citations (e.g., **?** or **?**). The evidence on strategic interactions among firms in innovation races is limited. **?** uses an event-study methodology to estimate the effect of a patent award on rival firms relative to the effect on its recipient. **?** estimate the winning probabilities of incumbents and entrants in pharmaceutical innovation races as a function of the incumbents' and entrants' financial wealth. This literature does not study firm's expected returns.

[3] A partial list of early work in this area includes **?**; **?**; **??**; **??**; **??**; and **?**.

[4] See, for example, **?**; **???**; **????**; **?**; **?**; **??**; **?**; and **?**. See **?** for a survey.

A small set of empirical papers in finance explores the link between industry technological characteristics and asset prices. **?** show that firms in highly concentrated industries are less risky and thereby command lower expected returns. They argue that this finding is either due to barriers to entry in highly concentrated industries which insulates firms from undiversifiable distress risk, or because firms in highly concentrated industries engage in less innovation. Recently, **?** finds that aggregate patent and R&D shocks have predictive power for market returns and premia in the U.S. as well as in other G7 countries. Following this result, **?** construct a technology factor which tracks the changes in technology prospects measured by U.S. patent shocks, and find that this factor helps to explain the cross-sectional variation of **?** portfolios. Unlike these papers, our analysis is at the firm level and hence directly investigates the link between firms' technological characteristics and expected returns.

Our paper makes four contributions. First, we formalize the link from the strategic investment decisions to the firms' systematic risk in industry equilibrium. Second, we quantify the effect of competition in innovation on firms' equity betas. To the best of our knowledge, we are the first to provide firm-level empirical evidence that strategic interactions among firms impact asset prices. Third, we develop a novel empirical methodology to identify innovation races using patent data. Fourth, despite the fact that our explanatory variables are, by construction, unconnected to financial market data, we show that their explanatory power for the cross-section of equity returns is robust and applies to a large part of the U.S. equity market.

The rest of the paper proceeds as follows. In Section 2, we develop a model of an innovation race and derive firms' values and betas. Section 3 describes our sample, defines main variables, outlines methodology, and presents the results. Section 4 concludes. Proofs of all propositions are in Appendix A, details of the empirical analysis are in Appendix B, and tables with robustness results are in Appendix C.

2 Model

In this section, we develop a model of competition in innovation between two firms and characterize their equilibrium investment strategies, values, and risk characteristics.

2.1 The innovation game

Two all-equity financed firms, $i = 1, 2$, have an opportunity to invest in innovation. The first firm to make a discovery is awarded a patent that guarantees exclusive monopoly profits from the commercialization of the innovation (winner-takes-all). Investment in innovation is risky—the discovery is a random event—and the market value of the patent evolves stochastically over time. The competing firms are therefore subject to both idiosyncratic (technological) and systematic (market-wide) risk.

We denote by $x(t)$ the market value of the monopoly profits protected by the patent at time t.[5] We take the process $x(t)$ as exogenous and assume that it evolves according to a geometric Brownian motion

$$dx(t) = \mu x(t)dt + \sigma x(t)dW(t), \tag{1}$$

where $dW(t)$ is the increment of a standard Brownian motion under the true probability measure, μ is the constant drift, and σ is the constant volatility.

Each firm has one option to invest in innovation. It exercises its option by deploying a fixed amount of capital $K > 0$. This investment is irreversible. Once the capital has been deployed, the discovery happens randomly according to a Poisson distribution with constant hazard rate $h_i > 0$. For simplicity, we assume that h_i is not a function of the investment K. Since the investment cost is the same for both firms, the hazard rate h_i measures each firm's efficiency of its innovation effort: the firm with a higher hazard rate is more efficient in innovation.

We model competition in innovation as a *stochastic stopping time game*.[6] Formally, if firm i invests K at time t, it can make a discovery at all dates $s > t$. Once firm i has invested in innovation it cannot make any other action. The game ends when either firm makes a discovery, i.e., acquires the patent. Unlike **???**, the firms in our model do not precommit ex-ante to a specific investment date. Instead, as in **?**, we allow firms to observe and respond immediately to their rivals' investment decisions. As we will show later, this can lead to (subgame perfect)

[5] One can think of $x(t)$ as the present value of the future cash flows from commercialization of the innovation, or, more generally, the market value of the opportunity to commercialize the patent (see **?** for a model in which innovation and commercialization are separate decisions).

[6] See **?** for a formal treatment of stopping time games. **?** and **?** are early applications of stochastic stopping time games to, respectively, foreign direct investments and real estate development.

equilibria in which firms may try to "preempt" each other, while preemptive strategies are not feasible when firms precommit to investment dates.

At each point in time, the *state* of the game is described by the history of the stochastic process $x(t)$ and by whether each firm has invested in innovation. In general, a strategy is a mapping from the set of states to the set of actions: invest or wait. We restrict our attention to *Markov strategies*, i.e., time invariant strategies in which actions depend only on the current level of the variable $x(t)$. Specifically, a firm's strategy is a *stopping rule* characterized by a threshold x^* for $x(t)$ such that the firm invests when $x(t)$ crosses x^* from below for the first time.[7]

A *Markov perfect equilibrium* is the set of strategies such that, in every state, each firm's strategy is value maximizing conditional on the rival's strategy. Section 2.2 characterizes firms' values and investment strategies and Section 2.3 characterizes the Markov perfect equilibria of the innovation game.

2.2 Firms' values and investment strategies

Firms' values are the net present values (NPV) of their risky profits. To evaluate the profits, we assume the existence of a pricing kernel. Following a standard argument (e.g., **?**), we construct a risk-neutral probability measure under which the process $x(t)$ evolves as

$$dx(t) = (r - \delta)x(t)dt + \sigma x(t)d\widehat{W}(t), \quad r > \delta > 0, \tag{2}$$

where $d\widehat{W}(t)$ is the increment of a standard Brownian motion under the risk-neutral probability measure implied by the pricing kernel, r is the risk-free rate, and δ is the opportunity cost of keeping the option to invest in innovation alive.[8] From (1) and (2) we infer that the constant risk-premium associated with the process $x(t)$ is $\lambda \equiv \mu - (r - \delta)$. We assume that $\mu > r - \delta$, implying a positive risk premium $\lambda > 0$.

To insure that no firm has already invested in innovation at the beginning of the game, we require that the initial value of the patent $x(0)$ is sufficiently low so that the NPV of investing

[7] Because $x(t)$ is a Markov process, Markov strategies contain all payoff-relevant information. In general, one cannot exclude the existence of non-Markovian strategies. However, if one firm follows a Markov strategy, the opponent's best response is also Markov (see **?**, Chapter 13, for a formal treatment of Markov equilibria).

[8] By assuming the existence of a pricing kernel exogenously, we implicitly rule out the possibility that any firm's innovation activity alters the state prices in the economy.

at time zero is negative (ignoring any strategic interactions among firms):[9]

$$E\left[\int_0^{\infty} e^{-(r+h_i)t} h_i x(t) dt\right] - K < 0, \quad \text{i.e.,} \quad \frac{h_i\, x(0)}{h_i + \delta} - K < 0, \;\; i = 1, 2. \tag{3}$$

This assumption rules out the multiplicity of equilibria with simultaneous immediate investment considered in **?** although, in general, it does not prevent the existence of other types of simultaneous equilibria, as we will show in Proposition 4.[10]

In principle, there are three possible outcomes of the innovation game: (i) Firm 1 invests first (leader) and firm 2 invests at a later date (follower); (ii) Firm 2 invests first as a leader and firm 1 follows; or, (iii) both firms invest simultaneously. As is standard in dynamic games, we first derive the firms' values and investment strategies associated with these three possibilities, taking the roles of leader and follower as given. We then endogeneize the firms' roles and construct equilibrium investment strategies by comparing the value of investing as a leader, the value of waiting and being a follower, and the value of investing simultaneously. As we will show, in the case of an asymmetric game in which firms differ in their hazard rates, $h_i \neq h_j$, the more efficient firm (high hazard rate) endogenously takes the leadership role, unless firms invest simultaneously.[11]

2.2.1 Follower

The problem of the follower is to determine the optimal time to invest, given that its opponent has already invested. Let firm i be the follower and firm j be the leader and denote by x_i^F the follower's investment threshold. The value of firm i is the solution of the optimal stopping time problem:

$$V_i^F(x) = \max_{\tau_i^F} E\left[e^{-(r+h_j)\tau_i^F}\left(\int_{\tau_i^F}^{\infty} e^{-(r+h_i+h_j)(t-\tau_i^F)} h_i x(t) dt - K\right)\right], \;\; i \neq j, \tag{4}$$

[9]Note that, because the discovery occurs according to the Poisson distribution with hazard rate h_i, its arrival time τ has a negative exponential distribution, $Pr(\tau < t) = 1 - e^{-h_i t}$. Hence, $e^{-h_i t} h_i$ in integral (3) is the density of a negative exponential random variable, i.e., it represents the probability at t of making the discovery in the next dt instant, conditional on no discovery occurring until time t.

[10]In particular, condition (3) rules out equilibria in which firms invest when $x(t)$ decreases, as in the "recession-induced construction booms" studied in **?**.

[11]In a symmetric innovation game, one needs a selection mechanism to determine the role of firms in the game. This selection mechanism requirement an enlargement of the strategy space to allow for mixed strategies in continuous time (see **?**).

where $\tau_i^F = \inf\{t > 0 : x(t) \geq x_i^F\}$ is the stopping time and the expectation is taken under the risk-neutral measure. Since the discovery by firm j occurs with a Poisson arrival rate h_j, $e^{-h_j \tau_i^F}$ in (4) represents the probability that firm j does not make the discovery in the time period $[0, \tau_i^F]$. Moreover, as the discoveries are independent, $e^{-(h_i+h_j)(t-\tau_i^F)} h_i$ is the probability that firm i makes the discovery in the next dt instant given that neither firm was successful before time t.

Notice that the hazard rate of the leader h_j augments the discount rate for the future profits of the follower. This means that higher probability of firm j successfully innovating before i reduces the value of the profits from i's discovery. This is common in R&D models involving a constant Poisson arrival process (e.g., **?**). Furthermore, the expected profits $h_i x(t)$ for firm i upon investing are discounted at a rate that includes the hazard rate of both firms, $r + h_i + h_j$. The following proposition characterizes the solution of the follower's stopping time problem.

Proposition 1. *Conditional on firm j having already invested in innovation, the optimal strategy of firm i is to invest at the threshold*

$$x_i^F = \frac{\phi_j}{\phi_j - 1} \frac{h_i + h_j + \delta}{h_i} K, \quad i \neq j, \tag{5}$$

where

$$\phi_j = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2(r + h_j)}{\sigma^2}} > 1. \tag{6}$$

The value of firm i acting as a follower is

$$V_i^F(x) = \begin{cases} \left(\frac{x}{x_i^F}\right)^{\phi_j} \left[\frac{h_i x_i^F}{h_i + h_j + \delta} - K\right] & \text{if } x < x_i^F \\ \frac{h_i x}{h_i + h_j + \delta} - K & \text{if } x \geq x_i^F \end{cases}. \tag{7}$$

The proposition highlights several aspects of the interactions between the leader's and follower's decisions. First, firm i's NPV, $\frac{h_i x}{h_i + h_j + \delta} - K$, is decreasing in firm j's hazard rate, h_j. Second, as shown in Lemma A.2 in Appendix A, firm i's investment threshold x_i^F is increasing in h_j, i.e., the more efficient the leader (firm j) is, the later the follower (firm i) invests. Finally, the value of the option to wait, determined via the price $(x/x_i^F)^{\phi_j}$ of the Arrow-Debreu security that pays one dollar when the process x first hits the threshold x_i^F, depends on the hazard rate h_j of the leader. These innovation technology "externalities" play a crucial role in determining the required rate of return of the follower, as we discuss later. Panel A of Figure 1 illustrates the results of the above proposition by displaying the value of a firm investing as a follower for two possible hazard rate values.

2.2.2 Leader

We now determine the value of a firm conditional on investing as the leader and anticipating that the follower will respond optimally according to Proposition 1. The value of firm i when it invests as a leader is

$$V_i^L(x) = E\left[\int_0^{\tau_j^F} e^{-(r+h_i)t} h_i x(t) dt\right] + E\left[e^{-(r+h_i)\tau_j^F} \int_{\tau_j^F}^{\infty} e^{-(r+h_i+h_j)(t-\tau_j^F)} h_i x(t) dt\right] - K, \quad (8)$$

where τ_j^F is the time at which the follower (firm j) invests in innovation. The first term captures the expected profits firm i receives before firm j invests, while the second term captures the expected profits it receives after firm j invests. The next proposition characterizes the value of the leader $V_i^L(x)$.

Proposition 2. *Conditional on firm j investing as a follower at the threshold x_j^F from Proposition 1, the value of firm i acting as a leader at the time it invests is*

$$V_i^L(x) = \begin{cases} \frac{h_i x}{h_i+\delta} - \left(\frac{x}{x_j^F}\right)^{\phi_i} \left[\frac{h_i x_j^F}{h_i+\delta} - \frac{h_i x_j^F}{h_i+h_j+\delta}\right] - K & \text{if } x < x_j^F \\ \frac{h_i x}{h_i+h_j+\delta} - K & \text{if } x \geq x_j^F \end{cases}. \quad (9)$$

When the follower has not invested yet, $x < x_j^F$, the value of the leader in (9) has three parts. The first part is the present value of a perpetuity with expected profits $h_i x$ and discount rate $h_i + \delta$. The second part can be thought of as the value of a short position in an option that pays $\left[\frac{h_i x_j^F}{h_i+\delta} - \frac{h_i x_j^F}{h_i+h_j+\delta}\right]$ when x first hits x_j^F. Intuitively, it is as if the leader is shorting this option to the follower who exercises it at the threshold x_j^F. The third part is the fixed investment cost. Lemma A.3 shows that the value of the leader is increasing in its innovation efficiency. Panel B of Figure 1 illustrates the results of the above proposition by displaying the value of a firm investing as a leader for two possible hazard rate values.

2.2.3 Simultaneous investment

A possible outcome of the investment timing game is simultaneous investment. Under the conditions discussed later in Proposition 4, it is possible to sustain equilibria in which firms agree to invest at the same threshold. This happens if the value of each firm from investing simultaneously dominates the value of investing as a leader. The following proposition characterizes the the value of firm i when both firms invest at a pre-specified threshold x^C.

Proposition 3. *The value of firm i when both firms invest at a given threshold x^C is*

$$V_i^C(x;x^C) = \begin{cases} \left(\frac{x}{x^C}\right)^{\phi_0}\left[\frac{h_i x^C}{h_i+h_j+\delta} - K\right] & \text{if } x < x^C \\ \frac{h_i x}{h_i+h_j+\delta} - K & \text{if } x \geq x^C \end{cases}, \quad i \neq j, \tag{10}$$

where

$$\phi_0 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{11}$$

The optimal joint threshold x_i^C from each firm's individual perspective is obtained by maximizing (10) with respect to x^C, yielding

$$x_i^C = \frac{\phi_0}{\phi_0 - 1}\frac{h_i + h_j + \delta}{h_i}K, \quad i,j = 1,2. \tag{12}$$

Notice that if $h_i \neq h_j$ the two firms disagree on the optimal joint threshold. For example, if $h_1 > h_2$ then, from (12), $x_1^C < x_2^C$. This implies that in an equilibrium with simultaneous investment and different levels of technological efficiency, one of the two firms adopts a strategy that is not value maximizing although, as discussed in Proposition 4, no profitable deviations exist.

2.3 Equilibrium investment strategies

Given the value of investing as the leader, the follower, and simultaneously, we can now characterize the set of Markov perfect equilibria of the innovation game. Proposition 4 below shows that there are three types of equilibria: (i) preemptive, (ii) sequential, and (iii) simultaneous. To understand the structure of the equilibrium, let us assume, without loss of generality, that $h_1 > h_2$. From Lemma A.3, this implies that $V_1^L(x) > V_2^L(x)$ and $V_1^F(x) > V_2^F(x)$ for all x.

When firm 2 has no incentive to become the leader, $V_2^F(x) > V_2^L(x)$ for all $x < x_2^F$, then the equilibrium of the *sequential* type. Firm 1 acts as a "designated leader", investing at the threshold x_1^D that it would have chosen if it had the exclusive right to invest first (see equation (A18) in Appendix A). Firm 2 optimally chooses the investment threshold x_2^F.[12]

When both firms have an incentive to become leaders, $V_i^L(x) > V_i^F(x)$, $i = 1, 2$, for some x, then the equilibrium is of the *preemptive* type, with firm 1 acting as the leader and firm 2 as the follower. The preemption threat of firm 2 (the less efficient) induces firm 1 to "retaliate" by investing at a lower threshold. The preemption threat of firm 2 vanishes when firm 2 is indifferent between being the leader and being the follower, which happens at the value x_2^P defined as

$$x_2^P = \inf\{x : V_2^L(x) = V_2^F(x)\} \tag{13}$$

Proposition 4 below shows that at x_2^P, $V_1^L(x_2^P) > V_1^F(x_2^P)$ and therefore firm 1 invests at the threshold equal to the minimum between x_2^P and the investment threshold x_1^D. In both these cases, in a preemption equilibrium, firm 2 optimally invests at the threshold x_2^F.

Finally, a *simultaneous* equilibrium can be sustained if $V_i^L(x) < V_i^C(x; x^C)$, for all x, $i = 1, 2$. These conditions insure that there is no unilateral incentive to deviate from the strategy to invest simultaneously at x^C and therefore an equilibrium involving a joint investment threshold can be sustained. There are infinitely many of these equilibria, depending on the pre-specified threshold x^C. As in **?** and **?**, we reduce the multiplicity of these equilibria by focusing on the Pareto-dominating one, which, as Proposition 4 below shows, is the equilibrium with the optimal joint investment threshold x_1^C for firm 1, derived in equation (12). In fact, if $h_1 > h_2$, the only sustainable joint investment threshold is $x_1^C < x_2^C$, because, under the condition (3) for $x(0)$, firm 1 would always have an incentive to deviate from the alternative joint threshold x_2^C that maximizes firm 2's value. Importantly, the existence of simultaneous equilibria does not rule out the existence of both preemptive and sequential equilibria and hence, in principle, there can be multiple equilibria when $V_i^L(x) < V_i^C(x; x^C)$, for all x, $i = 1, 2$.

The following proposition describes the regions of technological efficiencies h_1 and h_2 for which each of the three types of equilibria described above occur.

[12]Notice that, as shown in Lemma A.4 in Appendix A, if $h_1 > h_2$ there always exists a unique x^L such that $V_1^L(x) > V_1^F(x)$ for all $x \in [x_L, x_2^F]$, while it is possible that $V_2^L(x) < V_2^F(x)$ for all x. This implies that the more efficient firm has always the incentive to be the leader and rules out the case in which neither firms has incentive to lead.

Proposition 4. *Assume $x(0)$ satisfies condition (3) and let $h_1 > h_2$. Let x_2^F, x_2^P, and x_1^D, be the thresholds for the process $x(t)$ defined, respectively, by (5), (13), and (A18) in Appendix A. Then, for every h_1, there exist two thresholds for h_2, $J(h_1)$ and $S(h_1)$, such that the Markov perfect equilibrium is:*

1. *Preemptive, if $h_2 > S(h_1)$, with firm 1 investing at the threshold $x_1^P = \min\{x_1^D, x_2^P\}$ and firm 2 investing at the threshold $x_2^F > x_1^P$.*
2. *Sequential, if $h_2 < S(h_1)$, with firm 1 investing at the threshold x_1^D and firm 2 investing at the threshold $x_2^F > x_1^D$.*
3. *Simultaneous, if $h_2 > J(h_1)$, with both firms investing at the threshold x_1^C.*

We refer to equilibria of the preemptive and sequential types as *leader-follower equilibria*. Figure 2 depicts the regions of different types of equilibria from Proposition 4 in the (h_1, h_2) plane. The solid line is the threshold $J(h_1)$ and the dash-dotted line is the threshold $S(h_1)$. Since we assume $h_1 > h_2$, the relevant region in Figure 2 is the shaded area below the 45-degree line.

Panel A (B) of Figure 2 depicts the case when the volatility of the process x is low (high). Simultaneous equilibria are more likely to occur when volatility is high. Intuitively, the higher volatility, the more valuable is the option to wait, and the less incentive the leader has to preempt by investing early. For sufficiently low levels of volatility (Panel A), the threshold $J(h_1)$ is always above the 45-degree line and simultaneous equilibria do not occur. From the threshold $S(h_1)$, we infer that when h_2 is sufficiently smaller than h_1, firm 2 has no interest to become the leader and hence the equilibria are sequential. As h_2 increases and crosses the threshold $S(h_1)$, firm 2 has an incentive to become the leader, and preemptive equilibria ensue. Finally, as discussed in **?**, note that on the 45-degree line, when $h_1 = h_2$, there are only preemptive or simultaneous equilibria.

2.4 Equilibrium firm values and betas

Given the characterization of the equilibria in Proposition 4 we derive firm values and betas for each equilibrium type.

Proposition 5. *Let $V_i^F(x)$ and $V_i^L(x)$, $i = 1, 2$, be given by Propositions 1 and 2, respectively, and let $h_1 > h_2$.*

1. *In a leader-follower equilibrium, the value of the leader,* V_1^{LF} *and of the follower,* V_2^{LF} *are:*

   (a) *If* $x < x_1^P$

$$V_1^{\mathrm{LF}}(x) = \left(\frac{x}{x_1^P}\right)^{\phi_0} V_1^L(x_1^P) \ \ \textit{and} \ \ V_2^{\mathrm{LF}}(x) = \left(\frac{x}{x_1^P}\right)^{\phi_0} V_2^F(x_1^P), \tag{14}$$

   where ϕ_0 *is given in (11).*

   (b) *If* $x_1^P < x < x_2^F$

$$V_1^{\mathrm{LF}}(x) = \frac{h_1 x}{h_1+\delta} - \left(\frac{x}{x_2^F}\right)^{\phi_1}\left[\frac{h_1 x_2^F}{h_1+\delta} - \frac{h_1 x_2^F}{h_1+h_2+\delta}\right] \tag{15}$$

$$V_2^{\mathrm{LF}}(x) = \left(\frac{x}{x_2^F}\right)^{\phi_1}\left[\frac{h_2 x_2^F}{h_1+h_2+\delta} - K\right] \tag{16}$$

   (c) *If* $x > x_2^F$

$$V_1^{\mathrm{LF}}(x) = \frac{h_1 x}{h_1+h_2+\delta} \ \ \textit{and} \ \ V_2^{\mathrm{LF}}(x) = \frac{h_2 x}{h_1+h_2+\delta}, \tag{17}$$

   where ϕ_1 *is given in (6),* $x_1^P = \min\{x_1^D, x_2^P\}$ *in a preemptive equilibrium, and* $x_1^P = x_1^D$ *in a sequential equilibrium, with* x_2^P *given by (13) and* x_1^D *given by (A18) in Appendix A.*

2. *In a simultaneous equilibrium, the value of each firm* $V_i^{\mathrm{S}}(x)$ *is*

$$V_i^{\mathrm{S}}(x) = \begin{cases} \left(\frac{x}{x_1^C}\right)^{\phi_0}\left[\frac{h_i x_1^C}{h_1+h_2+\delta} - K\right] & \textit{if } x < x_1^C \\ \frac{h_i x}{h_1+h_2+\delta} & \textit{if } x > x_1^C \end{cases}, \quad i = 1, 2, \tag{18}$$

   where x_1^C *is defined in (12).*

Note that when the investment takes place the values of both firms increases discontinuously. This happens because at the time of investing the option to invest is converted into assets in place and we assume that the investment cost K is financed through influx of new equity capital.[13]

To determine the risk premium demanded by each competing firm, we use the fact that systematic risk β_i of firm i can be expressed as the elasticity of the firm's value with respect to

[13] If, for example, K were financed through debt, the firm values would need to be adjusted to incorporate the present value of the liability cash flows.

x,

$$\beta_i = \frac{dV_i(x)}{dx} \frac{x}{V_i(x)}. \tag{19}$$

Hence, the instantaneous expected return of firm i can be expressed as[14]

$$E[R_i] = r + \beta_i \lambda, \tag{20}$$

where λ is the risk premium of the process x. Note that β_i in expression (20) is *not* the CAPM beta because our model is silent about the systematic risk structure of the process x. For ease of exposition, we nevertheless refer to the quantity in (19) as the "equity beta" because, in our setting, this is the only determinant of equity risk.[15] In Section 3.1.5 we discuss how we measure the process x in our empirical analysis.

Using the expressions for the firms' values in the different equilibria from Proposition 5, we obtain the following characterization of the firms' betas.

Proposition 6. *Let β_i be the measure of systematic risk of firm i defined in equation (19).*

1. *In a leader-follower equilibrium, the beta of the leader, β_1^{LF}, and of the follower, β_2^{LF} are*

   (a) *If $x < x_1^P$, $\beta_1^{\mathrm{LF}}(x) = \beta_2^{\mathrm{LF}}(x) = \phi_0 > 1$, where ϕ_0 is given in (11).*

   (b) *If $x_1^P < x < x_2^F$,*

$$\beta_1^{\mathrm{LF}}(x) = 1 - \omega(x)(\phi_1 - 1) < 1 \tag{21}$$

$$\beta_2^{\mathrm{LF}}(x) = \phi_1 > 1, \tag{22}$$

where ϕ_1 is given in (6), $\omega(x) = \frac{b(x)}{a(x)-b(x)} > 0$, with $a(x) = \frac{h_1 x}{h_1+\delta}$, and $b(x) = \left(\frac{x}{x_2^F}\right)^{\phi_1} \left[\frac{h_1 x_2^F}{h_1+\delta} - \frac{h_1 x_2^F}{h_1+h_2+\delta}\right]$.

[14] Expression (20) is obtained from the evaluation equation under the risk-neutral measure, $E[dV_i(x)] = rV_i(x)dt$. Applying Itô's lemma we get $1/2V''x^2\sigma^2 + V'(r-\delta)x = rV$. The instantaneous expected return is $E[R_i] = E[dV_i]/V_i$, where the expectation is taken under the physical measure. Applying Itô's lemma and using the valuation equation to express $1/2V''x^2\sigma^2$ we obtain $E[R_i] = r + \beta_i\lambda$, where β_i is given by (19) and $\lambda = \mu - (r - \delta)$.

[15] Note that there is a one-to-one correspondence between the (conditional) CAPM beta and our equity beta measure β_i, which are linked through the covariance of the process x with the pricing kernel in the economy. The expected return on equity may then be further expressed as

$$E[R_i] = r + \beta_i \cdot \rho \cdot \sigma \cdot SR,$$

where SR is the maximum Sharpe ratio attainable in the economy and $-\rho$ is the correlation of the patent value process x with the stochastic discount factor in the economy. This implies that the risk premium λ associated with the process x is $\lambda = \rho \cdot \sigma \cdot SR$ (see **?**).

(c) If $x > x_2^F$, $\beta_1^{\text{LF}}(x) = \beta_2^{\text{LF}}(x) = 1$.

In a preemptive equilibrium, $x_1^P = \min\{x_1^D, x_2^P\}$*, and, in a sequential equilibrium,* $x_1^P = x_1^D$*, with* x_1^D *and* x_2^P *given by (A18) and (13), respectively.*

2. *In a simultaneous equilibrium, both firms have the same systematic risk*

$$\beta_i^{\text{S}}(x) = \begin{cases} \phi_0 & \text{if } x < x_1^C \\ 1 & \text{if } x > x_1^C \end{cases}, \quad i = 1, 2, \tag{23}$$

where x_1^C *is defined in (12).*

The proposition formalizes how equilibrium investment strategies affect firms' systematic risk. In leader-follower equilibria, the leader's beta *decreases* from $\phi_0 > 1$ to $1 - \omega(x)(\phi_1 - 1) < 1$ when it invests, while, at the same time, the follower's beta *increases* from ϕ_0 to $\phi_1 > \phi_0$. Note that both firms have identical beta before the leader invests. This is a consequence of the fact that there are no "assets in place" and the beta refers uniquely to the "growth option" associated with investing in innovation.[16]

The drop in the leader's beta is a familiar result: the option to innovate is a levered asset and by exercising it, its riskiness is reduced. In a non-strategic case, the leader's beta would drop from ϕ_0 to the beta of the underlying profit which is equal to 1. In the case of a leader-follower equilibrium, the leader's beta is affected by the follower's option to invest later at the threshold x_2^F. Specifically, for values of the process x between x_1^P and x_2^F the leader is in a de facto monopoly position because it can make a discovery while the follower, who has not invested yet, cannot. However, in the presence of technological uncertainty, the follower's equilibrium value V_2^{LF} is a *long* position in an innovation option (see equation (16)) while the leader's equilibrium value V_1^{LF} is a portfolio composed of asset in place, worth $a(x)$, and a *short* position in an innovation option, worth $b(x)$ (see equation (15)). Both option positions have identical betas (elasticities) equal to ϕ_1. The short position in the innovation option pushes the leader's beta below the beta of the underlying process x. Proposition 6 shows that, after firm 1 invests, the follower's beta is $\phi_1 > 1$ and is not affected by x while the leader's beta drops below 1 and is *decreasing* in x. This happens because, after firm 1 invests, changes in x affect the probability

[16]This is consistent with the model of product market competition studied by **?**: In the absence of assets in place, betas of leader and follower are identical before the leader exercises. In the presence of assets in place, the beta of the leader is higher that the beta of the follower before investing (see their Propositions 5 and 7).

of firm 2 to innovate, and hence succeed, while do not alter the probability of firm 1 to succeed. In other words, as x increases, the value $b(x)$ of the short option position of the leader increases. This results in a higher weight $\omega(x)$ and, from (21) a lower value of the leader's beta.

The follower's beta increases because its option to invest in innovation becomes less valuable and more "levered" when the competitor invests and starts the discovery process. This can be seen by analyzing the value of the follower in equation (16). As the process x crosses the threshold x_1^P at which the leader invests, the follower's value $V_2^{\mathrm{LF}}(x)$ becomes more convex in x (since $\phi_1 > \phi_0$). In other words, the increase in the follower's beta comes from an increase in the sensitivity of the follower's value to changes in the process x. The leader's decision to invest in innovation imposes an externality on the follower, and this externality takes the form of adding extra "leverage" to the follower's option to innovate.

Figure 4 plots an example of a simulated path of length $T = 240$ periods for the process x in (1) and the corresponding betas in a leader-follower equilibrium computed according to Proposition 6. The times τ_1 and τ_2 indicate the stopping times at which the process x first reaches the equilibrium investment thresholds of the leader, x_1^P, and the follower, x_2^F. The process x starts at a level below x_1^P. For $t < \tau_1$ the spread between betas is zero. For $t \in [\tau_1, \tau_2]$, the spread between betas is positive and for $t \in [\tau_2, T]$ the spread is again zero.

In summary, our model of a two-firm innovation race predicts that the beta of the leader is always (weakly) smaller than that of the follower in all equilibria we consider and for all x. Importantly, in leader-follower equilibria, the beta of the leader is strictly smaller for $x \in [x_1^P, x_2^F]$.

2.5 Innovation efficiency and firms' betas

In this subsection, we analyze the effect of a change in the firms' innovation efficiencies (hazard rate) on systematic risk. Figure 3 shows the betas for the leader and the follower in leader-follower equilibria, derived in Proposition 6. Panel A analyzes the case of the follower "catching up", i.e., the hazard rate of the leader is set to $h_1 = 0.1$ and we consider three levels of $h_2 = \{0.06, 0.08, 0.09\}$. Panel B analyzes the case of the leader "pulling ahead", i.e., the hazard rate of the follower is set to $h_2 = 0.1$ and we consider three levels of $h_1 = \{0.11, 0.13, 0.15\}$. In both panels, the bottom part of the graph plots the beta of the leader and the top part plots the beta of the follower (equation (21) in Proposition 6). Figure 3 highlights that as h_1 and h_2 change, the investment thresholds change. Specifically, both x_1^P and x_2^F (i) decrease with h_2 for a given level of h_1, and (ii) increase with h_1 for a given level of h_2.

Panel A of Figure 3 shows that, as h_2 increases, the upper bound of the beta of the follower is unaffected, while the lower bound of the leader's beta decreases. To understand the behavior of the leader's beta in Panel A for $x \in [x_1^P, x_2^F]$, note that, from equation (15), the leader's value $V_1^{\text{LF}}(x)$ is a portfolio of assets in place, $a(x)$, and a short position in the option to innovate, $b(x)$. From equation (21), the leader's beta for $x \in [x_1^P, x_2^F]$ is given by $1 - \omega(x)(\phi_1 - 1)$, where $\omega(x) = b(x)/(a(x) - b(x)) > 0$ stands for the fraction of the value V_1^{LF} represented by the innovation option. As h_2 increases, the follower's innovation option $b(x)$ becomes more valuable, and, since $a(x)$ does not depend on h_2, $\omega(x)$ increases. This, together with the fact that a change in h_2 does not affect ϕ_1 imply that an increase in h_2 reduces the beta of the leader.

Panel B of Figure 3 shows that, as h_1 increases, the upper bound of the follower's beta increases. According to Proposition 6, in leader-follower equilibria, the beta of the follower increases with the innovation efficiency of the leader because this makes the follower's option to innovate more sensitive to the underlying process x. This effect can be seen by inspecting the expression for the follower's beta β_2^{LF} in equation (21). From equation (6), we see that ϕ_1 increases with the leader's hazard rate h_1 and hence β_2^{LF} increases with h_1 as well.

The implications of a change in h_1 on the beta of the leader in Panel B are more subtle because a change in h_1 has both a direct and an indirect effect. An increase in h_1, keeping $\omega(x)$ in equation (21) fixed, implies an increase in ϕ_1 and hence a decline in β_1^{LF} (direct effect). However, as h_1 increases, $\omega(x)$ changes as well. In particular, a higher h_1 reduces the value of the innovation option $b(x)$ held by the follower and increases the value $a(x)$ of the leader's expected discounted profits from the patent, causing $a(x) - b(x)$ to increase and $\omega(x)$ to decrease. A decrease in $\omega(x)$ causes an increase in β_1^{LF} (indirect effect). It is not clear, a priori, which of the two effects prevails.

In summary, because changes in innovation efficiencies affect equilibrium investment thresholds and have mixed implications for the leader's beta, comparative statics on innovation efficiencies do not lead directly to testable predictions.

2.6 Values and betas in the N-firm case

Our goal is to assess the size of the expected return externality effect empirically. As innovation races have typically many participants, it is important to analyze the properties of betas in an innovation race with multiple firms. The construction of the full set of equilibria in an N-firm game provides little guidance for our empirical analysis because the solution involves identifying all the possible subsets of firms investing in either simultaneous of leader-follower equilibria.[17] Instead, we can obtain testable predictions by focusing on leader-follower equilibria. In the next proposition, we derive firms' betas in a N-firm game under the assumption that firms are in a leader-follower equilibrium.

Proposition 7. *Suppose N firms have hazard rates $h_1 > h_2 > \ldots > h_N$ and $x_1 < x_2 < \ldots < x_N$ are the investment thresholds in a leader-follower equilibrium of a N-firm game. The beta of firm $m = 1, \ldots, N$ is*

$$\beta_m^{\text{LF}}(x) = \begin{cases} \phi_0 & \text{if } x < x_1 \\ \begin{array}{ll} \phi_n & \text{for } n < m \\ 1 - \omega_{m,n}(x)(\phi_n - 1) & \text{for } n \geq m \end{array} & \text{if } x_{n-1} < x < x_n,\ n = 2, \ldots, N-1 \\ 1 & \text{if } x > x_N \end{cases} \tag{24}$$

where $\omega_{m,n}(x) > 0$ is defined in (A38), ϕ_0 is defined in (11), and

$$\phi_n = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2\left(r + \sum_{i=1}^{n} h_i\right)}{\sigma^2}} > 1,\ n = 1, \ldots, N-1. \tag{25}$$

Equation (24) is a generalization of equation (21). The proposition implies that, for any pair (i, j), of firms in leader-follower equilibria with $h_i > h_j$, the two firms (i) either have the same beta, $\beta_i^{\text{LF}} = \beta_j^{\text{LF}}$, or (ii) the more efficient firm has strictly lower beta $\beta_i^{\text{LF}} < \beta_j^{\text{LF}}$. This holds for any given value of x. Furthermore, from equation (25), the maximum beta across all possible realization of x of a firm with innovation efficiency h_n is increasing in n.

[17] For example, **?** characterize equilibria investment thresholds in an oligopoly product market games without technological uncertainty.

2.7 Model predictions

Our model explains how technological characteristics of firms in an innovation race determine the roles the firms play in the race and derives a relationship between firms' roles and betas associated to each role. Specifically, Proposition 4 shows that, unless firms invest simultaneously, the technologically more efficient firm takes a leadership role while the less efficient firm invests as a follower. In Proposition 6, we further show that the leadership role is always associated with (weakly) lower beta compared to the beta of the follower. Moreover, Proposition 7 shows that in leader-follower equilibria with N firms investing according to their innovation efficiencies, for any pair of firms, the more efficient one has always weakly lower beta.

These results lead to our main testable prediction: The spread between the betas of the firms in an innovation race increases in the distance between their relative positions in the race. In our empirical investigation, we examine this prediction controlling for some of the alternative determinants of firms' betas.

3 Empirical analysis

In this section, we describe our data, define the sample of innovating firms, explain how we measure the relative positions of firms in a race and their betas, introduce our regressions, and present summary statistics. Our main results are described in Section 3.2.

3.1 Sample formation, variable definitions, and methodology

3.1.1 Data sources

We rely on data from five sources: (i) the NBER Patent Data Project (January 2011), (ii) the Worldwide Patent Statistical Database (PATSTAT, April 2008) compiled by the European Patent Office, (iii) the CRSP/Compustat Merged Database, (iv) the CRSP Daily and Monthly Stock Files, and (v) the Trade and Quote (TAQ) database.

The NBER Patent Data Project provides data about all utility patents[18] awarded by the U.S. Patent and Trademark Office (USPTO) over the period 1976-2006. Among other variables, the NBER project contains, for each patent, a unique patent number, patent assignee names

[18]According to the U.S. Patent Law (35 U.S.C. §101) utility is a necessary requirement for patentability and is used to prevent the patenting of inoperative devices. In our analysis, we do not use plant patents, i.e., patents for new varieties of plants.

matched to firms in Compustat (a patent number-GVKEY link), and a patent's technology field defined according to the standards of the International Patent Classification (IPC) system. The original matching of patent assignees, by name, to firms in Compustat is done by **?**. Since then, the matching has been updated using multiple manual and computer generated matches (see **?** for details).

The PATSTAT database contains information from patent documents submitted to and issued by the USPTO including the exact day when an application for each patent was filed and the day when each patent was awarded. We merge the NBER and the PATSTAT databases to create, for each firm, a day-by-day time series of patent filing and award events.

The key advantage of the resulting dataset is that it allows tracking of innovation activity over time by technology fields as well as by firms. We use these two features to distinguish firms active in innovation and to measure the relative innovation efficiency of firms in a race.

3.1.2 Sample of innovating firms

[JB]• To test the prediction of our theory, we first need to form a sample of firms that are active in innovation. Specifically, our sample has to consist of firms with non-zero innovation efficiency (h_i from our model), while excluding firms inactive in innovation as the expected return externality effect cannot arise among such firms. Furthermore, from the set of firms active in innovation, we need to identify subsets of firms that compete for the same monopoly rents—patents. • [JB]

<JB> ~~When forming our sample~~ [replaced by] To this end <JB> , we use the fact that each patent is classified into the area of technology according to the IPC system. The hierarchical structure of IPC is made up of a section, class, subclass, main group, and subgroup. There are eight sections in the first-level of IPC, about 400 classes in the second-level, and about 650 subclasses in the third-level.[19] **[JB]• We assume that firms that actively pursue patents in a given technology field of innovation compete for the same monopoly rent as in**

[19]The IPC classification system is created under the Strasbourg Agreement (1971) and is updated on a regular basis by a committee of experts, consisting of representatives of the contracting states of that agreement. It is used as a search tool for the retrieval of patent documents by intellectual property offices and it serves as a basis for investigating the state of the art in a given area of technology by patent examiners. A patent examiner assigns a classification to the a patent (or a patent application) at the most detailed level of the IPC hierarchy which is applicable to its content. See **?** for details (`http://www.wipo.int/export/sites/www/classifications/ipc/en/guide/guide_ipc_2009.pdf`).

our model. • [JB] We use the second-level of the IPC classification as our main proxy for the technology field of innovation and we refer to it as 'technology class.'

<JB> In particular, <JB> we denote a firm to be 'innovating' in technology class k at month t if the firm has been awarded at least one patent in technology class k during the last τ months (including t), and has filed one or more patent applications in technology class k in at least θ percent of months over the same τ-months period. Specifically, we create a dummy variable $D_{ikt}^{\text{innovating}}$ that is equal to 1 if firm i is innovating in technology class k at month t and is equal to 0 otherwise. Formally,

$$D_{ikt}^{\text{innovating}} = \begin{cases} 1 & \text{if } \sum_{s=t-\tau+1}^{t} D_{iks}^{\text{application}} \geq \theta \times \tau \text{ and } \sum_{s=t-\tau+1}^{t} D_{iks}^{\text{patent}} \geq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

where $D_{iks}^{\text{application}}$ (D_{iks}^{patent}) is equal to 1 if firm i files one or more patent applications (is awarded one or more patents) in technology class k at month s and 0 otherwise. Definition (26) relies on a patent application date <JB> ~~, which we take to be a proxy for the date when a firm is active in innovation~~ [replaced by] as it is the earliest date at which one can observe that a firm is innovating in a given technology class <JB> . **[JB]• Since, in our data on average, patents are awarded in two to three years after applying, firms are actively pursuing patents for a long time before they are awarded.[20] Finally, as there are about 3 mil. patents issued by the USPTO over the period covered by the NBER patent data and hence filing a single patent application is not a significant event, we require a minimum intensity of filing, which we capture using threshold $\theta \times \tau$. This is to ensure that firms inactive in innovation or those with very low innovation efficiency do not enter our sample. • [JB]** <JB> ~~Patents are awarded and the monopoly rent is secured in two to three years after applying, on average, so there is a delay between investing in R&D and winning the patent, as in our model. Also, at the time of applying, the firm does not know if the patent will be awarded, which we modelled as innovation being subject to technological risk.~~ <JB>

To form our sample, we begin with all firms in the CRSP/Compustat Merged Database in the 1976-2006 period when NBER patent data are matched to firms in Compustat. We then form a subset of firms that are innovating according to definition (26) in at least one technology

[20] At the time of investing in R&D, a firm does not know if it will have enough subject matter to file a patent application and if the patent will be awarded, which we modelled as R&D being subject to technological risk.

class and month over this period. Using this subset of firms, we create firm-technology class-month dataset that contains, for each firm, all technology class-month pairs in which the firm is innovating.**[JB]• This footnote is new. • [JB]**[21] Following these steps, we create two samples of innovating firm: The *1-Year Sample* with parameters $\tau = 12$ and the *3-Year Sample* with $\tau = 36$. In both samples we take $\theta = 20\%$.

3.1.3 Firm's relative innovation efficiency and position in a race

Our goal is to investigate the effect of a firm's relative innovation efficiency and position in a race on its beta. We start by presenting empirical proxies for these concepts and we describe our regression specifications next.

In the sample of innovating firms, we refer to a situation when there are two or more innovating firms in the same technology class and at the same month as the innovation race in technology class k at month t. For brevity, we refer to such race as 'race k.'

To measure the relative innovation efficiencies of firms in race k, for each innovating firm i in technology class k, we compute the relative amount of patenting output firm i has achieved in technology class k during the last τ-months period. Specifically, for innovating firm i in technology class k at month t, we compute the fraction h_{ikt} of the total number of patents that have been awarded in technology class k during the last τ months to firm i. Formally,

$$h_{ikt} = \frac{s \sum_{s=t-\tau+1}^{t} P_{iks}}{\sum_{\{j: D_{jkt}^{\text{innovating}}=1\}} \sum_{s=t-\tau+1}^{t} P_{jks}}, \tag{27}$$

where P_{iks} is the number of patents awarded to firm i in technology class k at month s, and $D_{jkt}^{\text{innovating}}$ is defined in (26).

Using the fraction h_{ikt}, we order[22] the innovating firms in race k in a decreasing order and assign a race-month-specific percentile rank n_{ikt} to each innovating firm. Specifically, denoting the number of innovating firms in race k as N_{kt}, we assign rank $1/N_{kt}$ to the firm with the highest innovation efficiency in race k, rank $2/N_{kt}$ to the firm with the second highest innovation

[21] Note that the ability to use patent data is central to our empirical analysis. This is because the data allows to identify the field(s) of technology in which a firm is innovating and hence it allows to determine the sets of firms that are competing with each other for the same monopoly rents. This cannot be achieved using R&D spending data, which are reported as an aggregate dollar amount without distinguishing the nature and category of the investment. Also, as firms covered by Compustat do not have to disclose R&D, only about one third of firms do so and, as a result, there are firms that actively patent while do not report any R&D.

[22] In case two firms have the same h_{ikt}, we refine the ordering using the number of patent applications filed over the same τ-months period.

efficiency, and so on until the least efficient innovating firm is assigned rank 1. As a result, n_{ikt} denotes the relative (ordinal) position of firm i in race k computed using its recent patenting success. We use the percentile rank n_{ikt} as the key explanatory variable in our regressions.

[JB]• We prefer the percentile ranking of firms in a race to an analogous unscaled ranking so that we make sure that the effect of the relative position of a firm in a race on the firm's beta is fully separated from the possible effect of the number of firms in the race on the average beta of the firms in the race. While our theory predicts that the relative ranking of firms in the race affects the firms' betas, it is silent about how the average beta of the firms in the race depend on the number of race participants.[23] • [JB]

3.1.4 Beta and firm characteristics

Our regressions follow directly from the model's prediction summarized in Section 2.7. To account for the fact that the firms in our sample may be innovating in multiple races at the same time,[24] possibly with different positions in each race, we present two alternative regression specifications.

First, we estimate a firm-level regression in which the main explanatory variable is the average percentile rank $\overline{n_{it}}$ computed across all races in which firm i is innovating at month t

$$\beta_{it} = \lambda_0 + \lambda_1 \overline{n_{it}} + \lambda_2 X_{it} + FE_{(\cdot)} + \epsilon_{it}. \tag{28}$$

In equation (28), β_{it} represents the equity beta of firm i at month t, and hence we effectively think of a firm's equity beta as a portfolio of multiple betas, each corresponding to the specific position firm i has in each race. X_{it} stands for firm-level control variables and $FE_{(\cdot)}$ denotes either SIC 3-digit industry interacted with month fixed effects ($FE_{SIC\times t}$) or firm fixed effects (FE_i). Our choice of firm-level control variables is motivated by structural models of investment and asset pricing that theoretically link conditional betas to firm characteristics. The firm-level control variables X_{it} include size (market capitalization), growth options (book-to-market equity

[23]Note that, if not scaled, the rank of hindmost followers in the race mechanically increases with the number of race participants. Nevertheless, when we eliminate the scaling, all results we report later in the paper become stronger. This is because, in our data, the average beta of the firms in the race increases with the number of race participants, hence, scaling by N_{kt} biases our results downwards.

[24]For example, in the *3-Year Sample*, 41% of the number of innovating firms are active in two or more technology classes in at least one month over our sample period.

ratio and profitability), operating leverage (tangibility), financial leverage (book leverage and cash holding), and investment policy (capital and R&D expenditure). Detailed definitions of the control variables are provided in Appendix B.1.

Second, we estimate a firm-race-level regression where we have, for each firm-month, as many observations as is the number or races in which a firm is engaged in at that month. In this analysis, we effectively treat a firm in two or more races as separate observations, but with identical betas. This regression, which preserves all race-specific percentile ranks, is

$$\beta_{ikt} = \lambda_0 + \lambda_1 n_{ikt} + \lambda_2 N_{kt} + \lambda_3 X_{ikt} + FE_k + FE_t + \epsilon_{ikt}. \tag{29}$$

In equation (29), n_{ikt} is the percentile rank of firm i in race k, N_{kt} is the number of innovating firms in race k, and FE_k and FE_t denote technology class and month fixed effects, respectively.

We estimate both regressions above using the *1-Year Sample* focusing on races, i.e., observations with $N_{kt} \geq 2$. The reason for using monthly frequency in our analysis is that it allows for betas to be conditional on innovation race characteristics (n_{ikt} and N_{kt}), which do change month by month. The disadvantage is that monthly conditional beta estimates might be affected by market microstructure issues, can be noisy, or both. To address these concerns, we estimate regressions (28) and (29) also at an annual frequency using annual beta estimates. The additional benefit of using annual data is that the frequency of all variables in the regression matches that of the firm-level control variables X_{it}.

At annual frequency, the sample used in regression (28) is based on the *3-Year Sample* and consists of all firm-years in which a firm is innovating in at least one month in a given year. In this case, in each year, the variable $\overline{n_{it}}$ stands for the average percentile rank computed, first, across all races in which firm i is innovating at month t, and second, over months in which firm i is innovating. The sample for regression (29) is defined analogously, and variables n_{ikt} and N_{kt} are respective period-averages of monthly observations in which firm i is innovating. The next subsection introduces our dependent variables.

3.1.5 Beta measures

Expected returns in our model satisfy the simple one-factor beta-pricing relation,

$$E\left[R_i\right] = r + \beta_i(x)\lambda, \tag{30}$$

where the process x is the factor, λ denotes the risk premium of the process x, and $\beta_i(x)$ is the elasticity of firm i's value to changes in x. In the model, x represents the market value of a monopoly profit protected by a patent. We consider two proxies for the process x.

The first proxy is the <JB> ~~value-weighted~~ <JB> return on the market portfolio <JB> ~~, R_M~~ <JB> . <JB> ~~As discussed in footnote 15, there is a one-to-one mapping between CAPM betas and betas from our model.~~ [replaced by] There is a one-to-one mapping between CAPM betas and betas from our model (see discussion in footnote 15). <JB> Therefore, cross sectional variation in betas predicted by our model <JB> ~~should persist in~~ [replaced by] holds for <JB> CAPM betas as well. To estimate CAPM betas, we use time-series regressions $R_{it} = \alpha_i + \beta_i R_{Mt} + \zeta_{it}$, where R_{it} is the firm i's excess stock return and R_{Mt} is the excess return on the <JB> ~~market portfolio~~ [replaced by] CRSP value-weighted index <JB> . <JB> We calculate excess returns using the one-month T-bill rate obtained from Ken French's web page. <JB> We use daily returns from CRSP Daily Stock File to estimate the equity beta for each firm-month using separate short-window regressions (see **?**).[25] We refer to these estimates of betas as *Market Betas*. Next, we correct for the potential intervalling-effect bias <JB> introduced by **?** using the methodology <JB> <JB> ~~discussed in~~ [replaced by] of <JB> **?** <JB> ~~using their methodology~~ <JB> .[26] We refer to this refinement of <JB> ~~market~~ [replaced by] CAPM <JB> beta estimates as *Sum Betas*.

A possible concern with <JB> ~~the use of~~ [replaced by] using <JB> the market portfolio <JB> ~~R_M~~ <JB> as a proxy for the patent's market value x is the fact that not all firms benefit equally from the adoption of new technologies. For example, growth firms that successfully innovate <JB> may <JB> benefit more than value firms that do not innovate (see **?** or **?**). Therefore, although the value process x <JB> ~~from investing in new technology may have~~ [replaced by] has <JB> a systematic <JB> market-wide <JB> component, it is not clear whether <JB> ~~the return on market portfolio~~ [replaced by] a broad market index <JB> <JB> ~~best~~ <JB> captures the different degree <JB> ~~in~~ [replaced by] to <JB> which firms benefit from adoption of new technologies. To address this concern, we use the return on the spread portfolio of growth and value firms as our second proxy for x. <JB> ~~Our second proxy for x is therefore the return $-R_{HML}$, where HML stands for high-minus-low book to market.~~

[25] To reduce the impact of outliers, we use betas that are estimated using at least 19 daily observations and we also winsorize betas at the 1% level.

[26] We follow Proposition 3 in **?**. This involves summing up the contemporaneous with the one-day-lead and the one-day-lag equity betas, which we estimate, one at the time, using analogous short-window regressions.

[replaced by] Specifically, the return on the spread portfolio of growth and value firms we use is minus one times the return on high-minus-low book to market portfolio, $-R_{HML}$, obtained from Ken French's web page. <JB> [27] <JB> Using this proxy, we estimate *Growth-Value Beta* for each firm-month as a coefficient on $-R_{HML}$ in a one-factor model short-window regression analogous to the one introduced above for the market model. <JB> <JB> ~~We refer to these estimates of betas as *Growth-Value Betas.*~~ <JB> If growth firms benefit disproportionately more from innovation compared to value firms, the <JB> ~~excess~~ <JB> return on the market portfolio measures x with error and <JB> using *Market Betas* <JB> should lead to weaker results compared to those obtained using <JB> ~~betas with respect to $-R_{HML}$~~ [replaced by] *Growth-Value Betas* <JB> . For <JB> ~~annual regressions~~ [replaced by] estimating regressions (28) and (29) at an annual frequency <JB> , we use daily returns over calendar year periods to estimate <JB> annual <JB> *Market Betas* <JB> , *Sum Betas*, <JB> and *Growth-Value Betas* using the methodologies explained above.

Finally, in order to reduce the noise inherent in the estimation of monthly equity betas from daily returns, we also compute realized equity betas based on higher frequency returns obtained from the TAQ database (see Appendix B.2 for details).[28] We refer to these beta estimates as *High-Frequency Betas*.

3.1.6 Sample overview

We start by describing our samples of innovating firms. Table 1 provides summary statistics of innovating firms' characteristics in the *3-Year Sample* and compares them to those of all non-financial firms.[29] The *3-Year Sample* has 8,377 firm-year observations and the sample of all non-financial firms has 113,509 firm-year observations.

Relative to the median non-financial firm, we show that the median innovating firm is considerably bigger, more profitable, and holds more cash. Also, it has lower book-to-market equity and leverage ratios. Median Tangibility and capital expenditure-to-sales ratio are about the

[27] We thank Dimitris Papanikolaou for suggesting this beta measure.

[28] See **?** and **?** for econometric theory underlying the estimation of volatility and covariance using high frequency data and **?** for an application.

[29] We exclude firms with four-digit SIC codes between 6000 and 6999. When constructing Table 1, we require all reported characteristics to be non-missing. Market capitalization (item prcc_f×csho) and total assets (item at) are measured in USD billions. Market leverage is total long-term debt plus debt in current liabilities (item dltt+dlc) scaled by sum of total long-term debt, debt in current liabilities, and market value of common equity (item dltt+dlc+prcc_f×csho). The variable is winsorized at the 1% level. Definitions of the remaining characteristics are provided in Appendix B.1.

same, while the R&D expenditure-to-sales ratio is significantly higher. Innovating firms have both higher *Market Betas* and higher *Growth-Value Betas* with the difference being especially pronounced for the latter. The sample of innovating firms represents only 7.4% (8.7%) of all firm-year observations (firms) while the fraction of the entire market capitalization taken by the innovating firms is between 40% and 50% over our sample period. This means that the firms with high patenting intensity are those with the largest market capitalization.

Next, we report how are characteristics of innovating firms related to their standing in innovation races. Table 2 uses the *1-Year Sample* and reports summary statistics of the characteristics of innovating firms broken down by the number of innovating firms N_{kt} and the race-specific rank $n_{ikt}N_{kt}$. To ease presentation, we aggregate the number of innovating firms in a race variable into bins, $N_{kt} = \{1,\ 2\text{–}5,\ 6\text{–}10,\ 11\text{–}15,\ \ldots,\ 56\text{–}60,\ >60\}$. The rank variable is aggregated analogously.

Panel A of Table 2 reports the average value-weighted *Market Beta* and the average *Growth-Value Beta*. Using both measures, we show that beta increases with the firm's rank in a race as well as with the number of firms in a race. For example, using the entire sample the average value-weighted *Market Beta* of firms with rank 2–5 is 1.03, while it is equal to 1.18 for the firms with ranks 21–25.

Panel B of Table 2 reports the average market capitalization (taken from the CRSP Monthly Stock File, item abs(prc)×shrout) and book-to-market equity ratio of the innovating firms. We show that the firms closer to race leadership are significantly larger, especially in races with high number of firms. This pattern in firm size leads us to include market capitalization as our main control variable in all regressions. The last panel reports the average book-to-market equity ratio. The book-to-market equity ratio tends to decrease with the firm's rank in a race and also with the number of firms in a race. This suggests that followers' values contain relatively more growth options, as in our model.

Most of our regression specifications have a panel structure and rely on the time series variation in the firms' ranks. Table 3 uses the same sample and rank variable as Table 2 to describe this variation. It reports the transition probabilities between the rank of an innovating firm in race k at month t and its rank in the same race at month $t + 1$. The firm-race-month just before the one in which we observe the firm as innovating in this race for the first time is coded as 'Enter.' The firm-race-month immediately after the last one is coded as 'Exit.' The table shows that the firm's rank does often change over time and that, typically, the probability with which a firm improves its rank is higher than the probability with which its rank worsens. As we move towards races with a higher number of innovating firms, the probability of a rank change, both improvement and worsening, goes up.

3.2 Results

In this section, we present our main results. Panel A of Table 4 presents estimates of the firm-level regression (28) using the *3-Year Sample* at the annual frequency. The dependent variable is either the value-weighted *Market Beta* or the *Growth-Value Beta*. For each beta measure, we present four specifications that differ in the firm-level control variables and in the set of fixed effects we include in the regression. In one of the specifications, we also include the lagged values of the dependent variable as regressors. This proxies for any omitted autocorrelated variables, which might possibly exist in the context of our analysis. In all regressions, standard errors are clustered at the firm level.

Across all eight specifications, the positive and significant estimates suggest that the percentile rank in a race is associated with higher beta. **[JB]• To asses the economic magnitude of the estimates, consider the second specification in Panel A of Table 4 and a 5-firm race, for example. The difference between the beta of the leader and that of the hindmost follower is 0.13, which explains 0.63% of excess return (with 5% market risk premium). • [JB]** The estimated coefficients are about twice as large in the specifications in which we use the *Growth-Value Beta* as the dependent variable compared to the analogous specifications with the value-weighted *Market Beta*. These findings support the prediction of our theory that the more a firm is lagging behind in an innovation race, the higher its beta is. They are also consistent with arguments in **?** and **?** that growth firms benefit disproportionately more from innovation compared to value firms, suggesting that the portfolio spread $-R_{HML}$ is a less noisy measure of the process x from our model.

Panel B of Table 4 presents estimates of regression (28) using the *1-Year Sample* at monthly frequency. The dependent variable is the value-weighted *Market Beta*, *Growth-Value Beta*, value-weighted *Sum Beta*, and *High-Frequency Beta*. For each beta measure, we present the same four specifications as in Panel A, and we only report the coefficients on the firm's percentile rank in a race.[30] As before, standard errors are clustered at the firm level. In fifteen out of sixteen specifications, we estimate a positive and significant effect of the percentile rank in a race on beta. These results confirm that changing data frequency, introducing alternative measures of systematic risk, or modifying the definition of the rank in the race variable has little impact on the results.

The next table reports the results from estimating the firm-race-level regression (29) that explicitly allows for a firm to be innovating in multiple races, with possibly different percentile ranks, at the same time. Panel A of Table 5 is based on the *3-Year Sample* at the annual frequency and uses the value-weighted *Market Beta* and *Growth-Value Beta* as the dependent variables. For each of the two beta measures, we present a specification with and without firm-level control variables. Technology class and year fixed effects are included in all regressions. Standard errors are clustered at the firm level. With this alternative data structure, we continue to find a positive and significant effect of the firm's percentile rank in a race on beta. Also, the estimated coefficients are again about twice as large in the specifications in which we use the *Growth-Value Beta* compared to the analogous specifications with the value-weighted *Market Beta*.

Panel B of Table 5 presents estimates of regression (29) using the *1-Year Sample* at the monthly frequency with the value-weighted *Market Beta*, *Growth-Value Beta*, value-weighted *Sum Beta*, and *High-Frequency Beta* as the dependent variables. For each beta measure, we present the same two specifications as in Panel A, and we only report the coefficients on the firm's percentile rank in a race and on the number of innovating firms in a race variables. Technology class and month fixed effects are included in all regressions and standard errors are clustered at the firm level.

In all specifications, we estimate positive coefficients on the percentile rank variable as well as on the number of innovating firms variable. When we use the *Growth-Value Beta* as the dependent variable, the coefficients on the percentile rank are precisely estimated, while they are not significant when we use the *High-Frequency Beta*. When we use the value-weighted *Market Beta* and *Sum Beta*, the coefficient on the percentile rank is significant at the 1% level in our first specification and it becomes insignificant when we include the firm-level control

[30]The market capitalization control variable comes from the CRSP Monthly Stock File (item abs(prc)$\times$shrout) and is entered at monthly frequency. The other firm-level control variables come from Compustat and take the same value for all months in a given calendar year.

variables. Overall, the results in Table 5 demonstrate that our results are robust to explicitly allowing for possible heterogeneity of a firm's percentile ranks across multiple races.

In summary, our empirical findings support the predictions of our theory that there is an effect of a firm's relative position in an innovation race on its systematic risk.

3.3 Robustness

We conduct a battery of robustness checks on our results.

First, when estimating the firm-level regression (28), we consider two alternative ways of aggregating the firm's percentile ranks across multiple races. Our first approach is to compute the weighted average of the firm's percentile ranks across all races in which the firm is innovating. The weights are, for each race, the number of innovating firms in a given race divided by the sum of the number of innovating firms in all races in which the firm is innovating. This way, races with a higher number of innovating firms receive a higher weight. Our second approach is to use the percentile rank of the firm in the race with the highest number of innovating firms, i.e., the most populated race. One can argue that races with a higher number of firms are more important in the economy in terms of monopoly rents they generate, and hence standing of a firm in such races is a more important determinant of its beta. In both cases, we reestimate regression (28) using the *3-Year Sample* at annual frequency with the same dependent variables and specifications as those in Panel A of Table 4, as well as using the *1-Year Sample* at monthly frequency with the same dependent variables and specifications as those in Panel B of Table 4.

We present the results with the weighted average of percentile ranks in Panel A of Table C-1 and the results with the percentile rank in the most populated race in Panel B of Table C-1.[31] The results are practically the same as those reported in Table 4. When we use the weighted average of percentile ranks, the results are stronger, while when we use the percentile rank in the most populated race, the results are marginally weaker. This evidence suggests that our results do not depend on how we aggregate the relative positions a firm has across multiple races.

Second, we check whether our results are robust to changing the definition of the innovation race. In particular, our main proxy for a patent's technology class is the second-level of the IPC classification. This proxy is prominent in our empirical analysis as both the definition of whether a firm is innovating (26) as well as the definition of the relative innovation efficiency

[31] The tables that report results of our robustness checks are in Appendix C.

of a firm in a race (27) relies on technology class. To assess the extent to which our results are affected by changing this proxy, we also consider technology class defined based on a more aggregated 'sections of IPC' (the first-level of the IPC hierarchy) as well as the technology class defined based on a more disaggregated 'subclasses of IPC' (the third-level of the IPC hierarchy). In both cases, we reestimate regression (28) using the *3-Year Sample* at annual frequency with the same dependent variables and specifications as those in Panel A of Table 4, as well as using the *1-Year Sample* at monthly frequency with the same dependent variables and specifications as those in Panel B of Table 4. We present the results obtained using the first-level of the IPC hierarchy in Panel A of Table C-2 and the results obtained using the third-level of the IPC hierarchy in Panel B of Table C-2. The results are analogous to those reported in Table 4.

Third, to check robustness to changing the definition of the market portfolio, we reestimate regressions (28) and (29) using equally-weighted *Market Beta* and *Sum Beta* as the dependent variables (the specifications are otherwise identical to those in Tables 4 and 5). The results, reported in Table C-3, show that when we use the CRSP equally-weighted index, we obtain stronger results. The coefficients are bigger and are significant at lower levels.

Fourth, since firm volatility is equal to $\beta_i \times \sigma$ in our one-factor model, where σ is the volatility of x, our model also predicts that a firm's total risk increases in its relative position in an innovation race.[32] To see if this prediction is supported by the data, we reestimate regressions (28) and (29) using the firm's standard deviation of daily returns scaled by the market's standard deviation of daily returns as the dependent variable (again, the specifications are otherwise identical to those in Tables 4 and 5). The results are reported in Table C-4. We find strong support for this prediction using all our annual regression specifications. At monthly frequency, all estimated coefficients are positive, but only sometimes significant. This suggests that our monthly estimates of the total volatility are very noisy.

Finally, in our main tables, we present estimates with standard errors clustered at the firm level. As an alternative, Table C-5 presents estimates obtained using the Newey-West estimator that produces correct standard errors when the error term is autocorrelated in addition to being possibly heteroskedastic.[33] In our annual regressions, we allow for up to 3 lags, while we allow

[32] This is a simple application of Itô's lemma on the firm value $V(x)$.

[33] Specifically, for each dependent variable, we reestimate the second specification from Tables 4 and 5.

for up to 12 lags in the regressions at the monthly frequency. All the results become more significant when we use this alternative technique to compute standard errors.

4 Conclusion

This paper explores the link between competition in innovation and asset prices. We develop a model of an innovation race in which two firms compete for the acquisition of a patent whose value varies over time due to market-wide risk. Firms' investments are irreversible and a discovery is a random idiosyncratic event. The model predicts that when leader-follower equilibria emerge, the beta of the leader is always smaller than that of the follower and that the difference between the leader's and the follower's betas increases in the distance between the firms' relative positions in the race.

We test this prediction using a comprehensive panel of patent application filing and patent award events in the U.S. over 1976-2006 period. We find that the equity beta of a firm depends on its recent patenting success when benchmarked to that of its rivals in an innovation race: The equity beta decreases the closer the firm is to a leading position in the race.

Our finding that innovation race variables have a strong explanatory power for systematic risk in the cross-section of firms suggests that modelling industry rivalry is important for understanding the cost of capital of firms in different competitive environments. The pattern of within-industry heterogeneity of equity betas implied by our model challenges the commonly followed practice of using industry peer betas to estimate the cost of capital.

Consistent with earlier empirical work, we observe that only the firms with the largest market capitalization are highly active in innovation. Our model provides a possible answer to this empirical phenomenon. A firm that considers joining an innovation race may face a high cost of capital. This makes joining the race costly and constitutes a *de-facto* barrier to entry. A formal investigation of the effect of entry on the cost of capital has potentially important policy implications for the relationship between competition and innovation. To fully address this point, however, one would need to extend the current model to a general equilibrium context with entry and exit, a task we leave for future research.

Figure 1: Values of the follower and the leader

The figure reports the value of firm $i = 1, 2$ when it invests as the follower (Panel A) and when it is the leader (Panel B) assuming that $h_1 > h_2$. The values in Panel A (B) illustrate the results in Proposition 1 (Proposition 2). The optimal investment thresholds for the follower are x_1^F and x_2^F derived in Proposition 1. Parameter values: $h_1 = 0.5$, $h_2 = 0.2$, $\delta = 2\%$, $\sigma = 75\%$, and $K = 1$.

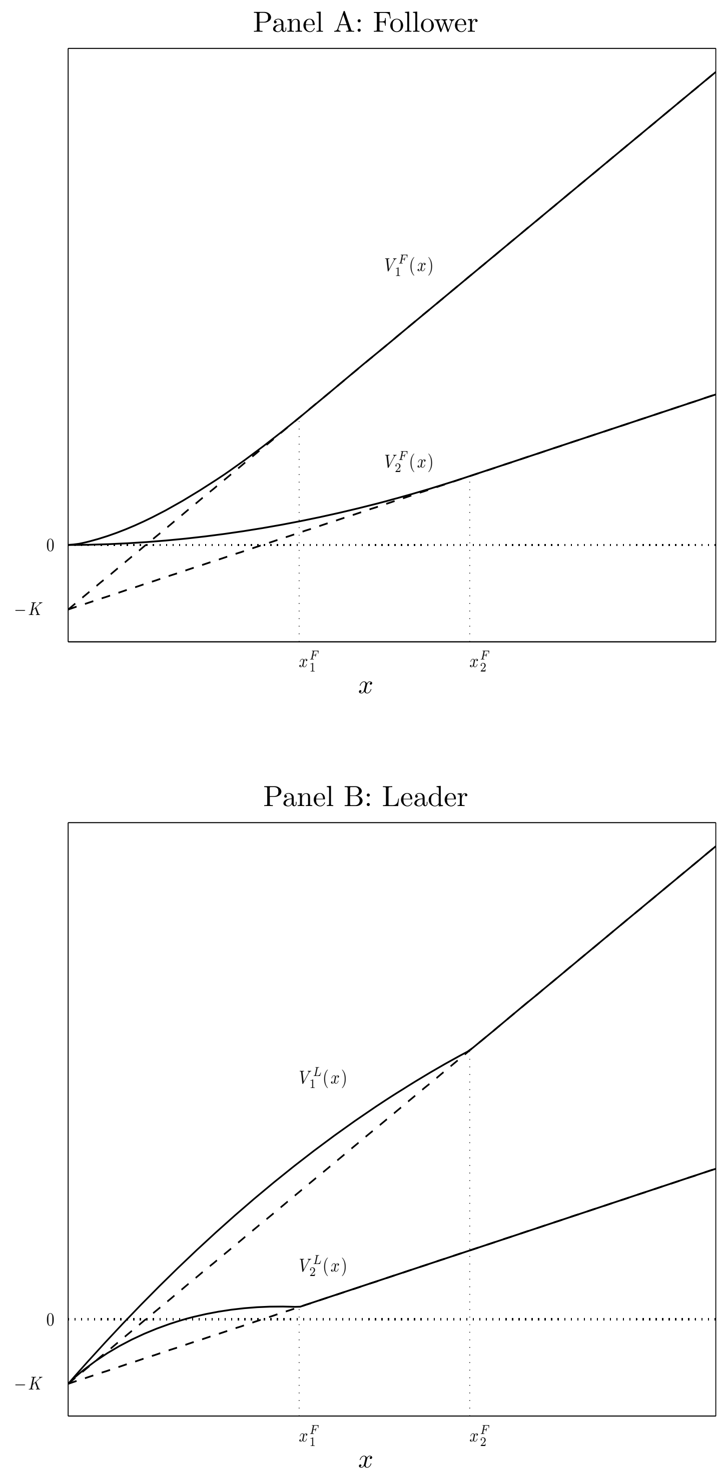
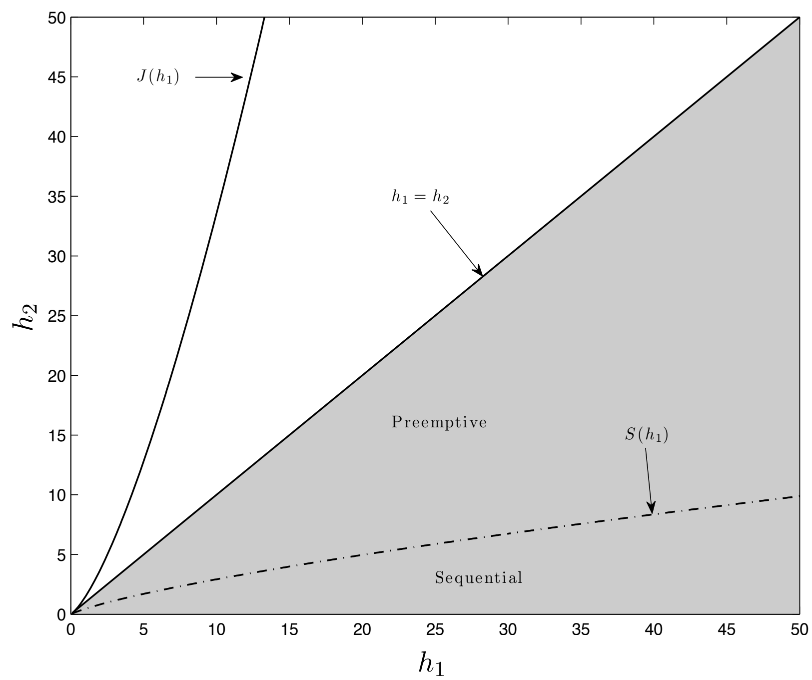


Figure 2: Equilibrium regions

The figure reports the thresholds $J(h_1)$ (solid line) and $S(h_1)$ (dash-dotted line) and the corresponding Markov perfect equilibrium regions derived in Proposition 4. Parameter values: $\delta = 2\%$ and $K = 1$.

Panel A: Low volatility ($\sigma = 30\%$)



Panel B: High volatility ($\sigma = 95\%$)

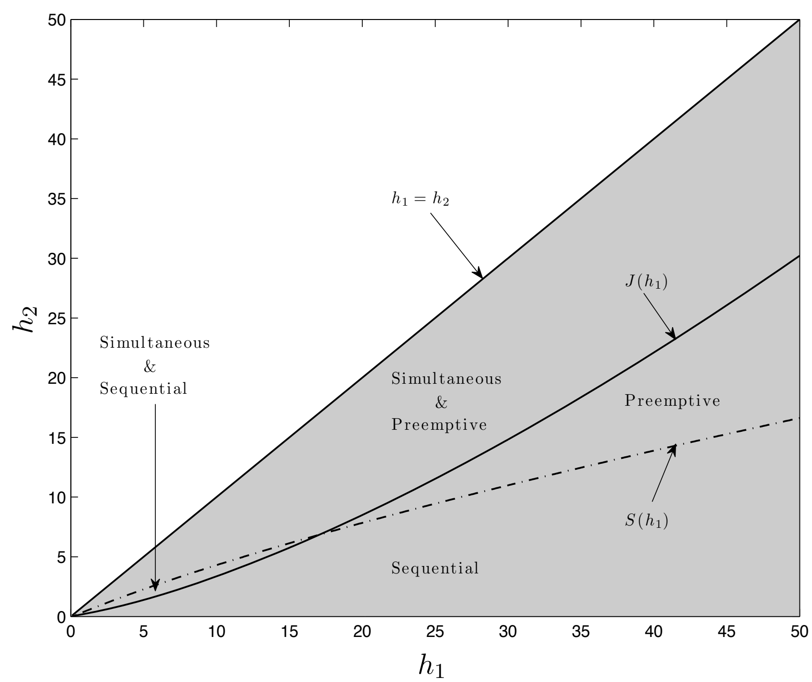
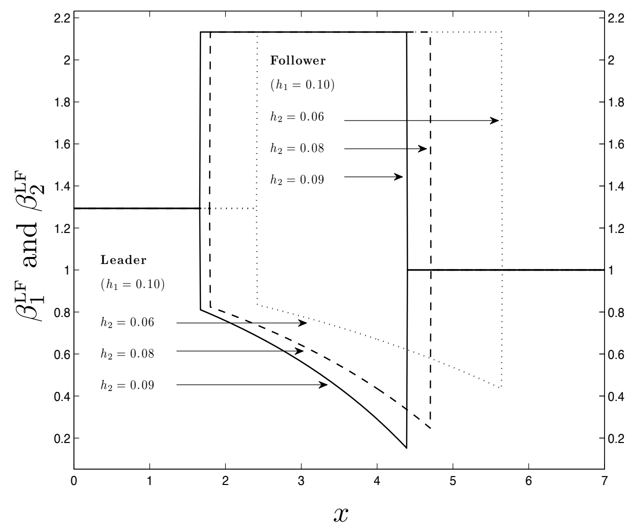


Figure 3: Leader's and follower's beta

The figure reports the beta of the leader, $\beta_1^{LF}(x)$, and the follower, $\beta_2^{LF}(x)$, in leader-follower equilibria derived in Proposition 6. Parameter values: $\delta = 2\%$, $K = 1$, and $\sigma = 30\%$.

Panel A: h_2 varies, $h_1 = 0.1$



Panel B: h_1 varies, $h_2 = 0.1$

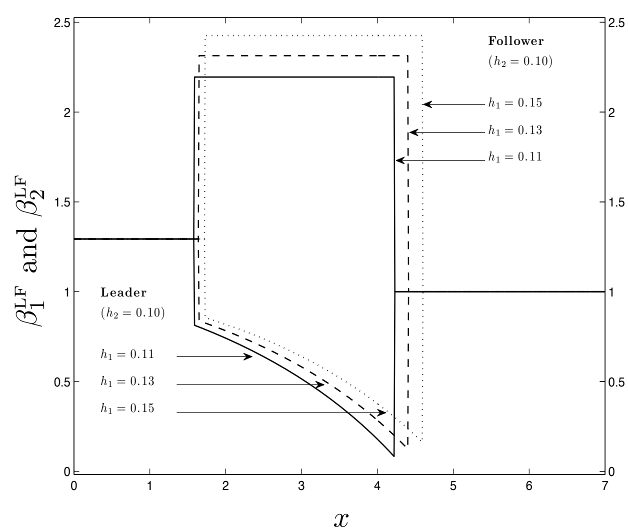


Figure 4: Leader's and follower's beta along a sample path of $x(t)$

The figure reports a sample path of the process $x(t)$ in (1) over $T = 240$ periods, the investment thresholds of the leader and the follower, and the corresponding betas in the case of a preemptive equilibrium. Parameter values: $h_1 = 1$, $h_2 = 0.5$, $\mu = 0.11$, $\delta = 2\%$, $\sigma = 30\%$, and $K = 1$.

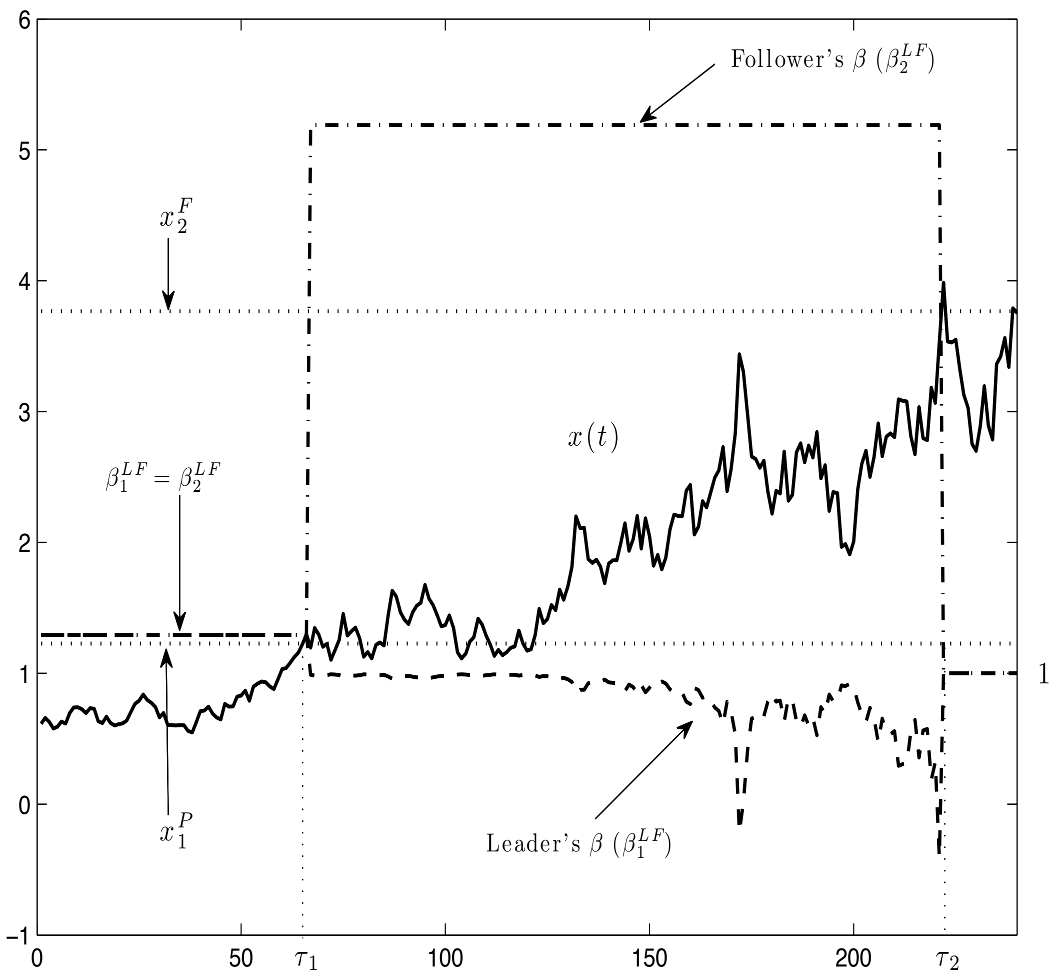


Table 1: Summary statistics of innovating firms, 1976-2006

The table reports summary statistics of the characteristics of innovating firms in the 1976-2006 period (the *3-Year Sample* with 8,378 firm-year observations, see section 3.1.2) and compares them to those of all non-financial firms (excluding firms with four-digit SIC codes between 6000 and 6999). Total assets (item at) and market capitalization (item prcc_f×csho) are measured in USD billions. Market leverage is total long-term debt plus debt in current liabilities (item dltt+dlc) scaled by sum of total long-term debt, debt in current liabilities, and market value of common equity (item dltt+dlc+prcc_f×csho). *Market Beta* is firm equity beta from the market model in which the CRSP value-weighted index proxies the market portfolio. *Growth-Value Beta* is estimated using a one-factor model as the coefficient on the return on the spread portfolio of growth and value firms (the factor). The return on the spread portfolio of growth and value firms is minus one times the return on high-minus-low book to market portfolio obtained from Ken French's web page. Definitions of the remaining characteristics are provided in Appendix B.1.

	Innovating Firms					All Non-Financial Firms				
	Mean	S.D.	10th Percentile	Median	90th Percentile	Mean	S.D.	10th Percentile	Median	90th Percentile
Market Cap	4.64	18.33	0.08	0.84	8.00	0.91	6.45	0.01	0.07	1.34
Total Assets	3.17	8.88	0.08	0.80	6.55	0.88	4.59	0.01	0.09	1.54
Profitability	0.11	0.17	-0.05	0.14	0.24	0.08	0.20	-0.10	0.12	0.24
B/M	0.58	0.45	0.16	0.46	1.12	0.77	0.64	0.18	0.59	1.55
Tangibility	0.28	0.16	0.08	0.25	0.50	0.31	0.23	0.06	0.25	0.69
Cash	0.19	0.21	0.01	0.10	0.52	0.15	0.19	0.01	0.07	0.43
Book Leverage	0.19	0.15	0.00	0.18	0.37	0.23	0.19	0.00	0.21	0.49
Market Leverage	0.18	0.18	0.00	0.13	0.44	0.25	0.24	0.00	0.19	0.62
R&D	0.17	0.42	0.01	0.05	0.29	0.07	0.28	0.00	0.00	0.14
CAPEX	0.08	0.11	0.02	0.05	0.15	0.11	0.27	0.01	0.04	0.23
Market Beta	1.12	0.60	0.42	1.03	1.99	0.75	0.63	0.04	0.67	1.59
Growth-Value Beta	1.46	1.20	0.15	1.27	3.17	0.92	1.19	-0.31	0.76	2.50

Table 2: Characteristics of firms in innovation races

The table reports summary statistics of the characteristics of innovating firms in the 1976-2006 period (the *1-Year Sample* with 88,765 firm-month observations, see section 3.1.2) broken down by the number of innovating firms in a race and the race-specific rank of a firm in a race. Panel A reports the average value-weighted *Market Beta* and the average *Growth-Value Beta* (defined in section 3.1.5), while Panel B reports the average market capitalization (taken from the CRSP Monthly Stock File, item abs(prc)×shrout) and the average book-to-market equity ratio (item ceq/(prcc_f×csho)).

Panel A: Beta

Number of innovating firms	Innovating firm's rank in a race: 1	2–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50	51–55	56–60	>60	Mean
Market Beta															
1	0.96														0.96
2-5	0.97	0.93													0.94
6-10	0.97	0.96	0.98												0.97
11-15	1.20	1.05	1.01	1.02											1.03
16-20	1.17	0.94	1.00	0.97	0.96										0.98
21-25	1.43	1.13	1.12	1.10	1.07	1.11									1.11
26-30	1.13	1.05	1.06	1.12	1.09	1.07	1.10								1.09
31-35	1.19	1.08	1.10	1.09	1.09	1.11	1.14	1.13							1.11
36-40	1.05	1.00	1.07	1.16	1.19	1.20	1.18	1.19	1.13						1.15
41-45	0.95	1.06	1.14	1.32	1.28	1.30	1.32	1.35	1.32	1.24					1.28
46-50	1.12	1.27	1.26	1.42	1.40	1.34	1.31	1.34	1.41	1.38	1.28				1.35
51-55	1.26	1.23	1.13	1.26	1.20	1.26	1.20	1.23	1.27	1.22	1.20	1.24			1.23
56-60	1.19	1.15	1.03	1.13	1.12	1.14	1.14	1.15	1.19	1.17	1.19	1.16	1.10		1.15
>60	1.31	1.26	1.15	1.16	1.19	1.24	1.20	1.21	1.21	1.21	1.24	1.19	1.23	1.28	1.24
Mean	1.05	1.03	1.06	1.12	1.14	1.18	1.19	1.21	1.23	1.22	1.23	1.19	1.22	1.28	1.17
Growth-Value Beta															
1	1.18														1.18
2-5	1.15	1.08													1.10
6-10	1.10	1.06	1.11												1.08
11-15	1.58	1.36	1.23	1.15											1.27
16-20	1.28	0.84	1.05	1.15	1.10										1.06
21-25	1.91	1.26	1.36	1.32	1.33	1.37									1.35
26-30	1.48	1.27	1.28	1.44	1.45	1.42	1.45								1.40
31-35	1.47	1.44	1.42	1.38	1.34	1.51	1.59	1.62							1.48
36-40	1.43	1.02	1.17	1.43	1.71	1.66	1.60	1.65	1.49						1.53
41-45	1.17	1.19	1.21	1.80	1.74	1.81	1.84	1.90	1.85	1.71					1.74
46-50	0.98	1.41	1.70	2.17	1.99	2.04	1.95	1.86	1.93	2.11	1.84				1.94
51-55	1.24	1.31	1.08	1.64	1.53	1.67	1.43	1.41	1.61	1.62	1.59	1.68			1.53
56-60	1.72	1.60	1.21	1.48	1.51	1.49	1.51	1.43	1.51	1.61	1.59	1.59	1.54		1.52
>60	1.77	1.79	1.54	1.46	1.61	1.63	1.57	1.62	1.62	1.60	1.63	1.61	1.65	1.74	1.66
Mean	1.29	1.22	1.28	1.40	1.49	1.58	1.59	1.63	1.64	1.65	1.63	1.62	1.64	1.74	1.53

Table 2 (cont.): Characteristics of firms in innovation races

Panel B: Size and book-to-market

		Innovating firm's rank in a race														
		1	2-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	>60	Mean
		Market capitalization (USD bil.)														
Number of innovating firms	1	6.3														6.3
	2-5	7.5	5.8													6.3
	6-10	7.2	9.4	8.4												8.8
	11-15	11.1	11.5	9.8	8.2											10.0
	16-20	9.7	11.3	8.6	8.6	10.5										9.5
	21-25	19.5	13.7	14.0	10.7	8.8	9.3									11.3
	26-30	17.3	16.9	11.5	9.8	8.0	8.0	7.1								9.8
	31-35	13.0	16.0	11.9	11.7	11.7	9.0	8.3	5.9							10.3
	36-40	12.3	19.0	18.3	23.1	17.3	15.3	9.7	8.3	7.1						13.8
	41-45	16.5	24.2	23.2	32.2	25.4	22.1	16.9	10.0	6.5	6.7					17.2
	46-50	21.1	45.7	35.4	24.2	16.2	11.2	8.2	8.0	8.9	7.7	4.8				13.6
	51-55	29.4	33.8	23.3	23.0	16.1	10.8	8.3	7.7	5.9	6.9	7.5	8.2			11.6
	56-60	29.2	16.3	12.1	14.4	13.3	11.8	9.5	11.1	8.7	6.7	8.1	5.7	7.5		10.0
	>60	26.0	39.2	21.6	19.0	18.3	18.0	14.0	12.7	9.7	7.9	7.2	6.6	8.1	7.1	10.8
	Mean	10.5	14.3	13.5	14.6	14.0	13.6	11.3	10.5	8.6	7.6	7.3	6.6	8.0	7.1	10.8
		Book-to-market equity ratio														
Number of innovating firms	1	0.69														0.69
	2-5	0.71	0.74													0.73
	6-10	0.61	0.70	0.74												0.71
	11-15	0.59	0.54	0.64	0.70											0.62
	16-20	0.44	0.62	0.63	0.66	0.63										0.63
	21-25	0.60	0.77	0.77	0.80	0.79	0.78									0.78
	26-30	0.53	0.70	0.74	0.77	0.73	0.68	0.71								0.72
	31-35	0.52	0.68	0.72	0.73	0.66	0.67	0.63	0.65							0.67
	36-40	0.60	0.58	0.62	0.60	0.55	0.55	0.55	0.58	0.60						0.58
	41-45	0.74	0.60	0.64	0.58	0.57	0.55	0.54	0.57	0.55	0.59					0.57
	46-50	0.59	0.49	0.50	0.48	0.53	0.55	0.58	0.56	0.52	0.53	0.54				0.54
	51-55	0.48	0.49	0.50	0.50	0.50	0.53	0.54	0.52	0.53	0.53	0.54	0.53			0.52
	56-60	0.70	0.64	0.60	0.61	0.57	0.56	0.57	0.55	0.56	0.59	0.60	0.60	0.58		0.58
	>60	0.58	0.49	0.56	0.56	0.56	0.58	0.60	0.59	0.60	0.61	0.60	0.60	0.57	0.49	0.55
	Mean	0.65	0.66	0.66	0.66	0.63	0.62	0.60	0.59	0.58	0.59	0.59	0.60	0.57	0.49	0.60

Table 3: Transition probabilities of firms' ranks in innovation races

The table reports the transition probabilities (in percentages) between the rank of an innovating firm in a given race at month t and the firm's rank in the same race at month $t+1$. The table uses the *1-Year Sample* at the firm-race-level, where we have, for each firm-month, as many observations as is the number or technology classes in which the firm is active in innovation as of month t. The firm-race-month just before the firm-race-month in which we observe the firm as innovating in a race for the first time is coded as 'Enter.' The firm-race-month immediately after the last firm-race-month in which we observe the firm as innovating in a race is coded as 'Exit.'

		Innovating firm's rank in a race ($t+1$)															
		1	2–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50	51–55	56–60	>60	Exit	Total
	Enter	2.8	9.2	7.7	5.7	5.2	5.8	5.0	4.6	4.0	3.6	3.3	3.7	4.2	35.0	0.0	100.0
Innovating firm's rank in a race (t)	1	84.1	13.0	0.1												2.7	100.0
	2-5	4.3	85.3	8.0	0.2	0.1										2.2	100.0
	6-10		9.6	76.5	11.0	0.6	0.1									2.1	100.0
	11-15		0.2	13.2	70.1	13.4	1.1	0.2	0.1							1.8	100.0
	16-20			0.7	15.0	66.1	14.8	1.3	0.2	0.1						1.8	100.0
	21-25			0.1	1.4	17.5	61.5	15.4	1.7	0.4	0.1	0.1				1.9	100.0
	26-30				0.2	1.9	18.3	57.7	16.9	2.4	0.5	0.1	0.1			1.9	100.0
	31-35				0.1	0.3	2.3	20.1	55.7	16.1	2.4	0.6	0.2	0.1	0.1	2.1	100.0
	36-40					0.1	0.4	3.2	19.7	52.4	18.4	2.7	0.6	0.3	0.2	1.9	100.0
	41-45						0.1	0.8	3.8	20.8	48.5	19.0	3.4	0.8	0.5	2.2	100.0
	46-50						0.1	0.2	0.8	4.4	21.4	47.3	18.9	3.8	1.1	1.9	100.0
	51-55							0.1	0.4	0.9	5.1	21.6	45.6	19.1	4.8	2.3	100.0
	56-60							0.1	0.1	0.4	1.3	5.9	22.0	43.9	23.4	2.9	100.0
	>60										0.1	0.3	1.1	4.1	91.1	3.3	100.0
	Total	3.2	10.2	9.0	8.0	7.6	6.9	6.2	5.6	4.9	4.4	4.1	3.7	3.3	20.6	2.3	100.0

Table 4: Beta and the firm's average rank in innovation races

The table presents estimates from firm-level OLS regressions of the equity beta on firm characteristics (equation (28) in section 3.1.4). Percentile Rank denotes the average of the innovating firm's relative (ordinal) positions in innovation races computed using, for each race, the firm's recent patenting output relative to that of the other firms active in the race. Definitions of the firm-level control variables are provided in Appendix B.1. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: 3-Year Sample at annual frequency

We use the *3-Year Sample* at annual frequency. The dependent variables are value-weighted *Market Beta* or *Growth-Value Beta* defined in section 3.1.5. We include the 3-digit-SIC industry interacted with year fixed effects or the firm fixed effects.

	Market Beta				Growth-Value Beta			
Percentile Rank	0.233***	0.158***	0.062**	0.096**	0.485***	0.326***	0.127**	0.286***
	(0.053)	(0.050)	(0.028)	(0.049)	(0.095)	(0.087)	(0.050)	(0.098)
Market Cap	0.075***	0.085***	0.020***	0.015	0.144***	0.151***	0.052***	-0.022
	(0.009)	(0.011)	(0.006)	(0.015)	(0.016)	(0.018)	(0.010)	(0.028)
Profitability		-0.074	0.092	0.319***		0.025	0.131	1.047***
		(0.085)	(0.065)	(0.107)		(0.137)	(0.103)	(0.225)
B/M		-0.035*	-0.051***	-0.037*		-0.105***	-0.109***	-0.006
		(0.018)	(0.012)	(0.019)		(0.032)	(0.023)	(0.037)
Tangibility		-0.063	-0.090	-0.246*		-0.022	-0.174	0.722**
		(0.128)	(0.066)	(0.133)		(0.233)	(0.129)	(0.285)
Cash		0.607***	0.248***	0.106		1.288***	0.595***	0.413**
		(0.078)	(0.049)	(0.093)		(0.138)	(0.093)	(0.188)
Book Leverage		0.235***	0.084*	-0.233***		0.275*	0.091	-0.178
		(0.088)	(0.048)	(0.084)		(0.157)	(0.090)	(0.173)
R&D		0.028	0.063**	-0.015		0.034	0.075	0.078
		(0.032)	(0.030)	(0.050)		(0.060)	(0.054)	(0.089)
CAPEX		0.299***	0.127*	0.293***		0.716***	0.422***	1.072***
		(0.094)	(0.069)	(0.110)		(0.178)	(0.132)	(0.215)
Beta 1-Lag			0.426***				0.422***	
			(0.017)				(0.018)	
Beta 2-Lag			0.188***				0.149***	
			(0.017)				(0.016)	
Industry × Year FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.392	0.448	0.636	0.554	0.482	0.547	0.665	0.437
N	9,061	8,378	7,551	8,378	9,061	8,378	7,551	8,378

Table 4 (cont.): Beta and the firm's average rank in innovation races

Panel B: 1-Year Sample at monthly frequency

We use the *1-Year Sample* at monthly frequency. The dependent variables are value-weighted *Market Beta*, *Growth-Value Beta*, value-weighted *Sum Beta*, or *High-Frequency Beta* defined in section 3.1.5. The regression specifications are identical to those in Panel A, and we do not report firm-level controls for brevity. We include the 3-digit-SIC industry interacted with month fixed effects or the firm fixed effects.

	Market Beta				Growth-Value Beta			
Percentile Rank	0.190***	0.142***	0.100***	0.069**	0.470***	0.378***	0.292***	0.189**
	(0.052)	(0.049)	(0.035)	(0.034)	(0.089)	(0.084)	(0.065)	(0.073)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.126	0.149	0.189	0.214	0.229	0.258	0.276	0.143
N	95,414	88,765	86,851	88,765	95,414	88,765	86,851	88,765
	Sum Beta				**High-Frequency Beta**			
Percentile Rank	0.244***	0.202***	0.171***	0.088**	0.160***	0.177***	0.062***	-0.051
	(0.061)	(0.057)	(0.050)	(0.044)	(0.054)	(0.053)	(0.017)	(0.036)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.040	0.053	0.062	0.109	0.434	0.469	0.669	0.591
N	95,414	88,765	86,851	88,765	49,326	44,654	42,985	44,654

Table 5: Beta and the firm's ranks in all innovation races

The table presents estimates from firm-race-level OLS regressions of the equity beta on firm and race characteristics (equation (29) in section 3.1.4). Percentile Rank denotes the relative (ordinal) position of a firm in an innovation race computed using the firm's recent patenting output relative to that of the other firms active in the race. Number of Innovating Firms is the number of firms that are active in innovation in a given race and year (month). Definitions of the firm-level control variables are provided in Appendix B.1. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: 3-Year Sample at annual frequency

We use the *3-Year Sample* at annual frequency. For each firm-year, we have as many observations as is the number or technology classes in which the firm is active in innovation as of that year. The dependent variables are value-weighted *Market Beta* or *Growth-Value Beta* defined in section 3.1.5. We include the technology class and year fixed effects in all specifications.

	Market Beta		Growth-Value Beta	
Percentile Rank	0.134***	0.054**	0.334***	0.114***
	(0.028)	(0.024)	(0.058)	(0.044)
Number of Innovating Firms	0.109***	0.001	0.240***	0.002
	(0.041)	(0.037)	(0.082)	(0.071)
Market Cap	0.042***	0.047***	0.076***	0.064**
	(0.012)	(0.013)	(0.027)	(0.027)
Profitability		0.042		0.411**
		(0.111)		(0.191)
Tangibility		-0.032		-0.134***
		(0.020)		(0.036)
B/M		-0.354**		-0.871***
		(0.138)		(0.309)
Cash		0.850***		1.820***
		(0.098)		(0.195)
Book Leverage		-0.045		-0.443**
		(0.108)		(0.214)
R&D		-0.020		-0.083
		(0.036)		(0.073)
CAPEX		0.714***		1.861***
		(0.164)		(0.367)
Technology, Year FE	Yes	Yes	Yes	Yes
R^2	0.247	0.339	0.356	0.476
N	22,702	21,509	22,702	21,509

Table 5 (cont.): Beta and the firm's ranks in all innovation races

Panel B: 1-Year Sample at monthly frequency

We use the *1-Year Sample* at monthly frequency. For each firm-month, we have as many observations as is the number or technology classes in which the firm is active in innovation as of that month. The dependent variables are value-weighted *Market Beta*, *Growth-Value Beta*, value-weighted *Sum Beta*, or *High-Frequency Beta* defined in section 3.1.5. The regression specifications are identical to those in Panel A, and we do not report firm-level controls for brevity. We include the technology class and month fixed effects in all specifications.

	Market Beta		Growth-Value Beta	
Percentile Rank	0.114***	0.037	0.276***	0.091**
	(0.026)	(0.024)	(0.053)	(0.042)
Number of Innovating Firms	0.207***	0.072*	0.290***	0.049
	(0.044)	(0.040)	(0.091)	(0.078)
Technology, Month FE	Yes	Yes	Yes	Yes
R^2	0.129	0.167	0.260	0.306
N	230,113	219,238	230,113	219,238
	Sum Beta		**High-Frequency Beta**	
Percentile Rank	0.115***	0.041	0.065	0.006
	(0.033)	(0.030)	(0.041)	(0.036)
Number of Innovating Firms	0.261***	0.130***	0.294***	0.218***
	(0.050)	(0.046)	(0.056)	(0.050)
Technology, Month FE	Yes	Yes	Yes	Yes
R^2	0.084	0.104	0.383	0.447
N	230,113	219,238	115,499	107,456

Table 6: Portfolio returns and the firm's average rank in innovation races

The table shows asset pricing tests for three portfolios sorted based on the 33rd and 66th percentiles of Percentile Rank and for a zero-investment portfolio that goes long on the portfolio of 'Followers' and short on the portfolio of 'Leaders'. Percentile Rank denotes the average of the innovating firm's relative (ordinal) positions in innovation races computed using, for each race, the firm's recent patenting output relative to that of the other firms active in the race. Portfolios are formed in June and rebalanced every year. We use monthly data from June 1976 to May 2006. The table reports the monthly average value-weighted excess return (in percentage) on these portfolios and the intercepts (α, in percentage) and slopes on risk factors obtained by regressing portfolio excess returns on factor returns. MKT refers to excess return on the value-weighted CRSP index, SMB and HML refer to **?** factors, and UMD to the momentum factor, available from Kenneth French's website. Excess returns are the difference between portfolio returns and one-month Treasury bill rate. Standard errors, reported in parenthesis, are computed using Newey-West estimator allowing for 1 lag of serial correlation. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Portfolio sorts on Percentile Rank	Excess return (%)	Market model		Fama-French three-factor model				Cahart four-factor model				
		α (%)	β_{MKT}	α (%)	β_{MKT}	β_{SMB}	β_{HML}	α (%)	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}
Leaders (L)	0.614***	0.016	1.001***	0.234**	0.932***	-0.170***	-0.286***	0.285**	0.927***	-0.157***	-0.293***	-0.060
	(0.270)	(0.115)	(0.030)	(0.109)	(0.025)	(0.041)	(0.060)	(0.115)	(0.025)	(0.041)	(0.060)	(0.042)
Middle	0.722***	-0.048	1.287***	0.346**	1.080***	-0.016	-0.610***	0.423***	1.073***	0.003	-0.621***	-0.092*
	(0.384)	(0.165)	(0.047)	(0.149)	(0.036)	(0.059)	(0.083)	(0.155)	(0.036)	(0.061)	(0.079)	(0.047)
Followers (F)	0.740***	-0.048	1.317***	0.176	1.126***	0.250***	-0.432***	0.259*	1.119***	0.271***	-0.444***	-0.099**
	(0.355)	(0.156)	(0.050)	(0.135)	(0.042)	(0.073)	(0.081)	(0.132)	(0.041)	(0.073)	(0.076)	(0.045)
$F - L$	0.126	-0.064	0.317***	-0.059	0.194***	0.420***	-0.146*	-0.026	0.191***	0.428***	-0.150*	-0.039
	(0.191)	(0.174)	(0.045)	(0.160)	(0.047)	(0.082)	(0.087)	(0.167)	(0.047)	(0.081)	(0.087)	(0.059)

A Appendix: Proofs

The following lemma contains two preliminary results that will be used extensively in the sequel.

Lemma A.1. *Let $x(t)$ be the stochastic process in (1) with $\mu < r$ and $\tau = \inf\{t > 0 : x(t) > x^*\}$, $x^* > x(0)$. Then*

$$E\left[e^{-r\tau}\right] = \left(\frac{x(0)}{x^*}\right)^{\phi}, \tag{A1}$$

$$E\left[\int_0^{\tau} e^{-rt}x(t)dt\right] = \frac{x(0)}{r-\mu}\left[1-\left(\frac{x(0)}{x^*}\right)^{\phi-1}\right], \tag{A2}$$

where $\phi = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ is the positive root of the quadratic equation

$$\frac{1}{2}\sigma^2\phi(\phi-1) + \mu\phi - r = 0 \tag{A3}$$

Proof: The proof is standard and can be found, for example, in **?**, Chapter 3, or **?**, pp. 315–316. ∎

Proof of Proposition 1

By the law of iterated expectations, we can express (4) for $x < x_i^F$ as

$$V_i^F(x) = \max_{\tau_i^F} E\left[e^{-(r+h_j)\tau_i^F} E_{\tau_i^F}\left[\int_{\tau_i^F}^{\infty} e^{-(r+h_i+h_j)(t-\tau_i^F)}h_i x(t)dt - K\right]\right], \quad x(0) = x, \tag{A4}$$

$$= \max_{x_i^F}\left(\frac{x}{x_i^F}\right)^{\phi_j}\left(\frac{h_i x_i^F}{h_i+h_j+\delta} - K\right), \tag{A5}$$

where the last equality follows from (A1) in Lemma A.1. Maximizing with respect to x_i^F, yields (5) and (7). ∎

Proof of Proposition 2

From (A2) in Lemma A.1, the leader's value (8) for $x < x_i^F$ can be written as

$$V_j^L(x) = \frac{h_j x}{h_j+\delta}\left[1-\left(\frac{x}{x_i^F}\right)^{\phi_j-1}\right] + \left(\frac{x}{x_i^F}\right)^{\phi_j}\frac{h_j x_i^F}{h_i+h_j+\delta} - K, \tag{A6}$$

from which (9) follows. ∎

In the following lemma we collect useful properties of the investment thresholds and value functions.

Lemma A.2.[34] *Let x_i^F defined as in (5), $i = 1, 2$. If $h_1 > h_2$ then $x_1^F < x_2^F$.*

Proof: Let us set $h_2 = h > 0$ and $h_1 = (1 + \alpha)h$, $\alpha \geq 0$. If $\alpha = 0$, $x_1^F = x_2^F$. Hence, to prove the lemma we need to show that $x_1^F \leq x_2^F$ for $\alpha \geq 0$.

From (6) we define the function

$$\phi(\alpha) = \gamma + \sqrt{\gamma^2 + k(\alpha)} > 1 \tag{A7}$$

where $\gamma \equiv \frac{1}{2} - \frac{r-\delta}{\sigma^2}$, $k(\alpha) = 2\xi(r + h(1 + \alpha)$ and $\xi \equiv \frac{1}{\sigma^2}$. By definition, $\phi(\alpha)$ is the positive solution of the quadratic equation

$$\frac{1}{2}\sigma^2\phi(\phi - 1) + (r - \delta)\phi - (r + h(1 + \alpha)) = 0. \tag{A8}$$

The thresholds x_1^F and x_2^F in (5) can then be expressed as follows

$$x_1^F = \frac{\phi(0)}{\phi(0) - 1}\frac{h(1 + \alpha) + \delta}{h(1 + \alpha)} \quad \text{and} \quad x_2^F = \frac{\phi(\alpha)}{\phi(\alpha) - 1}\frac{h(1 + \alpha) + \delta}{h}. \tag{A9}$$

Therefore to show that $x_1^F \leq x_2^F$ it is therefore sufficient to show that the function

$$f(\alpha) = \frac{\phi(\alpha)}{\phi(\alpha) - 1}(1 + \alpha), \quad \alpha \geq 0 \tag{A10}$$

is increasing in α. Taking the first derivative with respect to α we get

$$f'(\alpha) = \frac{-\phi'(\alpha)(1 + \alpha) + \phi(\alpha)(\phi(\alpha) - 1)}{(\phi(\alpha) - 1)^2} \tag{A11}$$

Because $\phi(\alpha)$ is the positive root of quadratic equation (A8) with μ replaced by $r - \delta$ and r replaced by $r+h(1+\alpha)$, we can write $\phi(\alpha)(\phi(\alpha)-1) = k(\alpha)+2(\gamma-1)$. Moreover, $\phi'(\alpha) = \frac{\xi h}{\phi(\alpha)-\gamma}$. Hence, $f'(\alpha) \geq 0$ if and only if

$$\xi h(1 + \alpha) \leq UB(\gamma), \tag{A12}$$

[34] We thank Alberto Romero for help with the proof of this lemma.

where $UB(\gamma) \equiv (k(\alpha)+(2\gamma-1)(\gamma+\sqrt{\gamma^2+k(\alpha)}))\sqrt{\gamma^2+k(\alpha)}$. We now show that $\inf_\gamma UB(\gamma) \geq \xi h(1+\alpha)$, thus proving that $x_1^F \leq x_2^F$. Simple algebraic manipulation allows us to rewrite $UB(\gamma)$ as

$$UB(\gamma) = \left(\sqrt{\gamma^2+k(\alpha)}+\gamma-1\right)\left(\sqrt{\gamma^2+k(\alpha)}\right)\left(\sqrt{\gamma^2+k(\alpha)}+\gamma\right) > 0. \tag{A13}$$

Taking the derivative of $UB(\gamma)$ with respect to γ we obtain

$$UB'(\gamma) = \left(\gamma+\sqrt{\gamma^2+k(\alpha)}\right)^2 + \frac{UB(\gamma)}{\gamma^2+k(\alpha)}\left(\gamma+\sqrt{\gamma^2+k(\alpha)}\right) > 0. \tag{A14}$$

Because, $\gamma \equiv \frac{1}{2} - \frac{r-\delta}{\sigma^2}$ and $\delta \geq r$, for any r, γ is minimum for $\delta = 0$. Let $\hat{\gamma} = \inf_\delta \gamma = \frac{1}{2} - \frac{r}{\sigma^2}$. Hence, for every $0 \leq \delta \leq r$, $UB(\hat{\gamma}) < UB(\gamma)$. From (A13), using the definition of $\hat{\gamma}$ we obtain that

$$UB(\hat{\gamma}) = \left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}+\hat{\gamma}-1\right)\left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}\right)\left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}+\hat{\gamma}\right), \tag{A15}$$

where $c(\alpha) \equiv 2\xi(1+\alpha)$. From (A14), $UB'(\hat{\gamma}) > 0$ and, from the definition of $\hat{\gamma} = \frac{1}{2} - \frac{r}{\sigma^2}$ we obtain that $\frac{\partial UB(\hat{\gamma})}{\partial r} = \frac{\partial UB(\hat{\gamma})}{\partial \hat{\gamma}}\frac{\partial \hat{\gamma}}{\partial r} < 0$. The lowest bound of $UB(\hat{\gamma})$ thus obtains when $r \to \infty$. Direct computation shows that $\lim_{\hat{\gamma}\to\infty} = \frac{c(\alpha)}{2} = \xi h(1+\alpha)$, thus verifying condition (A12). ∎

Lemma A.3. *Let $V_i^F(x)$ and $V_i^L(x)$, $i = 1, 2$, be defined as in (7) and (8), respectively. Then, if $h_1 > h_2$: (i) $V_1^F(x) > V_2^F(x)$, and (ii) $V_1^L(x) > V_2^L(x)$ for all x.*

Proof: Part (i) follows immediately from the convexity of $V_i^F(x)$, $i = i, 2$, and Lemma A.2. To prove (ii), from the definition of $V_i^L(x)$, $i = 1, 2$, in (8), and the fact that $x_1^F > x_2^F$ (Lemma A.2), it is immediate to see that $V_1^L(x) > V_2^L(x)$ for $x > x_2^F$. For $x \in [x_1^F, x_2^F]$ we have that the difference

$$\begin{aligned} V_1^L(x) - V_2^L(x) &= x\left(\frac{h_1}{h_1+\delta} - \frac{h_2}{h_1+h_2+\delta}\right) - \left(\frac{x}{x_2^F}\right)^{\phi_1} h_1 x_2^F \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) \\ &> h_1 x\left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) - \left(\frac{x}{x_2^F}\right)^{\phi_1} h_1 x_2^F \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) \\ &= \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right)\left(\frac{x\,(x_2^F)^{\phi_1} - x^{\phi_1} x_2^F}{(x_2^F)^{\phi_1}}\right) > 0, \end{aligned} \tag{A16}$$

where the first inequality follows from $h_1 > h_2$ and the last inequality follows from $x < x_2^F$ and $\phi_1 > 1$. For $x \in [0, x_1^F]$ we note that: (a) $V_1^L(x_1^F) > V_2^L(x_1^F)$ by (A16); (b) $\left.\frac{\partial V_1^L(x)}{\partial x}\right|_{x=0} = \frac{h_1}{h1+h_2+\delta}$

and $\left.\frac{\partial V_2^L(x)}{\partial x}\right|_{x=0} = \frac{h_2}{h1+h_2+\delta}$, thus $\left.\frac{\partial V_1^L(x)}{\partial x}\right|_{x=0} > \left.\frac{\partial V_2^L(x)}{\partial x}\right|_{x=0}$; and, (c) $V_1^L(x)$ and $V_2^F(x)$ are strictly concave in $x \in [0, x_1^F]$. These facts imply that $V_1^L(x)$ and $V_2^L(x)$ cannot cross for any $x < x_1^F$, and therefore $V_1^L(x) > V_2^L(x)$. ∎

Lemma A.4. *If $h_1 > h_2$ then: (i) there always exists a unique $\hat{x}$ such that $V_1^L(x) > V_1^F(x)$ for all $x \in [\hat{x}, x_2^F]$, and (ii) there exists a h_2 such that $V_2^L(x) < V_2^F(x)$ for all $x \in [0, x_2^F]$.*

Proof: (i) Let $D_1(x) = V_1^L(x) - V_1^F(x)$, $x \in [0, x_2^F]$. Note that: $D_1(0) = -K$, $D_1(x_2^F) = 0$, $\left.\frac{\partial V_1^L(x)}{\partial x}\right|_{x=x_2^F} = \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) h_1(1-\phi_1) < 0$. Hence, $D_1(x)$ has a unique root $\hat{x} \in [0, x_2^F]$. (ii) Let $D_2(x) = V_2^L(x) - V_2^F(x)$, $x \in [0, x_1^F]$. Note that, if $h_1 = h_2$, then $x_1^F = x_2^F$ and, by (i), there exists a $\hat{x} \in [0, x_1^F]$ such that $D_2(x) > 0$ for $x \in [\hat{x}, x_1^F]$. Moreover, $\lim_{h_2 \to 0} = -K$ for $x < x_2^F$. Hence, there exists a $h_2 > 0$ such that $V_2^L(x) < V_2^F(x)$ for all $x \in [0, x_2^F]$. ∎

Lemma A.5. *Suppose the roles of the firms are preassigned and firm j is the designated leader who cannot be preempted by firm i. The value of firm j is*

$$V_j^D(x) = \begin{cases} \left(\frac{x}{x_j^D}\right)^{\phi_0} V_j^L(x_j^D) & \text{if } x < x_j^D \\ V_j^L(x) & \text{if } x \geq x_j^D \end{cases}, \tag{A17}$$

where $V_j^L(x)$ is defined in Proposition 2 and x_j^D is implicitly determined by the smooth pasting condition

$$(\phi_j - \phi_0)\frac{h_j}{h_j+\delta}\frac{h_i x_i^F}{h_i+h_j+\delta}\left(\frac{x_j^D}{x_i^F}\right)^{\phi_j} + (\phi_0 - 1)\frac{h_j x_j^D}{h_j+\delta} - \phi_0 K = 0, \tag{A18}$$

with ϕ_0 given in equation (11), ϕ_j given in equation (6), and x_i^F given in equation (5).

Proof: The optimal value of firm j as the designated leader $V_j^D(x)$ is given by

$$V_j^D(x) = \max_{\tau_j^D} E\left[e^{-r\tau_j^D} V_j^L(x_j^D)\right], \tag{A19}$$

where $\tau_j^D = \inf\{t > 0 : x(t) \geq x_j^D\}$. From Lemma A.1, the value of the designated leader (A19) for $x < x_i^D$ can be written as

$$V_j^D(x) = \max_{x_i^D}\left[\left(\frac{x}{x_j^D}\right)^{\phi_0} V_j^L(x_j^D)\right]. \tag{A20}$$

Maximizing with respect to x_iD yields (A19) where x_i^D is implicitly defined by (A18). Because $V_i^L(x)$ is increasing and concave for $x \in [0, x_j^F]$, $x_i^D <_j^F$. ∎

Proof of Proposition 3

The value of firm i when both firms invest at a pre-specified threshold x^C is given by

$$V_i(x; x^C) = E\left[e^{-r\tau^C}\left(\int_{\tau^C}^{\infty} e^{-(r+h_i+h_j)(t-\tau^C)} h_i x(t)dt - K\right)\right], \quad i = 1, 2, \tag{A21}$$

where $\tau^C = \inf\{t > 0 : x(t) \geq x^C\}$. The proposition follows immediately from the law of iterated expectations and Lemma A.1. ∎

Proof of Proposition 4

From Lemma A.3, $h_1 \geq h_2$ implies $V_1^L(x) \geq V_2^L(x)$, $V_1^C(x; x^C) > V_2^C(x; x^C)$ and $x_1^C \leq x_2^C$. A simultaneous equilibrium can only occur when $V_1^C(x; x_1^C) > V_1^L(x)$ and $V_2^C(x; x_1^C) > V_2^L(x)$ for all x. The only sustainable joint investment threshold is x_1^C, because, given that $x(0) < x_1^C$, firm 1 will have always incentive to deviate from the alternative joint threshold x_2^C that maximizes firm 2's joint value. Because $V_1^L(x)$ is concave and decreasing in h_2 in $x \in [0, x_2^F]$ and $V_1^C(x)$ is convex, for every h_1 there exist a pair (x^*, h_2^*) such that $V_1^L(x^*) = V_1^C(x^*; x_1^C)$ and $\frac{\partial V_1^L(x)}{\partial x} = \frac{\partial V_1^C(x; x_1^C)}{\partial x}|_{x=x^*}$ Let $J(h_1) = h_2^*$. For $h_2 > J(h_1)$, $V_1^L(x^*) < V_1^C(x^*; x_1^C)$ and a simultaneous equilibrium is possible. For $h_2 < J(h_1)$, $V_1^L(x^*) > V_1^C(x^*; x_1^C)$ and no simultaneous equilibria are possible. If $J(h_1) > h_1$ then no simultaneous equilibria are possible when $h_1 \geq h_2$.

Similarly, for every h_1 there exist a pair (x^{**}, h_2^{**}) such that $V_2^L(x^{**}) = V_2^C(x^{**}; x_1^C)$ and $\frac{\partial V_2^L(x)}{\partial x} = \frac{\partial V_2^C(x; x_1^C)}{\partial x}|_{x=x^{**}}$ Let $\widehat{J}(h_1) = h_2^{**}$. For $h_2 > \widehat{J}(h_1)$, $V_2^L(x) > V_2^C(x; x_1^C)$ for some x and a simultaneous equilibrium is not sustainable. For $h_2 < \widehat{J}(h_1)$, $V_2^L(x) < V_2^C(x; x_1^C)$ for all x and simultaneous equilibria are possible. Furthermore, if $J(h_1) < h_1$ then there exist a unique $\tilde{h}_1$ such that $\widehat{J}(h_1) > h_1 > J(h_1)$ for all $h_1 < \tilde{h}_1$ and $J(\tilde{h}_1) = \widehat{J}(\tilde{h}_1) = \tilde{h}_1$. Hence, if $h_1 > h_2$ a simultaneous equilibrium can emerge only if $h_2 > J(h_1)$ and $h_1 \leq \tilde{h}_1$.

A sequential equilibrium emerges if $V_1^C(x; x_1^C) < V_1^L(x)$ for some x and $V_2^L(x) < V_2^F(x)$ for all x. Because $V_2^L(x)$ is concave and increasing in h_2, and $V_2^F(x)$ is convex in $x \in [0, x_1^F]$, for every h_1 there exists a pair (x', h_2') such that $V_2^L(x') = V_2^F(x')$ and $\frac{\partial V_2^L(x)}{\partial x} = \frac{\partial V_2^F(x)}{\partial x}|_{x=x'}$ Let $S(h_1) = h_2'$. For $h_2 < S(h_1)$, $V_2^L(x) < V_2^F(x)$ for all x and the equilibrium is of the sequential type. For $h_2 > S(h_1)$, $V_2^L(x') > V_2^F(x')$ and so no sequential equilibrium are possible. In the last case, firm 2 will attempt to preempt firm 1 as long as $V_2^L(x) = V_2^F(x)$. Let $x_2^P = \inf\{x \in [0, x_1^F] : V_2^L(x) = V_2^F(x)\}$. Then, if $h_2 > S(h_1)$, firm 2 will try to preempt firm 1 until $x \geq x_2^P$. Because $h_1 > h_2$, by Lemma A.3, $V_1^L(x_2^P) - V_1^F(x_2^P) > 0$. Hence, the optimal response of firm 1 is to ϵ-preempt firm 2 and invest at $x_1^P = \min\{x_2^P - \epsilon, x_1^D\}$, where $\epsilon > 0$ and x_1^D is the optimal investment threshold of firm 1 as a designated leader, defined in (A18) of Lemma A.5.

The two thresholds $J(h_1) < h_1$ and $S(h_1) < h_1$ partition the space (h_1, h_2), $h_1 > h_2$, into four regions, each characterized by a different equilibrium.

1. *Region 1:* $h_2 > J(h_1)$ and $h_2 > S(h_1)$. Two types of equilibria: Simultaneous and preemptive.

2. *Region 2:* $h_2 > J(h_1)$ and $h_2 < S(h_1)$. Two types of equilibria: Simultaneous and sequential.

3. *Region 3.* $h_2 < J(h_1)$ and $h_2 > S(h_1)$. Unique preemptive equilibrium.

4. *Region 4.* $h_2 < J(h_1)$ and $h_2 < S(h_1)$. Unique sequential equilibrium. ∎

Proof of Proposition 5

The firms' value in the case of preemptive or sequential equilibrium follow directly from Propositions 1 and 2 while the firms' value in the case of simultaneous equilibria follow from Proposition 3.

∎

Proof of Proposition 6

Immediate from the definition of beta in (19) and Proposition 5. ∎

Proof of Proposition 7

The proof is by induction. We consider the case of three firms first and then generalize to the case of N firms. Let $\tau_1 < \tau_2 < \tau_3$ be the investment times of firms 1, 2 and 3 respectively, corresponding to the thresholds $x_1 < x_2 < x_3$. Following the derivation of leader's and follower's payoff in Section 2.2, the value of firm 1 at threshold x_1 is given by

$$\begin{aligned} V_1(x_1) &= E\left[\int_0^{\tau_2} e^{-(r+h_1)t} h_1 x(t)dt + e^{-(r+h_1)\tau_2}\left(\int_{\tau_2}^{\tau_3} e^{-(r+h_1+h_2)(t-\tau_2)} h_1 x(t)dt + \right.\right. \\ &\quad \left.\left. e^{-(r+h_1+h_2)(\tau_3-\tau_2)} \int_{\tau_3}^{\infty} e^{-(r+h_1+h_2+h_3)(t-\tau_3)} h_1 x(t)dt\right)\right] - K \\ &= \frac{h_1 x_1}{H_1+\delta} - K - \left(\frac{x_1}{x_2}\right)^{\phi_1} h_1 \left(x_2\Delta_2 + \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3\right) \qquad \text{(A22)} \end{aligned}$$

where $H_k \equiv \sum_{i=1}^{k} h_i$, $\Delta_k \equiv \frac{1}{H_{k-1}+\delta} - \frac{1}{H_2 k+\delta}$, $k \geq 2$, and ϕ_k defined as in (25). Following similar construction we obtain

$$V_1(x_2) = \frac{h_1 x_2}{H_2+\delta} - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3 \qquad \text{(A23)}$$

$$V_1(x_3) = \frac{h_1 x_3}{H_3+\delta} \qquad \text{(A24)}$$

$$V_2(x_1) = \left(\frac{x_1}{x_2}\right)^{\phi_1}\left(\frac{h_2 x_2}{H_2+\delta} - K - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3\right) \qquad \text{(A25)}$$

$$V_2(x_2) = \frac{h_2 x_2}{H_2+\delta} - K - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3 \qquad \text{(A26)}$$

$$V_2(x_3) = \frac{h_2 x_3}{H_3+\delta} \qquad \text{(A27)}$$

$$V_3(x_1) = \left(\frac{x_1}{x_2}\right)^{\phi_1}\left(\frac{x_2}{x_3}\right)^{\phi_2}\left(\frac{h_3 x_3}{H_3+\delta} - K\right) \qquad \text{(A28)}$$

$$V_3(x_2) = \left(\frac{x_2}{x_3}\right)^{\phi_2}\left(\frac{h_3 x_3}{H_3+\delta} - K\right) \qquad \text{(A29)}$$

$$V_3(x_3) = \frac{h_3 x_3}{H_3+\delta} - K. \qquad \text{(A30)}$$

These quantities can be used to derive firm values in leader-follower equilibria for all x as follows

$$V_1^{\mathrm{LF}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_1(x_1) & \text{if } x < x_1 \\ \frac{h_1 x}{H_1+\delta} - \left(\frac{x}{x_2}\right)^{\phi_1} h_1 \left(x_2\Delta_2 + \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3\right) & \text{if } x_1 < x < x_2 \\ \frac{h_1 x}{H_2+\delta} - \left(\frac{x}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3 & \text{if } x_2 < x < x_3 \\ \frac{h_1 x}{H_3+\delta} & \text{if } x > x_3 \end{cases} \tag{A31}$$

$$V_2^{\mathrm{LF}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_2(x_1) & \text{if } x < x_1 \\ \left(\frac{x}{x_2}\right)^{\phi_1} V_2(x_2) & \text{if } x_1 < x < x_2 \\ \frac{h_2 x}{H_2+\delta} - \left(\frac{x}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3 & \text{if } x_2 < x < x_3 \\ \frac{h_2 x}{H_3+\delta} & \text{if } x > x_3 \end{cases} \tag{A32}$$

$$V_3^{\mathrm{LF}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_3(x_1) & \text{if } x < x_1 \\ \left(\frac{x}{x_2}\right)^{\phi_1} V_3(x_2) & \text{if } x_1 < x < x_2 \\ \left(\frac{x}{x_3}\right)^{\phi_2} V_3(x_3) & \text{if } x_2 < x < x_3 \\ \frac{h_3 x}{H_3+\delta} & \text{if } x > x_3 \end{cases} \tag{A33}$$

Similarly, for $V_2^{\mathrm{LF}}(x)$ and $V_3^{\mathrm{LF}}(x)$. Generalizing to the case of N firms we obtain

$$V_m^{\mathrm{LF}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_m(x_1) & & \text{if } x < x_1 \\ \left(\frac{x}{x_{n+1}}\right)^{\phi_n} V_m(x_{n+1}) & \text{for } n < m & \multirow{2}{*}{\text{if } x_n < x < x_{n+1},\ n = 1, \ldots, N-1} \\ \frac{h_m x}{H_n+\delta} - \left(\frac{x}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{for } n \geq m & \\ \frac{h_m x}{H_N+\delta} & & \text{if } x > x_N \end{cases}, \tag{A34}$$

with

$$\Gamma_{m,n} = \begin{cases} x_{n+1}\Delta_{n+1} + \prod_{k=n+1}^{N-1} \left(\frac{x_k}{x_{k+1}}\right)^{\phi_k} x_{k+1}\Delta_{k+1} & \text{if } m \leq n \\ 1 & \text{if } m > n \end{cases}, \tag{A35}$$

$\Delta_k = \frac{1}{H_{k-1}+\delta} - \frac{1}{H_k+\delta} > 0$, and, for $n = 1, \ldots, N-1$,

$$V_m(x_n) = \begin{cases} \frac{h_m x_n}{H_n+\delta} - \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{if } m < n \\ \frac{h_m x_n}{H_n+\delta} - K - \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{if } m = n \\ \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} V_m(x_{n+1}) & \text{if } m > n \end{cases}, \tag{A36}$$

and

$$V_m(x_N) = \begin{cases} \frac{h_m x_N}{H_N+\delta} & \text{if } m < n \\ \frac{h_N x_N}{H_N+\delta} - K & \text{if } m = N \end{cases}. \tag{A37}$$

Using the definition of beta in (19) and the equilibrium firm values in (A34) we obtain (24) where

$$\omega_{m,n}(x) = \frac{a_{m,n}(x)}{a_{m,n}(x) - b_{m,n}(x)} > 0 \tag{A38}$$

with $a_{m,n}(x) = \frac{h_m x}{H_n+\delta}$ and $b_{m,n}(x) = \left(\frac{x}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n}$. ∎

B Appendix: Details of empirical implementations

B.1 Definitions of firm control variables

Market Cap	The natural logarithm of market capitalization of a firm's common equity in USD millions. We use the market capitalization reported in Compustat as of the fiscal year end (item prcc_f×csho) in our annual regressions, while we use market capitalization from the CRSP Monthly Stock File (item abs(prc)×shrout) in our monthly regressions.
Profitability	The earnings before interest, taxes, depreciation, and amortization (item oibdp) scaled by total assets (item at). The variable is winsorized at the 1% level.
B/M	The natural logarithm of the book value of common equity (item ceq) scaled by the market value of common equity (item prcc_f×csho). The variable is winsorized at the 1% level.
Tangibility	The total net property, plant, and equipment (item ppent) scaled by total assets (item at).
Cash	Cash and short-term investment (item che) scaled by total assets (item at).
Book Leverage	Total long-term debt plus debt in current liabilities (item dltt+dlc) scaled by total assets (item at). The variable is winsorized at the 1% level.
R&D	Research and development expenses (item xrd) scaled by sales (item sale). The variable is winsorized at the 1% level.
CAPEX	Capital expenditures (item capx) scaled by sales (item sale). The variable is winsorized at the 1% level.

B.2 Estimation of high-frequency betas

For each stock, we use prices between 9:45am and 4:00pm, sampled every 25 minutes, to compute high frequency returns. We combine these returns with the overnight return, computed between 4:00pm on the previous day and 9:45am on the current day, yielding a total of 16 intra-daily returns. We choose a 25-minute frequency to balance the desire for reduced measurement error with the need to avoid the microstructure biases that arise at the highest frequencies (e.g., **?**, **?**, and **?**). The prices we use are the national best bid and offer (NBBO) prices, computed by examining quote prices from all exchanges offering quotes on a given stock. We use the exchange traded fund tracking the Standard & Poor's Composite Index (SPDR) traded on Amex with ticker SPY to measure the market return, as in **?**. We compute monthly equity betas as the ratio of a stocks' realized covariance with the fund to the realized variance of the fund over a given month and we refer to it as *High-Frequency Betas*. Since TAQ data are available starting 1994, the size of our sample is reduced to about half compared to when we use the other beta measures.

C Robustness tables

Table C-1: Alternative aggregation of the firm's ranks across multiple races

The table presents estimates from firm-level OLS regressions of the equity beta on firm characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 4, and we do not report firm-level controls (defined in Appendix B.1) for brevity. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Weighted average of the firm's ranks in innovation races

Percentile Rank denotes the weighted average of the innovating firm's relative (ordinal) positions in innovation races computed using, for each race, the firm's recent patenting output relative to that of the other firms active in the race. The race-specific weights are computed as the number of firms that are innovating in a given race-year (race-month) divided by the sum of the number of innovating firms in all races-year (races-month) in which the firm is innovating.

	Annual Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.253***	0.173***	0.064**	0.098**	0.522***	0.351***	0.133***	0.291***
	(0.052)	(0.049)	(0.027)	(0.048)	(0.094)	(0.086)	(0.048)	(0.098)
Industry × Year FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.394	0.448	0.636	0.554	0.483	0.548	0.665	0.437
N	9,061	8,378	7,551	8,378	9,061	8,378	7,551	8,378
	Monthly Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.226***	0.168***	0.118***	0.071**	0.518***	0.404***	0.312***	0.207***
	(0.051)	(0.048)	(0.035)	(0.034)	(0.090)	(0.084)	(0.065)	(0.072)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.126	0.149	0.189	0.214	0.229	0.258	0.276	0.143
N	95,414	88,765	86,851	88,765	95,414	88,765	86,851	88,765
	Sum Beta				High-Frequency Beta			
Percentile Rank	0.280***	0.230***	0.195***	0.090**	0.175***	0.182***	0.064***	-0.056
	(0.060)	(0.057)	(0.049)	(0.043)	(0.055)	(0.054)	(0.018)	(0.036)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.040	0.053	0.062	0.109	0.435	0.469	0.669	0.591
N	95,414	88,765	86,851	88,765	49,326	44,654	42,985	44,654

Table C-1 (cont.): Alternative aggregation of the firm's ranks across multiple races

Panel B: The firm's rank in the most populated innovation race

Percentile Rank is the innovating firm's relative (ordinal) position in the innovation race-year (race-month) with the highest number of innovating firms.

	Annual Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.237***	0.161***	0.055**	0.085**	0.469***	0.304***	0.111***	0.204**
	(0.045)	(0.043)	(0.023)	(0.041)	(0.084)	(0.077)	(0.042)	(0.085)
Industry ×Year FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.395	0.449	0.636	0.554	0.484	0.548	0.665	0.436
N	9,061	8,378	7,551	8,378	9,061	8,378	7,551	8,378
	Monthly Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.209***	0.150***	0.106***	0.053*	0.446***	0.330***	0.252***	0.113*
	(0.044)	(0.041)	(0.030)	(0.028)	(0.080)	(0.074)	(0.057)	(0.058)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.126	0.149	0.189	0.214	0.229	0.258	0.276	0.143
N	95,414	88,765	86,851	88,765	95,414	88,765	86,851	88,765
	Sum Beta				High-Frequency Beta			
Percentile Rank	0.251***	0.197***	0.168***	0.068**	0.151***	0.142***	0.051***	-0.058*
	(0.050)	(0.047)	(0.041)	(0.034)	(0.049)	(0.048)	(0.016)	(0.030)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.040	0.053	0.062	0.109	0.435	0.469	0.669	0.591
N	95,414	88,765	86,851	88,765	49,326	44,654	42,985	44,654

Table C-2: Alternative definitions of the patent's field of technology

The table presents estimates from firm-level OLS regressions of the equity beta on firm characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 4, and we do not report firm-level controls (defined in Appendix B.1) for brevity. Percentile Rank denotes the average of the innovating firm's relative (ordinal) positions in innovation races computed using, for each race, the firm's recent patenting output relative to that of the other firms active in the race. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Sections of the IPC classification

Innovation race is defined by taking technology classes to be sections (the first-level) of the IPC hierarchical classification of patents.

	Annual Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.205***	0.150***	0.069***	0.117**	0.449***	0.312***	0.146***	0.323***
	(0.049)	(0.046)	(0.025)	(0.048)	(0.088)	(0.080)	(0.045)	(0.101)
Industry × Year FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.386	0.442	0.632	0.550	0.485	0.547	0.650	0.424
N	11,159	10,331	9,263	10,331	11,159	10,331	9,263	10,331
	Monthly Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.160***	0.124***	0.092***	0.058	0.392***	0.303***	0.239***	0.150*
	(0.049)	(0.047)	(0.034)	(0.037)	(0.089)	(0.083)	(0.065)	(0.080)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.126	0.150	0.189	0.210	0.222	0.246	0.262	0.138
N	115,599	107,758	105,417	107,758	115,599	107,758	105,417	107,758
	Sum Beta				High-Frequency Beta			
Percentile Rank	0.236***	0.202***	0.173***	0.056	0.104*	0.141***	0.053***	-0.023
	(0.059)	(0.057)	(0.049)	(0.046)	(0.054)	(0.054)	(0.017)	(0.038)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.044	0.057	0.065	0.105	0.432	0.467	0.665	0.586
N	115,599	107,758	105,417	107,758	56,886	51,511	49,574	51,511

Table C-2 (cont.): Alternative definitions of the patent's field of technology

Panel B: Subclasses of the IPC classification

Innovation race is defined by taking technology classes to be subclasses (the third-level) of the IPC hierarchical classification of patents.

	Annual Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.239***	0.179***	0.053*	0.122***	0.479***	0.369***	0.122**	0.335***
	(0.060)	(0.057)	(0.031)	(0.042)	(0.108)	(0.099)	(0.057)	(0.097)
Industry × Year FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.386	0.445	0.638	0.565	0.481	0.551	0.676	0.448
N	7,436	6,871	6,221	6,871	7,436	6,871	6,221	6,871
	Monthly Frequency							
	Market Beta				Growth-Value Beta			
Percentile Rank	0.155***	0.111**	0.077**	0.054*	0.411***	0.325***	0.234***	0.183***
	(0.056)	(0.054)	(0.039)	(0.030)	(0.096)	(0.093)	(0.071)	(0.068)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.124	0.151	0.192	0.217	0.239	0.270	0.291	0.153
N	79,915	74,366	72,774	74,366	79,915	74,366	72,774	74,366
	Sum Beta				High-Frequency Beta			
Percentile Rank	0.186***	0.139**	0.115**	0.088**	0.131**	0.129**	0.044**	-0.050
	(0.062)	(0.060)	(0.052)	(0.041)	(0.057)	(0.055)	(0.018)	(0.036)
Industry × Month FE	Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.037	0.049	0.058	0.112	0.434	0.469	0.671	0.593
N	79,915	74,366	72,774	74,366	42,719	38,690	37,260	38,690

Table C-3: Equally-weighted market portfolio

The top panel of the table presents estimates from firm-level OLS regressions of equally-weighted *Market Beta* or equally-weighted *Sum Beta* on firm characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 4. The bottom panel of the table presents estimates from firm-race-level OLS regressions of equally-weighted *Market Beta* or equally-weighted *Sum Beta* on firm and race characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 5. We do not report firm-level controls (defined in Appendix B.1) for brevity. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Annual Frequency				Monthly Frequency							
	Market Beta				Market Beta				Sum Beta			
	Firm-Level											
Percentile Rank	0.351***	0.257***	0.108***	0.134**	0.270***	0.216***	0.165***	0.085*	0.296***	0.264***	0.236***	0.142**
	(0.070)	(0.063)	(0.036)	(0.055)	(0.072)	(0.068)	(0.052)	(0.045)	(0.079)	(0.074)	(0.067)	(0.056)
Industry × Year FE	Yes	Yes	Yes									
Industry × Month FE					Yes	Yes	Yes		Yes	Yes	Yes	
Firm FE				Yes				Yes				Yes
R^2	0.361	0.440	0.628	0.579	0.080	0.105	0.134	0.181	0.024	0.039	0.042	0.085
N	9,061	8,378	7,551	8,378	95,414	88,765	86,851	88,765	95,414	88,765	86,851	88,765
	Firm-Race-Level											
Percentile Rank	0.176***	0.077**			0.153***	0.053			0.143***	0.064*		
	(0.036)	(0.030)			(0.036)	(0.032)			(0.042)	(0.038)		
No. of Innovating Firms	0.204***	0.048			0.344***	0.134**			0.334***	0.147***		
	(0.051)	(0.045)			(0.062)	(0.055)			(0.063)	(0.057)		
Technology, Year FE	Yes	Yes										
Technology, Month FE					Yes	Yes			Yes	Yes		
R^2	0.276	0.386			0.127	0.168			0.084	0.106		
N	22,702	21,509			230,113	219,238			230,113	219,238		

Table C-4: Total risk and the firm's rank in innovation races

The top panel of the table presents estimates from firm-level OLS regressions of firm's total risk on firm characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 4. Firm's total risk is the firm's standard deviation of daily equity returns scaled by the market's standard deviation of daily returns. The bottom panel of the table presents estimates from firm-race-level OLS regressions of firm's total risk on firm and race characteristics. The samples and specifications are analogous to those in Panel A and Panel B of Table 5. We do not report firm-level controls (defined in Appendix B.1) for brevity. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Annual Frequency				Monthly Frequency			
	Firm-Level							
Percentile Rank	0.367**	0.326***	0.177***	0.455***	0.173	0.169	0.085	0.248**
	(0.147)	(0.125)	(0.066)	(0.154)	(0.141)	(0.121)	(0.055)	(0.104)
Industry × Year FE	Yes	Yes	Yes					
Industry × Month FE					Yes	Yes	Yes	
Firm FE				Yes				Yes
R^2	0.546	0.627	0.776	0.614	0.434	0.472	0.594	0.404
N	9,061	8,378	7,551	8,378	95,414	88,765	86,851	88,765
	Firm-Race-Level							
Percentile Rank	0.186***	0.111**			0.135**	0.024		
	(0.070)	(0.045)			(0.054)	(0.044)		
Number of Innovating Firms	0.311***	-0.007			0.497***	0.104		
	(0.084)	(0.065)			(0.089)	(0.072)		
Technology, Year FE	Yes	Yes						
Technology, Month FE					Yes	Yes		
R^2	0.470	0.571			0.402	0.460		
N	22,702	21,509			230,113	219,238		

Table C-5: Newey-West regressions

The top panel of the table presents estimates from firm-level OLS regressions of the equity beta on firm characteristics. For each dependent variable, we use the second specification from Panel A and Panel B of Table 4. The bottom panel of the table presents estimates from firm-race-level OLS regressions of the equity beta on firm and race characteristics. For each dependent variable, we use the second specification from Panel A and Panel B of Table 5. We do not report firm-level controls (defined in Appendix B.1) for brevity. Newey-West standard errors are reported in parentheses. At annual frequency, we allow for up to 3 lags, while we allow for up to 12 lags in the regressions at the monthly frequency. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Annual Frequency		Monthly Frequency			
	Market Beta	Growth-Value Beta	Market Beta	Growth-Value Beta	Sum Beta	High-Frequency Beta
			Firm-Level			
Percentile Rank	0.158***	0.326***	0.137***	0.364***	0.177***	0.136***
	(0.043)	(0.075)	(0.025)	(0.048)	(0.035)	(0.029)
Industry × Year FE	Yes	Yes				
Industry, Month FE			Yes	Yes	Yes	Yes
N	8,378	8,378	88,765	88,765	88,765	44,654
			Firm-Race-Level			
Percentile Rank	0.054***	0.114***	0.037***	0.091***	0.041***	0.006
	(0.017)	(0.032)	(0.012)	(0.023)	(0.016)	(0.016)
Number of Innovating Firms	0.001	0.002	0.072***	0.049	0.130***	0.218***
	(0.024)	(0.044)	(0.019)	(0.036)	(0.024)	(0.033)
Technology, Year FE	Yes	Yes				
Technology, Month FE			Yes	Yes	Yes	Yes
N	21,509	21,509	219,238	219,238	219,238	107,456