Innovation, Growth and Asset Pricing

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Abstract

We examine the asset pricing implications of innovation and R&D in a stochastic model of endogenous growth. In equilibrium, R&D endogenously drives a small, persistent component in productivity growth, in line with the data. These productivity dynamics induce long and persistent swings in macro growth rates and asset market valuations at medium and low frequencies. With recursive preferences, households are very averse to such movements in growth rates and command high risk premia in asset markets, helping the model to quantitatively rationalize a variety of asset pricing data. In the model, the resolution of these puzzles is inherently linked to the strong propagation mechanism the model exhibits, absent in standard macroeconomic models. We find strong empirical support for innovation driven low frequency movements in aggregate growth rates and asset market valuations in the data.

Keywords: Endogenous growth, asset pricing, medium term cycles, R&D, business cycle propagation, recursive preferences.

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1 Introduction

Innovation and the development of new technologies have long been identified as important determinants of economic growth. At the aggregate level, such development of new technologies is reflected in sustained growth of capital and labor productivity. An important stylized fact about innovation as measured by R&D expenditures is that it is itself quite volatile and fairly persistent, as well as quite procyclical. Such movements in innovating activity should then reflected in the dynamics of growth rates, and hence in asset market valuations. Indeed, US post-war data exhibit significant movements in aggregate growth rates and asset market valuations at higher and lower frequencies.

In this paper, we quantitatively examine the asset pricing implications of innovation. Specifically, we ask how the dynamics of innovation are themselves reflected in the movements of the aggregate economy and in asset prices. That is, we focus on the link between innovation, growth rate dynamics and asset prices. Our setup has two distinguishing features. First, we use a stochastic endogenous growth model in which, in contrast to standard macroeconomic models, firms need to engage in R&D in order to generate sustained growth. Second, we assume that households have recursive preferences, so that they care about long-term growth prospects.

Our results suggest that accounting for the endogeneity of innovation goes a long way towards a macroeconomic framework, which simultaneously quantitatively captures the joint dynamics of quantities and asset prices in a general equilibrium setting. More specifically, the model generates a strong propagation mechanism for shocks absent in standard macroeconomic frameworks. This propagation mechanism induces significant movements in aggregate quantities and asset market valuations at both medium and low frequencies, a prediction for which we find strong support in the data. With recursive preferences agents are very averse to such low frequency dynamics in both consumption and cash flows, hence they command high risk premia in asset markets. These innovation driven dynamics allow the model to be consistent with a variety of stylized facts about asset markets, such as a high equity premium and a low and smooth risk free rate. Our results imply that there is a tight connection between macroeconomic risk, growth and risk premia in asset markets. Put differently, we view stochastic models of endogenous growth as useful tools for quantitative macroeconomic modeling and general equilibrium asset pricing.

We first show that in this model innovation and R&D endogenously drive a small, but persistent component in the growth rate of aggregate productivity. More specifically, productivity will contain a high-frequency component driven by exogenous shocks, as well as an endogenous component that is linked to R&D activity in the economy. Crucially, this endogenous component operates at lower frequencies than the exogenous component, hence productivity endogenously exhibits high and low frequency movements. Interestingly, these dynamics arise in a model that is driven by a single exogenous shock. While this shock induces fluctuations at business cycle frequency comparable to standard macroeconomic settings, the innovation process in the model translates this disturbance into an additional, slow-moving component in productivity. Naturally, these productivity dynamics will be reflected in the dynamics of aggregate quantities. In other words, the innovation process provides a strong propagation mechanism for shocks. In contrast to standard macro models, our setting therefore endogenously generates fluctuations at various frequencies, including significant movements at medium frequency. Such persistent movements are inherently linked to predictability of aggregate growth rates. Empirically, we show that an innovation related quantity, that we will refer to as R&D intensity, has significant predictive power for productivity growth, consumption growth and output growth, as well as cash flows, as predicted by the model.

Such consumption and cash flow dynamics have important implications for risk premia in asset markets given our assumption of preference specification. In line with much of the recent asset pricing literature we assume that households have recursive Epstein-Zin preferences. This implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. Under the standard assumption that these preferences exhibit a preference for early resolution of uncertainty, households are strongly averse to the persistent innovations to expected growth rates implied by the propagation mechanism of the model. Quantitatively, this is reflected in a substantial equity premium and a low and stable risk-free rate. Moreover, persistence in cash flow dynamics will also be reflected in asset valuations. In this respect, the model predicts productivity driven low frequency cycles in stock market values and price-dividend ratios, in line with the empirical evidence.

At the heart of these dynamics of the model is the innovation process, which arises endogenously. In the model, the consumption good is produced with standard inputs such as labor or capital, and additionally, a bundle of intermediate goods. These intermediate goods are most readily thought of as patents or blueprints for machines that facilitate the production of the consumption good by making the production process more efficient. Consumption good producers purchase intermediate goods from intermediate goods firms, which possess monopoly rights on their respective products. In accordance with the real business cycle literature we assume that consumption good production is subject to a random exogenous disturbance (a technology shock), which in contrast, we assume to be stationary. Thus, long-run growth in aggregate output must result from growth in the number of intermediate goods. In order to create new patents, firms need to engage in R&D. The reward for creating a new patent is the stream of monopoly profits associated with supplying the intermediate good to the representative firm each period. Given that both demand for new patents and monopoly profits rise after a positive disturbance, incentives to engage in R&D rise as well. This raises the number of intermediate goods the representative firm utilizes, and given the production technology, leads to a sustained rise in the growth rate of aggregate output, which spills over into a sustained rise in the growth rate of dividends and output. Given a realistically persistent stationary specification of the exogenous random disturbance, this mechanism leads to long swings in the growth rate of productivity. Crucially, we show that that the resulting model resembles a version of the real business cycle model in which TFP has an endogenous component driven by R&D activity.

Our paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The economic mechanisms driving the asset pricing implications are closely related to Bansal and Yaron (2004). In a consumption-based model, Bansal and Yaron specify both consumption and dividend growth to contain a small, persistent component, which leads to long and persistent swings in the dynamics of these quantities. This specification along with the assumption of Epstein-Zin recursive utility with a preference for early resolution of uncertainty, allows them to generate high equity premia as compensation for these 'long-run risks'. The ensuing literature on long-run risk quantitatively explains a wide range of patterns in asset markets, such as those in equity, government, corporate bond, foreign exchange and derivatives markets [1]. While somewhat hard to detect in the data, we show that the growth rate dynamics that Bansal and Yaron specify directly follow naturally from agents' equilibrium R&D decisions in our endogenous growth model. In other words, the growth rate dynamics that a broad class of stochastic endogenous growth models generate are precisely what Bansal and Yaron refer to as long-run risk. In short, equilibrium growth is risky. Two observations are in order. First, long-run risk arises as agents' equilibrium choice. In this setting, agents have the option to effectively eliminate their exposure to long run risk by smoothing their R&D expenditures. While smoothing R&D activity (i.e. not being responsive to procyclical profit opportunities) would diminish the swings in growth rates and therefore the exposure to long run risk, it would however lead to inefficiently low growth rates. Therefore, in equilibrium, agents trade off inefficiently

[1] Bansal and Yaron (2004) address the equity premium and time series predictability of stock returns, Bansal and Shaliastovich (2008) address bond and foreign exchange markets, Bhamra, Kuehn and Strebulaev (2009) and Chen (2009) look at corporate bond markets, while Drechsler and Yaron (2008) investigate derivatives markets.

low growth against exposure to long run risk. Second, in fairly standard calibrations of our model, these effects are quantitatively significant.

Our paper is also very closely related to a number of recent papers seeking to understand how long run risks arise endogenously in production economies (Kaltenbrunner and Lochstoer (2008), Croce (2008), Campanale, Castro and Clementi (2009), Ai (2008), Kuehn (2008)). These papers typically work in versions of the standard real business cycle model, where growth is given exogenously. One important conclusion from calibrated versions of these contributions is that while long run risks do arise endogenously in such settings, they are typically not quantitatively sufficient to rationalize key asset market statistics. This is in contrast to our specification, where the incentives to engage in R&D deliver quantitatively significant long run risks in both consumption and dividend growth, suggesting that stochastic endogenous growth models provide a natural environment for general equilibrium asset pricing.

Similarly, the paper is closely related to recent contributions examining the link between technological innovation and asset pricing (Garleanu, Panageas and Yu (2009), Garleanu, Kogan and Panageas (2009), Pastor and Veronesi (2009)). While these models generate similar dynamics as our paper for key macroeocnomic variables, such as consumption growth, they focus on adoption of new technologies. In this sense, because we focus on the creation of new technologies, our approach is complementary. Moreover, in these aforementioned models of technology adoption, the arrival of new technologies, and therefore the growth rate dynamics, follow exogenously specified processes, whereas we endogenize these growth dynamics. Additionally, another key difference is that these papers use preference specifications such that these growth rate dynamics are not priced. Lin (2009) examines the link between endogenous technological change and the cross-section of returns in a partial equilibrium model.

Methodologically, our paper is a variation of recent contributions seeking to link the endogenous growth literature and the business cycle literatures (Jones, Manuelli, Siu, Stacchetti (2004), Jones, Manuelli, Siu (2004), Jones and Manuelli (2005)). Within this literature it is especially closely related by recent contributions identifying medium term business cycles (Comin and Gertler (2007), Comin, Gertler and Santacreu (2009)), which in turn build on the seminal contributions of Romer (1990). In these models endogenous growth is generated by increasing the number of intermediate goods used in the final good production, hence the moniker, expanding-variety models. This endogenous growth mechanism was first initiated successfully by Romer. In this context, the paper is also related to a recent set of papers by Bilbiie, Ghironi and Melitz (2007, 2008) who explore the business cycle implications of endogenous product variety. While these papers do address how creation and adoption of new technologies affect asset prices, they do not consider risk premia and the dynamics of consumption growth, which are central to asset pricing and which are the main contributions of our paper.

The paper is structured as follows. In section 2 we describe our benchmark model, and detail its relationship to the real business cycle model. We explore its quantitative implications for productivity, macroeconomic quantities and asset prices in section 3, along with a number of empirical predictions. In section 4 we provide two extensions the benchmark model, one motivated by the asset pricing literature, the other by the empirical evidence on R&D. Section 5 concludes.

2 Model

We start by describing out benchmark endogenous growth model. The model is a stochastic version of the seminal work in Romer (1990), to which we add capital accumulation subject to convex adjustment costs and assume that households have recursive Epstein-Zin preferences. To facilitate comparison with the workhorse stochastic growth model, we then describe we then show that under a number assumptions to be made precise our benchmark model collapses to the latter model. In the following, we will refer to our benchmark model as the ENDO model, while the EXO model refers to the version of the stochastic growth model that the benchmark model collapses to.

2.1 Benchmark Endogenous Growth Model (ENDO)

In our benchmark model, rather than assuming exogenous technological progress, growth arises through firms' R&D investment. R&D investment leads to creation of new patents or intermediate goods used in the production of a consumption good. An increasing number of intermediate goods is the ultimate source of sustained growth, hence the model is a version of an expanding-variety model of endogenous growth.

The model features a representative final good firm which produces the single consumption good and behaves competitively. The production of the consumption good requires capital, labor and a composite intermediate good. Furthermore production of the final good is subject to stationary exogenous shocks. Intermediate goods are produced by a continuum of monopolistic producers. As in Romer (1990) introduction of new intermediate goods is the ultimate source of sustained productivity growth. Creation of new intermediate goods depends on research and development activity. We assume that the representative household has

Epstein-Zin preferences, whose consumption and savings problem is fairly standard.

Household The representative household has Epstein-Zin preferences defined over consumption:

$$U_t = \left\{(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}},$$

where γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. In the long-run risks literature, the parametrization $\psi > \frac{1}{\gamma}$ is assumed, that is, the agent has a preference for early resolution of uncertainty, so that the agent dislikes shocks to long-run expected growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions Z_t in the stock market, which pays an aggregate dividend $\mathcal{D}_t$, and in the bond market, B_t. Accordingly, the budget constraint of the household becomes

$$C_t + Q_t Z_{t+1} + B_{t+1} = W_t L_t + (Q_t + \mathcal{D}_t)Z_t + R_t B_t$$

where Q_t is the stock price, R_t is the gross risk free rate, W_t is the wage and L_t denotes hours worked.

As described above, the production side of the economy consists of several sectors, so that the aggregate dividend can be further decomposed into the individual payouts of these sectors, in a way to be described below.

The setup implies that the stochastic discount factor in the economy is given by

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \frac{[\mathbb{E}_t(U_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\psi}{1-\gamma}}}{U_{t+1}^{\gamma-1/\psi}} . \quad (1)$$

Final Goods Sector There is a representative firm that uses capital K_t, labor L_t and a composite of intermediate goods G_t to produce the final (consumption) good. Also, assume that the final goods firm owns the capital stock and has access to the CRS production technology

$$Y_t = (K_t^{\alpha}(\Omega_t L_t)^{1-\alpha})^{1-\xi} G_t^{\xi} \quad (2)$$

where the composite G_t is defined according to the CES aggregator,

$$G_t \equiv \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} \, di \right]^{\nu}$$

and $X_{i,t}$ is intermediate good $i \in [0, N_t]$, where N_t is the measure of intermediate goods in use at date t. Furthermore, α is the capital share, ξ is the intermediate goods share, and ν is the elasticity of of substitution between the intermediate goods. Note that $\nu > 1$ is assumed so that increasing the variety of intermediate goods raises the level of productivity in the final goods sector. This property is crucial for sustained growth. The productivity shock Ω_t is assumed to follow a stationary Markov process. Because of the stationarity of the forcing process, sustained growth will arise endogenously from the development of new intermediate goods. We will describe the research and development process below.

Define dividends of the final goods firm as

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} \, di$$

where I_t is capital investment, W_t is the wage rate, and $P_{i,t}$ is the price per unit of intermediate good i, which the final goods firm takes as given. The capital stock evolves as

$$K_{t+1} = (1 - \delta) K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t$$

where the capital adjustment cost function $\lambda(\cdot)$ is specified as in Jermann (1998)

$$\Lambda \left(\frac{I_t}{K_t} \right) \equiv \frac{\alpha_1}{1 - \frac{1}{\zeta}} \left(\frac{I_t}{K_t} \right)^{1 - \frac{1}{\zeta}} + \alpha_2$$

The parameter ζ represents the elasticity of the investment rate; in particular the limiting cases $\zeta \to 0$ and $\zeta \to \infty$ represent infinitely costly adjustment and frictionless adjustment, respectively. The parameters α_1 and α_2 are set so that there are no adjustment costs in the deterministic steady state. Specifically,

$$\begin{aligned} \alpha_1 &= (\Delta N_{ss} - 1 + \delta)^{\frac{1}{\zeta}} \\ \alpha_2 &= \frac{1}{\zeta - 1}(1 - \delta - \Delta N_{ss}) \end{aligned}$$

Taking the pricing kernel M_t as given, the firm's problem is to maximize shareholder's wealth, which can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t\geq 0, i\in[0,N_t]}} E_0\left[\sum_{t=0}^{\infty} M_t D_t\right]$$

subject to

$$\begin{aligned} D_t &= Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t}\, di \\ K_{t+1} &= (1-\delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right) K_t \end{aligned}$$

where δ is the depreciation rate of capital. The Lagrangian for the firm's problem is

$$\mathcal{L} = E_0\left[\sum_{t=0}^{\infty} M_t \left\{Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t}\, di + q_t\left((1-\delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right)K_t - K_{t+1}\right)\right\}\right]$$

The corresponding first order conditions are

$[I_t] : -1 + q_t \cdot \Lambda'\left(\frac{I_t}{K_t}\right) = 0$

$[L_t] : (1-\alpha)(1-\xi)\frac{Y_t}{L_t} - W_t = 0$

$[K_{t+1}] : q_t = E_t\left[M_{t+1}\left\{\alpha(1-\xi)\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left((1-\delta) - \Lambda'\left(\frac{I_{t+1}}{K_{t+1}}\right)\cdot\left(\frac{I_{t+1}}{K_{t+1}}\right) + \Lambda\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right\}\right]$

$[X_{i,t}] : (K_t^{\alpha}(\Omega_t L_t)^{1-\alpha})^{1-\xi}\nu\xi\left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}}\, di\right]^{\nu\xi-1}\frac{1}{\nu}X_{i,t}^{\frac{1}{\nu}-1} - P_{i,t} = 0$

For notational simplicity, define: $\Lambda_t \equiv \Lambda\left(\frac{I_t}{K_t}\right)$ and $\Lambda'_t \equiv \Lambda'\left(\frac{I_t}{K_t}\right)$. Rewrite the four equations above as:

$$
\begin{aligned}
q_t &= \frac{1}{\Lambda'_t} \\
W_t &= (1-\alpha)(1-\xi)\frac{Y_t}{L_t} \\
1 &= E_t\left[M_{t+1}\left\{\frac{1}{q_t}\left(\alpha(1-\xi)\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\Lambda_{t+1}\right)\right\}\right] \\
P_{i,t} &= (K_t^{\alpha}(\Omega_t L_t)^{1-\alpha})^{1-\xi}\nu\xi\left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}}\,di\right]^{\nu\xi-1}\frac{1}{\nu}X_{i,t}^{\frac{1}{\nu}-1}
\end{aligned}
$$

Intermediate Goods Sector Intermediate goods producers have monopoly power. Given the demand schedules set by the final good firm, monopolists producing the intermediate goods set the prices in order to maximize their profits.Intermediate goods producers transform one unit of the final good in one unit of their respective intermediate good. In this sense production is "roundabout" in that monopolists take final good as given as they are tiny themselves. This fixes the marginal cost of producing one intermediate good at unity.

The first-order condition with respect to $X_{i,t}$ implicitly gives the demand schedule for intermediate good i as a function of the price $P_{i,t}$. The local monopolist producing $X_{i,t}$ takes the demand schedule $X_{i,t}(P_{i,t})$ as given and produces at unit cost using the final good input. Thus, the monopolist solves the following static profit maximization problem each period

$$
\max_{P_{i,t}} \Pi_{i,t} \equiv P_{i,t} \cdot X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t})
$$

The monopolistically competitive characterization of the intermediate goods sector a lá Dixit and Stiglitz (1977) results in the symmetric industry equilibrium conditions

$$
X_{i,t} = X_t
$$

$$
P_{i,t} = P_t = \nu
$$

That is, each intermediate goods producer produces the same amount and charges a markup $\nu > 1$ over marginal cost. Substituting these two equilibrium conditions into the definition for G_t and the F.O.C. w.r.t.

$X_{i,t}$ yields:

$$G_t = N_t^{\nu} X_t \quad (3)$$

$$X_t = \left(\frac{\xi}{\nu} \left(K_t^{\alpha} (\Omega_t L_t)^{1-\alpha} \right)^{1-\xi} N_t^{\nu\xi - 1} \right)^{\frac{1}{1-\xi}} \quad (4)$$

and hence

$$\Pi_t = (\nu - 1) X_t$$

Consequently, monopoly profits are procyclical.

The value of owning exclusive rights to produce intermediate good i is equal to the present discounted value of the current and future monopoly profits

$$V_{i,t} = \Pi_{i,t} + \phi E_t [M_{t+1} V_{i,t+1}]$$

where ϕ is the survival rate of an intermediate good. Imposing the symmetric equilibrium conditions, we can drop the i subscript and write

$$V_t = \Pi_t + \phi E_t [M_{t+1} V_{t+1}]$$

R&D Sector Innovators develop intermediate goods for the production of final output. They do so by conducting research and development, using the final good as input at unit cost. For simplicity, we assume that households can directly invest in research and development. They develop new intermediate goods, whose patents can then be sold in the market for intermediate goods patents. A new intermediate goods producer will buy the new patent. Assuming that this market is competitive, the price of a new patent will equal the value of the new patent to the new intermediate goods producer.

The R&D sector develops new intermediate goods and sells them to firms in the intermediate goods sector. It has access to linear technology and uses the final good as input to produce new varieties. Specifically, the law of motion for the measure of intermediate goods N_t is

$$N_{t+1} = \vartheta_t S_t + \phi N_t$$

where S_t denotes R&D expenditures (in terms of the final good) and ϑ_t represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In similar spirit as Comin and Gertler (2006), we assume that this technology coefficient captures a congestion externality effect so that higher R&D intensity leads to lower productivity in the innovation sector

$$\vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^{\eta}}$$

where $\chi > 0$ is a scale parameter and $\eta \in [0, 1]$ is the elasticity of new intermediate goods with respect to R&D. Since there is free entry into the R&D sector, the following break-even condition must hold:

$$E_t[M_{t+1} V_{t+1}](N_{t+1} - \phi N_t) = S_t$$

This says that the expected sales revenues equals costs. This condition can be equivalently formulated, at the margin, as

$$\frac{1}{\vartheta_t} = E_t[M_{t+1} V_{t+1}]$$

which states that marginal cost equals expected marginal revenue.

Forcing Process The exogenous productivity process Ω_t is assumed evolve as

$$\begin{aligned} \Omega_t &= e^{a_t} \\ a_t &= \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \end{aligned}$$

Market Clearing Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, and R&D:

$$Y_t = C_t + I_t + N_t X_t + S_t$$

Alternatively, this can be written as

$$Y_t = C_t + I_t + N_t^{1-\nu} G_t + S_t,$$

where the term $N_t^{1-\nu} G_t$ captures the costs of intermediate goods production. Given that $\nu > 1$ reflecting monopolistic competition, it follows that increasing product variety increases the efficiency of intermediate goods production, as the costs fall as N_t grows.

Balanced Growth Using the above expressions output can be rewritten as follows:

$$Y_t = \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{1-\xi}} K_t^{\alpha} (\Omega_t L_t)^{1-\alpha} N_t^{\frac{\nu\xi-\xi}{1-\xi}}$$

To ensure balanced growth, we need the aggregate production function to be homogeneous of degree one in the accumulating factors K_t and N_t. Thus, the following parameter restriction needs to be satisfied:

$$\alpha + \frac{\nu\xi - \xi}{1-\xi} = 1$$

Imposing this restriction, we have a production function that resembles the standard neoclassical one with labor augmenting technology:

$$Y_t = K_t^{\alpha} (Z_t L_t)^{1-\alpha}$$

where total factor productivity (TFP) is

$$Z_t \equiv \overline{A} \Omega_t N_t$$

and $\overline{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant. Hence, the fundamental difference between the ENDO model and the canonical real business cycle (RBC) framework is that the trend component of the TFP process, N_t, is endogenous and fluctuates in the ENDO model but exogenous and deterministic in the RBC model. This difference between the models leads to a very different propagation of the productivity shock a_t; namely, the ENDO model generates low- and high-frequency cycles whereas the RBC model only generates high-frequency cycles.

Since the agent has no disutility for labor, she will supply her entire endowment, which is normalized to one:

$$L_t = 1 \tag{5}$$

Imposing (4), the following equations can be simplified:

$$\begin{aligned} Y_t &= K_t^{\alpha} Z_t^{1-\alpha} \\ W_t &= (1-\alpha)(1-\xi)Y_t \\ X_t &= \left(\frac{\xi}{\nu}\Omega_t^{(1-\alpha)(1-\xi)} K_t^{\alpha(1-\xi)} N_t^{\nu\xi-1}\right)^{\frac{1}{1-\xi}} \end{aligned}$$

2.2 Exogenous Growth Model (EXO)

To contrast with our benchmark endogenous growth model (ENDO), we consider the neoclassical growth model with exogenous labor augmenting technology. In particular, we take the exact equilibrium production function (of the final goods sector) from the ENDO model, but make one modification: instead of having the trend growth component N_t be endogenously determined, we instead exogenously specify the evolution of N_t as an exponential time trend. Furthermore, there is no more role of innovation and production will simply be a one-sector representative firm that uses only capital and labor as factor inputs. This specification of production and technology for the EXO model is standard in the real business cycle (RBC) literature. The capital accumulation equation and capital adjustment cost function are exactly the same as in the ENDO model. Furthermore, the household's problem is unchanged, we will simply describe the production sector.

Production A representative firm produces the final (consumption) good using capital K_t and labor effort L_t with constant returns to scale technology

$$Y_t = K_t^{\alpha}(\tilde{Z}_t L_t)^{1-\alpha}$$

where productivity $\tilde{Z}_t$ is specified as a trend stationary process

$$\begin{aligned} \tilde{Z}_t &= \overline{A}\Omega_t N_t \\ N_t &= e^{\mu t} \end{aligned}$$

where the constant $\overline{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ and the productivity shock Ω_t are specified exactly as in the ENDO model:

$$\begin{aligned}
\Omega_t &= e^{a_t} \\
a_t &= \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)
\end{aligned}$$

Dividends are defined as

$$D_t \equiv Y_t - W_t L_t - I_t$$

where I_t is capital investment and W_t is the wage rate. The capital stock evolves as

$$K_{t+1} = (1-\delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right)K_t$$

Taking the pricing kernel M_t as given, the firm's problem is to maximize shareholder's wealth, which can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}\}} E_0\left[\sum_{t=0}^{\infty} M_t D_t\right]$$

subject to

$$\begin{aligned}
D_t &= Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t}\, di \\
K_{t+1} &= (1-\delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right)K_t
\end{aligned}$$

with corresponding first-order conditions are:

$$\begin{aligned}
q_t &= \frac{1}{\Lambda_t'} \\
W_t &= (1-\alpha)\frac{Y_t}{L_t} \\
1 &= E_t\left[M_{t+1}\left\{\frac{1}{q_t}\left(\alpha\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\Lambda_{t+1}\right)\right\}\right]
\end{aligned}$$

where $\Lambda_t \equiv \Lambda\left(\frac{I_t}{K_t}\right)$ and $\Lambda'_t \equiv \Lambda'\left(\frac{I_t}{K_t}\right)$. Output is used for consumption and investment:

$$Y_t = C_t + I_t$$

3 Quantitative Results

In this section we explore the quantitative implications of the model using simulations. We use perturbation methods to solve the model. To account for risk premia and potential time variation in them, we use a third order approximation around the stochastic steady state. The next section describes our calibration.

3.1 Calibration

In this section, we present the benchmark calibration used to assess the quantitative implications of the endogenous growth model (ENDO). The elasticity of intertemporal substitution ψ and the coefficient of relative risk aversion γ are set to standard values in the long-run risks literature.[2] Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor β has a large impact on the level of the riskfree rate and the growth rate, which are tightly linked variables. Thus, β is set to be consistent with both the level of the riskfree rate and average growth rate. The scale parameter χ is calibrated to help match balanced growth evidence. The depreciation rate of capital δ and capital share α are set to standard values in the real business cycle literature. The capital adjustment cost parameter ζ is set to match the relative volatility of consumption to output. The markup in the intermediate goods sector ξ and the elasticity of new intermediate goods with respect to R&D η are set to values similar to Comin and Gertler (2006).[3] Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret one minus the survival rate ϕ as the depreciation rate of the R&D stock. Hence, we set the parameter ϕ to match the depreciation rate the R&D stock assumed by the BLS. The persistence parameter ρ of the productivity shocks $a_t \equiv \log(\Omega_t)$ is calibrated to match the first autocorrelation of R&D intensity, which is the key driver of expected growth rates. Furthermore, this value for ρ allows us to be consistent with the first autocorrelations of the key quantity growth rates and

[2]See Bansal and Yaron (2004).

[3]Note that η is within the range of panel and cross-sectional estimates from Griliches (1990) and Comin and Gertler (2006).

productivity growth.[4] The volatility parameter σ is set to match output growth volatility. In addition, for the exogenous growth model (EXO) we keep the common parameters the same to facilitate the comparison of the economic mechanisms between the two models. The one parameter that is in the EXO model but not the ENDO model is the trend growth parameter μ, which is set to match average output growth.

3.2 Productivity Dynamics

Many of the key implications of the model can be understood by looking at the endogenous dynamics of total factor productivity (TFP), Z_t, which we derived in section 2:

$$Z_t \equiv \overline{A}\Omega_t N_t$$

with $\overline{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$, a constant. This implies that the log growth rate of productivity, ΔZ_t, can be decomposed as $\Delta Z_t = \Delta N_t \cdot \Delta\Omega_t$ Given a realistically persistent calibration of $\{\Omega_t\}$ in logs, we have $\Delta\Omega_t \approx e^{\epsilon_t}$. On the other hand, given the accumulation of N_t as $N_t = \theta_t \cdot S_{t-1} + \phi N_{t-1}$, the log growth rate of patents becomes $\Delta N_t = \theta_t \cdot \widehat{S}_{t-1} + \phi$, where we set

$$\widehat{S}_t \equiv \frac{S_t}{N_t}$$

We will refer to $\widehat{S}_t$ as the R&D *intensity*. Accordingly, we find $\Delta Z_t \approx (\vartheta_t \cdot \widehat{S}_{t-1} + \phi)(e^{\cdot\epsilon_t})$. Thus,

$$\begin{aligned} E_t[\Delta Z_{t+1}] &\approx E_t\left[(\vartheta_t \cdot \widehat{S}_t + \phi)(e^{\cdot\epsilon_{t+1}})\right] \\ &= (\vartheta_t \cdot \widehat{S}_t + \phi)E_t\left[e^{\cdot\epsilon_{t+1}}\right] \\ . &\approx \vartheta_t \cdot \widehat{S}_t + \phi \end{aligned}$$

[4] To provide further discipline on the calibration of ρ, note that Since the ENDO model implies the TFP decomposition, $\Delta z_t = \Delta a_t + \Delta n_t$, we can project log TFP growth on log growth of the R&D stock to back out the residual Δa_t. The autocorrelations of the extracted residual $\Delta\hat{a}_t$ show that we cannot reject that it is white noise. Hence, in levels, it must be the case that a_t is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of ρ is .95.

Qualitatively therefore, the dynamics of TFP are driven by the endogenous movements in R&D. This is in sharp contrast to the EXO specification, where productivity growth is

$$\begin{aligned}\Delta\tilde{z}_t &= \mu + \Delta a_t \\ &\approx \mu + \epsilon_t\end{aligned}$$

where the last approximate equality comes from the persistence of a_t. That is, as in the standard stochastic growth model, productivity growth is roughly *iid*.

Quantitatively, the implications of the model will thus depend on the ability of our calibration to match basic stylized facts about R&D activity and innovation. As tables 2 and 3 document, the model is quantitatively reasonably consistent with volatilities and autocorrelations R&D investment, the stock of R&D and R&D intensities in the data. Crucially, as in the data, the R&D intensity is a fairly volatile and persistent process. Specifically, we match its annual autocorrelation of 0.93.

The above decomposition of the expected growth rate of TFP therefore predicts a highly persistent component in TFP growth. Table 5 confirms this prediction, both in the data as well as in the model. While uncovering the expected growth rate of productivity as a latent variable in the data (as in Croce (2010)) suggests an annual persistence of 0.93, our model almost matches this number as 0.95. Moreover, the volatilities of expected TFP growth rates in the data and in the model roughly match. Naturally, matching the persistence properties of TFP in the data is important in a production economy, as this will affect the dynamics of macroeconomic variables. Note that in contrast to our benchmark model, the EXO specification implies that TFP growth is roughly iid, in contrast to the empirical evidence.

Qualitatively, the above decomposition suggests that the R&D intensity should track productivity growth rather well. This is confirmed in figures 1 and 2, both in the model as well as in the data. The plots visualize the small, but persistent component in TFP growth induced by equilibrium R&D activity.

On the other hand, from an empirical point of view, these results suggest, that R&D activity, and especially the R&D intensity should forecast productivity growth rather effectively. We confirm this prediction in table 10, which documents results from projecting productivity growth on R&D intensity, over several horizons. In the data, R&D intensity forecasts productivity growth over several years, significantly, and with R^2's increasing with horizons. Qualitatively, the model replicates this pattern rather well.

The intuition for these results comes from the endogenous R&D dynamics that the model generates. This can readily be gleaned from the impulse responses to an exogenous shock displayed in figure 7. It exhibits responses of quantities in the patents sector. Crucially, after a shock profits rise persistently. Intuitively, a positive shock in the final goods sector raises the demand X_t for intermediate goods, and with $\Pi_t = (\nu - 1)X_t$, this translates directly into higher profits. Naturally, given persistently higher profits the value of a patent goes up, as shown in the third panel. Then in turn, as the payoff to innovation is the value of patents, this triggers a persistent increase in the R&D intensity. This yields the persistent endogenous component in productivity growth displayed above. Crucially, the exogenous shock has two effects. It immediately raises productivity of the final output firm, leading to standard fluctuations at business cycle frequency, but it also induces more R&D which will be reflected in more patents with a lag. Importantly, the increases in R&D are persistent, leading to fluctuations at medium and lower frequencies. Intuitively, in this setting, an exogenous shock to the level of productivity endogenously generates a persistent shock to the growth rate of the economy.

3.3 Fluctuations

In the previous section, we documented that the benchmark model has rich implications for the dynamics of TFP, which will naturally be reflected in the quantity dynamics in our production economy. Table 2 reports basic statistics implied by the model. Concentrating on the quantities for the moment, the table shows that the model is reasonably consistent with basic quantity statistics. In particular, our benchmark model does just as well as the EXO model, which mimics the real business cycle model (this is further analyzed in tables 11 and 12, which provide some sensitivity analysis). One way of interpreting this finding is by saying that the ENDO model generates high-frequency dynamics or business cycle statistics in line with the canonical real business cycle model. On the other hand, both specifications predict investment to be too smooth. This is a general difficulty with production models with recursive preferences [5].

While table 2 suggests that our benchmark endogenous growth model performs equally well as the real business cycle model when it comes to basic quantity statistics, the endogenous productivity dynamics of the model lead to a strong propagation mechanism absent from the latter model, as we explore now.

Table 3 reports autocorrelations of basic growth rates, in the data, as well as in the ENDO and the EXO model. Note first that while all growth rates exhibit considerable positive autocorrelation at annual fre-

[5] In a companion paper, Croce, Kung and Schmid (2010), we show how to overcome this problem by explicitly modeling financial frictions. Therefore, we do not pursue this further in the present paper

quencies, the corresponding persistence implied by the EXO model is virtually zero, and sometimes even negative. This is one of the main weaknesses of the real business cycle model (as pointed out e.g. in Cogley and Nason (1995)). Quite in contrast, our ENDO model generates substantial positive autocorrelation in all quantities, sometimes qualitatively and sometimes quantitatively close to their data counterparts. Note that the exogenous component of productivity is the same in both model. Accordingly, the ENDO model possesses a strong propagation mechanism induced by the endogenous component of productivity, e.g. by the R&D.

The intuition for this endogenous propagation is of course simple, and tightly linked to the dynamics of TFP documented in the previous section. To the extent that innovation induces a persistent component in productivity, this will be reflected in quantity dynamics. Recall however, that the TFP dynamics implied by the model is consistent with the empirical evidence.

Such patterns are visualized in figures 1, 2 and 3, for consumption, and output growth, both in the model as in the data. A persistent component in TFP will induce a persistent component in output growth, hence R&D intensity should be expected to track expected output growth rather well. This is documented in Figure 3. The figure also shows that these dynamics naturally also arise in expected consumption growth. The same holds for realized consumption growth in the data as well as in the model, as shown in figures 1 and 2.

The propagation mechanism implies that macroeconomic quantities display markedly different behavior at different frequencies, in other words, it implies a rich intertemporal distribution of growth rates. This can be seen from tables 4, 6 and 7, which capture the intertemporal distribution of growth rates. While the implied volatilities of growth rates of the EXO and ENDO model are basically undistinguishable at short horizons, in the ENDO model they grow fairly quickly over longer horizons. Another way of interpreting this finding is that the ENDO model generates significant dynamics at medium and lower frequencies, while the EXO model does not. Empirically, this can be captured by looking at medium and low frequency components of growth rates. By doing so, we follow Comin and Gertler (2006) who used bandpass filters to identify medium term fluctuations in quantities. In their spirit, we identify high frequency or business cycle movements with a bandwidth of 2 to 32 quarters, medium frequency components with a bandwidth of 32 to 200 quarters, and low frequency movements with a bandwidth of 200 to 400 quarters. Table 6 shows that our benchmark model generates substantial movements in output growth at all frequencies in line with the data, while the EXO model does not. The correspondence of high and medium term movements consumption and output growth in both the data and the model is visualized in figure 4.

Another implication of the model is that it generates cash flow dynamics in line with the empirical evidence. First of all, it generates heavily procyclical profits. This can be seen from figure 7. This is in line with recent work on expanding variety models in Bilbiie, Ghironi, Melitz (2007), but typically presents a challenge to macro models. In our setting, this is driven by the procyclical demand for intermediate goods. Second, the model generates a persistent component in dividend growth. This can be seen in table 4, which documents considerable volatility in conditional expected dividend growth, which implied substantial variation in the conditional mean of cash flow growth. This is visualized in figure 8. Again, this is in stark contrast to the exogenous growth specification. This will be important from an asset pricing perspective, as only the benchmark model generates sufficient long-run uncertainty about dividend growth.

One way of summarizing the results of this section is by saying that while matching business cycle statistics well, the benchmark endogenous growth model generates substantial movements in quantities at medium and lower frequencies, in contrast to the real business cycle model. In other words, the endogenous growth model exhibits a strong propagation mechanism absent in the real business cycle model.

3.4 Asset Pricing Implications

In order to use our model to shed some light on the link between macroeconomic risk and growth via asset price data, we must first make sure that the model is quantitatively consistent with basic asset market statistics. As we will show shortly, the quantity dynamics discussed in the previous section will be key in generating high risk premia roughly in line with historical data. Since we assume that the agent has Epstein-Zin utility with a preference for an early resolution of uncertainty, this implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth.

Table 11 reports asset market statistics along with the quantity statistics reported earlier, both for the benchmark model and the exogenous growth specification. The benchmark model is quantitatively broadly consistent with basic asset price data: It generates a low and smooth risk-free rate and s sizeable equity premium. The equity premium is close to 4%. The volatility of the equity premium is a close to 7% annually, which seems to fall dramatically short of the historical volatility of the market return. However, when comparing the volatility in the model to the fraction of total volatility explained by productivity, as reported by Ai, Croce and Li (2010), then the model implication is reasonable.

It is instructive to compare the asset pricing implications of the benchmark model with those of the exogenous growth specification. While, as discussed previously, the quantity implications of the models are almost identical, the price implications are radically different. As can be seen from the table, the risk free rate is counterfactually high in the exogenous growth specification, and the equity premium is close to zero, a tiny fraction of what obtains in the benchmark model. These differences are intimately connected to differences in medium term dynamics that the two models generate, as reported in tables 6 and 7. Intuitively, in the setting with exogenous growth , average growth rates are roughly constant (as in the real business cycle model), therefore diminishing households' precautionary savings motive. In such a setting, households want to borrow against their future income, which in equilibrium can only be prevented by a prohibitively high interest rate. In the endogenous growth setting, however, taking advantage of profit opportunities in the intermediate goods sector leads to long and persistent swings in aggregate growth rates, and higher volatility over longer horizons. In this context, households optimally save for low growth episodes, leading to a lower interest rate in equilibrium. On the other hand, the model also generates a substantial equity premium. This means that in the model,in equilibrium, dividends are risky. The reason is, as discussed above, the cash flows naturally inherit a persistent component from the endogenous component of productivity. These cash flow dynamics not only affect risk premia, but naturally also asset market valuations. In particular, the model roughly matches the dynamics of stock market values, in the data, as measured by Tobin's Q. This can be seen from tables 2 and 3. Not only does the model generate considerably higher stock market volatility than its EXO counterpart, it also matches its autocorrelation. This suggests that innovation driven cycles rationalize long-term movements in stock market valuations.

These effects can also be seen in figures 6, 7 and 8. Figure 8 shows conditonal expected growth rates for macro variables. Specifically, the figure documents that following a good shock not only realized growth rates of quantities incur persistent movements, but also expected growth rates do, and these effects are much stronger in the benchmark model. Therefore innovations to realized consumption and dividend growth are coupled with innovations to expected growth, both of which are priced when agents have Epstein-Zin utility with a preference for early resolution of uncertainty. Therefore bad shocks are simultaneously bad shocks for the long run, thus rendering equity claims very risky. This can be seen in figures 6 and 7. While qualitatively in both benchmark and exogenous growth model the responses of prices to a shock go in the same directions, quantitatively the effects are much more pronounced in the former, again owing to the increased persistence and long term volatility that it displays. Another way of understanding the asset pricing implications of the models is to recall that the equity premium is $E[r_d - r_f] \approx -cov(m, r_d)$. This implies that equity must offer the higher a premium, the more equity returns and the discount factor move in opposite directions. We can see from figure 10 that the benchmark model displays stronger co-movements of equity returns and discount factor, leading to a higher equity premium. Another implication that the figure suggests is that the model generates a negative term premium. As the interest rate is procyclical, this means that after a bad shock interest rates fall and hence long-maturity bond prices rise, making longer maturity bonds effectively good hedges against shocks, which will be reflected in a negative term premium. While this may be counterfactual when applied to the nominal the structure, there is evidence that the real term structure is actually downward sloping.

Taken together, these results suggest that an endogenous growth model with recursive preferences is a natural environment to understand asset pricing in a general equilibrium setting. More specifically, the mechanisms that allow the model to generate high risk premia, namely long persistent swings in growth rates coupled with recursive preferences, are exactly those that Bansal and Yaron (2004) specify exogenously and refer to as long-run risks. While the dynamics they specify as somewhat hard to detect in the data, they are a natural implication of agents' optimal innovation and R&D decisions in our model. Another way to see this is depicted in figure 12, which plots the time series of the R&D intensity as well as the process of expected consumption growth as estimated in Bansal, Kiku and Yaron (2007). In an empirical approach, Bansal, Kiku and Yaron project consumption growth on lagged short rates and P/D ratios to extract the plotted process for expected consumption growth. The fact that their process and the R&D intensity qualitatively exhibit similar patterns, lends support to the model implication that innovation is a key driver of time variation in growth prospects. This corroborates our finding of R&D intensity as quite a powerful predictor of aggregate growth rates.

The model suggests that these effects are quantitatively significant. This also suggests that the growth rate dynamics that Bansal and Yaron specify naturally endogenously arise in a wide class of stochastic endogenous growth models, making them a very natural way to link asset prices to long-term growth prospects.

3.5 Medium Term Comovement

So far, we have discussed how the benchmark model generates fluctuations in quantities and prices at various frequencies. However, the model also has interesting and realistic implications between comovement between prices and quantities at medium frequencies. This is displayed in figures 11 and 12.

Figure 11 reveals that the model replicates the medium term comovements between productivity and quantities in the data. This is noteworthy, first because it reveals the significant variation macro data exhibit at medium frequencies and the significant comovement between productivity and quantities, and second how closely our model matches them. Note that, in contrast, productivity exhibits virtually no variation at medium frequencies in the EXO model, and hence little comovement as well.

Figure 12 shows the close match between the price-dividend ratio and productivity growth in the data and the benchmark model at medium frequencies. This strongly suggests productivity driven slow movements in asset market valuations in the data. In the model,these movements are driven by variation in expected cash flows, induced by time variation in R&D. This is because risk premia in the benchmark model are essentially constant. While there is evidence for time-variation in expected cash flows as discussed above, time variation in price-dividend ratios is often related to time-variation in risk premia. To allow for such predictability, we extend our model below to account for stochastic volatility.

At medium frequencies we also find strong cross-correlations between stock returns and consumption growth. This is displayed in figure 10, indicating the lead-lag structure between returns and consumption growth. Interestingly, our benchmark model closely replicates the patterns in the data, while the EXO counterpart does not. In particular, at medium frequencies returns lead consumption growth by several quarters. Intuitively, this is because the persistent endogenous component in productivity is reflected in asset markets and hence returns immediately, while these movements spill over into consumption growth only with a lag. Absent a persistent component in productivity growth, in the EXO model returns and consumption move together.

4 Extensions

In this section, we provide two extensions to our benchmark model, one motivated by the asset pricing literature, and the other by the empirical evidence on R&D. First, we introduce stochastic volatility into the model, that is, we consider the possibility that productivity may exhibit time-varying volatility (see e.g. Croce (2010) for empirical evidence). This is motivated by the benchmark long-run risk model by Bansal-Yaron which entails stochastic volatility in consumption growth, and extends it to a production economy with endogenous growth. Second, we consider the possibility that productivity in the R&D sector may be subject to shocks, and examine the quantitative implications of this assumption in the context of our model.

4.1 Stochastic Volatility (ENDO-SV)

We now extend our benchmark model to allow for stochastic volatility. We do this for several reasons. A number of papers have recently pointed to the importance of volatility shocks for macroeconomic dynamics. Justiniano and Primiceri (2008) by means of formal estimation establish that most types of shocks typically considered in the DSGE literature exhibit stochastic volatility, while Bloom (2009) argues that volatility shocks can lead to temporary recessions. On the other hand, from an asset pricing perspective, starting from Bansal and Yaron (2004) the long-run risk literature has usually appealed to stochastic volatility. As our model generates long-run risks endogenously, it is interesting to see whether the stochastic volatility intuition carries over to an endogenous growth setting.

For the model ENDO-SV, modify the forcing process so that volatility is time-varying:

$$\begin{aligned}
\Omega_t &= e^{a_t} \\
a_t &= \rho a_{t-1} + \sigma_{t-1}\epsilon_t \\
\sigma_t^2 &= \hat{\sigma}^2 + \lambda_1 \left(\sigma_{t-1}^2 - \hat{\sigma}^2\right) + \sigma_e e_t
\end{aligned}$$

where $\epsilon_t, e_t \sim N(0,1)$. The calibration is reported in table 14, with summary statistics displayed in table 15. Our main qualitative results are summarized in figure 9, where we report the responses of quantities and prices to a volatility shock. To interpret the results, first note that a volatility shock does not alter effective productivity, but only its distribution. As actual disposable resources are unchanged after the shock, the responses of consumption and investment (including R&D) go in opposite directions. Consistent with the earlier discussions, the strong precautionary savings motive in our calibration leads to an increase in R&D investment and hence to a persistent increase in the number of patents or intermediate goods. Accordingly, this implies a positive conditional relationship between volatility and growth. However, this implies a fall in consumption upon impact. Similarly, given the precautionary savings motive, the risk-free rate falls, and expected excess returns go up. The latter observation suggests that, as could be expected, introducing stochastic volatility leads to time-variation in risk premia and hence, at least qualitatively, to predictability. Will it necessarily lead to a higher unconditional equity premium? The answer is no, and the reason lies in the endogenous consumption dynamics of the model. While, as can be seen in the figure, a volatility shock leads a fall in realized consumption growth, it also leads to an increase in expected consumption growth.

Hence, a volatility shock is a bad shock for the short-run, but a good shock for the long-run in the language of Kaltenbrunner and Lochstoer (2008). As both of these effects are priced with Epstein-Zin preferences, with opposite signs, they tend to offset each other and the overall effect is small. Hence, in a production economy setting, with mean reverting volatility, the unconditional effects of introducing stochastic volatility are small. Quantitatively, as can be seen from table 15, the equity premium actually falls. On the other hand, the volatility of returns not surprisingly increases, so that the market price of risk actually falls. This is in stark contrast to the specification in Bansal and Yaron (2004), where stochastic volatility accounts for a considerable fraction of the unconditional equity premium. One interesting alternative would be to follow Jutiniano and Primiceri (2008) who assume volatility shocks to be permanent.

One interesting implication of introducing volatility shocks can be seen from table 15, namely the positive autocorrelation of returns, which was virtually absent in the benchmark model. While quantity growth rates could be predicted in the latter model, it essentially did not feature return predictability. This is different in the present extension with time-varying risk, as we explore in the next tables. First of all, as reported in table 16, and consistent with the empirical evidence, P/D ratios predict returns negatively, and increasingly so with increasing horizon as measured by the R^2. As a matter of fact, the population R^2 reported demonstrate considerable explanatory power for the P/D ratio in the model. On the other hand, in table 16 we report risk premia forecasts through two measures of R&D, namely R&D intensity and R&D growth. While the explanatory power is visibly lower, interestingly both these measures predict high risk premia going forward. This is consistent with the positive forecasting power of such measures for quantity growth. High R&D forecasts long growth episodes ahead, with associated high returns on the stock market. For a long-run investor with recursive preferences, this makes the stock market a risky investment, thus commanding high risk premia.

4.2 Stochastic R&D Productivity

5 Conclusion

We provide a quantitative analysis of a stochastic model of endogenous growth where households have recursive Epstein-Zin preferences. In the model, innovation and investment in research and development is the ultimate source of sustained growth. In the data, R&D is quite volatile, persistent and cyclical. Our model then predicts that these movements in the sources of growth should be reflected in the dynamics of the aggregate economy. More precisely, the model predicts a small, but persistent innovation-driven endogenous component in productivity growth leading to long and persistent swings in the macroeconomic quantities. Therefore, in spite of being a driven by a single exogenous shock, the model generates significant cycles at high, medium and low frequencies generated by the endogenous response of innovation to the shock. In other words, the innovation process generates a strong propagation mechanism absent in standard macroeconomic models. Empirically, we find strong support for innovation driven medium and low frequency fluctuations in aggregate growth rates in the data.

Under the assumption of recursive preferences these quantity dynamics have strong implications for asset prices and risk premia. With such a preference specification agents are very averse to the low-frequency variation in expected growth rates that the model generates, which yields high risk premia in asset markets and a low risk and stable free rate. As such the model provides a macroeconomic foundation for long run risks in asset markets, as pioneered by Bansal and Yaron (2004), and suggests that a strong propagation mechanism in macro models and high risk premia are inherently linked. Moreover, the model predicts innovation and productivity driven low frequency movements in stock market values and price-dividend ratios, in line with the empirical evidence.

In short, our model implies that there are tight links between macroeconomic risk, growth and risk premia in asset markets and hence suggests that stochastic models of endogenous growth are a useful framework for quantitative macroeconomic modeling and asset pricing.

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Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP).

Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation.[6]

[6]We model the monthly time series process for inflation using an AR(4).

Table 1: Calibration

Parameter	Description	ENDO	EXO
β^4	Subjective Discount Factor	0.984	0.984
ψ	Elasticity of Intertemporal Substitution	1.85	1.85
γ	Risk Aversion	10	10
ξ	Intermediate Goods Share	0.5	-
ν	Elasticity of Substitution Between Intermediate Goods	1.65	-
α	Capital Share	0.35	0.35
ρ^4	Autocorrelation of Ω	0.95	0.95
χ	Scale Parameter	0.332	-
ϕ	Survival Rate of Intermediate Good	0.9625	-
η	Elasticity of New Intermediate Goods wrt R&D	0.83	-
δ	Depreciation Rate of Capital Stock	0.02	0.02
σ	Volatility of Productivity Shock ϵ	1.75%	1.75%
ζ	Elasticity of Capital Investment Rate	0.70	0.70
$\mu * 4$	Trend Growth Rate	-	1.90%

This table reports the benchmark quarterly calibration used for the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 2: Summary Statistics

	Data	ENDO	EXO
First Moments			
$E[\Delta y]$	1.90%	1.90%	1.90%
$E[r_f]$	1.62%	1.21%	2.58%
$E[r_d^* - r_f]$	5.57%	4.10%	0.19%
Second Moments			
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	1.13
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	1.85	0.65
$\sigma_{\Delta s}/\sigma_{\Delta y}$	2.10	1.64	-
$\sigma_{\Delta z}/\sigma_{\Delta y}$	1.22	1.52	1.54
$\sigma_{\Delta c}$	1.42%	1.42%	2.58%
σ_{r_f}	0.67%	0.30%	0.09%
$\sigma_{r_d - r_f}$	14.98%	6.35%	4.10%
σ_Q	12.85%	16.46%	5.19%

This table presents annual first and second moments from the endogenous growth (ENDO) model, the exogenous growth (EXO) model, and the data. The models are calibrated at a quarterly frequency and the moments are annualized. Since the equity risk premium from the models is unlevered, we follow Boldrin, Christiano, and Fisher (2001) and compute the levered risk premium from the model as: $r^*_{d,t+1} - r_{f,t} = (1+\kappa)(r_{d,t+1} - r_{f,t})$, where r_d is the unlevered return and κ is the average aggregate debt-to-equity ratio, which is set to $\frac{2}{3}$. Annual macro data are obtained from the BEA, BLS, and NSF. Monthly return data are from CRSP and the corresponding sample moments are annualized. The data sample is 1953-2008.

Table 3: First Autocorrelations

	Data	ENDO	EXO
$AC1(\Delta c)$	0.40	0.46	-0.002
$AC1(\Delta y)$	0.37	0.21	0.001
$AC1(\Delta i)$	0.25	0.14	0.012
$AC1(\Delta z)$	0.09	0.11	-0.020
$AC1(Q)$	0.95	0.96	0.89
$AC1(\Delta s)$	0.21	0.06	-
$AC1(\Delta n)$	0.90	0.94	-
$AC1(S/N)$	0.93	0.93	-

This table reports first autocorrelations of annual variables. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF.

Table 4: Volatility of Expected Growth Rates

	ENDO	EXO
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.17%
$\sigma(E_t[\Delta y_{t+1}])$	0.42%	0.14%
$\sigma(E_t[\Delta i_{t+1}])$	0.37%	0.10%
$\sigma(E_t[\Delta d_{t+1}])$	0.92%	0.31%
$\sigma(E_t[\Delta z_{t+1}])$	0.38%	0.27%

This table reports annualized volatilities of expected growth rates from the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 5: Expected Productivity Growth Rate Dynamics

	Estimate	ENDO
$\rho_{\tilde{x}}$	0.93	0.95
$\sigma(\tilde{x})$	1.10%	1.20%

This table reports the annual persistence and standard deviation of the expected growth rate component of productivity growth from the data and from the endogenous growth (ENDO) model. The estimates are taken from Croce (2010), where the expected growth rate component of productivity $\tilde{x}_{t-1}$ is a latent variable that is assumed to follow an AR(1). In contrast, in the ENDO model the expected growth rate component is the growth rate of the variety of intermediate goods Δn_t, a endogenous structural variable of the model. In particular, since the shock Ω_t is persistent, log productivity growth can be written approximately as $\Delta z_t = \tilde{x}_{t-1} + \epsilon_t$, where $\tilde{x}_{t-1} \equiv \Delta n_t$ and ϵ_t is an iid disturbance. The ENDO model endogenously generates a productivity process that is the same as the exogenous specification of Croce (2010), which is supported empirically.

Table 6: Volatility of High-, Medium-, and Low-Frequency Components

	High			Medium			Low		
	Data	ENDO	EXO	Data	ENDO	EXO	Data	ENDO	EXO
$\sigma_{\Delta y}$	1.95%	2.17%	2.18%	0.69%	0.62%	0.53%	0.49%	0.30%	0.21%

This table reports the annualized volatilities of high-, medium-, and low-frequency components of output growth from the data and from the ENDO and EXO models. The bandpass filter from Christiano and Fitzgerald (1999) is used to isolate the components of the various frequencies. The high-frequency component is defined as a bandwidth of 2 to 32 quarters. The medium-frequency component is defined as a bandwidth of 32 to 200 quarters. The low-frequency component is defined as a bandwidth of 200 to 400 quarters. Quarterly output data is from the BEA.

Table 7: Growth Rate Volatility for Long-Horizons

	5-year		10-year		20-year	
	ENDO	EXO	ENDO	EXO	ENDO	EXO
$\sigma_{\Delta c}$	6.63%	5.63%	11.97%	7.70%	21.18%	10.22%
$\sigma_{\Delta y}$	7.54%	5.02%	12.77%	6.90%	21.76%	9.24%
$\sigma_{\Delta z}$	9.92%	7.45%	15.79%	9.92%	25.24%	12.67%

This table presents volatilities of log consumption growth, log output growth, and log investment growth from the endogenous growth (ENDO) and exogenous growth (EXO) models for horizons of 5, 10, and 20 years. The models are calibrated at a quarterly frequency and the quarterly observations are then time-aggregated to compute these lower frequency moments.

Table 8: Consumption Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	$\hat{R}^2$	β	R^2
1	0.017	0.006	0.070	0.104	0.320
2	0.034	0.012	0.105	0.201	0.402
3	0.048	0.017	0.131	0.290	0.430
4	0.062	0.023	0.163	0.374	0.439
5	0.077	0.030	0.202	0.453	0.440

This table presents consumption growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Consumption data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 9: Output Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	$\hat{R}^2$	β	R^2
1	0.020	0.013	0.040	0.085	0.105
2	0.046	0.022	0.084	0.163	0.161
3	0.068	0.029	0.119	0.236	0.195
4	0.089	0.041	0.158	0.306	0.217
5	0.114	0.051	0.210	0.372	0.231

This table presents output growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Output data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 10: Productivity Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	$\hat{R}^2$	β	R^2
1	0.014	0.009	0.031	0.075	0.039
2	0.031	0.015	0.080	0.142	0.062
3	0.049	0.024	0.120	0.204	0.077
4	0.069	0.032	0.174	0.261	0.088
5	0.091	0.041	0.232	0.314	0.095

This table presents productivity growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta z_{t,t+1} + \cdots + \Delta z_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Multifactor productivity data and R&D stock data are from the BLS. R&D flow data are from the NSF.

Table 11: Sensitivity Analysis

	ENDO	ENDO-CRRA	EXO	EXO-AC
First Moments				
$E[\Delta y]$	1.90%	0.51%	1.90%	1.90%
$E[r_f]$	1.21%	2.44%	2.58%	2.60%
$E[r_d^* - r_f]$	4.10%	0.61%	0.19%	1.44%
Second Moments				
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	1.28	1.13	0.63
$\sigma_{\Delta i}/\sigma_{\Delta c}$	1.85	0.29	0.65	2.86
$\sigma_{\Delta s}/\sigma_{\Delta y}$	1.64	0.82	-	-
$\sigma_{\Delta c}$	1.42%	2.92%	2.58%	1.42%
σ_{r_f}	0.30%	0.42%	0.09%	0.10%
$\sigma_{r_d - r_f}$	6.35%	2.07%	4.10%	1.43%
$AC1(\Delta c)$	0.46	0.006	-0.002	0.14
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.04%	0.17%	0.18%

This table compares key summary statistics from the benchmark ENDO and EXO models with alternative specifications. ENDO-CRRA is the same as the ENDO model but with the one modification that $\psi = \frac{1}{\gamma}$, so that the Epstein-Zin preferences collapse to CRRA utility. All the other parameters are kept the same as the benchmark calibration. EXO-AC is the same as the EXO model with the one modification that the adjustment cost parameter ζ is set to match consumption growth volatility, which corresponds to a value of 5.1. All other parameters are kept the same as the benchmark calibration.

Table 12: Sensitivity Analysis: Preference Parameters

	ENDO	$\gamma = 4$	$\gamma = 18$	$\psi = 0.5$	$\psi = 2.2$
First Moments					
$E[\Delta y]$	1.90%	1.89%	2.02%	0.86%	2.38%
$E[r_f]$	1.21%	2.11%	0.06%	2.23%	0.87%
$E[r_d^* - r_f]$	4.10%	1.55%	7.55%	1.28%	5.06%
Second Moments					
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	0.62	1.09	0.52
$\sigma_{\Delta i}/\sigma_{\Delta c}$	1.85	1.86	1.83	0.57	2.37
$\sigma_{\Delta s}/\sigma_{\Delta y}$	1.64	1.63	1.66	1.11	1.73
$\sigma_{\Delta c}$	1.42%	1.42%	1.43%	2.61%	1.21%
σ_{r_f}	0.30%	0.29%	0.28%	0.38%	0.27%
$\sigma_{r_d - r_f}$	6.35%	6.36%	6.36%	3.41%	6.95%
$AC1(\Delta c)$	0.46	0.47	0.46	0.07	0.62
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.52%	0.52%	0.19%	0.59%

This table compares key summary statistics from the benchmark calibration of the endogenous growth model (ENDO) with other calibrations that vary the preference parameters, risk aversion γ and the elasticity of intertemporal substitution ψ, one at a time while holding all other parameters fixed at the benchmark calibration. In the benchmark calibration for ENDO, $\gamma = 10$ and $\psi = 1.85$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).

Table 13: Sensitivity Analysis: Adjustment Cost and Persistence Parameters

	ENDO	$\xi = 0.40$	$\xi = 3.0$	$\rho^4 = 0.90$	$\rho^4 = 0.97$
First Moments					
$E[\Delta y]$	1.90%	1.20%	3.32%	1.76%	2.63%
$E[r_f]$	1.21%	1.10%	1.64%	2.07%	0.10%
$E[r_d^* - r_f]$	4.10%	4.90%	1.65%	1.55%	7.24%
Second Moments					
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.72	0.44	0.69	0.58
$\sigma_{\Delta i}/\sigma_{\Delta c}$	1.85	1.26	3.58	1.27	2.19
$\sigma_{\Delta s}/\sigma_{\Delta y}$	1.64	1.64	1.62	1.69	1.63
$\sigma_{\Delta c}$	1.42%	1.65%	1.03%	1.58%	1.38%
σ_{r_f}	0.30%	0.25%	0.36%	0.15%	0.39%
$\sigma_{r_d - r_f}$	6.35%	8.78%	2.16%	4.90%	7.26%
$AC1(\Delta c)$	0.46	0.36	0.76%	0.23	0.65
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.46%	0.65%	0.28%	0.72%

This table compares the key summary statistics from the benchmark calibration of the endogenous growth (ENDO) model with other calibrations that vary the capital adjustment cost parameter ξ and the persistence parameter ρ of the shock $log(\Omega_t)$ one at a time while holding all other parameters fixed at the benchmark calibration. In the benchmark calibration for ENDO, $\xi = 0.70$ and $\rho^4 = .95$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).

Table 14: Calibration of Stochastic Volatility Process

Parameter	Description	Value
$\hat{\sigma}$	Mean of σ	1.75%
λ_1^4	Autocorrelation of σ	0.98
σ_e	Volatility of shock e	8×10^{-4}%

This table reports the quarterly calibration of the stochastic volatility process for the ENDO-SV model.

Table 15: Annualized Summary Statistics: Stochastic Volatility

	ENDO	ENDO-SV
First Moments		
$E[\Delta y]$	1.90%	1.90%
$E[r_f]$	1.21%	1.24%
$E[r_d^* - r_f]$	4.10%	3.94%
Second Moments		
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.64
$\sigma_{\Delta i}/\sigma_{\Delta c}$	1.85	1.72
$\sigma_{\Delta s}/\sigma_{\Delta c}$	1.64	1.59
$\sigma_{\Delta c}$	1.42%	1.49%
σ_{r_f}	0.30%	0.32%
$\sigma_{r_d - r_f}$	6.35%	6.17%
$ACF_1(\Delta c)$	0.46	0.43
$\sigma(E_t[\Delta c_{t+1}])$	0.51	0.51

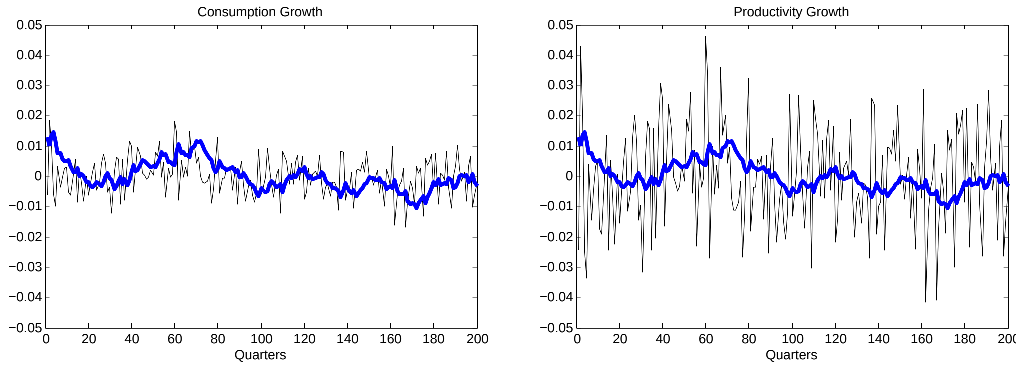
This table presents annual first and second moments from the endogenous growth (ENDO) model and endogenous growth with stochastic volatility (ENDO-SV) model, and the data. The models are calibrated at a quarterly frequency and the summary statistics are annualized. Annual macro data are obtained from the BEA and NSF. Monthly return data are from CRSP and the corresponding sample moments are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).

Table 16: Risk Premia Forecasting Regressions

Horizon (k)	Data			ENDO-SV	
	$\hat{\beta}$	S.E.	$\hat{R}^2$	β	R^2
1	-0.113	0.042	0.081	-0.033	0.029
2	-0.194	0.079	0.134	-0.067	0.057
3	-0.247	0.097	0.162	-0.099	0.082
4	-0.290	0.106	0.182	-0.131	0.104
5	-0.371	0.122	0.222	-0.162	0.124

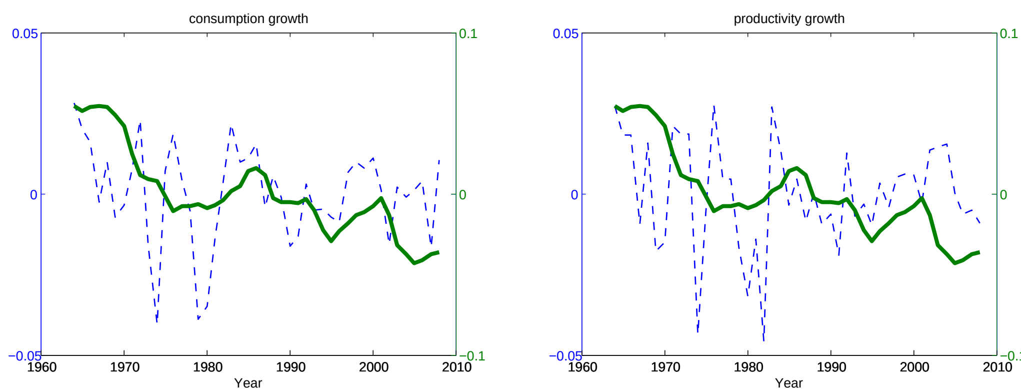
This table presents return forecasting regressions from the data and from the endogenous growth model with stochastic volatility for horizons (k) of one to five years with overlapping quarterly data. Specifically, we project risk premia on the log price-dividend ratio, $r^{ex}_{t,t+1} + \cdots r^{ex}_{t+k-1,t+k} = \alpha + \beta(p_t - d_t) + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $4 * k - 1$ lags. The model regression results correspond to the population values. Return, price and dividend data are from CRSP.

Figure 1: Growth Rates and R&D Intensity



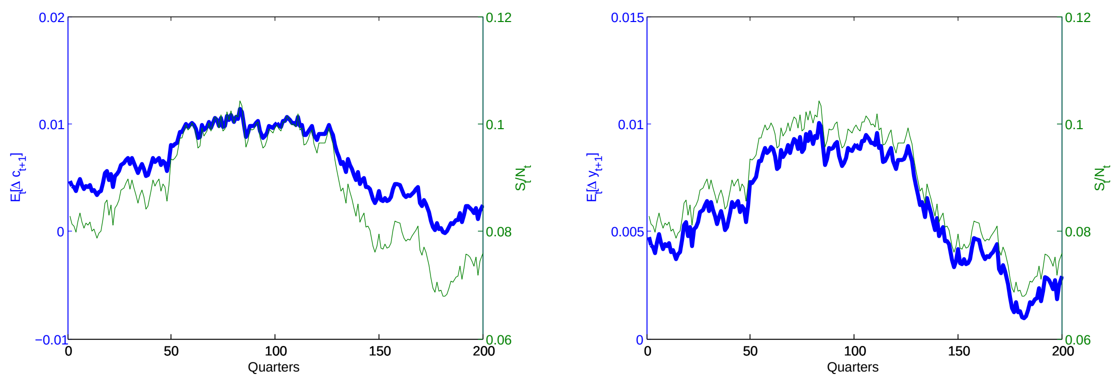
The left panel plots demeaned log consumption growth Δc_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. The right panel plots demeaned log output growth Δy_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 2: Growth Rates and R&D Intensity from Data



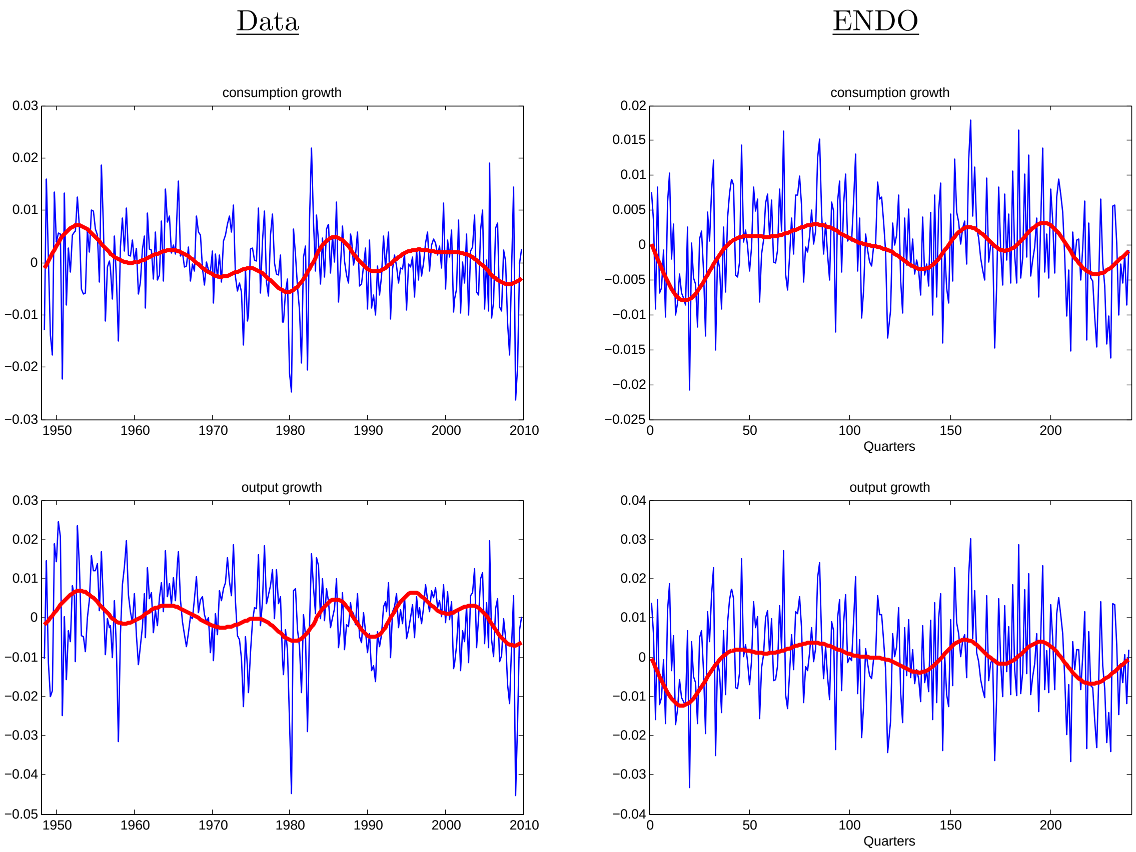
The left panel plots demeaned log consumption growth Δc_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. The right panel plots demeaned log output growth Δy_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. Annual data on aggregate output and consumption is from the BEA. Annual data on R&D expenditures are from the NSF and data on R&D stocks are from the BLS. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 3: Expected Growth Rates and R&D Intensity



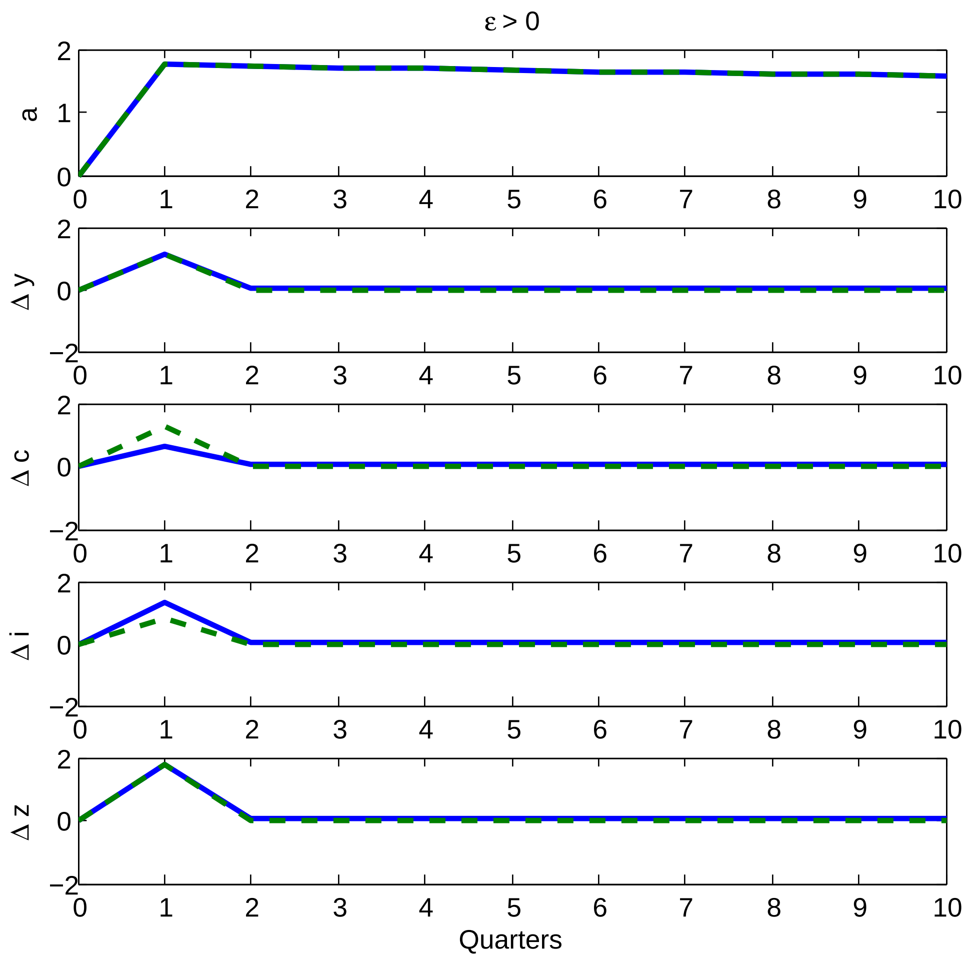
The left panel plots expected log consumption growth $E_t[\Delta c_{t+1}]$ (thick line) and R&D intensity $\frac{S_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. The right panel plots expected log output growth $E_t[\Delta y_{t+1}]$ (thick line) and R&D intensity $\frac{S_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 4: Growth Rate with Medium-Frequency Component



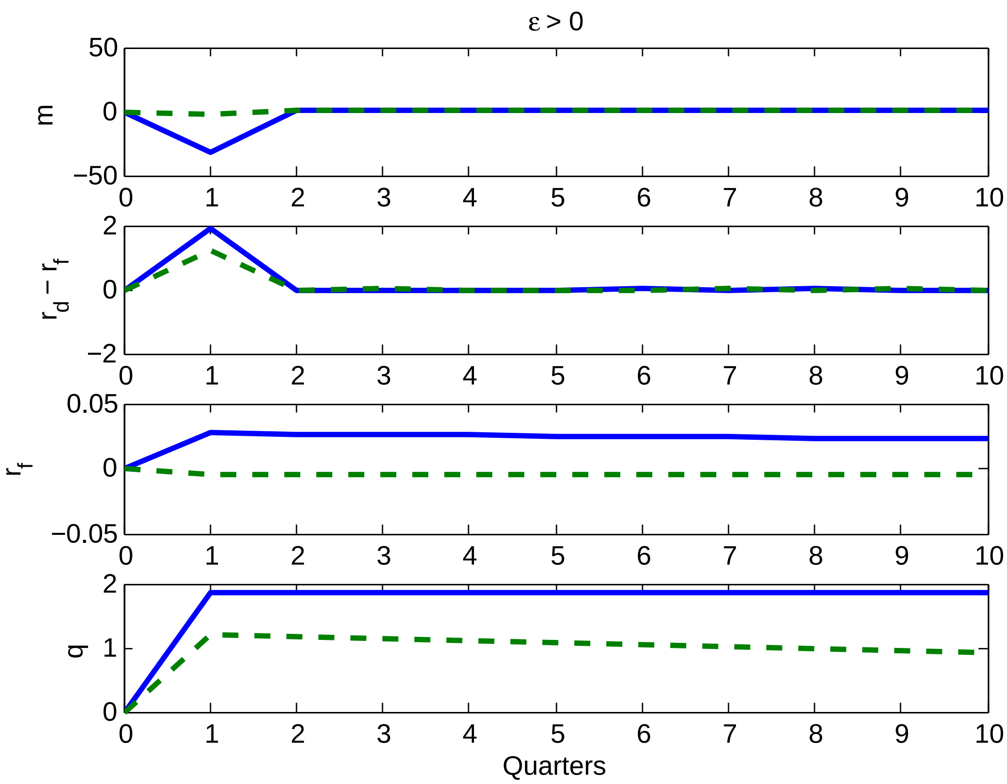
The top left panel plots the demeaned log consumption growth rate (thin line) with the medium-frequency component (thick line) from the data. The bottom left panel plots the demeaned log output growth rate (thin line) with the medium-frequency component (thick line) from the data. The top right panel plots the demeaned log consumption growth rate (thin line) with the medium-frequency component (thick line) from the ENDO model. The bottom right panel plots the demeaned log output growth rate (thin line) with the medium-frequency component (thick line) from the ENDO model. The medium-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (1999) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption and output data are obtained from the BEA.

Figure 5: Quantities



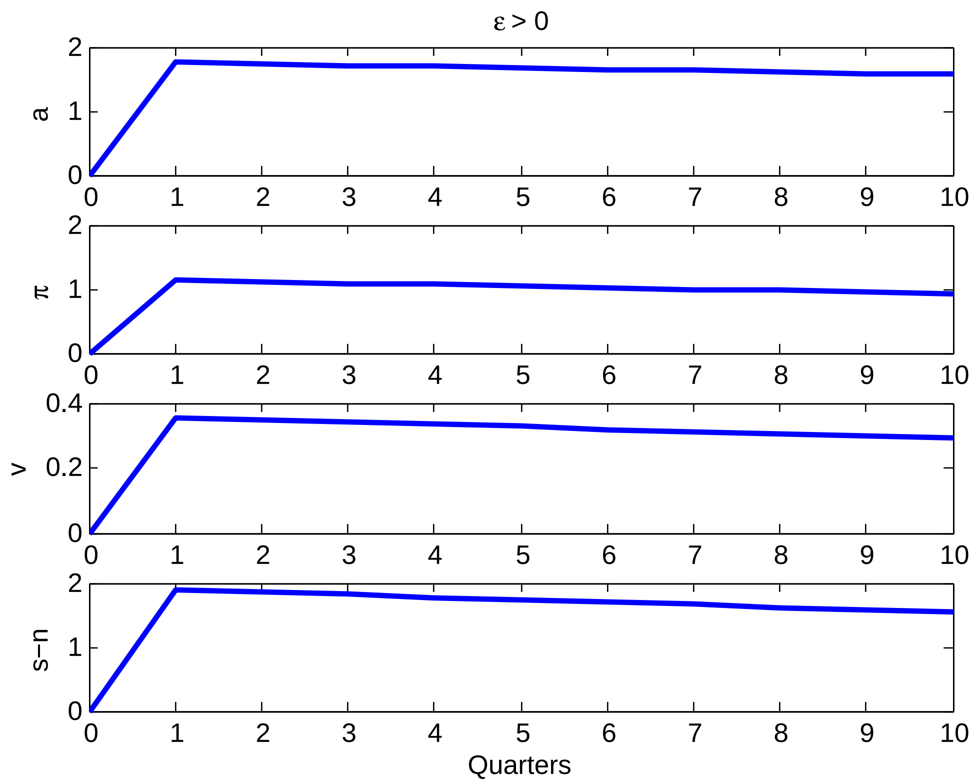
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 6: Asset Prices



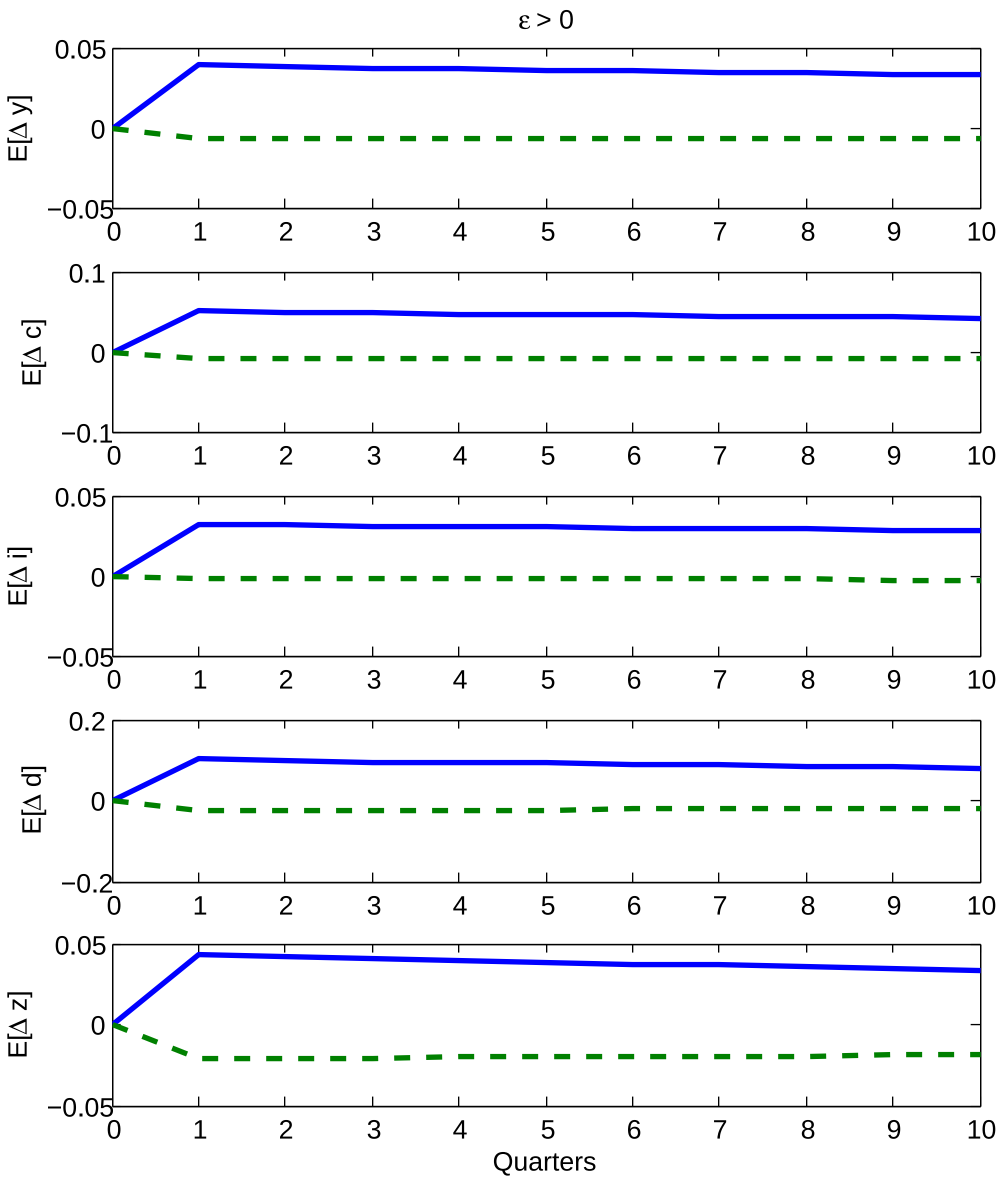
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 7: Endogenous Growth Mechanism



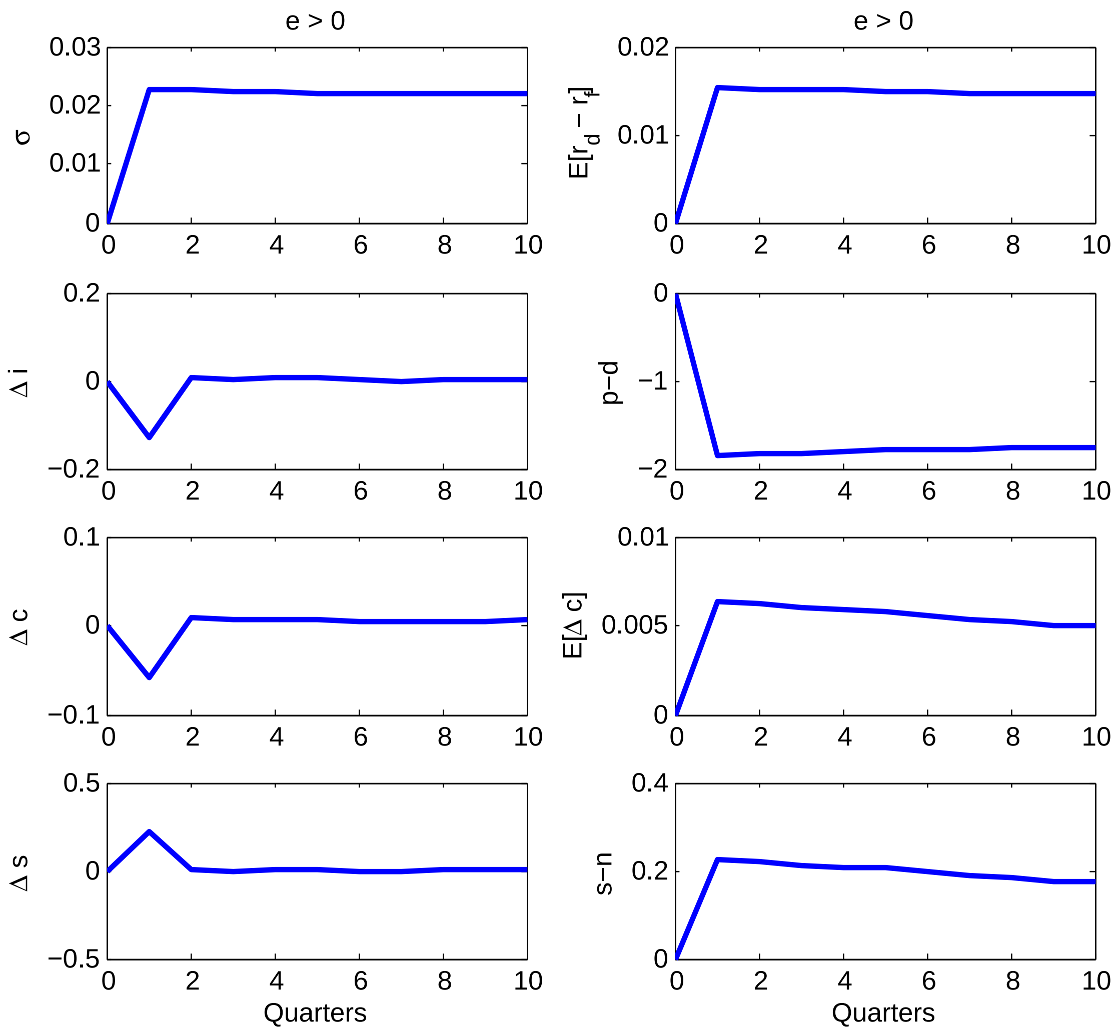
This figure shows quarterly log-deviations from the steady state for the ENDO model. All deviations are multiplied by 100.

Figure 8: Expected Growth Rates



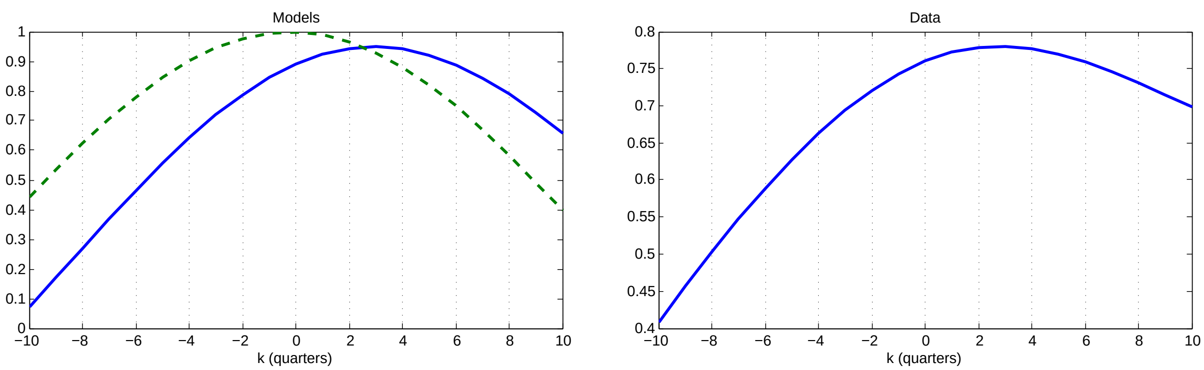
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 9: Stochastic Volatility



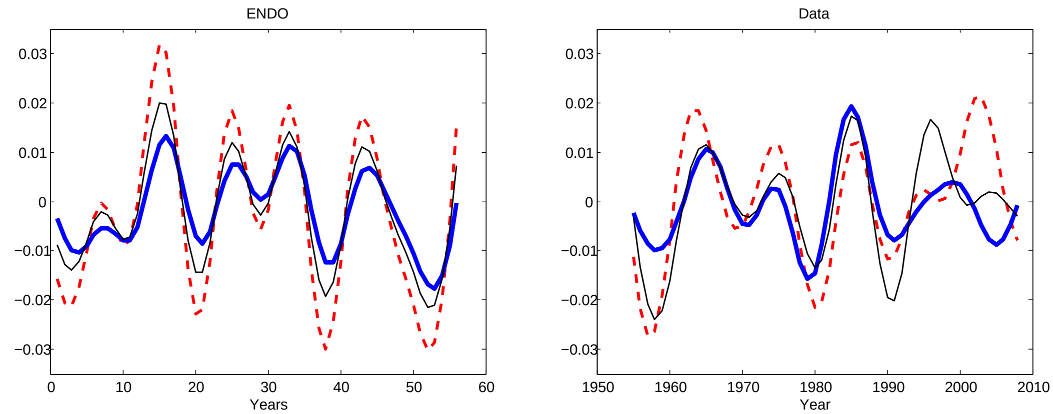
This figure shows quarterly log-deviations from the steady state from a one standard deviation shock to volatility for the endogenous growth model with stochastic volatility. All deviations are multiplied by 100.

Figure 10: Medium-Frequency Cross-Correlation of Returns and Consumption Growth



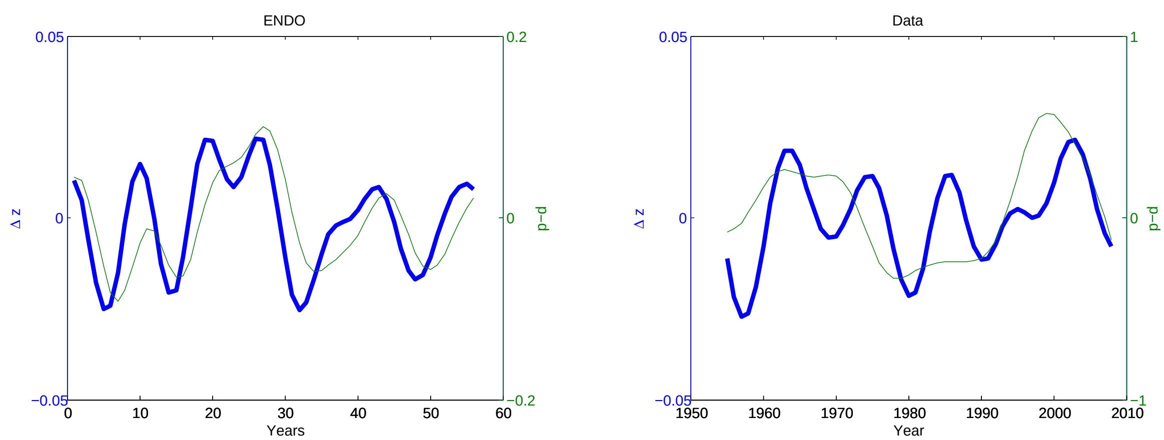
The left panel plots cross-correlations of the medium-frequency component of the equity return and the medium-frequency component of consumption growth for the ENDO (bold line) and EXO (dashed line) models: $corr(r_{d,t}, \Delta c_{t+k})$. The right panel plots the same cross-correlations from the data. The medium-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (1999) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption data is obtained from the BEA. Monthly return data is obtained from CRSP and then compounded to a quarterly frequency.

Figure 11: Medium-Frequency Growth Components



This figure plots the medium-frequency growth components for productivity (dashed line), output (thin line), and consumption (bold line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The medium-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (1999) to annual data and selecting a bandwidth of 8 to 50 years. Annual data on GDP and consumption are from the BEA and annual productivity data are from the BLS.

Figure 12: Medium-Frequency Component of Productivity Growth and Price-Dividend Ratio



This figure plots the medium-frequency components for productivity growth (bold line) and for the price-dividend ratio (thin line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The medium-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (1999) to annual data and selecting a bandwidth of 8 to 50 years. The correlation between the two series is 0.46 in the data and 0.67 in the model. Annual data on productivity are from the BLS and price-dividend data are from CRSP.