Tranching and Pricing in CDO-Transactions***

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Abstract

This paper empirically investigates the tranching and tranche pricing of European securitization transactions of corporate loans and bonds. Tranching allows the originator to issue bonds with strong quality differences and thereby attract heterogeneous investors. We find that the number of differently rated tranches in a transaction is inversely related to the quality of the underlying asset pool. Credit spreads on rated tranches in a transaction are inversely related to the number of tranches. For all rated tranches in a transaction, the average price for transferring a unit of expected default loss is inversely related to the default probability of the underlying asset pool. For a tranche, this price increases with the rating of the tranche; it is higher for the lowest rated tranche and very high for Aaa-tranches in true sale-transactions. It varies little across butterfly spreads obtained from rated tranches except for the most senior spread.

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1 Introduction

The financial crisis starting in 2007 is intimately related to the securitization of mortgage backed securities. The crisis triggered a general discussion on the future of securitization including the role of originators and rating agencies. Currently, banks and regulators attempt to revive securitization because it is considered an important instrument for trade and allocation of default risks. The purpose of this paper is to analyse the tranching and pricing of bond tranches issued in a sample of European CDO (collateralized debt obligation)-transactions. Investors and originators can evaluate the costs and benefits of securitization only if they understand tranching and pricing. These properties are also important determinants of the social costs and benefits of securitizations. Mispricing of tranches, for example, might lead to misallocation of risks and necessitate regulation. Hence it is important to know more about the determinants of tranching and pricing. We analyse tranching and pricing because both are interdependent. In a perfect market tranching would be irrelevant. Market imperfections drive tranching and pricing.

CDO-transactions can be split in CLO- and CBO-transactions. In a collateralized loan obligation (CLO)-transaction a bank usually securitizes part of its corporate loans. In a collateralized bond (CBO)-transaction, a bank or an investment company buys corporate bonds, pools them and securitizes the asset pool. The transaction is either a true sale or a synthetic transaction. In a true sale transaction the originator sells the asset pool without recourse to a special purpose vehicle which funds itself through issuing an equity tranche and rated tranches. The allocation of default losses to the issued tranches is governed by strict subordination. The equity tranche is usually not rated and called the First Loss Position (FLP). All default losses of the underlying asset pool are allocated exclusively to the FLP until it is exhausted. Losses exceeding the FLP are allocated exclusively to the tranche with the lowest rating until it is exhausted, then exclusively to the tranche with the second lowest rating and so on. Therefore, the rated tranches exhibit strong quality differences. Strict subordination can be characterized also by the attachment and detachment point of tranches. These points are defined by special portfolio loss rates, i.e. ratios of default losses over the par value of the underlying asset pool. The attachment point of a tranche defines the portfolio loss rate such that the tranche incurs (no) losses whenever the portfolio loss rate is above (below) the attachment point. The detachment point of a tranche equals its attachment point plus its size, with the size of a tranche being its par value divided by the par value of the asset pool. Whenever the portfolio loss rate exceeds the detachment point of a tranche, then this tranche is fully exhausted by default losses. The detachment point of a tranche also defines the attachment point of the tranche with the adjacent better rating. In a true sale-transaction the sizes of the First Loss Position and the rated tranches add up to about 1. The investors buying the rated tranches take the Second Loss Position.

In synthetic transactions loss allocation is somewhat different. The originator transfers the default risk of the underlying asset pool to a special purpose vehicle through a credit default swap. The special purpose vehicle also issues differently rated tranches, but usually only for a small portion of the par value of the asset pool. The size of the FLP is a threshold of the portfolio loss rate such that the credit default swap only covers losses beyond the FLP. The credit default swap covers losses only up to its par value. This par value defines a Second Loss Position taken by the investors who buy the rated tranches from the special purpose vehicle. Losses which exceed the sum of the FLP and the par value of the credit default swap are born by the originator unless she insures against these losses. These non-securitized losses on the super-senior claims define a Third Loss Position. Such a position does not exist in true sale transactions. The originator cannot use the proceeds from issuing tranches in a synthetic transaction for funding. Fig. 1 illustrates the relation between the tranche losses and the asset pool loss rate for a synthetic transaction, assuming a BB-, a A- and a Aaa-tranche.

- *Fig. 1* -

To our best knowledge, this paper is the first to look at the interdependencies between tranching, pricing and asset pool quality. In an imperfect market, tranching and pricing may matter because of regulatory costs, management and transaction costs, costs related to asymmetric information, illiquidity premiums. As the size of the FLP has been investigated in another paper (*Franke, Herrmann* and *Weber* 2011), this paper analyses the tranching and pricing of rated tranches. In our sample of CDO-transactions corporate loans and bonds are securitized, as opposed to mortgage-backed securities transactions. The rating agencies have been criticised for overly optimistic ratings of mortgage-backed securities, but not for their ratings of CDO-transactions. The ratings of corporate loans and bonds have been quite stable until 2008. This also holds for the securitization of these instruments (Newman et al (2008)). Therefore insights from CDO-transactions launched before the crisis still appear valuable for understanding securitizations.

The main findings of this paper can be summarized as follows. First, the number of differently rated tranches varies between 1 and 6. It tends to be higher for an asset pool of lower quality, i.e. a portfolio with a higher expected default loss or less diversification. For a lower asset pool quality a more differentiated tranching apparently pays for the originator.

Also, a larger asset pool appears to raise the number of differently rated tranches, indicating economies of scale effects. The tranche ratings are concentrated in the high rating classes. About 14 percent of the tranches are sub-investment grade. For lower quality-asset pools we often observe more tranches with lower ratings. Yet, within a transaction, tranches tend to be clustered more in the range of high than of low ratings.

In 88 percent of the transactions there exists a Aaa-tranche. In true sale-transactions, we observe very thick Aaa-tranches while in synthetic transactions the Aaa-tranche is usually very thin. Thickness or size of a tranche is defined by the par value of the tranche, divided by the par value of the underlying asset pool. In true sale-transactions the originator maximizes the size of the Aaa-tranche since the credit spread on this tranche is lower than that on any other tranche. In synthetic transactions the originator chooses a thin Aaa-tranche. This tranche may be an important quality signal for investors and regulators. But it is likely to be of little use in transferring default risk. Given high credit spreads of Aaa-tranches relative to corporate bonds and the restriction in synthetic transactions that the originator cannot use the proceeds from issuing tranches for her funding, presumably it does not pay to issue a volume of the Aaa-tranche higher than that needed for signalling purposes.

Second, given the asset pool and the FLP, the originator tranches so as to minimize the weighted average credit spread paid on the rated tranches plus her transaction costs, besides of other costs such as the costs of required equity capital. Since weighted average credit spreads cannot be reasonably compared across transactions, we analyse annual risk premiums. The annual risk premium of the rated tranches in a transaction is defined as the weighted average credit spread minus the expected annual default loss borne by the rated tranches, i.e. the difference between the expected annual loss under the risk-neutral and the true probability measure. To derive it, we approximate the probability distribution of the default loss of the asset pool by a lognormal distribution. This is clearly an approximation which could lead to biased results. We find for true sale-transactions that the annual risk premium of all rated tranches increases with the weighted average default probability of the underlying asset pool. This is not surprising since a higher weighted average default probability tends to raise the volume of transferred default losses which in turn should raise the risk premium. The risk premium declines in the number of rated tranches. More tranches apparently allow the originator to better tailor the tranches to differentiated investor needs so as to extract more investor rents. Also, a higher number of tranches provides more information to investors and, thus, may mitigate information asymmetry and thereby reduce credit spreads.

Third, consider the mean stochastic discount factors in securitization transactions. The mean stochastic discount factor of all rated tranches in a transaction is the weighted average of the stochastic discount factors, implicit in the tranche credit spreads. The weights are given by the default losses borne by the rated tranches, multiplied by the respective probability densities. The higher the mean stochastic discount factor, the higher is the average price for transferring default risk. Our findings indicate that this price *declines* in the weighted average default probability of the underlying asset pool and *increases* in the size of the FLP. A higher weighted average default probability tends to concentrate default losses in "cheap" macro states, i.e. in states with low stochastic discount factors. This property should also hold for all rated tranches together. Consistent with this, a higher share of expected default losses borne by the FLP raises the average price because it tends to take away default losses in the "cheap" macro states. A higher number of tranches lowers the average price, consistent with extracting more investor rents. The observed negative impact of asset pool diversification on the average price is consistent with an information asymmetry effect. More diversification tends to reduce information asymmetry which might reduce credit spreads.

Fourth, the analysis of individual tranches mostly confirms the findings for the risk premiums of all rated tranches. While the risk premium of a tranche *increases* with the weighted average default probability of the underlying asset pool, the mean stochastic discount factor *declines*. A higher number of subordinated tranches tends to lower the risk premium of a tranche. This finding is not surprising since more subordinated tranches indicate a higher attachment point of the tranche, reducing its expected default loss. Interestingly, the explanatory power of the number of subordinated tranches is stronger than that of the attachment point. The negative impact of the number of subordinated tranches is also consistent with an information asymmetry effect since more tranches mitigate this asymmetry.

Fifth, in general, the mean stochastic discount factor of a tranche increases with its attachment point, also with its rating. A higher attachment point tends to concentrate losses in the "expensive" macro-states. Against the rule, the mean stochastic discount factor is higher for the lowest rated tranche than for the mezzanine tranches. This suggests a complexity premium of the lowest rated tranche, perhaps for more expensive risk management of this tranche. Much more dramatic, however, is the very high mean stochastic discount factor of Aaa-tranches in true sale-transactions. Aaa-tranches tend to incur default losses only in states in which the aggregate default losses are very high indicating high stochastic discount factors. Since Aaa-tranches should be very information-insensitive, the high mean stochastic discount factor cannot be explained by information sensitivity. To check the pricing kernel effects more carefully, we also analyse butterfly spreads, similar to state-contingent claims. Surprisingly, we do not find a significant relation between the mean stochastic discount factor of a butterfly spread and its attachment point, with the exception of the top-butterfly spreads including Aaa-tranches in true sale-transactions. Hence the observation that the mean stochastic discount factor of a tranche increases with its attachment point should not be taken as evidence of a monotonically increasing pricing kernel.

These findings are new, to our best knowledge, and improve our understanding of tranching and pricing in securitization transactions. The evidence is based on a set of European securitization transactions which may not be representative for other parts of the globe.

The paper is organized as follows. The next section provides a literature review. Section 3 derives hypotheses about tranching and pricing. These hypotheses are tested and discussed in section 4. Section 5 concludes.

2 Literature Review

Various theoretical papers analyse the optimal design of financial contracts. Several papers advocate the benefits of tranching in the presence of information asymmetry between the seller and the buyer of a claim. Tranching allows to differentiate the degree of information-sensitivity of the issued securities. *Boot* and *Thakor* (1993) argue that a risky cash flow should be split into a senior and a subordinated security. The senior security is information-insensitive and can be sold to uninformed investors while the subordinated security is information-sensitive and should be sold to informed investors. This allows the seller of the cash flow to raise the sales revenue. *Riddiough* (1997) extends this reasoning by showing that loan bundling allows for portfolio diversification which mitigates information asymmetries. *DeMarzo* (2005) considers a bank which may securitize a portfolio of debt claims by issuing a collateralized debt obligation. The bank can sell claims separately and, thereby, signal information about the quality of the different loans. Pooling the claims precludes this, but leads to a well-diversified portfolio mitigating information asymmetries. This allows the originator to issue low-risk, information-insensitive tranches. *DeMarzo* argues that for large portfolios the diversification benefit of pooling outweighs the information destruction cost.

Malamud, Rui and *Whinston* (2009) analyse optimal tranching in a world with heterogeneous investor preferences. Investors buy different portfolios of tranches, the improved allocation of default risks lowers the average credit spread to be paid on tranches. *Brennan*, *Hein* and *Poon* (2009) assume that investors (erroneously) charge the same credit spread for a standard corporate bond and a securitization tranche, given the same rating. They show that tranching then lowers the credit spreads to be paid, regardless of whether ratings are based on default probabilities or on expected default losses.

Many papers discuss how to model the distribution of the portfolio default loss rate in securitizations. *Duffie and Garleanu* (2001) suggest a default risk model using obligor default intensities. They discuss Moody´s diversity score and illustrate the sensitivity of the portfolio loss rate distribution to various parameters including the weighted average default probability, the default correlation and the diversity score. *Krahnen and Wilde* (2008) simulate the portfolio loss rate distribution and the tranche loss rate distributions for CDO-transactions. The differences between the loss rate distributions of standard bonds and CDO-tranches are nicely illustrated[1]. *Duffie et al (2009)* argue that models underestimate the tail risks because they miss unobservable, non-stationary risk factors which raise default correlations. Also *Berndt, Ritchken* and *Sun* (2010) address default correlations in several ways. *Tarashew* (2010) shows that parameter uncertainty raises the tail risk. *Albrecher, Ladoucette* and *Schoutens* (2007) propose a generic one factor Lévy model for the portfolio loss rate distribution. *Burtschell, Gregory* and *Laurent* (2009) derive default intensities for CDOs using models with one latent factor and different copulas. *Krekel* (2008) proposes a Gaussian base correlation model with correlated recovery rates to improve the empirical model fit.

Several empirical papers investigate single name-corporate bond spreads and find high expected excess returns, e.g. *Driessen* (2005) and *Chen, Collin–Dufresne, Goldstein* (2009). This is often referred to as the ‘credit spread puzzle’. *Elton et al* (2001) find that only a small fraction of credit spreads is explained by expected default losses, substantial fractions are explained by tax and liquidity effects. Other empirical papers study the tranching and pricing in securitizations. *Amato and Remolona* (2003) analyze the credit spread puzzle in collateralized bond obligations and find very high mean stochastic discount factors. They argue that, due to the strong skewness of the default loss distribution, idiosyncratic default risk cannot be fully diversified in typical bond portfolios, and therefore earns a significant premium.

Childs, *Ott and Riddiough* (1996) investigate the pricing of Commercial Mortage-Backed securities (CMBS) and conclude that the correlation structure of the asset pool and the tranching are important determinants of the credit spreads of the tranches. *Maris and Segal* (2002) examine credit spreads in CMBS-transactions and document the impact of several macro-variables, similar to *Duffee* (1998). *Titman, Tompaidis and Tsyplakov* (2005) analyse determinants of credit spreads in MBS-transactions and find that spreads widen after poor performance of real estate markets. *Cuchra* and *Jenkinson* (2005) conclude that the number of tranches in securitizations increases with sophistication of investors, with information asymmetry and with the volume of the transaction. Finally, *Cuchra* (2005) finds that ratings are important determinants of the tranche credit spreads. *Longstaff and Rajan* (2008) analyze the market prices of tranches on the CDX credit index. They find a three-modal loss rate distribution and attribute about two thirds of the CDX spread variations to firm specific risk, one fourth to market expectations about joint defaults of firms in an industry, the remaining small rest to systemic default risk.

The paper which is closest to ours is *Weber* (2008). He uses credit spreads of tranches and information on the underlying portfolio quality to derive implied levels of relative risk aversion in synthetic CDO-transactions. He finds significantly higher levels of relative risk aversion for better rated tranches indicating high stochastic discount factors for defaults in states with high aggregate default losses. Deteriorating portfolio quality lowers relative risk aversion. *Weber* also finds that the lowest rated tranche of a transaction earns an additional risk premium and that lower portfolio diversification increases risk premiums for the two lowest rated tranches. He interprets this as evidence for additional costs due to information asymmetries. Finally, several recent papers address moral hazard issues in securitizations[2], for example *Purnanandam* (2008), *Loutskina and Strahan* (2009), (2011) and *Piskorski, Seru* and *Vig* (2010).

3 Derivation of Hypotheses

This section derives hypotheses about tranching and pricing in securitization transactions. Given the underlying asset pool in a transaction, tranching is defined by the size of the non-rated FLP, the number and the properties of the rated tranches, in particular their attachment and detachment points and their ratings. The originator decides about tranching and credit spreads of rated tranches, in cooperation with rating agencies and major investors. Rating agencies play an important role in this process. Their main tool for determining the attachment and detachment points of the tranches and their ratings are simulation models, supplemented by stress tests and the analysis of various transaction characteristics. We derive hypotheses about tranching and pricing by analysing, first, effects of market imperfections, and, second, pricing kernel effects.

3.1 Market imperfections and tranching

Tranching and pricing are driven by market imperfections. They include transaction and management costs, incompleteness of capital markets, costs of regulatory equity capital, illiquidity and liquidity risks of securities, taxes and information asymmetries. Investors buy rated tranches to optimize their expected net income and their risk. Net income is defined as the gross income (= interest income - funding costs - default losses) minus other costs including transaction costs and taxes. Investors demand credit spreads which compensate them for their costs and the tranche risks. The level of these costs and risks may depend on tranching.

3.1.1 Market incompleteness, portfolio quality and information asymmetry

If markets are incomplete, then adding new securities which are not spanned by existing securities is mostly beneficial (*Marin* and *Rahi (*2000)). New securities enlarge the set of trading opportunities for investors and, thus, allow them to put together portfolios which better fit their needs. This should lower credit spreads and, thus, motivate the originator to issue many rated tranches (*Malamud, Rui* and *Whinston* (2009))[3].

Arguments for tranching can also be derived from information asymmetry between the originator and investors. (1) According to *Boot* and *Thakor* (1993) and *DeMarzo* and *Duffie* (1999) the originator should sell the information-insensitive senior bonds and perhaps retain the information-sensitive junior bonds. This idea can be extended to the subset of rated tranches. More differently rated tranches allow for more differentiation of information sensitivity and, thus, may better fit investor needs. (2) A higher number of rated tranches provides more information to investors. For each rated tranche, the attachment and the detachment points together with the rating and the credit spread are published. This additional information helps investors to more reliably infer the parameters of the portfolio loss rate distribution. (3) Suppose that tranche-ratings are governed by tranche-default probabilities as is true of S&P and Fitch. Split one tranche with a given size and a given rating into two tranches which together have the same size. Hence the new senior tranche has a better rating. As ratings affect credit spreads in the presence of information asymmetries, tranche splitting should reduce the overall credit spread paid on both tranches (see also *Brennan, Hein* and *Poon* 2009). (4) A higher number of rated tranches may provide investors of senior tranches with more early warning signals. Each time a subordinated tranche is hit by default losses for the first time, a signal is sent. The more rated tranches exist, the more signals are sent, the smaller information asymmetry may be, perhaps leading to smaller credit spreads. The preceding arguments motivate

Hypothesis 1: Given the underlying portfolio quality and the size of the equity tranche, a higher number of differently rated tranches reduces the weighted average credit spread of the rated tranches.

The optimal number of tranches chosen by the originator depends on marginal costs and benefits of tranches. These costs increase in the number of tranches. As the marginal benefit declines, an interior optimal number of tranches should exist. Due to economies of scale-effects, a higher transaction volume should raise the marginal tranche benefit, and, thus, the optimal number of tranches.

The marginal benefit of a tranche should be inversely related to the quality of the underlying asset pool for two reasons. First, given a very good quality, there is little to be gained from tranching. Hence we should observe only a few tranches. For a low asset pool quality, the loss rate distribution would have a high mean and be broad providing more room for differently rated tranches.

Second, the marginal benefit of a tranche should be higher, the stronger is the information asymmetry between the originator and investors. This asymmetry cannot be observed directly. We proxy it by the asset pool quality. We conjecture an inverse relation between asset pool quality and information asymmetry. Rating agencies publish information on the asset pool quality. It can be measured by the weighted average default probability of the asset pool (WADP) and its diversity score (DS). WADP is an average of the default probabilities of all assets, weighted by the par values of assets. Since the loss given default for each asset is often not available, assume that it is a constant being the same for all assets. Then the expected portfolio loss rate equals WADP, multiplied by this constant. The second measure of asset pool quality is asset pool diversification given by Moody´s Diversity Score (DS) or, in a refined version, the adjusted diversity score (ADS). An increase in WADP lowers asset pool quality while an increase in DS improves it. Errors in estimating WADP are likely to be proportional to the true WADP, implying a positive relation between WADP and information asymmetry. But a high DS reduces information asymmetries because the idiosyncratic risks of the assets are diversified away (*DeMarzo* (2005)). This inverse relation between asset pool quality and information asymmetries is also consistent with the empirical findings about asset pool quality and loss allocation in *Franke, Herrmann* and *Weber* (2011). They find that a better quality reduces the size of the FLP. The FLP is the most important credit enhancement in a securitization to mitigate problems of adverse selection and moral hazard. Hence we use asset pool quality as an inversely related proxy for information asymmetry. The preceding arguments suggest

Hypothesis 2:

a) *The optimal number of differently rated tranches is inversely related to the quality of the asset pool.*
b) *The optimal number of differently rated tranches grows with the transaction volume.*

3.1.2 Costs of Equity Capital and Funding Costs

Within a transaction the credit spread is lower for a tranche with a better rating. Hence the originator minimizes the attachment point of the Aaa-tranche to minimize credit spreads. Also regulators require less equity capital for a better rated tranche under Basel II[4]. Therefore, the originator maximizes the size of the Aaa-tranche in a true sale-transaction.

As shown by *Franke, Herrmann* and *Weber* (2011), synthetic transactions are preferred to true sale transactions by banks with a strong rating. For them funding through standard bank bonds appears to be cheaper than funding through a Aaa-tranche. Also a Aaa-tranche does not allow a substantial default risk transfer so that we should not observe Aaa-tranches in synthetic transactions. Yet, most synthetic transactions include a Aaa-tranche. The purpose of this tranche may be to provide a quality signal to investors and regulators. Also the quality of the non-securitized super-senior tranche should be even better than that of the Aaa-tranche, implying very little regulatory capital and low cost of insuring against the default risk for the super-senior tranche. For these purposes, a very small Aaa-tranche should suffice in synthetic transactions. This motivates

Hypothesis 3: In true sale-transactions the originator chooses a very large Aaa-tranche, in synthetic transactions a very small Aaa-tranche.

3.2 Risk Premiums of Rated Tranches and Stochastic Discount Factors

3.2.1 Some Foundations

Credit spreads of bonds are driven by expected default losses, pricing kernel effects, tax, liquidity, maturity and other effects (*Driessen* 2005). This should also be true for initial credit spreads of tranches in securitization transactions. We do not have data on taxes, transaction costs and liquidity premiums for our set of European transactions. Therefore we refrain from modelling these determinants of credit spreads. We assume that they are taken into account in the pricing kernels which determine the credit spreads. To investigate the pricing of default losses in securitization transactions, we focus on pricing kernel effects, on the effects of differences in the quality of the asset pools in securitization transactions, and on information asymmetry effects. We start with the (annualized) risk premium of a rated tranche,

risk premium of tranche = tranche credit spread

- expected tranche loss per € invested /transaction maturity.

Since the credit spread is the spread earned per € invested, we subtract the annualized expected tranche loss per € invested. It equals the expected default loss borne by the tranche divided by its par value and by the maturity of the transaction. Without loss of generality, we assume a par value of 1 € for the transaction volume. Then the default loss of the asset pool equals its loss rate and the par value of a tranche equals its size. For simplicity, the tranche loss per € invested is denoted the tranche loss.

Let l_τ and l_i denote the annualized random default loss of asset pool τ resp. tranche i of transaction τ. $\Psi(l_\tau)$ is the pricing kernel for transaction τ, i.e. all rated tranches of transaction τ are priced by this transaction-specific pricing kernel. Then the risk premium of tranche i, RP_i, is

$$RP_i = E[l_i \Psi(l_\tau)] - E(l_i) = cov[l_i, \Psi(l_\tau)]. \qquad (1)$$

Equation (1) assumes $E(\Psi(l_\tau)) = 1$ as in a perfect capital market. Since the tranche loss l_i weakly increases in the asset pool loss l_τ, the risk premium should be positive given that $\Psi(l_\tau)$ increases with the asset pool loss.

The properties of the pricing kernel, or equivalently, the stochastic discount factor (SDF), can be seen more clearly by dividing the risk premium of tranche i by its expected loss,

$$RP_i / E(l_i) = E[l_i (\Psi(l_\tau) - 1)] / E(l_i) =: E^{Qi}(\Psi(l_\tau) - 1) \qquad (2)$$

Hence this ratio gives the weighted average of the stochastic discount factor minus 1 with the weight being the loss of tranche i multiplied by its probability density, divided by the expected tranche loss. We call $E^{Qi}(\Psi(l_\tau))$ the *mean tranche-stochastic discount factor*.

Many papers study the stochastic discount factors implicit in corporate bonds and find that they tend to be lower for bonds and loans with a lower rating. For European Aaa, Aa, A and Baa rated bonds *Amato/Remolona* (2003) find average ratios of credit spreads over annualised expected losses of 210, 35, 6.7 and 1.6, respectively. For the US, they report 625 for Aaa-bonds and 2.2 for Baa-bonds. *Berndt et al* (2005) find that the ratio of risk neutral over actual default intensities is higher for safer firms. Similarly, *Chen et al* (2009) find much higher values for Aaa- than for Baa-firms. Similar effects should also be observed for securitizations with different WADPs of the asset pools. *Weber* (2008) looked into synthetic transactions and derived the portfolio loss rate distribution by a simulation model. Using a pricing kernel with constant relative risk aversion, he finds that the estimated relative risk aversion is highest for the Aaa-tranche and lower for lower rated tranches except for the lowest rated tranche. It should be kept in mind that the estimation of the tail risk of Aaa-tranches is particularly sensitive to estimation errors. Therefore the estimates of the Aaa-tranche risk premiums should be interpreted with caution.

The intuition for all these findings is that default losses of the better rated bonds are more heavily concentrated in bad macro-states with high stochastic discount factors. To model the pricing kernel effect, we assume that the stochastic discount factor depends on a macro-factor and, perhaps, an orthogonal industry-factor. The macro-factor might be the aggregate default loss rate in the economy. This would be true in a world in which investors only bear default risk. But even if investors also invest in stocks, it is likely that stock prices are depressed when default losses in the economy are high and vice versa. Therefore the stochastic discount factor is likely to be an increasing function of the aggregate default loss rate. It might also increase in industry-specific default rates. Given a rather well-diversified asset pool in a securitization transaction, it is likely that the asset pool-loss rate is strongly positively correlated with the aggregate default rate. Hence the transaction-specific stochastic discount factor should also increase with the aggregate default rate.

For illustration, consider a standard linear *Gordy*-type model (2003) and combine it with the KMV-approach (see also *Tarashew* (2010)). We use this simple model to illustrate the basic idea which also holds in more sophisticated models. In this model, the date t - total market value of an obligor firm in industry j, $V(t)$, divided by the current market value $V(0)$, is driven by the aggregate default rate m, an orthogonal industry-factor n_j and an idiosyncratic risk factor ε, all standardized to zero expectation and unit variance,

$$V(t)/V(0) = a + \sigma[- m \sqrt{\rho} - n_j\sqrt{\underline{\rho}} + \varepsilon \sqrt{(1- \rho- \underline{\rho})}] \tag{3}$$

$-\sqrt{\rho}$ is the correlation coefficient between the firm value and the macro-factor, $-\sqrt{\underline{\rho}}$ the correlation coefficient between the firm value and the orthogonal industry-factor. $V(t)/V(0)$ has an expectation of a and a standard deviation of σ. Suppose that the firm defaults when $V(t)$ falls below a given trigger D. Then, assuming that the industry-factor and the idiosyncratic factor are normally distributed, the physical default probability PD of the firm, conditional on m, is given by

$$PD(m) = Prob(V(t) \leq D \mid m) = N((-\Delta + m\sqrt{\rho})/\sqrt{(1-\rho)}) \qquad (4)$$

with $\Delta = (a-D/V(0))/\sigma$ being the distance to default and $N(.)$ the cumulative standard normal distribution. Clearly, the conditional PD increases with the aggregate default rate if $\rho > 0$. It should be noted that we do not impose any distributional assumptions on the aggregate default rate.

To see the impact of the macro-factor on tranche pricing, first consider a standard corporate bond. Then a lower obligor quality, measured by a smaller distance to default, should raise the risk premium, because default losses increase. Hence the covariance-term in (1) should increase. But, given a negative correlation between firm value and macro-factor, the mean bond-stochastic discount factor should decline because default losses should be concentrated relatively more in the "cheap" macro-states where the macro-factor, say the aggregate default rate, is low. To see this, differentiate $ln\ PD(m)$ with respect to $-\Delta$ and then with respect to m

$$\partial^2 \ln (PD(m)))/\partial(-\Delta)\, \partial(m\sqrt{\rho}) = - [N(y_m)y_m + n(y_m)]\, n(y_m)/[\,(1-\rho)N^2(y_m)], \qquad (5)$$

with $y_m =: (-\Delta + m\sqrt{\rho})/\sqrt{(1-\rho)}$. This derivative is negative whenever the first term in brackets on the right hand side is positive. This is true whenever $y_m \geq -4$, i.e. $PD(m) = N(y_m) \geq N(-4) \approx 0.$. Hence, for $\rho > 0$, a lower obligor quality implies that default losses tend to grow faster in states with a lower aggregate default rate, i.e. in "cheaper" macro-states. Therefore, the mean bond-stochastic discount factor should decline with declining obligor quality. If the orthogonal industry-factor is also priced, this might reinforce the decline in the mean bond-stochastic discount factor. This is in line with the empirical findings for corporate bonds. Similarly, the mean stochastic discount factor for a securitized asset pool should decline with declining pool quality.

3.2.2 Implications for Securitizations

a) Risk Premiums

The preceding results allow us to derive hypotheses for securitization transactions. To see the effects of changes of the underlying asset pool on single tranches, we first repeat some basics. We model the effect of a higher weighted average default probability WADP of the asset pool as a first order stochastic dominance shift in the loss distribution. Hence a higher WADP raises the expected loss of the FLP and that of every tranche, given the attachment points of all tranches. We model the effect of a lower diversity score DS on the loss distribution of the asset pool as a second order dominance shift so that its risk increases. Hence a lower DS implies a redistribution of default losses from the FLP to the rated tranches, given the size of the FLP.

Hypothesis 4 (loss volume effect):

a) *Given the loss share of the First Loss Position, the risk premium of all rated tranches together increases with the weighted average default probability of the asset pool.*

b) *An increase in the annualized expected loss of a tranche raises its risk premium.*

Hypothesis 4a) addresses the risk premium of all rated tranches together in a transaction. Assuming the same loss given default across transactions, the expected default loss rate of the asset pool is determined by its WADP. A higher expected loss rate of the asset pool raises the expected loss of the rated tranches, given the loss share of the First Loss Position. This share is defined as the expected default loss borne by the First Loss Position, divided by the expected loss of the asset pool. As shown in *Franke, Herrmann* and *Weber* 2011, the loss share is quite stable in our set of transactions. Hence, the expected losses of the rated tranches together should increase with WADP. Also, $cov(\sum_i l_i , \Psi(l_\tau))$ should increase so that, by equation (1), the risk premium should increase. This is not precisely true for synthetic transactions because the loss share of the non-securitized Third Loss Position might change. But this share is very small anyway so that we ignore it here.

Hypothesis 4a) is driven by the volume of default losses borne by all rated tranches together. Note that a change in the diversity score of the asset pool would not change the expected loss of all rated tranches together, given a constant loss share. A volume effect should also exist for single tranches as stated in Hypothesis 4b. Whether an increase in the WADP of the asset pool raises the expected loss of a tranche, depends, however, also on the impact of WADP on the number of tranches.

b) *Mean Stochastic Discount Factors*

Next, we consider the mean tranche-stochastic discount factors for all rated tranches together and for single tranches. An increase in the attachment point of a tranche should, ceteris paribus, raise the mean tranche-stochastic discount factor because a higher attachment point tends to concentrate tranche losses in the range of “expensive” macro-states with high aggregate default rates. This motivates Hypothesis 5a.

Next, an increase in the WADP of the underlying asset pool has two opposing effects. First, as shown above, the mean transaction-stochastic discount factor should decline because a higher WADP concentrates default losses more in the “cheap” macro-states. But a higher WADP also raises the FLP so that losses tend to be concentrated more in the “expensive” macro-states. The net effect on the mean stochastic discount factor for all rated tranches together is open. A similar argument holds for the mean stochastic discount factor of a tranche with a given rating since the tranche-attachment point should also increase with WADP.
To motivate Hypothesis 5b, assume a pricing kernel which is relatively flat for a wide range of macro-states and increases strongly in the bad macro-states. Then the attachment point effect should be relatively small for tranches with low attachment points. Therefore the mean tranche-stochastic discount factor of the lower rated tranches should decline with increasing WADP. For tranches with high attachment points, we might see the opposite result because the default losses are concentrated in the very bad macro states. Then the attachment point effect might dominate the WADP-effect.

Hypothesis 5 (pricing kernel and attachment point effects):

a) The mean tranche-stochastic discount factor increases with the tranche-attachment point.

b) An increase in the weighted average default probability of the asset pool lowers the mean tranche-stochastic discount factor of tranches with a low rating and raises that of highly rated tranches.

c) An increase in the diversity score of the asset pool lowers the mean tranche-stochastic discount factor of all rated tranches.

Hypothesis 5c relates to asset pool diversification. A higher diversity score concentrates asset pool-default losses around the expected loss rate. Assuming value additivity, this should have no effect on the mean transaction-stochastic discount factor. Hence the effect of the DS on the mean tranche-stochastic discount factor should be driven mainly by the attachment point effect. Suppose that the attachment point is higher than the expected portfolio loss rate[5]. Then a higher DS should lower the attachment point of a rated tranche to preserve its default probability or its expected loss. As a consequence, according to Hypothesis 5a, the mean tranche-stochastic discount factor should decline.

c) Information Asymmetry Effects

So far we have excluded information asymmetry effects. Since investors are averse to information asymmetry, this asymmetry should raise tranche-risk premiums. As compared to a perfect capital market with $E(\Psi(l_\tau)) = 1$, information asymmetry might raise this expectation above 1 so that credit spreads increase. This motivates

Hypothesis 6 (information asymmetry effect): The risk premiums and mean stochastic discount factors of rated tranches increase with information asymmetry. Hence they are inversely related to asset pool quality, to the loss share of the First Loss Position and the number of rated tranches.

The impact of asset pool quality and the number of rated tranches on information asymmetry has been discussed before. A higher loss share of the First Loss Position discourages the originator from adverse selection and moral hazard so that it should reduce information asymmetry problems.

A final hypothesis concerns the importance of ratings. Rating agencies claim to have very good information so that information asymmetries have only small effects on ratings. This suggests that investors rely more on tranche rating than on asset pool quality and observable tranche properties. This motivates

Hypothesis 7: The risk premium and the mean stochastic discount factor of a tranche are better explained by the tranche rating than by the underlying portfolio quality, the size and the attachment point of the tranche.

4 Empirical Findings

4.1 Summary Statistics

These hypotheses will be tested in the following. Our empirical analysis is based on 167 European CDO-transactions. Except for two poorly documented transactions, these transactions include all European CDO-transactions between the end of 1997 and the end of 2005, for which we know Moody´s diversity score and for which we can derive WADP. These transactions represent about half of the European CDO-transactions over this time period. The data are collected from offering circulars, Moody´s presale reports on CDO-transactions and from the Deutsche Bank Almanac.

Moody´s diversity score DS was criticized as a diversification measure because it ignores correlations between obligors of different industries. The adjusted diversity score is a more sophisticated diversification measure; it assumes an asset correlation ρ_{ex} between all obligors in the underlying asset pool and an additional correlation (ρ_{int} - ρ_{ex}) for obligors within the same industry. Rating agencies use these correlations in their simulation models to derive the loss rate distributions of asset pools. We use additional information on the industry structure of asset pools from securitization documents to derive the adjusted diversity score ADS. This is possible only for 92 transactions. We assume an intra-industry correlation $\rho_{int} = 0.2$ and an inter-industry correlation of $\rho_{ex} = 0.00$, 0.02 or 0.04. There is no agreement on the "correct" asset correlations. As illustrated in *Fender* and *Kiff* (2004) and confirmed in informal discussions with the rating agencies, the assumed correlations appear to be roughly in line with those used by the agencies.

First, we present some summary statistics (Table 1). About 48 % are true sale-transactions, about 52 % are synthetic. About 44 (56) % are CLO (CBO)-transactions. Most of the transactions were set up between 2000 and 2004.

- Table 1-

As shown in Table 2, the weighted average default probability (WADP) is, on average, much smaller for synthetic than for true sale-transactions. As expected, the FLP shows a similar pattern. CLO-transactions tend to be much better diversified than CBO-transactions.

- Table 2 -

To estimate the expected default loss rate of an asset pool, multiply WADP by the loss given default LGD. With few exceptions, we assume LGD = 50 percent, due to a lack of transaction specific information. *Acharya et al (2007)* find an average LGD of slightly below 50 percent for loans and bonds in the US. For 2 transactions with secured bonds we use LGD = 25 percent, for some mezzannine transactions with subordinated, unsecured loans we use LGD = 100 percent, in line with the rating agencies.

Table 3 provides an overview of the tranching. In many transactions, there are some rated tranches which are not subordinated to each other. For example, two equally rated tranches are denominated in different currencies or one tranche pays a fixed coupon while the other tranche pays a floating rate. We count tranches of equal rating as one tranche because we focus on quality differentiation in tranching. As the upper and the lower panel in Table 3 show, the number of tranches with different ratings varies between 1 and 6. Most transactions have 3 to 5 differently rated tranches. 14 transactions have only one rated tranche (single-tranche deals[6]) while 6 transactions have 6 tranches. In 9 transactions with 4 or 5 tranches there are two strictly subordinated Aaa-tranches which we count as two differently rated tranches. The total number of tranches with the same rating is, by far, the highest for Aaa. 88 % of all transactions have at least one Aaa-tranche. Also, there is a concentration of tranches in Aa2, A2, Baa2 and Ba2 which correspond to the "even" S&P-ratings AA, A, BBB and BB. The tranches in the few-tranche-deals are mostly concentrated in the good rating classes. The more tranches are issued, the more likely low rated tranches are issued, too. About 14% of the tranches are subinvestment-grade. They exist mostly in 4, 5- and 6-tranche deals. B-rated tranches are issued only in synthetic deals with at least 4 tranches.

- Table 3 -

Additional evidence on tranche rating can be obtained by looking at the rating range and the average rating gap for transactions with at least two rated tranches. The *rating range* of a transaction, i.e. the range between the lowest and the highest rating within a transaction, is defined as *(1 + the difference between the lowest and the highest rated tranche measured in rating notches)*. We assign 1 to the rating Aaa, 2 to Aa1, 3 to Aa2 and so forth until 16 to B3. Given at least two rated tranches, the minimum rating range is 2 while the maximum rating range is 16 (for a transaction with a Aaa- and a B3-tranche). Table 4 displays the medians of the rating range. It supports the visual impression of Table 3 that a higher number of tranches is associated with a wider rating range. Since most transactions have a Aaa-tranche, a higher number of tranches tends to add tranches with lower ratings.

- Table 4 -

In addition, define for each transaction
average rating gap = (rating range - number of rated tranches) /(number of rated tranches - 1).
It indicates how many notches between two adjacent tranche ratings are missing on average within a transaction.While the median rating gap is 4 given 2 rated tranches, it declines to 2, given 4 or more rated tranches. Hence even though a higher number of tranches includes tranches with lower ratings, the median rating gap declines, indicating a stronger clustering of ratings.

If there are more than two rated tranches, then the question arises whether these tranches are clustered more in the higher or the lower rating range. Define for each transaction

$$\textit{rating asymmetry measure} = \sum_i (\textit{rating gap i/average rating gap of transaction})\ (i/\sum i) \quad - 1$$

The rating gap between two adjacent tranches is the difference between their numerical ratings minus 1, i.e. the number of missing notches between the two tranches. The rating gap between the two highest rated tranches is indexed by i = 1, the rating gap between the second and third best rated tranches is indexed i = 2 and so on. Rating gaps are attached a higher weight (i/∑i), the lower are the ratings of the adjacent tranches. The rating asymmetry measure is 0 when the rating gaps are the same for tranches with high and low ratings. The measure is positive when the rating gaps are larger between tranches with low ratings. In our sample the rating asymmetry measure has a mean of 0.087, a standard deviation of 0.20, a minimum of -1/3and a maximum of 1.17. This indicates that rating gaps tend to be stronger between tranches with lower ratings, i.e. ratings are clustered more in the higher rating range. Possibly a more differentiated tranching does not pay in the range of lower ratings because investors buying these tranches are more sophisticated and do not rely much on ratings. Another potential explanation might be that tranche sizes tend to be smaller in the range of low ratings so that it does not pay to split a small tranche. Therefore we next look at the tranche sizes (Table 5).

- Table 5 -

In true sale transactions the average size of the Aaa-tranche is large. It declines from 81% for single-tranche deals to 68% for deals with 6 rated tranches, with an average of about 76 %. In synthetic deals the average size of the Aaa-tranche varies between 3 % for single-tranche deals and 5 % for deals with 3 or 4 tranches, with an average of about 4.5 %. Thus, Hypothesis 3 is clearly supported. The small average size of Aaa-tranches in synthetic deals can be understood better in relation to the non-securitized super-senior tranche (Third Loss Position) which is on average about 87 % (excluding 3 atypical fully funded synthetic transactions). Adding this and the average size of the Aaa-tranche yields 91.5 % for synthetic deals. Hence the attachment point of Aaa-tranches is about 15 % higher in synthetic than in true sale transactions. This is due to the better quality of the asset pools in synthetic deals. The quality difference materializes also in the size of the FLPs. The average size of the FLP is about 8.7 % in true sale and 3.3 % in synthetic transactions. The aggregate size of all rated non-Aaa-tranches is 100 – 8.7 – 76 = 15.3 % in true sale transactions, but only 100 – 3.3 - 87-4.5 = 5.2 % in synthetic transactions. This explains why the average size of the non-Aaa-tranches tends to be much smaller in synthetic deals. For true sale transactions, Table 5 shows several examples of average tranche sizes above 10 % in the A-range, but for synthetic transactions only single tranche-deals provide one example.

The average size of tranches rated below A3 is small. In most transactions, the size of a tranche tends to decline with its rating. But the originator cannot choose the tranche sizes arbitrarily because they are constrained through the default probabilities resp. the expected losses as defined by the rating agencies. As an example[7] demonstrates, the tranche size does not decline monotonically with the tranche rating. Also, if two tranches with adjacent ratings are merged, then the merged tranche is rather thick. In our transaction sample, within a transaction the size of a tranche is larger than that of the adjacent lower rated tranche in about 75 % and smaller in about 25 % of the cases. Surprisingly, in synthetic transactions we observe rather thin tranches with a rating below Ba3, which are absent in true sale-transactions. Possibly there is more investor demand for low rated tranches in high quality synthetic transactions.

A puzzling finding relates to the initial credit spreads of rated tranches. Within a transaction, the credit spread always increases from one tranche to the adjacent tranche with a lower rating. Fig. 2 plots the credit spreads of all rated tranches for all transactions on a logarithmic scale, differentiated with respect to the issuance quarter. For tractability, Fig. 2 differentiates only among the rating classes Aaa, Aa, A, Baa, Ba and B. Although the credit spreads for the A-related ratings are mostly below those for the B-related ratings, there is surprisingly much overlap in several issuance quarters. These overlaps can be explained neither by variations over time nor by different transaction maturities. *Berndt et al* (2005) also find a rather strong overlap of credit spreads for corporate bonds.

- Figure 2 -

4.2 Regression results

Next we run regressions to test the hypotheses stated before. In all regressions we control for originator characteristics which may affect credit spreads and tranching decisions. We distinguish banks and investment firms as originators and include various characteristics of originating banks in our empirical analysis. In the regressions we include as controls

- the tier 1-capital ratio and the total capital ratio,
- capital structure: equity/total assets,

- asset structure: loans/total assets,
- profitability: return on average equity capital in the transaction year, average return over the years 1994 to 2004, and the standard deviation of these returns as a proxy for profitability risk,
- Tobin´s Q to proxy for the bank´s profitability and also for its growth potential,
- the bank´s rating. Rating is captured by an integer variable which equals 1 for a Aaa-rating and increases by 1 for every notch, with 16 for a rating of B3. A higher integer indicates a lower rating.

These bank data are obtained from the Bank Scope Database. These data are not available for investment firms, some data are also missing for some banks. We subtract from each bank control variable its average to eliminate effects of averages. The standardized bank control variable then is multiplied by a dummy which is 1 for a bank if this variable is available and 0 otherwise. In the regressions, reported in the following sections, insignificant regressors are mostly excluded and, thus, not shown.

4.2.1 The Number of Tranches

The first regression is a probit regression explaining the number of tranches in a transaction by several characteristics of the transaction and of the originator. For banks, the average return on equity and the loans/assets ratio have some explanatory power and therefore are included. We find

$$\textit{Number of tranches} =$$
$$\underset{(0.0000)}{.19\,\textit{WADP}} + \underset{(0.0377)}{9.6\;\textit{1/ln ADS}} - \underset{(0.0002)}{.19\,\textit{FLP}} + \underset{(0.0004)}{.58\,\textit{ln vol}} \underset{(0.0101)}{-.70\,\textit{CBO}} \underset{(0.0353)}{-.04\,\textit{return on equity}} + \underset{(0.0068)}{.02\,\textit{loans/assets}}$$

$$\text{Pseudo-}R^2 = 0.238$$

The regression coefficients are shown together with their p-values (in parentheses). The regression is based on the 92 transactions for which we can derive the adjusted diversity score (*ADS*), based on asset correlations $\rho_{int} = 0.20$ and $\rho_{ex} = 0.02$. *FLP* is the size of the FLP, defined as a fraction of the transaction volume. *Vol* is the €-volume of the transaction. *CBO* is a dummy which is 1 for a CBO-transaction and 0 otherwise. The regression indicates that the number of tranches is inversely related to the portfolio quality as stated in Hypothesis 2a. Not surprisingly, the FLP-size has a negative impact on the number of tranches because a larger FLP reduces the space for rated tranches. The positive impact of the transaction volume on the number of tranches clearly supports Hypothesis 2b. The negative sign of the CBO-dummy indicates that the number of tranches tends to be smaller in CBO-transactions. This is not surprising since 13 out of 14 single tranche-deals are CBO-transactions.

While the regression coefficient of *WADP* is stable in various regressions, the regression coefficient of *1/ln ADS* depends on the regressors included. The regression coefficient for *1/ln ADS* is positive even though the correlation coefficient between the number of tranches and *ln ADS* is 0.31.This sign reversal is intuitive since well diversified asset pools tend to have a high transaction volume and a small FLP, both suggesting a high number of tranches. Controlling for transaction volume and *FLP* reveals a negative impact of asset pool diversification on the number of tranches.

Only two originator characteristics have a significant impact on the number of tranches. For banks as originators, a lower return of equity and a higher loan to assets ratio tend to raise the number of tranches. The positive impact of the loans to assets-ratio might indicate more securitization activity of the originating bank and, thus, a need for a broader investor base which might be easier to attract through more differentiated tranches. A lower return on equity might intensify a bank´s securitization activities to raise bank profits quickly as found by *Titman* and *Tsyplakov* (2010). The impact of these originator characteristics on the explanatory power of the regression is quite limited, however. If we exclude them, the pseudo-R^2 decreases to 0.201.

4.2.2 Aggregate Risk Premiums

Next, we analyse the aggregate risk premiums, i.e. the sum of the risk premiums of all rated tranches in a transaction. The originator of a transaction minimizes the credit spreads on the rated tranches, given the asset pool quality and the FLP. The weighted average spread of all tranches should be inversely related to the number of differently rated tranches (Hypothesis 1). Given the loss rate distribution of the asset pool, the FLP, and, in a synthetic transaction, the Third Loss Position, minimizing the weighted average credit spread is equivalent to minimizing the aggregate risk premium (ARP).

As Moody´s does not publish expected tranche losses, we need to estimate them. The rating agencies use multi-period simulation models to derive the loss rate distribution of the asset pool. These models produce unimodal loss rate distributions. Given the strong impact of rating agencies in securitization, we also use a unimodal distribution. This may be dangerous. *Longstaff/Rajan* (2008) analysed the loss rate distribution of CDX-tranches and find a three-modal loss rate distribution where the second (third) mode has a much smaller density than the first (second).

From Moody´s we know for each transaction WADP and DS of the asset pool and, for a restricted sample, the more refined adjusted diversity score (ADS). These two parameters allow us to use a two-parameter distribution. We assume that the loss rate distribution can be reasonably approximated by a lognormal distribution[8]. Also Moody´s (2000) used this distribution. The two parameters of this distribution are inferred from WADP and ADS as explained in Appendix 1. The lognormal distribution approximates the distribution, obtained from the simulation models of the rating agencies, reasonably well. This approximation allows us to use the Black-Scholes framework, and hence, to use analytic expressions for the expected tranche losses and the share of expected default losses of the asset pool borne by the FLP (see Appendix 2).

Table 6 shows the regression results. Regressions are based on 37 true sale- and 45 synthetic observations. We have assigned the atypical 3 fully funded synthetic transactions to the true sale-sample. We distinguish true sale and synthetic transactions because the average aggregate risk premium (ARP) for true sale transactions is 39 basis points while it is 102 bp for synthetic transactions. This difference is due to the large non-sold super-senior tranche in synthetic transactions which is not included in ARP. In true sale-transactions the large Aaa-tranches earn an average credit spread of 35 basis points, but they bear an annual expected loss per € invested of only 2.5 basis points so that the risk premium is about 32 bp. Hence the large Aaa-tranche strongly pulls down the ARP in true sale-transactions.

- Table 6 -

The first regression shows that in *true sale-transactions* the aggregate risk premium ARP increases with WADP. This supports the loss volume effect stated in Hypothesis 4a. A higher WADP imposes more default losses on investors and, hence, they charge a higher risk premium. A higher WADP is also associated with stronger information asymmetry which should also raise ARP, consistent with Hypothesis 6. According to this hypothesis, a lower diversity score should also raise ARP. But the regression coefficient of ln ADS is far from being significant so that we exclude this regressor.

To account for different conditions in credit markets, we include the IBOXX-spread at the issuance date as a regressor. This spread is the difference between the BBB- and the government credit spreads for a maturity between 3 and 5 years. Not surprisingly, a higher IBOXX-spread raises ARP. More importantly, a higher number of rated tranches lowers ARP, supporting Hypothesis 1. Issuing more tranches allows the originator to better signal the asset pool quality and to better exploit heterogeneous investor preferences. One might expect also a negative influence of the FLP-loss share. But this variable has no significant impact. This may be due to the observation that the loss share varies only little across transactions (*Franke, Herrmann and Weber* (2011)).

The second regression indicates that a higher rating asymmetry measure tends to lower ARP. This measure is higher in a transaction where the tranche ratings are clustered more in the high rating range. This tends to reduce the weighted average credit spread and, hence, ARP. Originator characteristics do not play a significant role.

Regressions (3) and (4) in Table 6 analyse ARP for *synthetic* transactions. Surprisingly, regression (3) shows a negative regression coefficient of WADP, significant at the 5%-level. When we include in regression (4) the number of tranches and the rating asymmetry measure, the coefficient of WADP is no longer significant. Hence the puzzling sign of the WADP-coefficient in regression (3) may be driven by tranching effects. Moreover, in synthetic transactions WADPs are, on average, quite small and have small standard deviations (Table 2) so that WADPs may convey little information. Again, a higher number of rated tranches and a higher rating asymmetry measure lower ARP, supporting Hypothesis 1. Now, adjusted diversity scores are significant. The negative coefficient suggests that investors may prefer highly diversified transactions because of smaller information asymmetry (Hypothesis 6).

4.2.3 Mean Stochastic Discount Factor of all Rated Tranches in a Transaction

To analyse pricing kernel effects, we first study the mean stochastic discount factor of all rated tranches in a transaction, ASDF. Since ASDF is 7.8 for synthetic transactions and 27.5 for true sale-transactions, due to the expensive Aaa-tranche in true sale transactions, we run regressions on ASDF first separately for true sale and synthetic transactions, then jointly for all transactions. The results are shown in regressions (5) to (8) in Table 6.

In all these regressions WADP and ADS have a clearly negative impact on ASDF as suggested by the pricing kernel effect. As argued before, an increase in WADP should reduce the mean stochastic discount factor of the asset pool, and hence ASDF. Similarly, Hypothesis 5 c predicts a negative effect of the diversity score.

The FLP-loss share has a significant positive impact on ASDF in all four regressions. A higher loss share suggests a higher attachment point for the lowest rated tranche which, by a pricing kernel effect, should raise ASDF (Hypothesis 5 a). But a higher loss share also supports investor confidence and thus should *lower* ASDF (Hypothesis 6). Apparently, the pricing kernel effect dominates. As expected, the IBOXX-spread has a positive impact on ASDF. In line with Hypothesis 1, a higher number of rated tranches has a clearly negative impact on ASDF in synthetic transactions (regression (6)), but not in true sale transactions (regression (5)).

The synthetic dummy has a strongly negative impact on ASDF. By the pricing kernel effect, the better quality of asset pools in synthetic transactions suggests a higher ASDF, but the exclusion of the non-securitized super-senior tranche suggests a lower ASDF. Also, less information asymmetry due to the better quality of the asset pool suggests a lower ASDF. Apparently, the latter two effects dominate. Finally, originator characteristics are mostly irrelevant. But a bank with a higher total capital ratio and lower return variability tends to pay a smaller ASDF. Investors may have more confidence in an originator with better capitalization and less profitability risk.

It is worth noting that the adjusted R^2s are much higher for the ASDF- than for the ARP-regressions. This indicates that the pricing of risk transfer in securitizations follows a common logic making it difficult to earn arbitrage profits by trading securitization tranches against each other.

4.2.4 Pricing of Individual Tranches

So far, we analysed properties of all rated tranches in a transaction. Next, we analyse properties of individual rated tranches. There are 4 tranches with an annual risk premium below -1 %. We exclude these tranches because of potential data errors. Since most transactions have more than one tranche, the residuals of tranches belonging to one transaction might be clustered. Therefore the p-values are estimated using a clustered residual robust variance matrix[9]. Originator characteristics play a small role in the aggregate risk premium and the mean stochastic discount factor of all rated tranches as shown above. Their empirical impact on properties of individual tranches is even smaller because tranche characteristics play a more important role. Therefore originator characteristics do not show up as regressors in the following regressions.

First, we run OLS-regressions to explain the (annual) tranche risk premium (TRP). In the first regression of Table 7, WADP/maturity, a proxy for the expected annual asset pool loss, has a significant, positive impact on TRP, supporting a loss volume effect (Hypothesis 4). This is also consistent with an information asymmetry effect (Hypothesis 6). But asset pool diversification measured by the adjusted diversity score has no impact. The loss volume effect is supported by the FLP-impact. Since an increase in WADP/maturity raises the FLP in a linear regression with the coefficient 2.72, we use (FLP - 2.72 WADP/maturity) as a regressor. Its negative coefficient indicates that a higher FLP takes away more default losses from the rated tranches and therefore reduces TRP.

Surprisingly, the number of subordinated tranches adds more to the explanatory power of the regression than the strongly correlated attachment point. Therefore, regression (1) of table 7 reports the regression coefficient of the number of subordinated tranches, but not that of the attachment point. A higher number of subordinated rated tranches reduces TRP. More subordinated tranches signal a higher attachment point and, thus, a smaller expected tranche loss which should reduce the annual tranche risk premium (Hypothesis 4 b). Also more subordinated tranches may provide more information about the tranche risk, thereby reducing risk premiums. There is a strong additional risk premium for the lowest rated tranche as shown by the positive regression coefficient for the lowest rated tranche-dummy. This tranche is particularly risky, it is the most information-sensitive rated tranche, also managing its risk is most complex (see also *Weber* 2008). As expected, the IBOXX-spread has a positive impact on the tranche risk premium. Tranche size and originator characteristics do not help explaining TRP.

- Table 7 -

Next we ask whether the tranche risk premium can be better explained by the tranche rating than by the observable economic properties used in regression (1). As shown in regression (2), substituting tranche rating for economic tranche properties slightly increases the explanatory power of the regression from 39.1 to 42.6 %. This supports the strong impact of ratings and, thus, Hypothesis 7. Yet, the R^2 of only 42.6 % indicates that investors do not blindly trust ratings, but also evaluate other tranche properties.

While the tranche risk premium appears to be driven primarily by volume and pricing kernel effects, the *mean tranche stochastic discount factor (TSDF)* should be driven primarily by pricing kernel effects. To check for this, first consider *TSDF* for different ratings classes. Senior tranches generate losses primarily in the "expensive" macro-states and therefore should pay a high risk premium per unit of expected loss. This should be particularly true for the thick Aaa-tranches in true sale transactions, not so much for the thin Aaa-tranches in synthetic transactions. This conjecture is strongly supported by the following table which shows the averages of the mean stochastic discount factors for different rating classes in our sample. Since the estimation of the very small expected loss of the Aaa-tranche is subject to strong estimation error implying a strong effect on *TSDF*, we winsorize the mean stochastic discount factor of Aaa-tranches assigning a cap of 200 to all tranches with a higher value.

rating class	TS Aaa	SYN Aaa	Aa and A	Baa	Ba and B
mean tranche stochastic discount factor (average)	47.7	20.39	7.97	5.22	3.31

The mean tranche stochastic discount factors in the table are in line with the spread ratios for corporate bonds derived by *Amato/Remolona* (2003). The pricing kernel effect clearly dominates the potential information asymmetry effect. The senior tranches are less information-sensitive which should reduce their mean tranche stochastic discount factors.

Regressions (3) to (6) in Table 7 display our findings on mean tranche stochastic discount factors. Since WADP and the attachment point are strongly correlated, we regress on *WADP* and on *(WADP – 0.424 attachment point)* with 0.424 being the coefficient of the attachment point on WADP in a linear regression. As a primer, the R^2 of regression (3) is 77% which is surprisingly high. This indicates that the pricing of securitization tranches is more homogeneous than suggested by Fig. 1. The attachment point has a clearly positive impact on *TSDF*, supporting Hypothesis 5a. The regression coefficient of *WADP* is negative, in line with Hypothesis 5b for lower rated tranches. We will see, however, that this result needs to be differentiated for rating classes. The IBOXX spread has a positive impact as usual.
The dummy "TS-Aaa", being 1 for the Aaa-tranche in true sale transactions and 0 otherwise, has a very strong impact on *TSDF*. This is not surprising, given the very high mean stochastic discount factor of 47.7 in the previous table. The dummy "lowest" also has a strong positive regression coefficient indicating a complexity premium for the lowest rated tranche. Regression (4) in Table 7 excludes true sale-Aaa-tranches, otherwise being the same. The R^2 strongly goes down from 77 to 48.6%. This once more illustrates the strong effect of the true sale-Aaa-tranches.
Again, we check the explanatory power of ratings by substituting the tranche rating for WADP and attachment point. Since rating agencies do not care about pricing kernel effects, ratings should do a poor job in explaining mean stochastic discount factors of tranches. Regression (5), which includes true sale Aaa-tranches, shows that the substitution clearly reduces the explanatory power from 77 to 57.9 %. The regression coefficient of the tranche

rating is negative as expected. The dummies for the true sale-Aaa-tranche and the lowest rated tranche remain significant. In regression (6), we exclude the true sale Aaa-tranches. The decline of R^2 from 48.6 to 10.1 % is dramatic. This confirms that ratings ignore pricing kernel effects. Yet, the rating coefficient remains strongly significant. This is in line with Hypothesis 5 a), since a better rating indicates a higher attachment point.

The minor importance of ratings in explaining mean stochastic discount factors of tranches is also supported by another regression excluding true sale-Aaa-tranches (not shown). If we add the tranche rating as a regressor in regression (4), the R^2 increases only from 48.6 to 52.3%. These findings indicate that investors rely less on ratings and more on economic tranche properties in pricing trranches. The observed tranche pricing is consistent with pricing kernel effects predicted by theory.

4.2.5 Mean Stochastic Discount Factors of Butterfly Spreads

The previous findings for the mean stochastic discount factors of tranches document a positive attachment point effect as well as a positive impact of the tranche rating. This suggests that the stochastic discount factor increases with the aggregate default rate in the economy. It is, however, also possible that the pricing kernel is flat over a large range of aggregate default rates and only increases in the range of high rates. The findings for the mean stochastic discount factors of tranches would be similar because the tranche losses occur not only between the attachment and the detachment point of the tranche, but more so in the range of very high loss rates of the asset pool, i.e. in the range of very “expensive” macro-states.

To obtain a more precise picture of the pricing kernel, we check the mean stochastic discount factors of butterfly spreads. Consider two adjacent tranches i and $(i+1)$ in a transaction with sizes s_i and s_{i+1} and tranche i having the better rating. A butterfly spread is a portfolio of investing 1 € in tranche $(i+1)$ and selling short s_{i+1}/ s_i € of the better rated tranche i. The butterfly spread generates triangular default losses for a portfolio loss rate between the attachment point of tranche $(i+1)$ and the detachment point of tranche i. Otherwise the payoff of the spread is zero. The loss of the butterfly spread is highest at the attachment point of tranche i. Hence a butterfly spread is similar to a state-contingent default loss if both tranches have a small size. Thus, butterfly spreads should reveal pricing kernel effects more clearly than tranches with default losses in a wide range of states.

15 (14) out of 78 (90) butterfly spreads in true sale- (synthetic) transactions have a mean stochastic discount factor *BSDF* below 1. This is to be expected whenever the (forward) stochastic discount factor is mostly below 1 in the range between the attachment and the

detachment point of the butterfly spread. We would expect that for butterfly spreads with low attachment points. But mean stochastic discount factors below 1 might also indicate a puzzle similar to that in option pricing discovered by *Jackwerth* (2000) and later confirmed by *Rosenberg/Engle* (2002) as well as *Barone-Adesi et al* (2008). The puzzle is that the aggregate relative risk aversion implicit in option prices, appears to be negative in some moneyness-range. A similar phenomenon might explain negative mean stochastic discount factors of some butterfly spreads in securitizations.

- Fig. 3 -

Plotting the mean stochastic discount factors (minus 1) of butterfly spreads *BSDF* against their attachment points reveals no systematic pattern as shown in Fig. 3a) for true sale-transactions and in Fig. 3b) for synthetic transactions. Only the mean stochastic discount factor of the most senior butterfly spread in true sale-transactions which includes the large Aaa-tranche tends to be higher. This is also confirmed in the regressions reported in Table 8.

- Table 8 -

Regression (1) in Table 8 shows the determinants of *BSDF* for butterfly spreads in *synthetic transactions*. The attachment point of the butterfly spread has *no* significant effect. The dummy lowest equals 1 for the butterfly spread with the lowest attachment point within a transaction and 0 otherwise. Its regression coefficient of 1.5 indicates a complexity premium for this butterfly spread, but it is insignificant, in contrast to the findings for the lowest rated tranche. Besides of the IBOXX-spread, *WADP* and *ADS* have strong negative effects, supporting Hypothesis 5b and c. The ADS-effect may be driven by information asymmetry as stated in Hypothesis 6 rather than an attachment point effect. This is likely since the attachment point effect is insignificant even in the absence of the ADS-regressor.

For *true sale-transactions,* log ADS turns out to be insignificant (regression (2)). But the attachment point effect is positive and strongly significant, as expected. This finding disappears, however, if we exclude the most senior butterfly spread of each transaction (regression (3)). Now the attachment point effect is clearly insignificant. This indicates that the pricing kernel is flat in the range of asset pool loss rates below the attachment points of Aaa-tranches and strongly increases above these points. A much higher R^2 can be obtained by including log ADS as a regressor, again excluding the most senior butterfly spreads (regression (4)). The strong impact of ADS may be due to an information asymmetry effect. These findings correspond to those in regression (1) for synthetic transactions. Overall, these results indicate that the attachment point effect disappears if we exclude the most senior butterfly spreads. The pricing kernel appears to be rather flat in a wide range of asset pool loss rates.

4.3 Checks and Robustness Tests

4.3.1 Mean Stochastic Discount Factors and Ratings of Tranches

In the following, we check our findings by additional regressions and by robustness tests. First, since a better rating tends to be associated with a higher attachment point, we also check for the attachment point effect by analysing the impact of WADP and ADS on the mean stochastic discount factors *TSDF* of tranches separately for each rating class. Besides of the Aaa-tranches in true sale- and synthetic transactions, we do not differentiate within the rating classes Aa, A, Baa, Ba and B. We use a dummy variable which is 1 if the tranche has a specific rating and 0 otherwise. In an OLS–regression we find for 301 observations (p-values in parentheses)

TSDF =	82.1	(0.0000)
	349 D(TS Aaa) *WADP*	(0.0050)
	375 D(SYN Aaa) *WADP*	(0.0652)
	-21.5 D(Aa and A) *WADP*	(0.3534)
	-32.0 D(Baa) *WADP*	(0.0002)
	-24.9 D(Ba and B) *WADP*	(0.0000)
	22.8 D(TS Aaa) *ln ADS*	(0.0000)
	-5.01 D(SYN Aaa) *ln ADS*	(0.0603)
	-3.86 D(Aa and A) *ln ADS*	(0.0477)
	-4.74 D(Baa) *ln ADS*	(0.0112)
	-5.40 D(Ba and B) *ln ADS*	(0.0002)

Adj $R^2 = .58$

These results corroborate our finding that Aaa-tranches are special. A higher WADP tends to raise the mean stochastic discount factor of a Aaa-tranche, but to lower that of a lower rated tranche. This supports Hypothesis 5b. While a higher WADP appears to lower the mean stochastic discount factor of the asset pool, it raises the attachment point of a tranche concentrating default losses in the more expensive macro-states. If the pricing kernel increases weakly with the aggregate default rate in the range of low aggregate default rates, but strongly in the range of high rates, then the mean stochastic discount factor of a tranche with a weak (strong) rating should decline (increase) with WADP. These findings are consistent with those derived from butterfly-spreads.

A stronger asset pool diversification should lower the mean stochastic discount factor of all tranches because it may reduce information asymmetry (Hypothesis 6) and lower the attachment point of a tranche (Hypothesis 5c). This ADS-effect is supported by the regression, with the exception of Aaa-tranches in true sale-transactions. A possible explanation for this exception might be that a higher diversity score concentrates default losses of the Aaa-tranches in the very expensive macro-states.

4.3.2 Different Measures of Diversification

One controversial issue is the correlation between debtor defaults in an asset pool. Therefore we check our findings by using different diversity scores. So far, we used the adjusted diversity score based on an asset correlation of debtors within the same industry of 0.2 and of debtors in different industries of 0.02. First, we replace the adjusted diversity score by Moody´s traditional diversity score which ignores correlations of debtors in different industries. But this should be viewed with caution because the allocation of default losses to individual tranches reacts quite sensitively to the input parameters (*Duffie* and *Garleanu* (2001)). The difference between the adjusted diversity score and the diversity score can be quite large. While the diversity score has a mean of 63.6 in all 167 transactions, the mean adjusted diversity score in the subset of 92 transactions is 25.3. This suggests that the adjusted diversity score is less than half the diversity score. For the 92 transactions, the mean difference (0.5 diversity score - adjusted diversity score) is 12.3, the difference is above 40 for 8 transactions. Therefore it would be surprising if regressions based on 0.5 diversity score would provide answers similar to those obtained with the adjusted diversity score. This conjecture is verified. Even though the number of observations almost doubles when we use 0.5 diversity score instead of the adjusted diversity score, the adjusted R^2 almost invariably goes down, sometimes dramatically. Also in many cases the coefficients of regressors which are significant in our previous regressions, show higher p-values or lose significance.

In addition, we run the regressions with the adjusted diversity score based on asset correlations of $\rho_{ex} = 0.00$ or 0.04 for debtors in different industries. This has little effect on the significance of the regressors. But the adjusted R^2 s are clearly smaller for $\rho_{ex} = 0.00$ than for $\rho_{ex} = 0.02$. Replacing $\rho_{ex} = 0.02$ by $\rho_{ex} = 0.04$ sometimes raises and sometimes lowers the adjusted R^2. This suggests that a range between 0.02 and 0.04 was considered realistic by market participants.

4.3.3 The Impact of Diversification Across Industries

Our findings on the tranche pricing depend on the underlying assumptions, in particular on approximating the asset pool loss rate distribution by a lognormal distribution. Many papers use more sophisticated distributions like Levy-distributions, based on more complicated default factor structures. But we cannot obtain the information required for more sophisticated distributions from the offering circulars and other available documents. Also, the simulation models used by the rating agencies generate loss rate distributions which are reasonably approximated by a lognormal distribution. To check for default clustering effects found by *Longstaff* and *Rajan* (2008), we include the regressor *(1/ADS 4- 1/ADS0)* with *ADS4* based on $\rho_{ex} = 0.04$ and *ADS0* based on $\rho_{ex} = 0.00$. Given an asset pool with equally sized loans, *m* industries and the same number of loans in each industry, *(1/ADS 4- 1/ADS0) = 0.04 (1- 1/m)* (see *Fender* and *Kiff* (2004)). Hence it would monotonically increase in the diversification of the asset pool across industries. This regressor has no significant impact in our regressions. Hence, interindustry diversification does not seem to have an impact on tranche pricing beyond what is captured in the adjusted diversity score.

In all loss rate models, the upper tail reacts very sensitively to the parameter input. Therefore, the expected losses and the mean stochastic discount factors derived for Aaa-tranches should be viewed with caution. *Duffie et al* (2009) argue that most models miss unobservable default risk factors which may imply a much higher tail risk.

4.3.4 CBO-versus CLO-Transactions

Another issue might be our joint analysis of CLO- and CBO-transactions. *Franke,Herrmann and Weber* (2011) found that the FLP-loss share tends to be higher in CBO- than in CLO-transactions. In our regressions, we check for potential differences between CLO- and CBO-transactions by including a dummy which is 1 for CBO-transactions and 0 otherwise. This dummy always turns out to be insignificant; therefore we do not report it in our regressions. The difference between true sale and synthetic transactions, however, is important as shown before.

4.3.5 Originator Characteristics and Endogeneity Issues

The impact of originator characteristics on our findings is very limited. These characteristics are apparently of little concern to investors and rating agencies. This suggests that tranching and pricing mainly depend on transaction characteristics. We cannot rule out that the originator chooses these characteristics taking into consideration their impact on tranching and pricing. But this potential endogeneity is of little concern because we try to find out the effects of transaction characteristics on tranching and pricing. Thus, we take the transaction characteristics as exogenous. We account for interdependencies betweeen them by avoiding strongly correlated regressors or by using regressors adjusted for linear dependencies.

5 Conclusion

This paper analyses the tranching and pricing of securitized pools of corporate bonds and loans, using a sample of European securitizations. The number of issued bond tranches with different ratings varies between 1 and 6. 88 percent of the transactions have at least one Aaa-tranche, about 14 percent of the tranches are subinvestment grade. Given a transaction with at least three differently rated tranches, tranche ratings tend to be clustered more in the good rating range. Since credit spreads of Aaa-tranches are lower than those of lower rated tranches, the originator issues very large Aaa-tranches in true sale-transactions. In synthetic transactions, Aaa-tranches are very thin. This is presumably explained by two reasons. First, in synthetic transactions the originator cannot use the funds from issuing bond tranches. Second, the transfer of default losses through Aaa-tranches is expensive.

The number of tranches tends to be higher for asset pools with larger volume and those with lower quality. More tranches provide more signals about asset pool quality and permit the originator to extract more investor rents. In line with this, the risk premium paid on all rated tranches in a transaction tends to decline if the number of tranches increases.

The risk premium, paid on all rated tranches in a transaction, increases with the expected default loss of the asset pool in true sale-transactions, presumably because a larger volume of losses is transferred. For synthetic transactions, the expected default loss has no systematic risk premium-effect when we include the positively correlated number of tranches. The mean stochastic discount factor of all rated tranches in a transaction declines with a higher expected loss of the asset pool, as suggested by a pricing kernel effect. Apparently this effect dominates the opposite information asymmetry effect. A higher loss share of the First Loss Position raises the mean stochastic discount factor of all rated tranches, again indicating the dominance of the pricing kernel over the information asymmetry effect.

Our findings for the risk premiums and mean stochastic discount factors of individual tranches mostly confirm the results obtained for all rated tranches in a transaction. The analysis of individual tranches is, however, complicated by the fact that their attachment points depend on the quality of the asset pool. A decline in asset pool quality should reinforce the information asymmetry effect and, thus, raise the risk premium, while the pricing kernel effect is less clear. The relative strength of theses effects appears to vary across tranches. The risk premium of the lowest rated tranche is upward biased indicating a complexity premium, perhaps because this tranche is very information-sensitive and requires more skills of risk management. The mean stochastic discount factor tends to increase with a better tranche rating, consistent with a pricing kernel effect dominating the information asymmetry effect. The mean stochastic discount factor is very high for Aaa-tranches in true sale–transactions.

The mean stochastic discount factors of butterfly-spreads which are obtained from two adjacent tranches, provide a more precise picture of the pricing kernel. We find that the attachment point of a butterfly spread has almost no effect on the mean stochastic discount factor except for the most senior butterfly spread. This suggests that the pricing kernel may be rather flat below the attachment point of the Aaa-tranche, but then increases strongly with the loss rate of the asset pool.

The finding that tranche ratings have very little power explaining the mean stochastic discount factor of non-Aaa-tranches, confirms that ratings ignore pricing kernel effects. Yet the economic properties of tranches explain their mean stochastic discount factors reasonably well. This indicates that investors do not blindly rely on ratings, but adjust required credit spreads to pricing kernel and information asymmetry considerations.

This paper is a first step to better understand tranching and pricing in securitizations. More research is needed. The paper uses particular measures of expected losses and diversification of asset pools and postulates a lognormal loss rate distribution. It is necessary to check the sensitivity of findings to the model setup in further studies. Also, the findings are based on a European sample of securitization transactions and might differ for US-transactions. Moreover, we need to better understand the interdependence between the pricing kernels in credit and in stock markets. Since tranching is often considered a driver of the subprime crisis, new insights on tranching should also be helpful in improving regulations of securitization transactions.

Appendix

1. The Parameters of the Lognormal Loss Rate Distribution

We estimate the portfolio loss rate distribution assuming a lognormal distribution. We simplify the analysis by a two date-analysis so that all default losses occur at date 1. The expected portfolio loss rate equals $E(l) = \lambda\pi$ with

π = *WADP*,

λ = loss given default, assumed to be non-random.

Hence we need to know the standard deviation of the loss rate of the asset pool to obtain the parameters of the lognormal distribution. Let denote

S = standard deviation of the loan loss rate ,

$\sigma = \sigma(\ln l)$ = standard deviation of the lognormally distributed asset pool loss rate,

μ = expectation of the lognormally distributed asset pool loss rate, $\mu = E(\ln l)$,

P_i = par value of loan *i*, divided by the par value of all loans; *i = 1,…, n,*

ρ_{ij} = asset correlation between loan i and loan j.

Assuming identical default properties of all loans, the variance of the loan loss rate is

$$S^2 = (0 - \lambda\pi)^2 (1 - \pi) + (\lambda - \lambda\pi)^2 \pi = \pi(1 - \pi)\lambda^2 .$$

Then the variance of the asset pool loss rate is

$$S_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} S^2 \quad \rho_{ij} \quad P_i P_j = S^2 \sum_{i=1}^{DS} \left(\frac{1}{ADS} \right)^2 = S^2 / ADS .$$

The latter part of the equation follows from the definition of the adjusted diversity score. It is the number of equally sized loans whose defaults are uncorrelated which generates the same variance of the asset pool loss rate.

For a lognormally distributed asset pool loss rate,

$$S_P^2 = [E(l)]^2 \left[\exp \sigma^2 - 1\right]$$

so that

$$\sigma^2 = ln\left[1 + \left(\frac{S_P}{E(l)}\right)^2\right] = ln\left[1 + \frac{1/\pi - 1}{DS}\right].$$

For μ we obtain

$$\mu = lnE(l) - \sigma^2/2 = ln(\lambda\pi) - \frac{1}{2} ln\left[1 + \frac{1/\pi - 1}{DS}\right] .$$

2. Expected Default Losses of Tranches

Given strict subordination of tranches, the expected default loss of a tranche with attachment point *a* and detachment point *d* equals

$\int_a^d (l-a)\, dF(l) + (d-a)\,(1 - F(d)).$

F(l) is the cumulative lognormal distribution function of the portfolio loss rate *l*. Since the expected tranche loss equals the expected loss of a call with strike price *a* minus the expected loss of a call with strike price *d*, we derive the expected loss of the tranche analytically, as in the Black-Scholes world. The expected tranche loss per € invested is the expected tranche loss, divided by *(d-a),* the par value of the tranche which equals its market value at the issuance date.

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Tables and Figures

	True sale	Synthetic	∑
CLO	35	38	73
CBO	45	49	94
∑	80	87	167

Table 1: The table shows the number of transactions in the sample differentiating CLO- and CBO-transactions as well as true sale- and synthetic transactions.

	CLO – ts	CLO – synth	CBO – ts	CBO – synth
WADP – mean	7.2 %	3.5 %	14.0 %	1.9 %
WADP – std.	7.3 %	2.3 %	9.6 %	3.1%
DS – mean	89	88	32	56
DS – std.	45	29	10	26
FLP – mean	5.6 %	2.9 %	11.2 %	3.6 %
FLP std.	4.7 %	1.4 %	5.7 %	2.5 %

Table 2: The table presents averages and standard deviations of WADP (weighted average default probability of the asset pool in a transaction), DS (Moody's diversity score of the asset pool) and FLP (the initial size of the first loss position as a percentage of the volume of the asset pool). The data are presented separately for the four subsets of CLO-true sale, CLO-synthetic, CBO-true sale and CBO-synthetic transactions.

Absolute frequencies of tranches. Total number of tranches is 594

# tr. per transact.	# transact.	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3
1	14	6	2	2		1	1			1		1					
2	17	14	2	1	2	2	6	1		3	3						
3	47	37	8	17	8	11	10	7	5	20	6	4	6	2			
4	46	46	8	16	5	9	19	11	10	29	6	5	13	6	1		
5	37	44	10	23	3	5	23	7	7	19	12	8	16	7		1	
6	6	5	4	4	1	1	5			4	1		5	1		2	3
∑	*167*	***152***	*34*	***63***	*19*	*29*	***64***	*26*	*22*	***76***	*28*	*18*	***40***	*16*	*1*	*3*	*3*

Relative frequencies of tranches.

# tr. per transact.	# transact.	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3
1	14	43 %	14%	14%	-	7%	7%			7%		7%					
2	17	**82**%	12%	6%	12%	12%	35%	6%		18%	18%						
3	47	**79**%	17%	36%	17%	23%	21%	15%	11%	43%	13%	9%	13%	4%			
4	46	**100%**	17%	35%	11%	20%	41%	24%	22%	**63%**	13%	11%	28%	13%	2%		
5	37	**119%**	27%	**62%**	8%	14%	**62%**	19%	19%	**51%**	32%	22%	43%	19%		3%	
6	6	**83%**	**67%**	**67%**	17%	17%	**83%**			**67%**	17%		**83%**	17%		33%	**50**%

Table 3: In each panel, the first column classifies transactions by the number of differently rated tranches. The second column shows the observed number of transactions for each class. The top line indicates the rating. In the upper (lower) panel, the following columns display for each rating the absolute (relative) frequencies of issued tranches. Relative frequency is the number of tranches with a given rating, divided by the number of transactions in the same line (2nd column).The last line in the upper panel displays the total number of transactions resp. of tranches with a given rating. Bold figures in the upper (lower**)** panel indicate local maxima in the last line (relative frequencies ≥ 50 %).

Number of differently rated tranches in a transaction	2	3	4	5	6
Median of rating range	6	9	10	12	15
Median of average rating gap	4	3	2	2	2

Table 4: It shows the median of the rating range (= 1+ difference between highest and lowest rating) and the median of the average rating gap for transactions, given the number of differently rated tranches.

Table 5: The table displays the average tranche size separately for true sale (upper panel) and synthetic transactions (lower panel). The first column classifies transactions by the number of differently rated tranches, the second column shows the total number of rated tranches in this class in parentheses, the following columns display for each rating the average tranche size (= tranche volume/transaction volume, in percent), below in parentheses the number of tranches with the indicated rating. The last column gives the average size of the FLP, below the number of transactions in parentheses. Average tranche sizes of at least 10% are in bold numbers. The last line in each panel displays the number of rated tranches in parentheses, i.e. the sum of the numbers in parentheses in the respective column. In the FLP-column, last line, 80 is the number of true sale transactions.

The lower panel displays the same for synthetic transactions, with the third loss position TLP (non-securitized super senior tranche) in the third column, below the number of transactions in parenthesis. The number of synthetic transactions is 87.

True Sale Transactions: average tranche size

# tr. per transact.	Total # rated tranches	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	FLP
1		**81%**		**88%**			**68%**											18%
	(6)	(4)		(1)			(1)											(6)
2		**79%**			**14%**	**15%**	**10%**			**11%**								10%
	(14)	(7)			(1)	(1)	(4)			(1)								(7)
3		**78%**	**31%**	**38%**	6%	5%	8%	5%	3%	6%	5%	3%	2%	8%				8%
	(65)	(18)	(6)	(5)	(3)	(6)	(6)	(3)	(1)	(9)	(4)	(2)	(2)	(1)				(22)
4		**77%**	**14%**	**13%**	**10%**	3%	5%	**10%**	4%	4%	7%	1%	2%	3%				8%
	(104)	(28)	(2)	(9)	(1)	(4)	(12)	(6)	(5)	(18)	(3)	(2)	(8)	(6)				(26)
5		**71%**	6%	9%	2%		7%	8%	3%	5%	5%	1%	3%	2%				9%
	(90)	(22)	(4)	(12)	(1)		(12)	(5)	(1)	(9)	(8)	(2)	(8)	(6)				(18)
6		**68%**	1%	7%			7%				4%			4%				9%
	(6)	(1)	(1)	(1)			(1)				(1)			(1)				(1)
∑	(286)	(80)	(13)	(28)	(6)	(11)	(36)	(14)	(7)	(37)	(16)	(6)	(18)	(14)				(80)

Synthetic Transactions average tranche size

# tr. per transact.	Total # rated tranches	TLP	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	FLP
1		89%	3%	5%	**17%**		3%				6%		3%						6%
	(8)	(8)	(2)	(2)	(1)		(1)				(1)		(1)						(8)
2		89%	4%	6%	4%	4%	3%	**10%**	1%		5%	5%							3%
	(20)	(10)	(7)	(2)	(1)	(1)	(1)	(2)	(1)		(2)	(3)							(10)
3		88%	5%	1%	3%	3%	3%	3%	2%	3%	2%	2%	2%	1%	1%				3%
	(75)	(25)	(19)	(2)	(12)	(5)	(5)	(4)	(4)	(4)	(11)	(2)	(2)	(4)	(1)				(25)
4		87%	5%	2%	3%	3%	1%	2%	2%	1%	1%	2%	1%	2%		1%			3%
	(80)	(20)	(18)	(6)	(7)	(4)	(5)	(7)	(5)	(5)	(11)	(3)	(3)	(5)		(1)			(20)
5		86%	4%	2%	3%	1%	1%	1%	1%	2%	1%	1%	1%	1%	1%		0.5%		3%
	(95)	(19)	(22)	(6)	(11)	(2)	(5)	(11)	(2)	(6)	(10)	(4)	(6)	(8)	(1)		(1)		(19)
6		82%	4%	2%	3%	0.5%	3%	2%			2%			2%			0.5%	2%	4%
	(30)	(5)	(4)	(3)	(3)	(1)	(1)	(4)			(4)			(5)			(2)	(3)	(5)
∑	(308)	(87)	(72)	(21)	(35)	(13)	(18)	(28)	(12)	(15)	(39)	(12)	(12)	(22)	(2)	(1)	(3)	(3)	(87)

Explained variable	ARP True sale (%) (1)	ARP True sale (%) (2)	ARP Syn (%) (3)	ARP Syn (%) (4)	ASDF True Sale (5)	ASDF Syn (6)	ASDF All (7)	ASDF All (8)
Constant	0.442 (0.0335)	0.484 (0.0235)	4.986 (0.0186)	4.669 (0.0209)	-20.90 (0.2269)	53.34 (0.0064)	55.96 (0.0006)	27.04 (0.0806)
WADP (%)	0.0233 (0.0000)	0.0228 (0.0000)	-0.074 (0.0337)	-	- 0.67 (0.0002)	-1.18 (0.0000)	- 0.35 (0.0021)	- 0.57 (0.0000)
ln ADS	-	-	-1.50 (0.0319)	-1.20 (0.0707)	-16.42 (0.0000)	-19.3 (0.0003)	-27.12 (0.0000)	-20.58 (0.0000)
Rating asymmetry	-	-0.350 (0.0245)	-	-1.339 (0.0746)	-	-	-	-
FLP- loss share (%)	-	-	-	-	84.5 (0.0000	24.5 (0.0646)	45.4 (0.0028)	50.88 (0.0029)
Spread35	0.137 (0.0290)	0.120 (0.0622)	0.656 (0.0319)	0.647 (0.0139)	14.83 (0.0000)	2.88 (0.2788)	7.25 (0.0004)	9.60 (0.0000)
No. of tranches	-0.115 (0.0145)	-0.0115 (0.0135)	-	-0.195 (0.0655)	-	-0.89 (0.0696)	-1.53 (0.0440)	-
SYN-dummy	-	-	-	-	-	-	-	-9.34 (0.0001)
Total capital ratio	-	-	-	-	-	-	-	-1.34 (0.0271)
Var of return on equity	-	-	-	-	-	.0332 (0.0001)	-	-
Adj. R^2	0.538	0.586	0.248	0.356	0.833	0.573	0.633	0.729
Observations	37	37	45	45	37	45	82	82

Table 6: The table shows the regression coefficients from OLS-regressions with Newey-West heteroscedasticity adjusted p-values (in parentheses), explaining the aggregate risk premiums ARP and the mean stochastic discount factor of all rated tranches in a transaction, ASDF. ARP is measured in percent. Rating asymmetry is the rating asymmetry measure explained above. FLP-loss share is the share of expected losses of the asset pool borne by the FLP. Spread 35 is the IBOXX-spread difference between BBB- and government bonds for a maturity of 3-5 years at the issue date. No. of tranches is the number of differently rated tranches. SYN is a dummy variable which is 1 for a synthetic transaction and 0 otherwise. Total capital ratio and var of return on equity are the originating banks´s total capital ratio and variance of its return on equity.

explained variable	TRP (%) (1)	TRP (%) (2)	TSDF (3)	TSDF w/o TS-Aaa (4)	TSDF (5)	TSDF w/o TS-Aaa (6)
constant	.0867 (0.742)	-1.008 (0.001)	-33.04 (0.000)	-16.51 0.000)	2.378 (0.756)	13.70 (0.016)
WADP/ maturity (%)	0.253 (0.023)	-	-	-	-	-
FLP – 2.72 WADP/mat. (%)	- 0.072 (0.003)	-	-		-	-
WADP -0.424 attachment point (%)	-	-	-1.43 (0.000)	-1.68 (0.000)	-	-
attachment point (%)	-	-	3.18 (0.000)	2.73 (0.000)	-	-
no of sub tranches	-0.262 (0.000)	-	-	-	-	-
spread 35	0.754 (0.000)	0.714 (0.000)	13.03 (0.000)	5.22 (0.016)	10.53 (0.013)	4.05 (0.114)
dummy TS-Aaa	-	-	60.39 (0.000)	-	101.26 (0.000)	-
dummy lowest	1.067 (0.000)	0.881 (0.000)	7.26 (0.001)	6.47 (0.000)	3.99 (0.021)	5.50 (0.000)
Tranche rating	-	0.117 (0.000)	-	--	-1.59 (0.000)	-1.80 (0.000)
Adj R^2	.391	0.426	.770	.486	.579	0.101
Observations	298	298	298	261	298	261

Table 7: The table shows the regression coefficients from OLS-regressions with heteroscedasticity, clustered residual adjusted p-values (in parentheses) explaining the tranche-risk premiums TRP and the mean-tranche stochastic discount factors TSDF. Attachment point is the attachment point of the tranche. No of sub tranches is the number of subordinated rated tranches. Spread 35 is the IBOXX-spread difference between BBB- and government bonds for a maturity of 3-5 years at the issue date. Dummy TS-Aaa is 1 for a Aaa-tranche in a true sale transaction and 0 otherwise. Dummy lowest is 1 if the tranche is the tranche with the lowest rating in a transaction, and 0 otherwise. Tranche rating is captured by an integer variable which is 1 for Aaa and 16 for B3.

explained variable	*BSDF* synthetic tr. (1)	*BSDF* true sale-tr. (2)	*BSDF* w/o top BS true sale-tr. (3)	*BSDF* w/o top BS true sale-tr (4)
constant	64.07 (0.001)	-18.29 (0.000)	-5.393 (0.034)	59.23 (0.002)
Attachment point (%)	-	2.16 (0.0000)	0.67 (0.2149)	-
Dummy lowest	1.508 (0.1189)	-	-	-
WADP (%)	-0.820 (0.0000)	-	-	-0.175 (0.0306)
WADP-0.54 attachment point (%)	-	-1.38 (0.0000)	-0.71 (0.0779)	-
Log ADS	-19.43 (0.0000)	-	-	-19.50 (0.0006)
Spread 35	4.52 (0.0002)	9.55 (0.0108)	5.92 (0.0087)	5.95 0.0193)
Adj R^2	0.467	0.578	0.353	0.506
Observations	90	78	48	48

Table 8: The table shows OLS-regressions explaining the mean stochastic discount factors of butterfly spreads *BSDF* in synthetic and true sale-transactions, with heteroscedasticity, clustered residual adjusted p-values (in parentheses). In regressions (1) and (4) *WADP* is a regressor, excluding the attachment point. In regressions (2) and (3) the attachment point is a regressor so that *(WADP – 0.54 attachment point)* is used to neutralize the dependence between the attachment point and WADP as given by a linear regression. Dummy lowest is 1 for the butterfly spread with the lowest attachment point in a transaction and 0 otherwise. Spread 35 is the IBOXX-spread difference between BBB- and government bonds for a maturity of 3-5 years at the issue date. In the true sale-regressions (3) and (4), the top butterfly spread including the large Aaa-tranche is excluded.

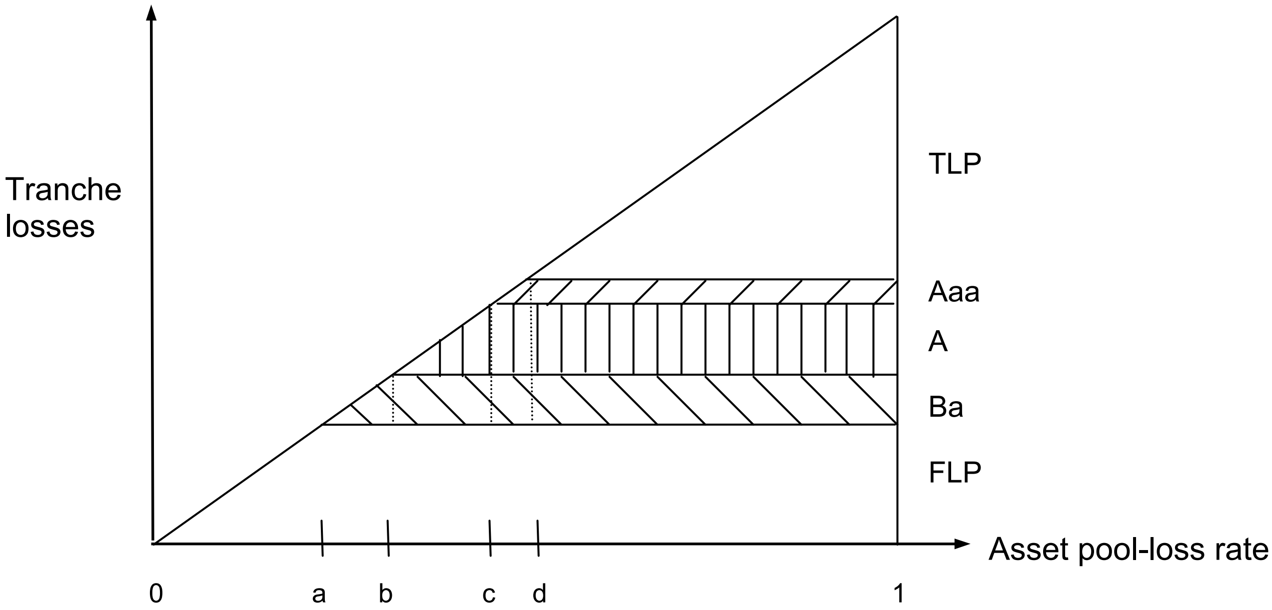


Figure 1: This figure shows the allocation of the asset pool loss in a synthetic transaction to the First Loss Position (FLP), the Ba-, the A-, the Aaa-rated tranche and the non-securitized super-senior tranche (Third Loss Position TLP). a, b, c and d denote the attachment points of the Ba-, the A-, the Aaa- and the Third Loss Position. The losses of the FLP and the TLP are given by the lowest resp. the top area, the losses of the Ba-tranche by the downward hatched area, those of the A- and the Aaa-tranche by the vertically resp. the upward hatched area.

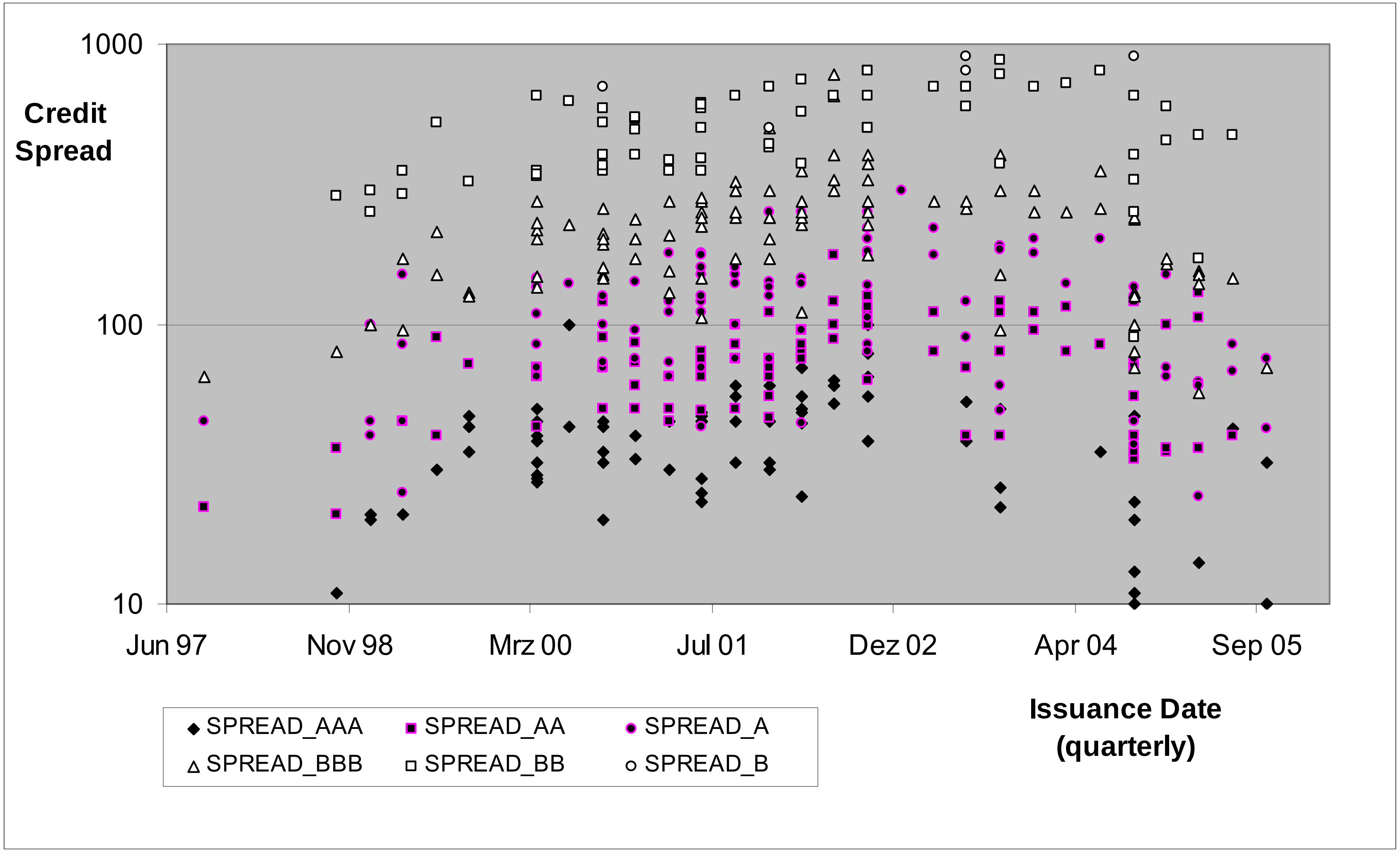
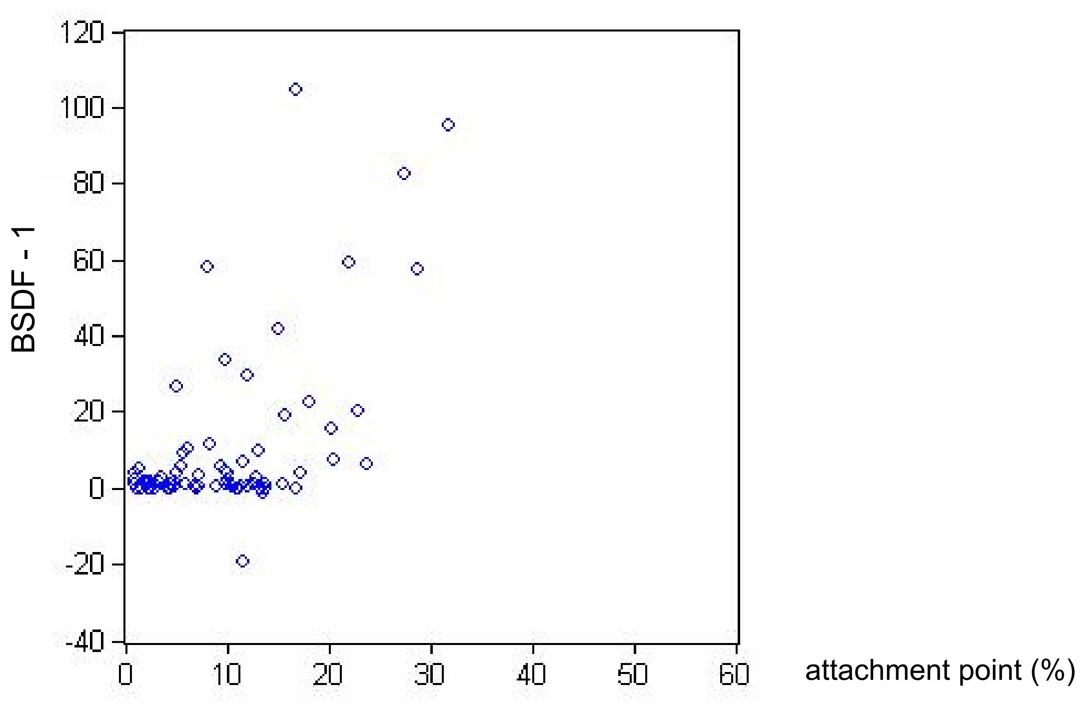
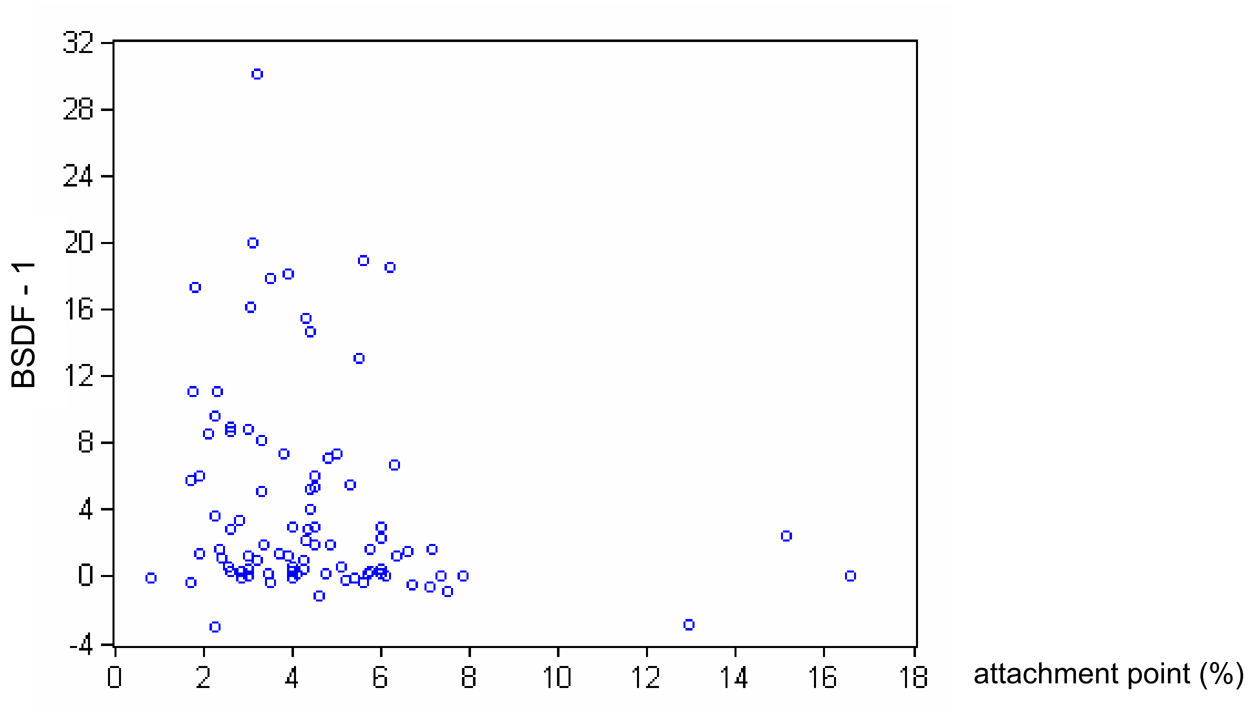


Figure 2: It displays the credit spreads (logarithmic scale) of rated tranches across issuance quarters differentiated for the main rating classes.

Figure 3: Mean stochastic discount factors of butterfly spreads in securitization transactions plotted against their attachment points. The means are reduced by 1 so that zero is the expectation of the (stochastic discount factor -1).



a): *True sale transactions*



b): *Synthetic transactions*

Footnotes

[1] *David* (1997) argues that tranches are sold to individual and institutional investors. The latter buy tranches to hedge their endowment risk. Hence tranches should be differentiated so as to allow the different groups of investors an effective hedging.

[2] *Cebenoyan* and *Strahan* (2004) document that banks securitizing loans have less capital than other banks and more risky assets relative to total assets.

[3] The benefits of quality differences between tranches derive from investor heterogeneity. Institutional investors may have statutes which allow them to invest in Aaa-tranches only. Regarding the capacity of analysing und managing default risks, investors with low capacity may prefer tranches with low default risk. Sophisticated investors like hedge funds may prefer high risk tranches.

[4] Under Basel I, the standard risk weight of 100 percent applied to all rated tranches, so that tranching had little impact on regulatory equity capital. But originators and investors anticipated capital requirements differentiated to tranche ratings according to Basel II. Regulators in some countries required little equity capital for the most senior tranche if it was insured against default risk.

[5] *Franke, Herrmann* and *Weber* (2011) find in their European sample that the FLP always exceeds the expected portfolio loss rate.

[6] Single-tranche deals are often initiated by investors looking for an investment in a diversified asset pool with a prespecified tranche rating.

[7] To provide an intuition, consider a lognormal distribution of the portfolio loss rate and derive the attachment points for a B, BB, BBB, A, AA and AAA tranche in a true sale transaction according to the idealized probabilities of default according to S&P. Assume a transaction with 6 years maturity, WADP = 6 %, loss given default $\lambda = 50\%$ and an adjusted diversity score of 40. Then the tranche sizes are depicted in the following Table.

Tranche rating	S&P Tranche -PD	Tranche size
AAA	0.19%	83.88%
AA	0.51%	2.77%
A	1.01%	1.75%
BBB	3.61%	3.02%
BB	13.31%	2.80%
B	29.73%	1.64%
FLP	100 %	3.45%

[8] Theoretically, a lognormal distribution allows for portfolio loss rates above 1. But for all realistic parameter values, the cumulative probability of these loss rates is negligible.

[9] The differences in p-values between the Newey-West and the cluster robust estimates are very small.