Implied Risk Aversion in Lottery Bond Prices

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Abstract

Both the equity premium puzzle and the credit spread puzzle address the problem of estimating the size of agents' risk aversion. The estimation of risk aversion parameters is impeded by the fact that observed prices depend on two unobservables; risk preferences and probability beliefs. The market for German redemption lottery bonds constitutes a clean environment to estimate risk aversion coefficients from transaction prices as the probabilities of price changes caused by redemption lotteries are objectively known. For our empirical analysis we built up a new, partly hand-collected data base for prices and terms of lottery bonds. We first contribute to the literature through a detailed analysis of price changes and risk premia at lottery dates. Second, we estimate implied relative risk aversion parameters from bond market data using a dynamic equilibrium model. Our most important findings are (1) relative risk premia estimated from lottery bond prices are lower than those typically reported for stock markets, (2) relative risk aversion parameters are of moderate magnitude and are in line with the findings in laboratory experiments rather than in stock markets, and (3) relative risk aversion parameters vary strongly over time in line with interest rates at the beginning of the oil price driven recession in 1980/81.

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1 Introduction

Market participants' risk preferences and probability beliefs are the main components of asset prices. Therefore, the size and dynamics of the representative investor's risk aversion are essential in understanding the price dynamics of financial assets. Mehra and Prescott (1985) started an ongoing discussion on the size of the relative risk aversion (RRA) in the stock market.[1] A related subject in the credit-risk literature is the credit spread puzzle. This puzzle relates to the relatively large part of the bond yield spread generally interpreted as risk premium, which cannot be explained by expected default loss.[2] Both the equity premium and the credit spread puzzle address the problem of estimating the magnitude of investors' risk aversion.

Even assuming that asset prices are determined by the investment and consumption decisions of a representative agent, the estimation of risk aversion is impeded by the fact that observed prices depend on probability beliefs. Hence, most RRA estimates are conditional on distributional assumptions about the state variables. The German bond market had a segment of bond issues that were redeemed by a sequence of lotteries (*Tilgungsanleihen*), providing us with an exceptional environment to study investors' risk preferences independent of subjective probability beliefs. These bonds, subsequently referred to as lottery bonds, were typically issued by the Federal Republic of Germany, German states, and government-owned enterprises, and may be considered free of default risk. The issuer was obligated to redeem a certain fraction of the outstanding debt at predefined dates and redemption values. According to the redemption amounts, the total issue was split into series of equal size. At each redemption date, one series was randomly drawn by a lottery; with the probability of been drawn equal to one over the number of outstanding series. Therefore, German redemption lottery bonds differ from amortizing bonds since a certain amount of outstanding bonds is repaid *in full*. They differ from bonds with a typical sinking fund provision as the issuer has *no repayment option*, and they differ from Swedish and Danish coupon lottery bonds since the lottery refers to *redemption* and not to the *coupon*.

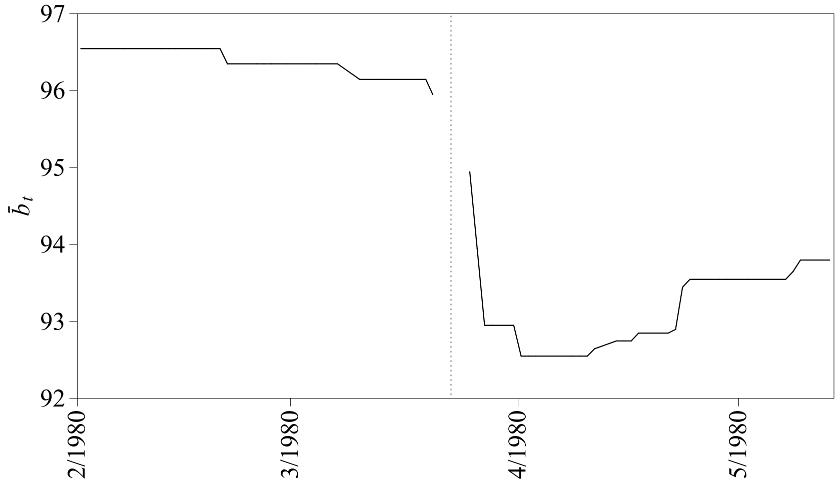
It is important to note that there is only one price for the lottery bond, and no individual price for each series, as the maturity of the outstanding series has the same uniform

[1] For comprehensive surveys on the equity premium puzzle, see Kocherlakota (1996), Mehra and Prescott (2003), and Mehra (2009).

[2] See Amato and Remolona (2003) for a literature overview pertaining to the credit spread puzzle.

Figure 1: **Price Jump**

This figure shows the time series of the clean price for the 6% lottery bond issued by the Federal Republic of Germany in 1963. The graph shows a time interval including the last redemption lottery on March 24, 1980 (dotted line). The bonds final maturity was July 1, 1982, and redemption lotteries were conducted biennially. Prices are reported in percentage terms.



probability distribution. Prices of the outstanding series usually jump at the drawing date: An explanation for this effect is provided by institutional facts in the German bond market. Since it was not possible to place orders for individual series of a lottery bond, a purchase of these bonds typically results in a non diversified portfolio w.r.t. to the lottery risk. If a bond is part of the series that is drawn, the bond holder receives the face value and the coupon at the next redemption date. In addition, this series no longer takes part in future lotteries. The traded bond after a drawing date consists of the non-drawn series only. If, for example, the lottery bonds trade below par immediately before the drawing date, its price will drop, as the bond no longer has the chance of being repaid at the next redemption date; this possibility is postponed until the next drawing date, typically by one year.

Figure 1 illustrates the price path for the 6% lottery bond issued by the Federal Republic of Germany in 1963. The last transaction price before the lottery on March 24, 1980 was 95.90. The first transactions after the lottery occurred on March 25, 26 and 27 with clean prices of 94.90, 93.90, and 92.90 respectively. As March 24 was the last lottery date, and the bond consisted of two outstanding series, the probability of a downward jump is 1/2.

It is important to note that the probability of a price change caused by the redemption lottery is objectively known but *cannot be arbitraged*, even though it is public information. Hence, this segment of German default-free lottery bonds presents an exceptionally clean market setting to implicitly estimate risk preferences from observed transaction prices. Since the issuers are considered to have no default risk the risk of lottery bonds differs from the simultaneously traded straight bonds of the same issuers due only to the lottery risk.

To the best of our knowledge, this study is the first to estimate implied RRA parameters from lottery bond prices in a dynamic equilibrium setting. One of the first papers dealing explicitly with lottery bonds is Schilbred (1973). He studies annuities issued by an Italian government enterprise that were redeemed by lottery. The study focuses on estimating the market price of risk in a mean-variance equilibrium model. Green and Rydqvist (1997) evaluate Swedish lottery bonds for which the coupons and not the maturity are determined by lotteries. By construction, the lottery risk is idiosyncratic and should not result in a risk premium. Nevertheless, the authors find empirical evidence that transaction prices include a premium for lottery risk. In two follow-up studies, Green and Rydqvist (1999) and Florentsen and Rydqvist (2002) analyze abnormal ex-day returns of Swedish and Danish coupon lottery bonds and find tax clientele effects. Lobe and Hölze (2007) study British perpetual premium bonds which are, in fact, non tradeable lottery-linked deposit accounts. The monthly interest rates for the accounts is determined by a lottery such as that described in the survey of lottery-linked deposit accounts described by Guillen and Tschoegl (2002). Our study is closest to the paper by Ukhov (2005). Ukhov uses price data of two coupon and redemption lottery bonds issued by the Russian government between 1864 and 1866 to estimate the Arrow-Pratt measure of absolute risk aversion with respect to the coupon lottery risk. In contrast to our study, Ukhov does not employ a dynamic asset pricing framework and estimates risk aversion under the assumption that bond prices after the lottery are perfectly known immediately before the lottery.

In the last four decades numerous studies have estimated RRA coefficients. These studies can be categorized into three groups. Table A, in the appendix, gives an overview of the range of RRA parameters reported in these studies. The first group of papers extracts risk preferences through direct assessments or from cross-sectional household survey data. Most of these studies find low levels of risk aversion between zero to approximately four. The second group of papers employs the framework of consumption-based asset pricing models introduced by Lucas (1978) and Breeden (1979) and their extensions. Using time series data on asset returns, and aggregate consumption, they report RRA estimates ranging from zero to above 60. The large variation in the estimates is caused by the differing characteristics of the underlying valuation model (in particular with respect to the preference specification), alternative econometric methods, and differing properties of the underlying dataset. A third body of literature employs option price data to obtain market risk preferences. For example, Bliss and Panigirtzoglou (2004) assume a parametric stationary form for the utility function, which they use to adjust the risk-neutral probability distribution function to deduce implied risk preferences. The estimated values of CRRA parameters in these studies also vary between 0 and 60.

Our empirical findings contribute to the dispute in the literature on estimating the size of risk premia and relative risk aversion coefficients. The results are obtained by building up a new, and partly hand-collected, dataset of German redemption lottery bonds. They allow us to put the empirical results obtained from stock markets, bond markets and laboratory experiments into perspective. In our analysis we concentrate on lottery bonds quoted below par. Between the last trading day before, and the first trading day after, a lottery a highly significant average return of -26 bp is observed. This first price decline is followed by a sequence of significant negative returns during the subsequent three trading days. This result can be explained mainly by the relatively low liquidity of the German lottery bond market. The average Sharpe ratio of lottery driven price jumps is 0.11 and significant on the 1% level. An application of the Hansen/Jagannathan bounds, with an assumed volatility of consumption growth of 1% per year, results in an upper bound for the RRA parameter of 10. The estimated Sharpe ratio is relatively stable for different groups of issuers. Surprisingly, the average Sharpe ratio is largest for the bonds with the smallest risk, i.e. the smallest probability of not being drawn. This effect can be explained by the observation that the average loss if the bond is not drawn is almost independent of the lottery probability, whereas the average gain if the bond is drawn strongly decreases if the probability to be drawn increases.

We find an average RRA parameter of 1.8. A bootstrap analysis shows a range for the sampled RRA parameters of between 1.66 and 1.89. This finding is in line with the estimates obtained in laboratory experiments and not with the majority of studies that use market prices. We trace this result back to the fact that obtaining RRA parameters from market prices are affected twofold by problems in estimating the probabilities of future prices. Ex ante market participants are ambiguous about these probabilities, ex post these probabilities are contaminated by sample errors. Therefore, available objective probabilities on future prices result in RRA parameters as they have been documented by Constantinides (1990) using habit formation and partly by Epstein/Zin (1991) applying recursive utility functions. Therefore, our study offers the explanation that the equity premium puzzle may be driven by the uncertainty regarding the distribution of consumption growth or stock returns.

The estimates of the RRA parameters vary in a non-systematic way with the lottery probabilities; between 0.6 for the smallest and 6.2 for the highest probability of a bond to be drawn. This variation can partly be explained by the time variation of RRA parameters. In the second half of the observation period the estimates of RRA parameters are higher than in the first half. Owing to maturity effects in the second half the sample also has more bonds with a high probability to be drawn.

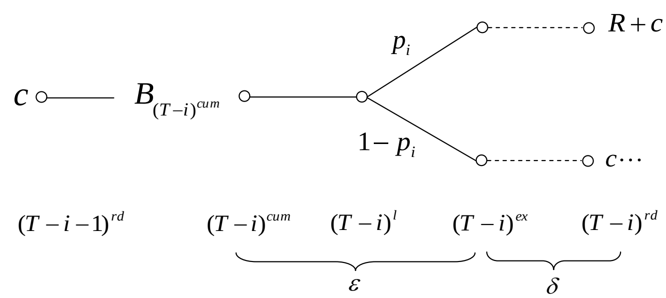
The outline of this paper is as follows. Section 2 describes the institutional details of the German redemption lottery bond market. Section 3 introduces our standard dynamic equilibrium valuation model for redemption lottery bonds, and provides comparative static results. Section 4 presents the dataset. In Section 5, we analyze the ex-day price behavior of lottery bonds, and determine absolute and relative risk premia within a standard event study framework. In Section 6 we estimate equilibrium RRA parameters. We report RRA estimates for several segments of the data panel, test for robustness, and analyze their time series behavior. Section 7 concludes.

2 Institutional Facts on German Redemption Lottery Bonds

German redemption lottery bonds are fixed coupon bonds redeemed by lotteries. Before issuance, a bond is split into series of equal size identified by a series number. Approximately three months before a partial redemption, the series to be redeemed is determined by a random drawing of series numbers. The series that are not drawn for redemption will participate in future lotteries. A sequence of lotteries is conducted until all but one series are drawn. A trustee monitors the indenture and calls the drawn bonds at the prearranged call price, which is usually the face value. Figure 2 characterizes the time and information structure at an arbitrary lottery date $(T-i)$, $\forall i \in \mathbb{N}$ and $1 \leq i < T$. Note that we assume a lottery bond with annual coupon payments and a time difference of one year between subsequent lotteries. With probability p_i the holder of the lottery bond will receive the coupon and redemption payment $R + c$ at the redemption date $(T-i)^{rd}$. Otherwise, he holds at the first trading date after the lottery $(T-i)^{ex}$ the

Figure 2: **Structure of Redemption Lottery**

This figure depicts the time and information structure at an arbitrary lottery date $(T-i)$. We consider a lottery bond with annual coupon payments and redemption lotteries. The time structure is as follows: (i) at $(T-i-1)^{rd}$, the last coupon before the lottery is paid, (ii) $(T-i)^{cum}$ denotes the last trading date of $(t-i)^{l}$ lottery bond before the lottery, (iii) between $(T-1)^{cum}$ and $(T-1)^{ex}$ the lottery takes place, (iv) $(T-i)^{ex}$ denotes the first trading date after the lottery of those series not drawn, (v) at $(T-i)^{rd}$, the lottery bond series which is drawn is redeemed, and the first coupon after the lottery is paid. The dirty lottery bond price before the lottery is denoted by $B_{(T-i)^{cum}}$ and immediately after the lottery by $B_{(T-i)^{ex}}$. The redemption probability is p_i, the annual coupon of the lottery bond is c, and the redemption value is R. ϵ is the time span between $(T-i)^{cum}$ and $(T-i)^{ex}$, δ the time span between $(T-i)^{ex}$ and $(T-i)^{rd}$.



lottery bond with dirty price $B_{(T-i)^{ex}}$ which, in general, will differ from the dirty price $B_{(T-i)^{cum}}$ immediately before the lottery.

For the issuer of a lottery bond, the redemption payments are deterministic. The bearer of a single bond series, however, does not know when his bond will be redeemed. Therefore, the maturities of outstanding bond series are uncertain as long as more than one series is outstanding. However, the bearer knows the conditional redemption probabilities of outstanding bond series at all future lottery dates. These probabilities are determined by the ratio of the actual number of series to be redeemed to the total number of outstanding series. Hence, the future cash flows of buying and holding a lottery bond are characterized by an objective probability distribution. Therefore, lottery bonds provide an exceptional environment to study investors' risk aversion. Furthermore, redemption risk is by construction independent of other risk sources in the economy.

It is important to note that individual series were not traded on organized exchanges. As a consequence, there is only one price for a lottery bond, rather than a price for each outstanding series. The bearer of a lottery bond faces the risk or chance that his series will be drawn before the final maturity of the bond. If a lottery bond is traded below par value immediately before the drawing date, the price of the undrawn series will jump downwards, since the chance of being repaid at the subsequent redemption date is gone.

A basic problem for our study refers to the question whether investors can diversify lottery risk. The following rules applied during the period in which lottery bonds were traded in Germany. In the primary market, the issuer assigned perfectly diversified portfolios of lottery bond series to retailers, i. e. portfolios with identical numbers of bonds from each series. Trading in secondary markets took place in the over-the-counter market and on organized exchanges such as the Frankfurt or Düsseldorf Exchange. It is central to our study that in both markets it was virtually impossible to trade diversified portfolios of the outstanding series or to place series-specific orders. This trading usance was confirmed independently by two former lottery bond traders. Therefore, lottery risk was not diversifiable, buyers of lottery bonds had to bear the future lottery risks, and prices should reflect this risk component. One main reason for this trading usance was brought forward by market participants; traders did not want to bear the costs for searching for individual series or conversely to hold inventories for each of the outstanding series.

3 Valuation of Lottery Bonds and Implied Risk Aversion

3.1 Model Setup and Assumptions

We consider a pure exchange economy without market frictions. The market consists of two types of bonds, a lottery bond and a set of zero bonds with maturities related to the payment dates of the lottery bond. The lottery bond is assumed to have annual redemption lotteries, annual coupon payments c, longest maturity T, and a time-independent redemption value R.[3] We assume that the economy is only exposed to lottery risk and abstract from interest rate and default risk. These assumptions allow us to model the term structure of interest rates at a flat interest level r.[4]

Trading is possible at any point of time $0 \leq t \leq T$. As no new information occurs between an ex-day and the subsequent cum-day, and as the dirty prices of all bonds increase with the risk-free rate r, it is sufficient to consider the ex- and the cum-days as possible trading days. A critical assumption refers to the investment opportunities of the investors. To exclude diversification strategies by splitting an order into many suborders we assume that there are as many homogeneous investor groups with identical constant relative risk aversion γ as there are series of the lottery bond. Each series is traded by one investor group in a separate market segment, and individual investor groups have access only to their group specific segment. This assumption ensures that lottery risk is of systematic nature and reflects the institutional setting that an individual investor can generally not trade multiple series in order to diversify lottery risk. Note that the assumption does not imply that an investor in one specific group knows when the series he trades will be drawn. Except for the differing series numbers, lottery bonds traded in all market segments are identical and will have the same equilibrium price. Hence, it is sufficient to focus on one segment. Whenever a specific series is drawn, the corresponding segment consists of zero-bonds only. Investors in such a segment are forced to invest their entire wealth into zero-bonds. This setting is the most simple to study whether prices of bonds which are exposed to risk can be explained in a standard expected utility framework.

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[3] It is straightforward to consider biennial lotteries, semi-annual coupon payments, and time dependent redemption values.

[4] In the empirical sections, we relax the assumption of a flat risk-free term structure of interest rates. We estimate the term structure from prices of straight default-free government bonds.

We assume homogeneous investors with constant relative risk aversion. In the model we abstract from interest rate risk. In the empirical study we consider interest rate risk as far as it is reflected in the current term-structure of interest rates.

3.2 Last-period Model

To simplify the exposition we consider the last-period problem first. At $(T-1)^{cum}$, shortly before the last lottery date, the investor optimally distributes his wealth between the lottery bond series and the risk-free instrument. If the agent invests in the lottery bond, he has to consider two disjoint states of the world at $(T-1)^{ex}$, shortly after the drawing. In state d, the investor holds a zero bond with maturity δ and in state n, he holds a coupon bond with two coupon payments and maturity at $1+\delta$.

The length of the time interval between $(T-1)^{cum}$ and $(T-1)^{ex}$ is equal to ε, and the length of the interval between $(T-1)^{ex}$ and the redemption date $(T-1)^{rd}$ is equal to δ. The difference between $(T-1)^{rd}$ and final maturity T is one year.

$$v_1^d = \frac{R+c}{(1+r)^\delta} \quad and \qquad v_1^n = \frac{c}{(1+r)^\delta} + \frac{R+c}{(1+r)^{1+\delta}} \tag{1}$$

denote the present value at $(T-1)^{ex}$ of future cash flows if the series is drawn (d) or not drawn (n) at $(T-1)^l$. Note that, for $c/R = r$, i.e. $v_1^d = v_1^n$, the economy is free of redemption risk, and the representative agent is indifferent between investing in the lottery bond or risk-free instrument. For ease of notation, we assume $c/R \neq r$ throughout this section.

We assume that a representative investor exists and has a state-independent power utility function of terminal wealth:

$$u\left(w_T^s\right) = \frac{\left(w_T^s\right)^{1-\gamma} - 1}{1-\gamma} \quad \text{for } \gamma \neq 1,\ u\left(w_T^s\right) = \ln w_T^s \text{ for } \gamma = 1,\ s = d, n. \tag{2}$$

$\gamma > 0$ is the RRA parameter and w_T^s denotes the investor's wealth at T in the state d or n.

The investor's utility maximization problem at $(T-1)^{cum}$ is characterized by:

$$\begin{aligned} & \max_{x_{cum}} \left\{p_1 \cdot u\left(w_T^d\right) + (1-p_1) \cdot u\left(w_T^u\right)\right\} \qquad (3) \\ = \; & \max_{x_{cum}} \left\{p_1 \cdot u\left(w_{ex}^d \cdot (1+r)^{1+\delta}\right) + (1-p_1) \cdot u\left(w_{ex}^n \cdot (1+r)^{1+\delta}\right)\right\}. \end{aligned}$$

In (3) x_{cum} denotes the proportion of wealth w_{cum} at $(T-1)^{cum}$ invested in the lottery bond, $(1-x_{cum})$ the proportion of wealth invested in the risk free zero bond with maturity $1+\delta+\epsilon$. Investors state-dependent wealth w_{ex}^s immediately after the drawing depends on the wealth w_{cum} immediately before the drawing, the present value v_1^s, the dirty price B_{cum} immediately before the drawing and the portfolio decision x_m as follows

$$w_{ex}^s = w_{cum} \cdot \left(x_{cum} \cdot \frac{v_1^s}{B_{cum}} + (1-x_{cum}) \cdot (1+r)^{\varepsilon}\right), \quad s = d, n. \qquad (4)$$

Solving the first order condition for the maximization problem (3) to (4) using the market clearing condition, $x_{cum} \equiv 1$, for B_{cum}, we obtain the equilibrium price (note $p_1 = 1/2$)

$$B_{cum}^* = \frac{\left(v_1^d\right)^{1-\gamma} + \left(v_1^n\right)^{1-\gamma}}{\left(v_1^d\right)^{-\gamma} + \left(v_1^n\right)^{-\gamma}} \cdot \frac{1}{(1+r)^{\varepsilon}} \quad \text{for } \gamma > 0. \qquad (5)$$

For $\gamma = 0$, B_{cum}^* equals the discounted expected value of the present values v_1^s $(s = d, n)$.

It can be shown that B_{cum}^* is strictly monotonic decreasing in γ, and that it is restricted by the no-arbitrage bounds

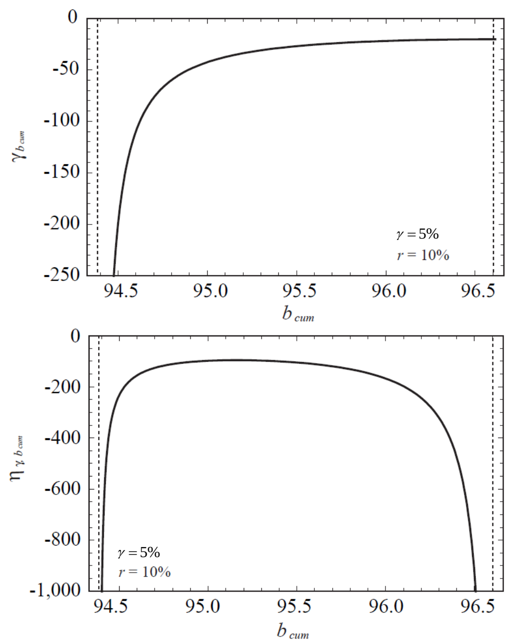
$$\min\left(v_1^d; v_1^n\right) < B_{cum}^* \cdot (1+r)^{\epsilon} < \max\left(v_1^d; v_1^n\right). \qquad (6)$$

B_{cum}^* converges to the lower bound for $\gamma \to +\infty$ and to the upper bound for $\gamma \to -\infty$. Therefore, as long as an observed bond price B_{cum}^{obs} lies in the no-arbitrage interval (6) there exists, for given terms of the bond and r, a unique γ such that $B_{cum}^{obs} = B_{cum}^*$:

$$\gamma = -\frac{\log\left[\frac{B_{cum} - \frac{v_1^n}{(1+r)^{\varepsilon}}}{\frac{v_1^d}{(1+r)^{\varepsilon}} - B_{cum}}\right]}{\log\left[\frac{v_1^d}{v_1^n}\right]}. \qquad (7)$$

Figure 3: **Sensitivity and Elasticity of RRA Parameter**

This figure plots the derivative $\gamma_{b_{cum}}$ and elasticity $\eta_{\gamma,b_{cum}}$ as a function of the clean lottery bond price b_{cum}. We consider a lottery bond with two outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The left dashed lines in the graphs depict the lower no-arbitrage bound for the clean lottery bond price, and the right dashed lines depict the maximum price a risk-neutral investor is willing to pay for the lottery bond. The risk-free rate is $r = 10\%$, and the coupon rate is $c = 5\%$.



The sensitivity of the implied RRA parameter γ is of major importance for our empirical study. The objective of the subsequent comparative static analysis is to provide theoretical evidence on the robustness of the implied RRA parameter helping us to assess the precision of our estimations. Figure 3 shows the partial derivative $\gamma_{b_{cum}}$ and the elasticity $\eta_{\gamma,b_{cum}}$ of γ as a function of the arbitrage-free clean lottery bond price b_{cum}. If b_{cum} converges to its lower no-arbitrage bound characterized by the left dashed line, $\gamma_{b_{cum}}$ converges to minus infinity. The RRA parameter is most sensitive to price changes close to the no-arbitrage bounds, and least sensitive to price changes close to the risk-neutral price which is characterized by the right dashed line. Since the elasticity of γ is absolutely large in the neighborhood of these two clean bond prices we control for this in our empirical analysis.

3.3 Dynamic Equilibrium Prices

The one-period model extends immediately to a multi-period setting with the possibility to trade the bond at every point in time. As in Section 3.2, the representative investor has a state-independent power utility function on terminal wealth at T. We determine equilibrium lottery bond prices through backward induction by exploiting the special structure of our problem.[5] In the first step, we characterize the equilibrium cum prices $B^*_{(T-i)^{cum}}$ immediately before the lottery date $(T-i)^l$ $(i = 1, \ldots, T-1)$ as a function of future equilibrium cum prices of the lottery bond. In the second step we derive the equilibrium ex prices $B^*_{(T-i)^{ex}}$ by the subsequent cum price $B^*_{(T-i+1)^{cum}}$ $(i = 1, \ldots, T-1)$.

The investor's utility maximization problem at $(T-i)$ is defined analogously to (3) as

$$\max_{x_{(T-i)^{cum}}} \left\{ p_i \cdot u\left(w^d_{(T-i)^{ex}} \cdot (1+r)^{i+\delta}\right) + (1-p_i) \cdot J_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right)\right\}. \tag{8}$$

In (8) $J_{(T-i)^{ex}}(\cdot)$ denotes the optimal value of the investor's optimization problem at $(T-i)^{ex}$, $p = 1/(1+i)$ is the probability that the bond is drawn at $(T-i)^l$, $x_{(T-i)^{cum}}$ the proportion of wealth invested in the lottery bond at $(T-i)^{cum}$, and $w^d_{(T-i)^{ex}}$, $w^n_{(T-i)}$ the investor's wealth in case that the bond is drawn and not drawn respectively.

Expression (8) is maximized subject to the dynamics:

$$\begin{aligned} w^d_{(T-i)^{ex}} = {} & w^n_{(T-i)^{cum}} \cdot \\ & \left(x_{(T-i)^{cum}} \cdot \frac{v^d_i}{B_{(T-i)^{cum}}} + \left(1 - x_{(T-i)^{cum}}\right) \cdot (1+r)^{\varepsilon} \right), \end{aligned} \tag{9}$$

$$\begin{aligned} w^n_{(T-i)^{ex}} = {} & w^n_{(T-i)^{cum}} \cdot \\ & \left(x_{(T-i)^{cum}} \cdot \frac{B^*_{(T-i)^{ex}}}{B_{(T-i)^{cum}}} + \left(1 - x_{(T-i)^{cum}}\right) \cdot (1+r)^{\varepsilon} \right). \end{aligned}$$

In (9) $w^n_{(T-i)^{cum}}$ is the investor's wealth immediately before the lottery at $(T-i)^l$ given that his series was not drawn in any of the previous lotteries. v^d_i is equal to v^d_1 as defined in Equation (16).

[5] See Ingersoll (1987), Chapter 11.

The functional form of the indirect utility function $J_{(T-i)^{ex}}(\cdot)$ is given by

$$J_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right) = \frac{1}{i} \cdot \left(w^n_{(T-i)^{ex}}\right)^{1-\gamma} \cdot \frac{a_{(T-i)^{ex}}}{\left(B^*_{(T-i)^{ex}}\right)^{1-\gamma}}, \tag{10}$$

where $a_{(T-i)^{ex}}$ is defined by

$$a_{(T-i)^{ex}} \equiv \sum_{j=1}^{i-1} u\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right) + u\left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right), \tag{11}$$

and $y_{(T-k)^{cum}}$ is given by

$$y_{(T-k)^{cum}} \equiv 1 + \frac{c \cdot (1+r)^{1-\epsilon-\delta}}{B^*_{(T-k)^{cum}}}. \tag{12}$$

Solving the first order condition of the general utility maximization problem (8) to (9) with market clearing, $x_{(T-i)^{cum}} \equiv 1$, we obtain the equilibrium price $B^*_{(T-i)^{cum}}$ as

$$B^*_{(T-i)^{cum}} = \frac{1}{(1+r)^{i+\varepsilon+\delta}}. \tag{13}$$

$$\frac{\sum_{j=1}^{i}\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right)^{1-\gamma} + \left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right)^{1-\gamma}}{\sum_{j=1}^{i}\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right)^{-\gamma} + \left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right)^{-\gamma}}.$$

For $i = 1$ equation (13) simplifies to the equilibrium pricing Equation (5) derived for the last drawing date.

The equilibrium price $B^*_{(T-i)^{ex}}$ can be determined completely analogous to the cum price. As expected, we obtain

$$B^*_{(T-i)^{ex}} = \frac{B^*_{(T-i+1)^{cum}}}{(1+r)^{1-\varepsilon}} + \frac{c}{(1+r)^{\delta}}. \tag{14}$$

Equilibrium prices at $(T-i)^{ex}$ are the sum of the discounted equilibrium price $B^*_{(T-i+1)^{cum}}$ and the present value of interim coupon payments. The general structure of equilibrium prices in the interval $[(T-i)^{ex}, (T-i+1)^{cum})$ is equal to that of $B^*_{(T-i)^{ex}}$ defined in Equation (14). After the redemption lottery at $(T-i)^l$ and before the subsequent lottery, the economy is risk-free. Hence, equilibrium prices are determined by simply accumulating the cum-lottery price $B^*_{(T-i+1)^{cum}}$ and interim coupon payments, and discounting the cash flows at the risk-free rate to the valuation date.

The numerical comparative-static analysis of $B^*_{(T-i)^{cum}}$, performed for reasonable values of the coupon c and the interest rate r, shows that the clean price $b^*_{(T-i)^{cum}}$ is strictly monotonically decreasing in γ and converges for $\gamma \to +\infty$ to the lower no-arbitrage limit. Furthermore, the price range $b^*_{(T-i)^{cum}}(\gamma = 0) - b^*_{(T-i)^{cum}}(\gamma = \infty)$ increases strictly in i. As a consequence $b^*_{(T-1)^{cum}}$ is much flatter than, e.g., $b^*_{(T-9)^{cum}}$ as a function of γ, and γ estimated from $b^*_{(T-1)^{cum}}$ is much more sensitive to noisy bond prices than the RRA parameter determined from a lottery bond with a much longer time to maturity.

4 Data

Our empirical study covers the period from January 1974 until December 1987. The analysis starts in 1974, when daily bond price data become available, and it ends in 1987 when the last lottery of the considered issuer groups was carried out. The terms of the lottery bonds, coupon, maximum maturity, lottery dates and volumes are hand-collected from print media, particularly the bond guide published by Hoppenstedt.

We focus on lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government-owned enterprises (GE). The issuers structure is sufficiently homogeneous with respect to credit-risk and taxation, although they differ by their issue volume and, therefore, possibly by liquidity. Market participants considered the bonds of these issues to be credit-risk free. Our dataset contains 83 lottery bonds for which complete datasets are available. For these issues, 483 redemption lotteries were played. Table 1 compiles the sample of lottery bonds segmented by issuer groups.

Table 2 gives an overview of the basic lottery bond characteristics. The outstanding volume of lottery bonds has constantly decreased over time. The FRG issued its last lottery bond in 1965, and GS and GE issued their last lottery bonds in 1973 and 1972,

Table 1: **Lottery Bond Issuers**

This table reports the number of lottery bonds issued by the Federal Republic of Germany, German states, and government enterprises as well as the number of lotteries related to these bonds.

Issuers	Bonds	Lotteries
Federal Republic of Germany	7	27
German States	55	361
Baden-Württemberg	3	18
Bavaria	4	35
Berlin	5	35
Bremen	3	26
Hamburg	4	28
Hesse	3	24
Lower Saxony	9	57
North Rhine-Westphalia	1	3
Rhineland-Palatinate	9	58
Saarland	8	51
Schleswig-Holstein	6	26
Government Enterprises	21	95
Deutsche Bundesbahn	10	46
Deutsche Bundespost	11	49
Total	83	483

respectively. The mean nominal volume per issue is DEM 392.86 million for FRG lottery bonds, DEM 114.45 million for GS bonds, and DEM 241.90 million for GE bonds included in our sample. During the same period (until 1973), straight bonds have slightly higher mean nominal volumes per issue of DEM 437.83 million for FRG bonds, DEM 167.67 million for GS bonds, and DEM 293.75 million for GE bonds. Lottery bonds were long term debt contracts with maturities between ten to 25 years, and an average maturity of 16.37 years. About one third of the lottery bonds paid annual coupons, and the remainder paid coupons semi-annually. Coupon rates ranged from 5% to 9% and averaged 6.55%.

Each lottery bond contained a redemption schedule specifying the lottery dates, repayment dates, the number of series to be redeemed, and the redemption values. Table 3 details the composition of lottery bonds regarding these redemption features. All redemption lotteries by the FRG and GE and about 90% of the lotteries by GS were redeemed at par. The remaining GS lotteries were redeemed at either 101, 102, or 103. Redemption lotteries were conducted on an annual or a biennial basis. More than 70% of the lottery bonds by the FRG had biennial drawings, whereas more than 90% of the lotteries by GS and more than 70% of the lotteries by GE were conducted annually. The majority of the lottery bonds was split into five to 15 series, and the average bond was

Table 2: **Basic Lottery Bond Characteristics**

This table shows descriptive statistics for the basic lottery bond characteristics of issues by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column reports the characteristics. The second to fourth columns report the absolute frequencies for bonds. The fifth to seventh columns report the absolute frequencies for lotteries.

Characteristics	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Issue Years						
1958 to 1960	0	6	6	0	28	36
1961 to 1965	7	15	13	27	65	47
1966 to 1970	0	19	1	0	151	9
1971 to 1973	0	15	1	0	117	3
Issue Volume (mDEM)						
< 50	0	4	0	0	19	0
50 to < 100	0	14	0	0	76	0
100 to < 200	0	28	3	0	195	15
200 to < 300	0	8	14	0	62	60
300 to < 400	2	1	1	5	9	3
400 to 500	5	0	3	22	0	17
Maximum Maturity (years)						
10 to < 15	3	32	9	7	209	19
15 to < 20	4	20	10	20	132	58
20 to 25	0	3	2	0	20	18
Coupon Frequency						
Annual	5	18	6	17	133	23
Semi-annual	2	37	15	10	228	72
Coupon Rate						
5 to < 6	0	4	7	0	21	36
6 to < 7	6	25	12	24	146	52
7 to < 8	1	15	1	3	106	4
8 to 9	0	11	1	0	88	3

Table 3: **Redemption Features**

This table shows descriptive statistics for the redemption features of lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column shows the features. The second to fourth columns report the absolute frequencies for bonds. The fifth to seventh columns report the absolute frequencies for lotteries.

Redemption Features	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Redemption Value						
Par	–	–	–	27	336	95
Above Par	–	–	–	0	25	0
Redemption Frequency						
Annual	2	50	15	10	345	80
Biennial	5	5	6	17	16	15
Number of Series[a]						
3 to < 5	0	2	1	0	5	3
5 to 9	5	8	6	17	31	15
10	1	35	5	2	268	18
11 to 15	1	6	8	8	31	55
16 to 20	0	2	1	0	15	4
100	0	2	0	0	11	0
Redemption Probabilities						
1/2	–	–	–	7	55	21
1/3	–	–	–	7	52	19
1/4	–	–	–	5	51	15
1/5	–	–	–	4	45	12
1/6	–	–	–	1	39	8
1/7	–	–	–	1	37	6
1/8	–	–	–	1	33	6
1/9	–	–	–	1	29	5
1/10	–	–	–	0	20	3
< 1/10	–	–	–	0	(6)[b]	0
Lag between Issuance and First Lottery (years)						
3 to < 5	0	7	1	0	53	3
5 to < 6	2	29	10	10	190	46
6 to < 10	2	16	4	5	101	8
10 to 12	3	3	6	12	17	38
Lag between Lottery and Redemption (days)						
< 100	–	–	–	2	39	10
100 to < 110	–	–	–	19	101	76
110 to < 130	–	–	–	6	153	9
130 to < 150	–	–	–	0	47	0
150 to 200	–	–	–	0	21	0

[a] For three lottery bonds, more than one series is regularly redeemed. The issues by the state of Hamburg (WKN 136510, 136511) consisted of 100 series that were redeemed in 12 and 15 stages, respectively. The issue by the Deutsche Bundesbahn (WKN 115003) consisted of 20 series that were redeemed in 14 stages.

[b] Six lottery observations of two bond issues by the state of Bavaria (WKN 105024, 105025) with redemption probabilities below 1/10 are excluded to simplify our analysis.

composed of twelve series. At each redemption date, one series was drawn by lottery and repaid. Since all series of one bond issue were approximately of the same size, the redemption probabilities can easily be determined by calculating the ratio of the actual number of redeemed series to the total number of outstanding series. Within the empirical analysis, we consider the first nine redemption probabilities (1/2 to 1/10). Lotteries with higher probabilities appear more frequently, since lottery bonds were issued between 1958 and 1973, and we disregard lotteries before 1974. After issuance, three to twelve years passed until the first redemption lottery was played. The average lag between issuance and the first drawing was 6.61 years. The lag between the lottery and the redemption payments ranged from 68 to 186 calendar days and averaged 115 days.

Apart from the scheduled redemption by lottery, most of the lottery bond indentures contained early or increased redemption options; during the entire period of our analysis the FRG and GE did not exercise any and GS exercised eight of the additional embedded options. Within the subsequent empirical analysis, we account for the early and increased redemption options by focusing on clean bond prices quoted below the discounted redemption value. We assume that these prices are not influenced by the out-of-the-money redemption options. A total of 18 lottery bonds (130 lotteries) by GS, and one issue (three lotteries) by GE, do not contain early or increased redemption options. These lottery bonds will be used as a control group to test for possible biases caused by redemption options.

German lottery bonds and straight coupon bonds were traded on organized exchanges and over-the-counter. Our empirical analysis is based on bond market prices (*amtlicher Kurs*) fixed once each trading day at organized exchanges such as the Frankfurt Stock Exchange.[6] These prices are determined by an official auctioneer according to the principle of maximum execution. In addition, it is also reported whether all orders, a fraction of the orders or whether no transactions were executed at the reported price. We use transaction prices only. Daily clean market prices for most of the outstanding lottery and straight coupon bonds are available as of January 2, 1974. The prices are restricted to changes in discrete ticks of DEM 0.05.

In addition to lottery bonds there are also traded straight bonds of the same issuers at the same organized exchanges. From these bonds with fixed maturities we estimate risk-free term structures of interest rates for each of the three issuer groups separately. Estimations

[6]See Deutsche Bundesbank (2000), pp.49, for details on the fixing of the official bond market price.

are based on the Nelson and Siegel (1987) approach extended by Svensson (1994). On average, 45 bond prices are available to estimate the term-structure of interest rates in the FRG market segment, 19 for the GS, and 35 for GE market segment. The overall mean squared estimation error is 11 basis points for the FRG, 13 basis points for GS, and 10 basis points for GE.[7] The mean of the term-structures of interest rates between the GS and the FRG market segment is 146 bp, between the GE and FRG segment 12 bp. These spreads can be attributed to liquidity differences between these segments. Unless stated otherwise, we use these term structures of interest rates to value lottery bonds in the three market segments. We take interest-rate risk into account in so far as it is reflected in the current term-structure of interest rates.

5 Ex-day Price Behavior and Risk Premia Under Perfect Foresight

In a first step we determine in a standard event study framework the ex-day price reaction of lottery bonds and risk premia. Our primary focus is the price behavior. In a first step, we discuss the returns of buying the lottery bond at point in time t, up to 5 trading days before $(T-i)^{cum}$ at the dirty price B_t, and liquidate the bond at point in time $\bar{t}$, up to 5 trading days after $(T-i)^{ex}$. The liquidation value is equal to the dirty price $B_{\bar{t}}$ if the bond is not drawn and $(R+c)/(1+r(\delta-\eta))^{\delta-\eta}$ in case that it will be redeemed at the next coupon date. R and c are defined as in Section 3.2. η is the time span between $(T-i)^{ex}$ and $\bar{t}$. $r(\delta-\eta)$ is the yield to maturity of a zero bond with maturity $\delta-\eta$. It is determined by estimating the term-structure of interest rates for the relevant market segment at the point in time of liquidation. Of special interest are the results for $\eta=0$, i.e. buying the bond at $(T-i)^{cum}$ and liquidating it at $(T-i)^{ex}$ at $B_{(T-i)^{cum}}$ and $B_{(T-i)^{ex}}$ respectively.

The holding period returns are defined as $(B_{\bar{t}}-B_t)/B_t$. Its mean, standard deviation, minima and maxima for varying t and η are shown in Table 4. Note that, on average, trading was suspended for two trading days before the drawing date and started again one trading day after. Hence the interval $[-1,+1]$ covers at least three calendar days. If at a

[7] The statistics of the error term are similar to those found in studies using a comparable dataset of straight German government bonds to estimate the term structure of interest rates. See e. g. Schich (1997).

day before $(T-i)^{ex}$ no transaction is available we use the transaction price of an earlier day closest to that day. In the cases that no transactions occur a day after $(T-i)^{cum}$ we take a later day.[8]

Daily mean returns shortly after the drawing are significantly negative. The mean return at the drawing date is -26 basis points. Prices continue to fall significantly over the three subsequent days with means of -10, -8, and -4 basis points. This result is surprising as it shows that public information is not priced immediately. The most convincing explanation of this observation is the relatively low liquidity of the market for lottery bonds. The small positive returns are within transaction costs for institutional investors of about 5 bp.

In the next step we analyse absolute and relative risk premia. We define the absolute risk premia simply as the difference of the expected value of the liquidation value of the lottery bond after the drawing and the bond's price before the lottery using those transaction prices that are closest to the lottery date. If we denote these two dates by t ($t \leq (T-i)^{cum}$) and $\bar{t}$ ($\bar{t} \geq (T-i)^{ex}$), the risk premium on the price level equals

$$\pi_i = p_i \cdot [B_{\bar{t}} - B_t] + (1-p) \cdot \left[(R+c)/(1+r(\delta-\gamma))^{\delta-\gamma} - B_t\right]. \tag{15}$$

Again, $\delta - \eta$ denotes the time span from $\bar{t}$ until the next coupon date. We do not correct for the interest rate effect from t to $\bar{t}$ as this is very small and can be neglected. Note that π_i is also equal to the expected value of the gains

$$g_i^n = B_{\bar{t}} - B_t, \quad g_i^d = (R+c)/(1+r(\delta-\gamma))^{\delta-\gamma} - B_t. \tag{16}$$

The relative risk premium is defined as π_i/σ_i, where δ_i is the standard deviation of the lottery's gains. This ratio is basically the Sharpe ratio.

In Tables 5 and 6 we report the mean lottery gain in states d and n, the mean (absolute) risk premium, and the mean relative risk premium for lottery observations resulting in a redemption gain. Results are given for the full dataset as well as for each issuer group and for different redemption probabilities. We first consider the results on the aggregate and issuer group level. The mean redemption gain is DEM 4.69, and the mean loss in state n is DEM 0.80. The mean risk premium is DEM 0.28[9], and the mean relative risk premium

[8]Note, however, that central results remain unchanged if observations, except those at trade resumption, for which the previous or current transaction price is unavailable, are excluded.

[9]The mean dirty lottery bond price $\bar{B}_{(T-i)^{cum}}$ is equal to 97.55 for the 187 observations in the full sample.

Table 4: **Time Series of Daily Returns**

This table shows daily returns of observed, clean lottery bond prices around lottery dates for lotteries resulting in a redemption gain. Mean returns, standard deviations, minima, and maxima are reported in basis points for the interval spanning five trading days before the lottery-related price suspension and five trading days after the lottery drawing. The first row depicts the lag in trading days, where lag -5 equals five days before the price suspension and lag 1 the date of trade resumption. The results are based on 252 to 266 observations per lag. The t-values are given in parentheses. We report the level of significance based on the two-sided t-test for time lags -5 to -1 and based on the one-sided t-test for time lags 0 to 5. *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Time Lag	−5	−4	−3	−2	−1	1	2	3	4	5
Mean Return	3**	2*	1	2*	1	−26***	−10***	−8***	−4**	−2
t-value	(2.59)	(1.78)	(0.98)	(1.71)	(0.75)	(−8.08)	(−3.80)	(−3.22)	(−2.46)	(−1.06)
Std. Dev.	17	19	16	18	16	53	44	38	29	26
min	−52	−152	−78	−110	−110	−323	−312	−244	−220	−178
max	137	113	105	111	77	184	153	197	102	108

Table 5: **Risk Premia and Prices of Risk**

This table shows mean gains and losses in states d and n, mean risk premia, and mean relative risk for lottery observations resulting in a redemption gain. Results are given for the full dataset, the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The gains and losses in states d and n as well as the risk premia are measured in German Mark per bond with face value 100. We report t-values in parentheses for risk premia and relative risk premia as well as standard deviations, minima, and maxima. The level of significance is based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Risk Measures	Overall	FRG	GS	GE
Observations	187	20	112	55
Gain in State d	4.69	4.39	4.71	4.74
Gain in State n	−0.80	−1.30	−0.66	−0.90
Risk Premium	0.28***	0.24***	0.28***	0.27***
t-value	(7.02)	(2.91)	(5.03)	(4.34)
Std. Dev.	0.54	0.38	0.60	0.47
min	−1.87	−0.15	−1.87	−1.68
max	2.30	1.35	2.30	1.29
Relative Risk Premium	0.11***	0.13***	0.09***	0.13***
t-value	(4.98)	(2.77)	(2.91)	(4.15)
Std. Dev.	0.30	0.21	0.33	0.24
min	−0.88	−0.22	−0.88	−0.67
max	0.69	0.69	0.66	0.59

is 0.11. Both measures are positive and highly significant. Mean risk premia range from DEM 0.24 for the FRG to DEM 0.28 for GS, the mean relative risk premia from 0.09 to 0.13 and are of moderate magnitude compared to Schilbred (1973) who estimates a market price of risk within a mean-variance equilibrium model in a bond market setting of the order of 0.50. If a typical volatility of consumption growth of 1% p.a. is assumed, the Hansen/Jagannathan bounds result in an upper bound for the RRA parameter of 10. The estimated relative risk premium (Sharpe ratio) is relatively stable for the different group of issuers.

Second, we consider the results for the redemption probability segments. The mean redemption gain ranges from DEM 9.48 for $p = 1/10$ to DEM 2.08 for $p = 1/2$ and documents the pull-to-par effect as bond maturity approaches. The mean loss in state n ranges from DEM 0.58 to 1.27 and is largest for $p = 1/2$. Mean risk premia and the relative risk premia are significantly positive for redemption probabilities above 1/7. Whereas the risk premia do not show a clear pattern if p changes, the relative risk premia, surprisingly, tends to increase with the redemption probability. This effect can be explained by the observation that the average loss if the bond is not drawn is almost independent of p, whereas the average gain strongly decreases in p.

The magnitude and significance of risk premia in Tables 5 and 6 show that bond market participants are risk-averse and demand compensation for redemption risk. This observation underlines our discussion that the lottery risk was not diversifiable and their value are different for the issuer of a lottery bond and the investors. In the second step of our analysis, we estimate bond market participants' risk aversion using the dynamic equilibrium valuation model derived in Section 3.

6 Estimation of Implied RRA Parameters

6.1 Implied RRA Parameters

As outlined in Section 3, RRA parameters can be extracted from prices of lottery bonds conditional on the appropriateness of our single dynamic model. The pooled, implied RRA estimate is obtained by minimizing the sum of squared deviations between the clean equilibrium prices b_t^* and the clean observed market prices b_t. We estimate γ by a

Table 6: **Risk Premia and Prices of Risk**

This table shows mean gains in states d and n, mean risk premia, and mean prices of risk for lottery observations resulting in a redemption gain. Results are segmented by redemption probabilities. The gains in states d and n as well as the risk premia are reported in German Mark. We report t-values in parentheses for risk premia and relative risk premia as well as standard deviations, minima, and maxima. The level of significance is based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Risk Measures	1/10	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2
Observations	10	16	21	19	16	27	25	30	23
Gain in State d	9.48	8.73	6.20	5.12	6.13	4.69	2.85	2.37	2.08
Gain in State n	−0.72	−0.73	−0.71	−0.78	−0.58	−0.90	−0.58	−0.77	−1.27
Risk Premium	0.30	0.32^{**}	0.15	0.06	0.54^{***}	0.22^{***}	0.28^{***}	0.28^{***}	0.41^{***}
t-value	(1.34)	(2.14)	(1.06)	(0.40)	(3.16)	(2.70)	(3.05)	(4.20)	(4.63)
Std. Dev.	0.71	0.60	0.66	0.68	0.68	0.42	0.45	0.36	0.42
min	−0.76	−0.52	−1.68	−1.87	−0.42	−0.73	−0.78	−0.53	−0.16
max	1.32	1.29	1.13	1.00	2.30	1.06	1.22	0.94	1.35
Relative Risk Premium	−0.02	0.00	0.05	0.00	0.12^{**}	0.08^{*}	0.10^{*}	0.18^{***}	0.31^{***}
t-value	(−0.18)	(0.00)	(0.98)	(0.02)	(1.88)	(1.51)	(1.39)	(3.61)	(5.52)
Std. Dev.	0.27	0.31	0.25	0.28	0.26	0.28	0.36	0.28	0.27
min	−0.46	−0.88	−0.65	−0.64	−0.62	−0.84	−0.78	−0.67	−0.18
max	0.31	0.33	0.33	0.37	0.39	0.36	0.50	0.53	0.69

non-linear least square procedure

$$\hat{\gamma} = \arg\min_{\gamma} \left\{ \sum_{n,t} \left(b^*_{n,t} - \bar{b}_{n,t} \right)^2 \right\}, \tag{17}$$

where n specifies the lottery bond and t the observation date. After each estimation, we apply an outlier detection rule to avoid model misspecifications that may result in distorted RRA estimates and invalid inferences. Bond prices are classified as outliers if their price residual deviates more than three times of the price residuals standard deviation from the average price residual. Outliers are excluded from the sample, and the estimation routine is run again.[10]

Bond prices around lottery dates may be contaminated by noise. To reduce this effect on our estimates of risk aversion parameters we use price data from an interval that covers 10 days after the last lottery date $(T-i-1)^{ex}$ til one day before the lottery date $(T-1)^{cum}$. We use this procedure even though no new information regarding the lottery risk has been revealed to the market. We utilize bond market data at a weekly frequency using clean Wednesday transaction prices.[11] Table 7 reports the number of weekly observations for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. For the final estimation, we only include lottery bond prices resulting in a redemption gain and located inside the no-arbitrage bounds. On the aggregate level, we exclude 971 observations due to no-arbitrage violations and classify 417 observations as outliers. Our filtered and pooled sample contains 12,019 observations in the full sample. The number of observations in each probability bracket varies from 364 for $p = 1/10$ to 2,055 for $p = 1/2$.

Table 8 shows the distribution of price observations for the full sample across issuer groups, and for the lottery probabilities. Because the last lottery bond has been issued in 1973 it is obvious that there are no observations of lotteries with low redemption probabilities at the end of the observation period. Furthermore, as we restrict our sample to bonds with a redemption gain, only few observations are available in the low interest-rate period 1977 to 1978.

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[10]We restrict the number of re-estimation loops and apply the outlier detection rule at most ten times. However, this limit was attained only for the 1/6 redemption probability segment based on the entire sample of bond price observations.

[11]If a Wednesday transaction price is missing we use the next available transaction price within the calendar week.

Table 7: **Number of Observations**

This table shows the number of price observations for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second and third column report the total number of observations and the number of observations resulting in a redemption gain. The fourth column reports the number of observations in the redemption gain segment excluded because the transaction price is located outside the no-arbitrage bounds. The fifth column reports the number of observations classified as outliers. The last column reports the number of filtered observations used in the estimations.

	Total	Red. Gain	No-arb.	Outl.	Clean
Overall	18,612	13,407	(971)	(417)	12,019
Issuer Group					
FRG	1,832	1,610	(52)	(19)	1,539
GS	12,462	8,164	(712)	(319)	7,133
GE	4,318	3,633	(207)	(122)	3,304
Redemption Probability					
1/2	3,368	2,464	(375)	(34)	2,055
1/3	3,231	2,367	(225)	(28)	2,114
1/4	2,872	2,267	(202)	(34)	2,031
1/5	2,238	1,530	(55)	(14)	1,461
1/6	1,743	1,286	(30)	(94)	1,162
1/7	1,629	1,315	(25)	(59)	1,231
1/8	1,504	1,099	(20)	(41)	1,038
1/9	1,222	701	(28)	(14)	659
1/10	805	378	(11)	(3)	364

Table 8: **Distribution of Price Observations**

This table shows the relative frequency of filtered price observations per observation year. Results are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Relative frequencies are reported in percentage points.

Observation Year	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985
Overall	16.0	18.3	18.0	8.4	2.3	9.2	10.3	9.1	5.6	1.5	1.2	0.2
Issuer Group												
FRG	17.9	22.0	20.7	9.6	4.4	11.6	10.2	3.5	–	–	–	–
GS	12.9	16.6	17.0	7.8	1.9	9.1	10.9	12.2	8.2	1.9	1.4	0.1
GE	21.2	20.2	18.8	9.2	2.2	8.4	9.2	5.5	2.7	1.1	1.2	0.3
Redemption Probability												
1/2	4.2	11.5	13.8	4.1	1.5	9.8	23.0	20.3	9.7	1.1	0.6	0.3
1/3	14.4	14.7	16.5	9.0	3.3	16.3	13.2	5.8	1.9	1.2	3.3	0.5
1/4	13.7	24.9	18.4	7.2	3.7	11.8	5.8	3.6	4.2	4.0	2.4	0.2
1/5	24.7	14.9	9.9	8.8	3.8	7.7	1.7	9.0	16.0	2.7	0.8	–
1/6	7.1	7.1	27.8	20.0	2.6	1.1	4.7	20.2	8.4	0.9	–	–
1/7	5.7	24.9	31.9	10.7	0.7	2.4	15.7	7.1	0.9	–	–	–
1/8	28.6	35.2	14.8	1.8	0.6	10.3	7.7	1.0	–	–	–	–
1/9	56.3	22.6	9.3	3.8	–	6.2	1.8	–	–	–	–	–
1/10	31.0	25.5	25.8	14.3	–	3.3	–	–	–	–	–	–

Table 9: **Implied RRA Estimates for Entire Sample**

This table shows the pooled, implied RRA estimates based on observations from the entire sample. The statistics are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second column reports the least squares RRA estimate $\hat{\gamma}$ of the respective segment. Columns three to seven report the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

	$\hat{\gamma}$	ME	SE	min	max	MAE
Overall	1.78	0.23	0.66	−1.76	2.22	0.55
Issuer Group						
FRG	1.17	0.14	0.45	−1.19	1.43	0.37
GS	2.93	0.28	0.70	−1.82	2.37	0.60
GE	0.98	0.11	0.54	−1.51	1.58	0.44
Redemption Probability						
1/2	6.17	0.09	0.36	−0.98	1.15	0.29
1/3	2.68	0.15	0.47	−1.27	1.55	0.40
1/4	1.75	0.16	0.60	−1.59	1.83	0.49
1/5	0.87	0.21	0.73	−1.96	2.22	0.59
1/6	6.37	0.17	0.72	−2.03	2.35	0.59
1/7	5.08	0.12	0.80	−2.27	2.47	0.66
1/8	2.04	0.18	0.94	−2.65	2.42	0.77
1/9	0.96	0.01	1.10	−3.28	2.64	0.85
1/10	0.61	0.33	0.90	−1.98	2.18	0.80

Table 9 summarizes the estimation results. The least squares RRA estimate for the full sample is 1.78 with a mean absolute error between theoretical and observed prices of DEM 0.55 or 0.58% of the mean lottery bond price of the full sample. The results from a bootstrap analysis presented in Table 10 shows a range for the sampled RRA parameters between 1.66 and 1.89. Our pooled results, therefore, provide evidence of a moderate risk aversion in the bond market. We find no evidence of the "puzzling" RRA coefficients found by Mehra and Prescott (1985) and others based on equity, bond, and consumption indices. On the contrary, our estimations suggest a level of risk aversion of about one for the FRG and GE and of about three for GS. These findings are in line with the estimates obtained in laboratory experiments, as summarized in the Appendix. We trace this result back to the issue that many estimates of RRA parameters are contaminated by incompletely known probabilities of fundamentals that affect asset prices. As a consequence, estimated RRA parameters reflect both risk aversion and possible estimation errors.

A somewhat puzzling result is the large differences of the estimated RRA parameter for the three issuer groups. In particular, the results for the German states differ strongly from those for the other two groups. This can be partly traced back to the lower liquidity of the GS lottery bond market compared to the other two markets when measured by the average issue volume. This is highest (DEM 345m) for the FRG lottery bonds, followed by the GE bonds (DEM 237m) and the GS bonds (DEM 110m). The spreads between the term-structures of interest rates reflect this difference to some extent. Attempts to explain the differences using issuer group specific redemption probabilities, redemption values or additional redemption options were not successful. Therefore, the observed differences between FRG and GE lottery bonds on one side and GS lottery bonds remain a puzzle.

To obtain further insight into the estimation errors of the estimated RRA parameters we perform a bootstrap analysis. Table 10 reports the bootstrap statistics. The standard deviation of the bootstrap estimation are low relative to the average values and allow for the following conclusions: (i) the pooled RRA estimates on the aggregate and issuer group level are significantly positive, (ii) RRA estimates for the FRG and GE segments are of similar magnitude at a level below two, (iii) relative to the FRG and GE results, RRA estimates for the GS segment are significantly higher.

6.2 Robustness Analysis

From the valuation model in Section 3 we know that the price sensitivity of γ is largest for bond prices close to the lower no-arbitrage boundary, i.e. a small noise driven change of the bond price result in large changes of the estimated RRA parameter. We control for this model related estimation problem by deleting bond prices from the sample that are closer to the no-arbitrage bound than a given threshold. Table 6.1 presents the result for thresholds of DEM 0.25, 0.50, 1.50 and 2.50.

Typically, the estimated RRA parameters decrease if the threshold increases. This result was to be expected as the implied RRA parameter increases strictly if the bond prices decreases. Therefore, if the sample is reduced as described, those bond prices are excluded which contribute to higher γ estimates. Less expected was the relatively small effect of

Table 10: **Bootstrap Statistics for Entire Sample**

This table shows the bootstrapped standard errors of pooled, implied RRA estimates based on observations from the entire sample. Bootstrap statistics are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Between 500 and 1,000 bootstrap samples are randomly drawn with replacement from the original dataset, and implied RRA coefficients are estimated. The size of the bootstrap reflects to some extent the different size of subsamples. Columns three to eight report the mean, standard deviations, minimum, maximum, 1% quantile, and 99% quantile of the estimated RRA parameters. We exclude observations classified as outliers when estimating the bootstrap statistics.

	Bootstr. Sample	Mean	Std. Dev.	min	max	1% Quant.	99% Quant.
Overall	1,000	1.78	0.03	1.66	1.89	1.71	1.86
Issuer Group							
FRG	500	1.18	0.04	1.07	1.30	1.08	1.27
GS	1,000	2.93	0.05	2.72	3.09	2.80	3.05
GE	500	0.98	0.04	0.86	1.15	0.89	1.08
Redemption Probability							
1/2	500	6.17	0.23	5.43	6.83	5.67	6.75
1/3	500	2.69	0.12	2.32	3.21	2.41	2.99
1/4	500	1.75	0.07	1.56	1.97	1.59	1.92
1/5	500	0.88	0.05	0.73	1.04	0.77	1.00
1/6	500	6.38	0.27	5.61	7.53	5.78	6.95
1/7	500	5.09	0.17	4.56	5.61	4.71	5.51
1/8	500	2.05	0.09	1.83	2.29	1.85	2.26
1/9	500	0.97	0.08	0.65	1.22	0.77	1.13
1/10	500	0.60	0.10	0.31	0.93	0.40	0.83

Table 11: **Impact of Thresholds for Bond Prices relative to the lower no-arbitrage Bounds**

This table shows the pooled, implied RRA estimates for various thresholds on the difference between bond prices and lower no-arbitrage bounds. RRA estimates are given for the overall dataset and for market segments of bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Columns two to six report the least squares RRA estimate $\hat{\gamma}$ based on the unrestricted sample and on samples containing observations with a minimum distance to the no-arbitrage bounds of DEM 0.25, 0.50, 1.50, and 2.50. The numbers of filtered observations are given in parentheses.

	Unrestr.	DEM 0.25	DEM 0.50	DEM 1.50	DEM 2.50
Overall	1.78 (12,019)	1.74 (11,350)	1.69 (10,599)	1.57 (7,637)	1.44 (5,461)
Issuer Group					
FRG	1.17 (1,539)	1.16 (1,473)	1.16 (1,401)	1.15 (998)	1.13 (770)
GS	2.93 (7,133)	2.89 (6,634)	2.81 (6,096)	2.65 (4,269)	1.97 (3,080)
GE	0.98 (3,304)	0.97 (3,182)	0.97 (3,033)	0.93 (2,289)	0.86 (1,603)
Redemption Probability					
1/2	6.17 (2,055)	5.90 (1,787)	5.70 (1,514)	4.41 (561)	2.68 (97)
1/3	2.68 (2,114)	2.69 (1,928)	2.67 (1,728)	2.43 (972)	1.98 (512)
1/4	1.75 (2,031)	1.72 (1,915)	1.70 (1,768)	1.67 (1,338)	1.62 (1,024)
1/5	0.87 (1,461)	0.87 (1,420)	0.87 (1,362)	0.85 (1,018)	0.81 (710)
1/6	6.37 (1,162)	6.17 (1,135)	5.91 (1,111)	1.57 (947)	1.24 (606)
1/7	5.08 (1,231)	5.06 (1,204)	5.03 (1,179)	5.02 (1,018)	4.87 (874)
1/8	2.04 (1,038)	2.03 (1,022)	1.96 (1,012)	1.88 (953)	1.88 (891)
1/9	0.96 (659)	0.96 (654)	0.96 (653)	1.00 (634)	0.96 (592)
1/10	0.61 (364)	0.61 (350)	0.60 (325)	0.56 (269)	0.55 (210)

imposing thresholds on bond prices in the full sample, the FRG and the GE sample. The largest threshold of DEM 2.50 reduces the sample size by about 50% relative to the original sample. The estimates of RRA parameters are reduced by 19% for the full sample, by 3% for the FRG bonds and by 12% for the GE bonds. The largest reduction is obtained for the lottery bonds issued by the German States. The estimated γ of 1.97 is now much closer to the value of the full sample and the FRG bonds. From this result we conclude that the relatively large value of $\widehat{\gamma} = 2.93$ for this group if no threshold is imposed is to a large extent a consequence of prices close to the lower no-arbitrage bound.

A second remarkable result is the lower variation of the estimated RRA parameters for the redemption probabilities if the threshold increases. In particular the high estimates are reduced considerably with one exception, the estimate for $p = 1/7$. Therefore, the puzzling observation of the large, non-systematic variation of the estimates of γ for different redemption probabilities can be partly traced back to the high price sensitivity of γ for low bond prices.

6.3 Time Variation of Relative Risk Aversion

In this Section we provide some preliminary results on the time variation of th estimated RRA parameters. Table 12 shows the estimates on a yearly basis. Since we restricted our sample to bonds trading below par, i.e. those bonds that result in a gain if drawn, the number of observations per year depends on the interest rate level. Interest rates in Germany decreased from a first post war peak during the first oil price stock in 1973/74 to very low interest rates in 1977/78. A second post war peak was observed in 1980/81 followed by very low interest rates from 1983 till 1987. The dotted line in Figure 4 shows the three-month money market rate from 1974 till the end of 1982.

Table 12 reports the annual estimates of γ for all years during the sample period in which more than 500 bond prices below par are available; as a result the years 1978 and 1983-1987 were excluded from the analysis. The results of Table 12 are obtained without imposing a threshold on bond prices. The estimates of γ fluctuate considerably over time. The large values in 1980 to 1982 reflect the challenging economic situation during and after the oil crisis in 1979. This crisis was followed by high interest rates and a severe recession in Germany. The negative RRA parameter in 1977 demonstrates that the bond prices in this year can not be explained by risk averse investors.

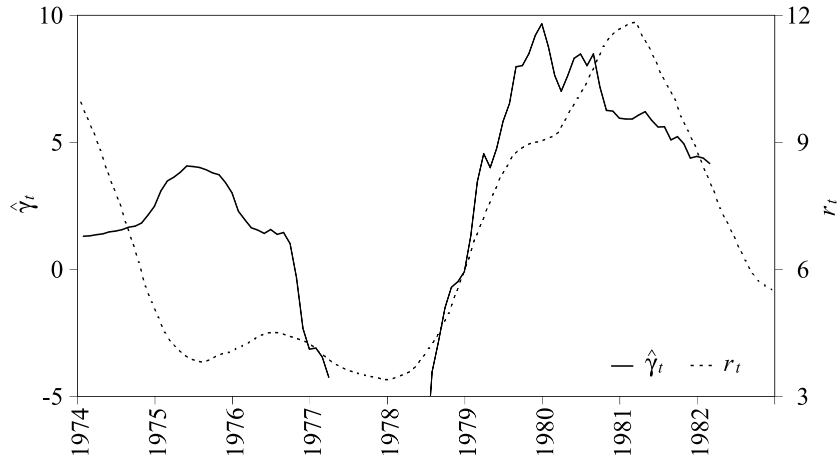
Table 12: **Annual Implied RRA Estimates**

This table shows implied RRA estimates for the full sample for disjoint annual time intervals from January 1 until December 31. We only list time intervals with more than 1,000 observations. The second column reports the least squares RRA estimate $\hat{\gamma}_t$ for the respective time interval. Columns three to eight report the number of observations, the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

	$\hat{\gamma}_t$	Obs.	ME	SE	min	max	MAE
Time Interval							
1974	1.20	2,154	0.18	0.73	−1.98	2.14	0.60
1975	2.99	2,268	0.25	0.83	−2.21	2.47	0.70
1976	2.18	2,122	0.25	0.77	−2.05	2.21	0.65
1977	−3.20	882	0.12	0.36	−0.95	1.15	0.30
1979	1.22	1,117	0.06	0.37	−1.05	1.09	0.30
1980	8.65	1,230	0.12	0.41	−1.09	1.34	0.34
1981	5.81	1,083	0.18	0.75	−2.07	2.42	0.59
1982	4.27	590	0.19	0.45	−1.14	1.53	0.39

Figure 4: **Time Series of Implied RRA Estimates**

The panels show the time series of implied RRA estimates (solid lines) and the three-month money market rate (dotted lines). RRA coefficients and money market rates are determined using annual forward looking rolling windows at a monthly frequency. We only include RRA parameters which are estimated using more than 500 observations.



The high time variability of the estimated RRA parameters becomes even more transparent if γ is estimated for a monthly rolling window of one year. The result is shown by the solid line in Figure 4.

The time series of estimated RRA parameters (solid line) and the three-month interest rate are positively correlated with a correlation coefficient of 0.64. A positive relationship between γ and the three-month interest rate exists also for the first differences. Regressing the monthly changes of γ on the changes of the interest rate for the period 1974 to 1982 yields a coefficient of 0.98, significant on the 1% level. These results together with Figure 4 indicate that risk aversion parameters may have some predictive power for macroeconomic variables.[12]

[12]See e.g. Gai and Vause (2004) , Deutsche Bundesbank (2005), or Illin and Meyer (2005) for a comprehensive overview of risk aversion indicators.

7 Concluding Remarks

To the best of our knowledge this is the first paper that estimates RRA parameters from a lottery played in German bond market. This segment of the German bond market provides a laboratory experiment with bond prices from transactions.

Using a unique dataset, containing transaction prices of 83 redemption lottery bonds traded between 1974 and 1987, we analyze risk preferences in the German bond market. Our empirical results describe the magnitude and evolution of risk aversion in the bond market. The pooled, implied RRA estimates are consistent with the moderate level of risk aversion found in most of the recent studies.[13] We obtain a robust pooled, implied RRA estimate of 1.78 and find no evidence of the extreme level of risk aversion suggested by Campbell and Cochrane (1999) and Kandel and Stambaugh (1990, 1991). Rather, the estimations indicate that the pooled, overall RRA coefficient is below two and robust across the length of estimation intervals as well as for restrictions on the proximity of price observations to the no-arbitrage bounds. We also obtain preliminary results on the dynamics of implied risk aversion. Implied risk aversion is time-dependent and attains its maximum in 1980 and 1981, reflecting the challenging economic situation after the second oil crisis in 1979. The time series properties of RRA coefficients provide further evidence that severe economic crises coincide with periods of high risk aversion.

There are a number of possibilities to extend the current study. On the theoretical side it is of interest to extend the dynamic equilibrium by using a recursive utility. On the empirical side it is interesting how this type of a utility function affects the estimates of risk aversion parameters. More work has to be done in the analysis of the interplay between RRA parameters and macroeconomic variables. Of special interest is the question whether RRA parameters determined in the clean environment of German lottery bonds have predictive power for future crises.

[13]See Table A for an overview of RRA parameters reported in the literature.

Appendix

Table A: **Range of RRA Parameters**

This table shows a selection of RRA parameters reported in literature. For further details, see (1) Arrow (1970), p. 98, (2) Friend and Blume (1975), pp. 920, (3) Wolf and Pohlman (1983), p. 848, Table II, (4) Barsky et al. (1997), p. 563, Table XI (harmonic mean (4.2) and arithmetic mean (12.1)), (5) Holt and Laury (2002), p. 1649, Table 3, (6) Grossman and Shiller (1981), p. 226, (7) Hansen and Singleton (1983), p. 258, Table I, (8) Ferson (1983), p. 492, Table 5, (9) Brown and Gibbons (1985), p. 374, Table III, (10) Mehra and Prescott (1985), p. 155, Footnote 5, (11) Grossman et al. (1987), p. 324, Table 5 (datasets 1 to 4), (12) Weil (1989), p. 413, Table 1 and 2, (13) Constantinides (1990), p. 532, Table 1, (14) Epstein and Zin (1991), pp. 277, Tables 2 to 5, (15) Ferson and Constantinides (1991), pp. 216, Table 4, (16) Kandel and Stambaugh (1991), pp. 50, (17) Mankiw and Zeldes (1991), p. 109, Table 6, (18) Cochrane and Hansen (1992), p. 124, Figure 1, (19) Jorion and Giovannini (1993), pp. 1092, Table 2 and 3, (20) Cecchetti et al. (1994), p. 135, Table II and p. 149, (21) Campbell and Cochrane (1999), p. 244, (22) Guo and Whitelaw (2006), pp. 1447, Tables II, IV, V, and VI, (23) Bartunek and Chowdhury (1997), p. 121, Table I, (24) Aït-Sahalia and Lo (2000), p. 35, (25) Bliss and Panigirtzoglou (2004), p. 431, Table VI (all observations and power utility).

	Study	RRA parameter
Direct Assessments and Survey Data		
1	Arrow (1970)	1
2	Friend and Blume (1975)	~ 2
3	Wolf and Pohlman (1983)	2 to 4.5
4	Barsky et al. (1997)	4.2 or 12.1
5	Holt and Laury (2002)	0.3 to 0.5
Consumption-based Asset Pricing		
6	Grossman and Shiller (1981)	~ 4
7	Hansen and Singleton (1982, 1983)	0 to 2
8	Ferson (1983)	-1.4 to 5.4
9	Brown and Gibbons (1985)	0 to 7
10	Mehra and Prescott (1985)	55
11	Grossman et al. (1987)	> 20
12	Weil (1989)	45
13	Constantinides (1990)	2.8
14	Epstein and Zin (1991)	0.4 to 1.4
15	Ferson and Constantinides (1991)	0 to 12
16	Kandel and Stambaugh (1991)	29
17	Mankiw and Zeldes (1991)	35
18	Cochrane and Hansen (1992)	40 to 50
19	Jorion and Giovannini (1993)	5.4 to 11.9
20	Cecchetti et al. (1994)	≥ 6
21	Campbell and Cochrane (1999)	≥ 60
22	Guo and Whitelaw (2006)	1.6 to 7.8
Option Data		
23	Bartunek and Chowdhury (1997)	0 to 1
24	Aït-Sahalia and Lo (2000)	1 to 60
25	Bliss and Panigirtzoglou (2004)	2 to 9.5

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